Examples for chapter 6 and 7

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Risk premium and certainty equivalent

Consider an investor with utility function $u(c) = \ln(c - 80)$ and degree of risk aversion given by $A = 4.605$. The investor has an initial wealth of \$100 and faces the following investment opportunity (I):

Outcome	Probability
\$85	30%
\$95	35%
\$125	35%

(i) The mean, variance and standard deviation of the investment are:

$$\begin{aligned} E(I) &= 0.3 \cdot 85 + 0.35 \cdot 95 + 0.35 \cdot 125 = \$102.5, \\ \sigma_I^2 &= 0.3(85 - 102.5)^2 + 0.3(95 - 102.5)^2 + 0.3(125 - 102.5)^2 = 288.75, \\ \sigma_I &= \sqrt{\sigma_I^2} = 16.99. \end{aligned}$$

(ii) The expected utility derived from this investment is

$$\begin{aligned} E[u(I)] &= 0.3u(85) + 0.35u(95) + 0.35u(125) \\ &= 0.3\ln(85 - 80) + 0.35\ln(95 - 80) + 0.35\ln(125 - 80) \\ &= 2.76. \end{aligned}$$

(iii) The certainty equivalent is given by

$$u(c^e) = E[u(I)] \quad \Rightarrow \quad \ln(c^e - 80) = 2.76 \quad \Rightarrow \quad c_e = e^{2.76} + 80.$$

Hence, the investment opportunity is equivalent to a risk-free investment that yields \$95.85.

(iv) The risk premium is then

$$E(I) - c^e = 102.5 - 95.85 = \$6.65.$$

(v) Alternatively, we can calculate the certainty equivalent using the "expected utility" formula:

$$U(I) = E(I) - 0.005A\sigma_I^2 = 102.5 - 0.005 \cdot 4.605 \cdot 288.75 = 95.85.$$

The certainty equivalent would be the risk-free investment that yields the same "expected utility", that is

$$U(c^e) = E(c^e) - 0.005A\sigma^2_{c^e} = c^e = 95.85.$$

So, we get again that the risky investment is equivalent to a risk-free investment that yields \$95.85.