# Econ306–Intermediate Microeconomics
Fall 2007
Final exam — Solutions

Question 1 (*20 points*)

John loves root beer floats, which he makes by combining one glass of root beer and two scoops of ice cream. Jane is on a diet that assigns points to food and drinks. One glass of root beer is one point and one scoop of ice cream is one point. She can consume any combination of root beer and ice cream as long as the total number of points is two.

(i) (*4 points*) What kind of goods are root beer and ice cream for John? Show his indifference curves on a graph.

Answer: Root beer and ice cream are perfect complements for John. His indifference curves will look like in the graph below, with the kinks lying on a line starting from the origin and having a slope of 0.5.
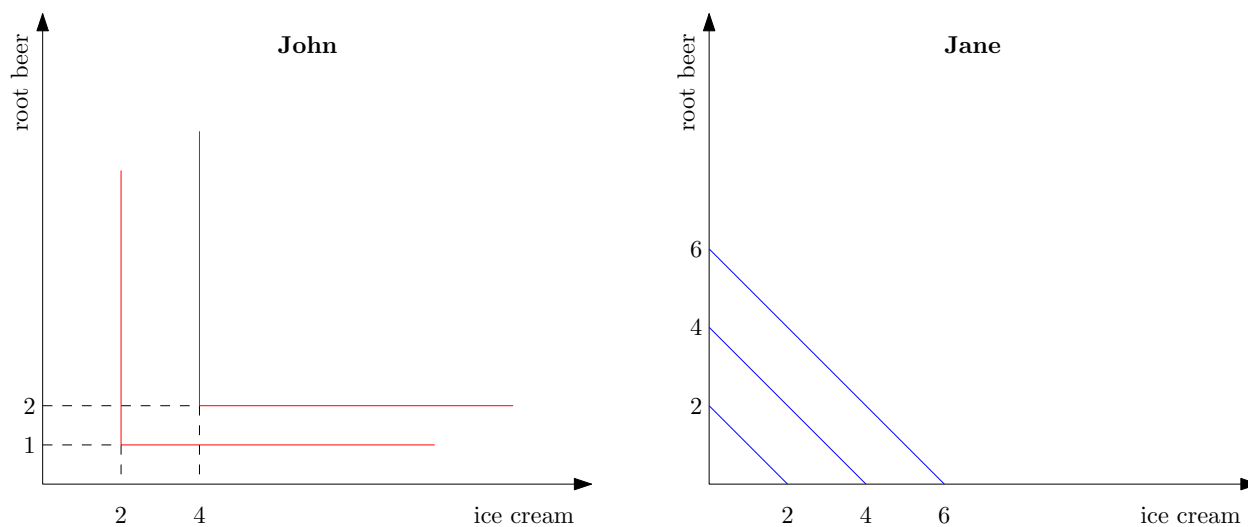

(ii) (*4 points*) What kind of goods are root beer and ice cream for Jane? Show her indifference curves on a graph.

Answer: Root beer and ice cream are perfect substitutes for Jane. Her indifference curves will look like in the graph above, with a slope of -1 (the negative of the marginal rate of substitution between the two goods).

(iii) (*4 points*) Suppose that Jane and John decide to have a private party. John brings one bottle of root beer (8 glasses) and Jane brings one box of ice cream (16 scoops). Represent this two-consumer, two-good exchange economy in an Edgeworth box, remembering to indicate the endowment point.

Answer: The Edgeworth box will look like in the graph below, where A represents the endowment point.
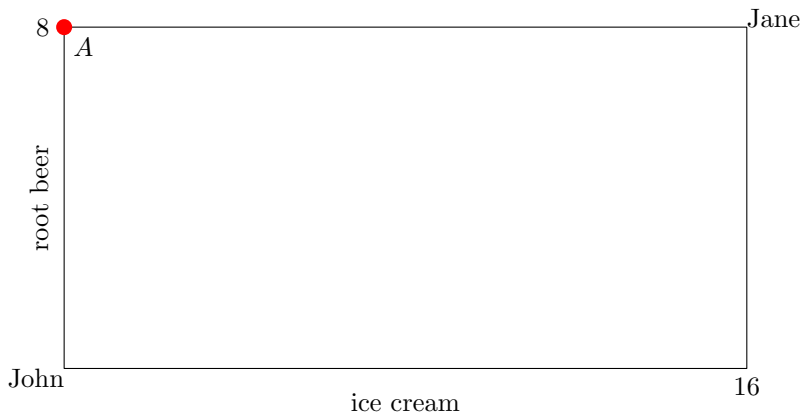
(iv) (*4 points*) Find the contract curve in this economy.

Answer: The contract curve is given by the points of "tangency" of the two sets of indifference curves. Note that these points will always be the kinks of John's indifference curves. For simplicity, we can just take the contract curve to be the set of all kinks, that is, the line starting from John's origin and having a slope of 0.5, as shown in the graph below (to be strict, it would be only the points "right before" a kink, but taking the whole line would be a perfectly fine answer).
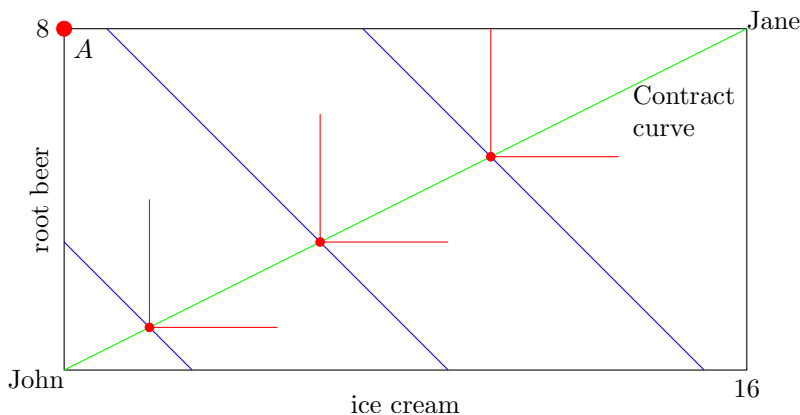
(v) (*4 points*) What is the equilibrium point in this Edgeworth box? What is the price ratio? (Hint: At the equilibrium, the slope of the budget line is equal to the slope of the indifference curves or is "tangent" to them, and the budget line has to pass through the endowment point.)

Answer: At the equilibrium, the budget line is has the same slope as the two indifference curves at their point of "tangency". John's indifference curves do not have a "slope" at the kink (or you can think of them as having potentially any slope), while Jane's have a slope of -1. Hence, the budget constraint should have a slope of -1, meaning that it basically overlaps Jane's indifference curve. Also, the budget lines should pass through the endowment point. This means that the equilibrium is given by the "tangency point" between Jane's indifference curve that passes through the endowment point and John's indifference curves, as shown in the graph below.

   Although you didn't need to, you could actually calculate the quantities that John and Jane get at the equilibrium. Jane's indifference curve passes through the endowment point, meaning that the sum of her allocation of ice cream and root beer is equal to 16. So, the sum of John's quantities is 8 (since the sum of the total quantities is 16 scoops of ice cream plus 8 glasses of root beer $= 24$). Also, John consumes root beer and ice cream in a proportion of $1:2$, so this means that he will have $\frac{8}{3}$ glasses of root beer and $\frac{16}{3}$ scoops of ice cream. This, in turn, means that Jane will get $8 - \frac{8}{3} = \frac{16}{3}$ glasses of root beer and $16 - \frac{16}{3} = \frac{32}{3}$ scoops of ice cream (again, abstract from the fact that this is not a multiple of two).

   Finally, the price ratio is 1 (the negative of the slope of the budget constraint).
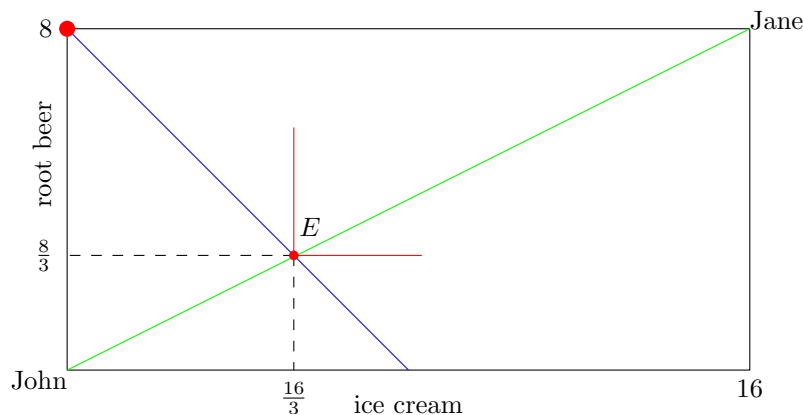
Question 2 (*23 points*)

In 1990, the town of Ham Harbor had a more-or-less free market in taxi services. Any respectable firm could provide taxi service as long as the drivers and cabs satisfy certain safety standards. Suppose that the marginal cost per trip of a taxi ride is constant, $MC = \$5$, and that each taxi has a capacity of 20 trips per day. Let the demand function for taxi rides be given by $D(p) = 1,200 - 20p$, where demand is measured in rides per day. Assume that the industry is perfectly competitive.

(i) (*2 points*) What is the competitive equilibrium price per ride? (Hint: Use the marginal output rule and remember that the marginal revenue equals the price in a competitive market.)

Answer: Firms choose the optimal number of rides provided according to the marginal output rule: $MR = MC$. In a competitive market, $MR = p$, which means that $p = MC = \$5$.

(ii) (*6 points*) What is the equilibrium number of rides per day? How many taxicabs will there be in equilibrium?

Answer: Since we know the equilibrium price, we can substitute in the demand function to find the equilibrium number of rides per day: $D(5) = 1,200 - 20 \cdot 5 = 1,100$ rides per day. Each taxicab has a capacity of 20 rides per day, which means that there will be $\frac{1,100}{20} = 55$ taxicabs in equilibrium.

(iii) (*4 points*) In 1990, the city council of Ham Harbor created a taxicab licensing board and issued a license to each of the existing cabs. The board stated that it would continue to adjust the taxicab fares so that the demand for rides equals the supply of rides, but no new licenses will be issued in the future. In 1995, costs had not changed, but the demand curve for taxicab rides had become $D(p) = 1,220 - 20p$. What was the equilibrium price of a ride in 1995?

Answer: The fact that no new licenses are issued means that the quantity supplied stayed at the same level as before, $1,100$ rides per day. The new equilibrium price can be derived from the new demand equation:

$$1,100 = 1,220 - 20p \quad \Rightarrow \quad 20p = 1,220 - 1,100 \quad \Rightarrow \quad p = \frac{120}{20} = \$6.$$

(iv) (*3 points*) What was the profit per ride in 1995, neglecting any costs associated with acquiring a taxicab license?

Answer: The marginal cost of a ride is constant, so it is also the cost per ride. In this case, the profit per ride is $p - MC = 6 - 5 = \$1$.

(v) (*4 points*) What was the profit per taxicab license per day in 1995?

Answer: Each taxicab runs 20 trips per day, so the profit per taxicab (license) is $20 \cdot 1 = \$20$.

(vi) (*4 points*) How much money would each current taxicab owner be willing to pay to prevent any new licenses from being issued?

Answer: In a competitive equilibrium, each taxicab owner would earn zero profits. Therefore, they would be willing to pay the difference between the current profits ($20) and the competitive equilibrium profits ($0), i.e., $20, to avoid new licenses being issued.

Question 3 (*27 points*)

A firm has a short-run variable cost function $VC_{SR}(Q) = Q^3 - 10Q^2 + 30Q$.

(i) (*3 points*) Derive the firm's short-run average cost function.

Answer: The short-run average cost of the firm is:

$$AC_{SR}(Q) = \frac{VC_{SR}(Q)}{Q} = \frac{Q^3 - 10Q^2 + 30Q}{Q} = Q^2 - 10Q + 30.$$

(ii) (*3 points*) Derive the firm's short-run marginal cost function.

Answer: The short-run marginal cost of the firm is:

$$MC_{SR}(Q) = \frac{\partial VC_{SR}(Q)}{\partial Q} = 3Q^2 - 20Q + 30.$$

(iii) (*3 points*) Find the optimal (cost-minimizing) level of production for the firm.

Answer: At the optimal production level, the short-run marginal and average costs are equal:

$$MC_{SR}(Q) = AC_{SR}(Q) \quad \Rightarrow \quad 3Q^2 - 20Q + 30 = Q^2 - 10Q + 30 \quad \Rightarrow$$
$$2Q^2 - 10Q = 0 \quad \Rightarrow \quad 2Q(Q - 5) = 0 \quad \Rightarrow \quad Q = 5.$$

(iv) (*3 points*) Calculate the short-run marginal and average cost at this production level.

Answer: The short-run marginal and average cost when $Q = 5$ are:

$$MC_{SR}(5) = AC_{SR}(5) = 3 \cdot 5^2 - 20 \cdot 5 + 30 = 75 - 100 + 30 = 5.$$

(v) (*3 points*) Find the profit-maximizing level of production for the firm.

Answer: The cost-minimizing and profit-maximizing levels of production are the same, so $Q = 5$ is also the profit-maximizing level of production.

(vi) (*6 points*) If the market price is $p = \$18$, how much would the firm decide to produce? (Note: If you obtain two possible values for Q, choose the highest.)

Answer: The firm will choose the level of output according to the marginal output rule, i.e. it will choose to produce at the level that equates the marginal cost and the price:

$$MC_{SR}(Q) = p \quad \Rightarrow \quad 3Q^2 - 20Q + 30 = 18 \quad \Rightarrow \quad 3Q^2 - 20Q + 12 = 0 \quad \Rightarrow$$
$$Q = \frac{20 \pm \sqrt{20^2 - 4 \cdot 3 \cdot 12}}{2 \cdot 3} = \frac{20 \pm \sqrt{256}}{6} = \frac{20 \pm 16}{6} \quad \Rightarrow \quad Q = 6.$$

(vii) (*6 points*) Calculate the economic profit of the firm when the market price is $p = \$18$ (remember that only variable costs are economic costs). How much will the profit be in the long run if the firm operates in a perfectly competitive market?

Answer: The economic profit of the firm is

$$\pi(Q) = pQ - VC_{SR}(Q),$$

so first we need to calculate the variable cost when $Q = 6$:

$$VC_{SR}(6) = 6^3 - 10 \cdot 6^2 + 30 \cdot 6 = 216 - 10 \cdot 36 + 180 = 36.$$

Then, the economic profit of the firm is

$$\pi(6) = 18 \cdot 6 - 36 = 72.$$

In a perfectly competitive market, new firms will enter as long as there are profits to be made. Hence, in the long run, profits will be driven down to zero.

Question 4 (*15 points*)

Furry Toys Co. uses robots and workers to produce Teddy-bears. The production function is "nice", in the sense that the isoquant lines are smooth curves. The rental price of a robot is \$200 per day and the wage of a worker is \$150 per day.

(i) (*3 points*) Draw the isocost line corresponding to an expenditure of \$4,200 (remember to label the intercepts and to calculate the slope of the isocost line).

Answer: If the firm spent all its money on robots, then it would be able to use $\frac{4200}{200} = 21$ robots. If it spent all its money on workers, it would be able to hire $\frac{4200}{150} = 28$ workers. The slope of any isocost line is the negative of the price ratio, $-\frac{150}{200} = -0.75$. The isocost line will look like in the figure below.
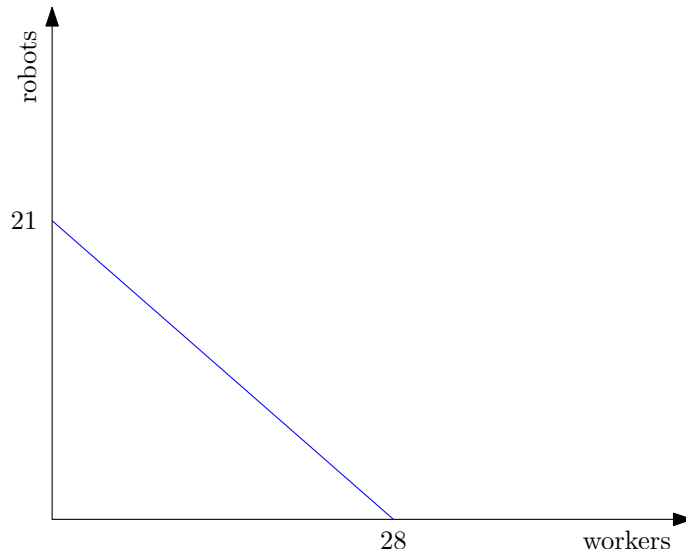
(ii) (*3 points*) Suppose that the combination of 10 robots and 8 workers is minimizing the cost of producing 100 Teddy-bears. Is this combination on the $4,200 isocost line? Show on a graph how this optimal mix of inputs would be found by the firm.

Answer: The cost of 10 robots and 8 workers is $10 \cdot 200 + 8 \cdot 150 = \$3,200$, so this point is not on the $4,200 isocost line, but on the $3,200 line. The firm would find this optimal input mix as the tangency point of the 100 Teddy-bears isoquant to the $3,200 isocost, as shown in the graph below.
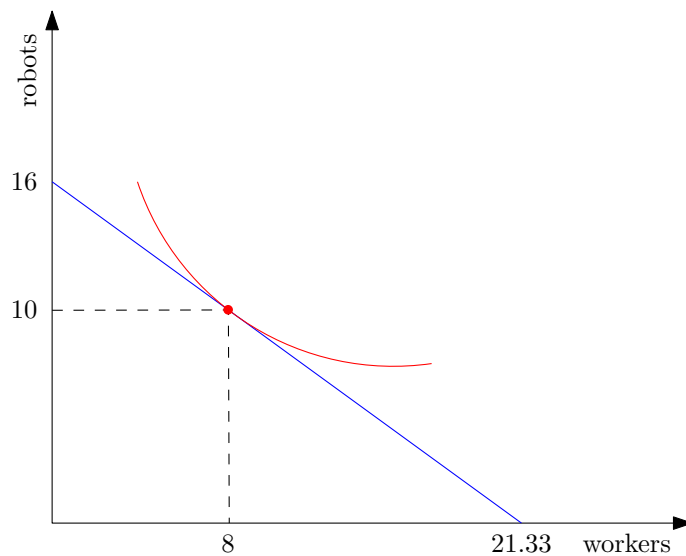
(iii) (*3 points*) If the marginal physical product of labor is 5 at the cost minimizing input bundle in the previous part, solve for the marginal physical product of capital.

Answer: At the optimum, the ratio of the marginal physical products is equal to the ratio of the corresponding prices:

$$\frac{MPP_L}{MPP_K} = \frac{w}{r} \quad \Rightarrow \quad \frac{5}{MPP_K} = \frac{150}{200} \quad \Rightarrow \quad MPP_K = 5 \cdot \frac{200}{150} = 6.67.$$

(iv) (*2 points*) For the next questions, suppose that the wage of a worker increases to \$200. How do the isocost lines change?

Answer: The slope of the isocost lines is given by the negative of the price ratio. If the price of labor (the wage) changes, then the slope of the isocost lines becomes $-\frac{200}{200} = -1$, meaning that the isocost lines become steeper.

(v) (*2 points*) How does the total cost change if the firm decides to keep producing 100 Teddy-bears?

Answer: When the isocost lines become steeper, the \$3,200 isocost line does not touch the 100 Teddy-bears isoquant anymore. The isocost line needs to shift out, as shown in the graph below. This, in turn, means that the total cost has to increase.
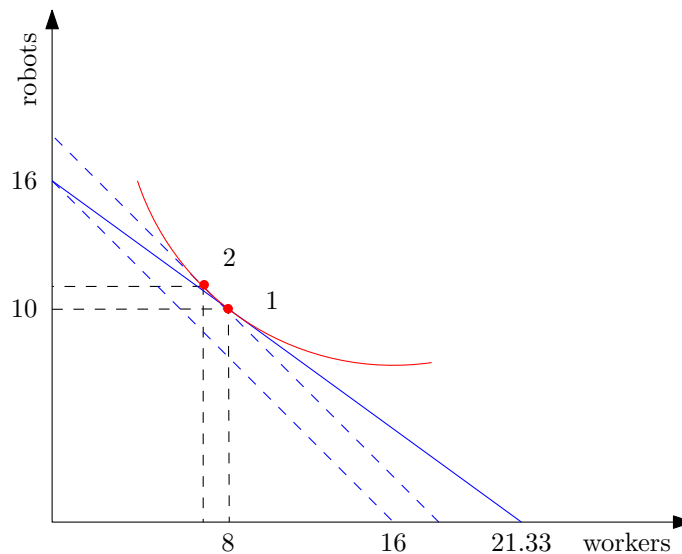
(vi) (*2 points*) How does the optimal mix of inputs change?

Answer: As shown in the graph above, the optimal input mix shifts from point 1 to point 2, which involves the use of less workers (the inputs experiencing a price increase) and more robots.

Question 5 (*15 points*)

Suppose that the demand curve for apples is given by $Q^d = 200 - 5p$, where Q^d is the number of pounds demanded per year and p is the price per pound. The supply of apples can be described by $Q^s = 40 + 3p$, where Q^s is the number of pounds provided.

(i) (*3 points*) What is the equilibrium price? (Hint: At the equilibrium, quantity demanded and quantity supplied are equal, $Q^d = Q^s$.)

**Answer:** At the equilibrium, quantity supplied equals quantity demanded:

$$Q^d = Q^s \quad \Rightarrow \quad 200 - 5p = 40 + 3p \quad \Rightarrow \quad 8p = 160 \quad \Rightarrow \quad p = \$20.$$

(ii) (*2 points*) What is the equilibrium quantity supplied and demanded?

Answer: The equilibrium quantity supplied and demanded is $Q^s = Q^d = 200 - 5 \cdot 20 = 100$.

(iii) (*3 points*) Calculate the consumer surplus at the equilibrium price.

Answer: To calculate the consumer surplus, let us first graph the demand and the supply (see graph below).
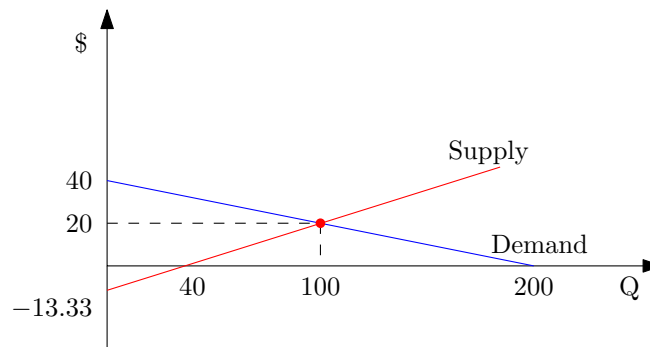
The consumer surplus can be calculated as the area under the demand curve and above the price: $CS = \frac{1}{2} \cdot 20 \cdot 100 = 1000$.

**(iv)** (*3 points*) Calculate the producer surplus at the equilibrium price.

Answer: The producer surplus is the area above the supply curve and below the price. Technically, it should be only the area above the horizontal axis, which can be calculated as the difference between the areas of the big and the small triangles in the graph above: $PS = \frac{1}{2} \cdot (20 + 13.33) \cdot 100 - \frac{1}{2} \cdot 13.33 \cdot 40 = 1,400$. However, the area of the big triangle is an acceptable answer as well: $PS = \frac{1}{2} \cdot 33.33 \cdot 100 = 1,666.67$.

(v) (*1 point*) Calculate the total surplus at the equilibrium price.

Answer: The total surplus is the sum of the consumer and the producer surplus: $TS = CS + PS = 1,000 + 1,400 = 2,400$ (or, if you used the area of the big triangle as the measure of producer surplus, $TS = 1,000 + 1,666.67 = 2,666.67$.

(vi) (*3 points*) Now suppose that the government imposes a tax of $8 per each pound sold, paid by the consumers, which effectively reduces the quantity sold and demanded to 85 pounds. In this case, what are the price and the consumer surplus?

Answer: If consumers pay the tax, that effectively acts as to shift the demand curve down by $8, as shown in the graph below.
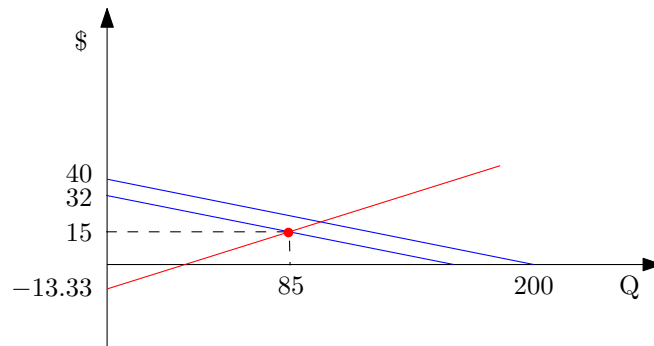
Since the consumers pay the tax, the new demand function is actually:

$$Q^d = 200 - 5(p + 8) = 200 - 5p - 40 = 160 - 5p,$$

which means the intercept with the vertical axis is now $\frac{160}{5} = 32$.

As we know the equilibrium quantity demanded, this gives us the equilibrium price:

$$85 = 160 - 5p \quad \Rightarrow \quad p = \$15.$$

Finally, the consumer surplus is now the area of the smaller triangle: $CS = \frac{1}{2} \cdot (32 - 15) \cdot 85 = 722.5$.

Question 6 (*extra 8 points*)

This comes from an actual newspaper story. "The average price of a home in W County rose more than 12% last year but the number of sales fell nearly 15%. 'It's the old law of supply and demand,' said a spokesman for the Board of Realtors. 'The number of sales is down because there's a higher demand for properties but there isn't a corresponding number to sell.'"

(i) (*4 points*) What does the "old law of supply and demand" predict would happen to price and quantity if the demand curve shifts outward and the supply curve does not change?

Answer: According to the law of demand, if the demand curve shifts out, the price increases as in the newspaper story. However, unlike in the story, the equilibrium quantity increases as well, as shown in the graph below.
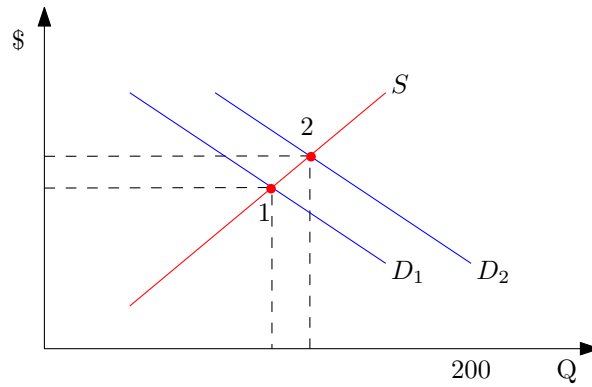
(ii) (*4 points*) Draw a diagram to illustrate the case of a shift in demand and/or supply curves that is consistent with the observed change in prices and quantities.

Answer: In order for the equilibrium price to rise and the equilibrium quantity to fall, what would need to happen is for the *supply* curve to shift *inward*, as in the graph below.

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