Econ306 – Intermediate Microeconomics
Solutions to Problem Set 4

Question 1 (*1.5 points*)

(i) (*1 point*) The marginal physical products for labor and capital are

$$MPP_L = \frac{\partial F(L,K)}{\partial L} = 3K,$$

$$MPP_K = \frac{\partial F(L,K)}{\partial K} = 3L.$$

The marginal product of labor is constant with respect to the number of workers hired, therefore the production function exhibits constant marginal returns for labor. The situation is similar for capital, meaning that the production function exhibits constant marginal returns for capital as well.

(ii) (*0.5 points*) When the amount of capital used in production changes, so does the marginal physical product of labor. This shows that, in the short run, the firm's production decision depends on the amount of "fixed" capital that it has.

Question 2 (*3 points*)

(i) (*1 point*) The cost of production is given by

$$Cost = wL + rK \quad \Rightarrow \quad 150 \cdot L + 100 \cdot K = 1,500.$$

The intercepts with the two axes are then $\frac{1,500}{150} = 10$ for labor and $\frac{1,500}{100} = 15$ for capital. The isocost lines have a slope equal to $-\frac{150}{100} = -1.5$. The isocost line corresponding to a cost of $\$1,500$ is plotted in the graph below.
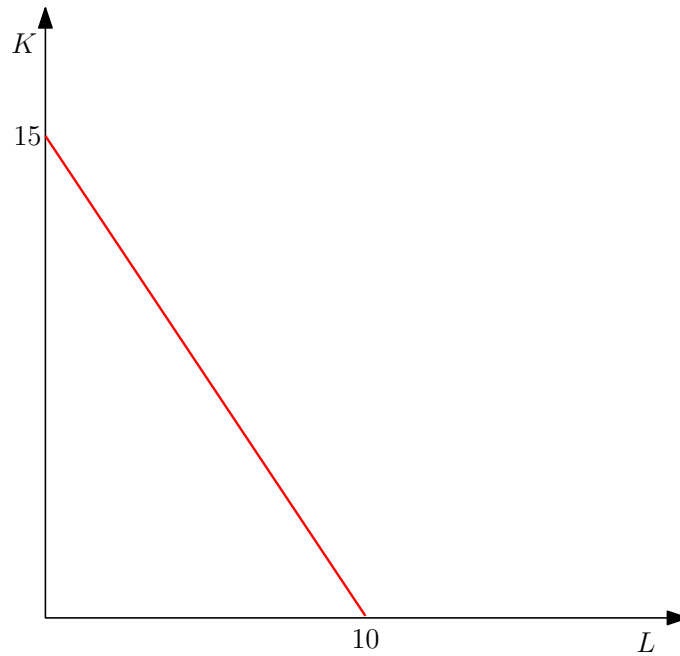
(ii) (*0.5 points*) Assuming that the production function is "nice", the isoquant and the optimal input bundle will look like in the graph below.
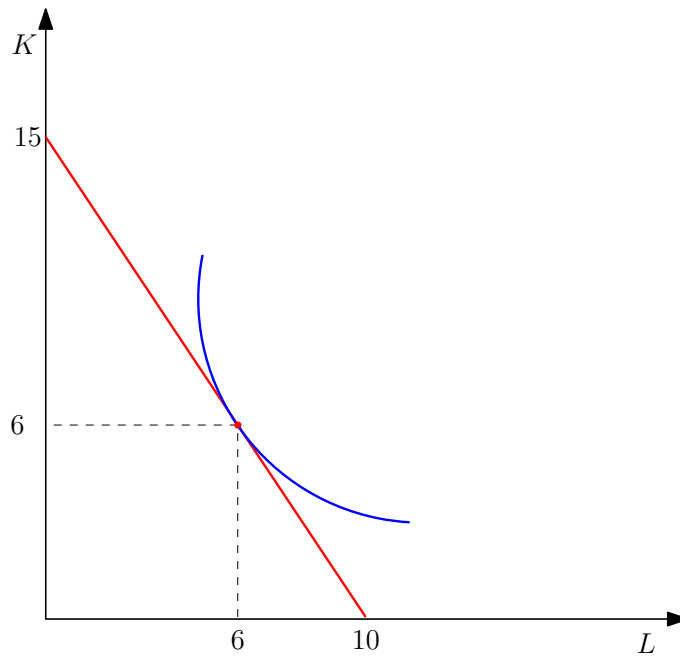
(iii) (*1 point*) At the optimal point, the slopes of the isoquant and of the isocost lines are equal. This means that

$$MRTS = \frac{w}{r} \quad \Rightarrow \quad \frac{MPP_L}{MPP_K} = \frac{w}{r} \quad \Rightarrow \quad \frac{10}{MPP_K} = \frac{150}{100} \quad \Rightarrow \quad MPP_K = \frac{100}{150} \cdot 10 = 6.67.$$

(iv) (*0.5 points*) If the price of robots increased, the isocost line will tilt inward around the horizontal intercept. The slope of the new isocost lines will be $-\dfrac{150}{150} = -1$. If the firm continues to produce 100 units, the total cost has to increase and the optimal input mix will include less robots and more workers.
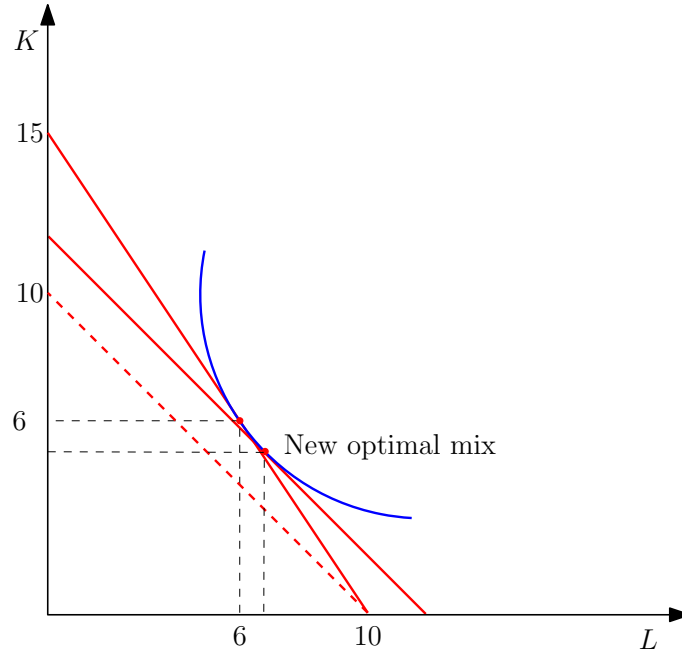
Question 3 (*3 points*)

(i) (*1 point*) The short-run average variable cost is

$$AVC_{SR}(Q) = \frac{VC_{SR}(Q)}{Q} = \frac{Q^3 - 20Q^2 + 220Q}{Q} = Q^2 - 20Q + 220.$$

(ii) (*1 point*) The short-run marginal cost is

$$MC_{SR}(Q) = \frac{\partial VC_{SR}(Q)}{\partial Q} = 3Q^2 - 20 \cdot 2Q + 220 = 3Q^2 - 40Q + 220.$$

(iii) (*1 point*) The optimal production decision of the firm is to produce at the level that equates the short-run average variable and marginal costs:

$$AVC_{SR}(Q) = MC_{SR}(Q) \quad \Rightarrow \quad Q^2 - 20Q + 220 = 3Q^2 - 40Q + 220 \quad \Rightarrow \quad 2Q^2 - 20Q = 0 \quad \Rightarrow$$
$$2Q(Q - 10) = 0 \quad \Rightarrow \quad Q = 10.$$

Hence, the optimal output for the firm is 10 units (or zero, but this would mean the firm shuts down and thus we ignore this solution).

Question 4 (*2.5 points*)

(i) (*1 point*) Since any combination of beet sugar and corn syrup can be used, the two inputs are perfect substitutes (at a ratio of one-for-one), in which case the isoquants will look like in the graph below. The slope of the isoquants is given by the negative of the rate of substitution between the two inputs, which is $\frac{1}{1} = 1$.
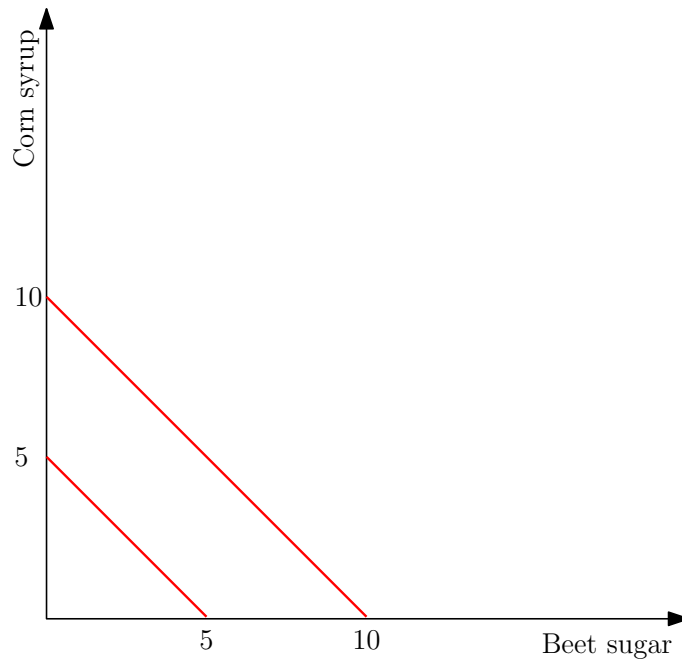
(ii) (*0.75 points*) The slope of an isocost line is given by the negative of the price ratio: $-\frac{1}{1.5} = -0.67$, which is less (in absolute value) than the slope of the isoquant. In this case, the isocost lines are flatter than the isoquant and the lowest attainable isocost line has only the horizontal intercept in common with the isoquant, as shown in the graph below. This means that the optimal input mix will include only beet sugar (the cheaper input), 200 pounds of it, and no corn syrup.

Isoquant

200

Isocost line

200   Beet sugar

Corn syrup
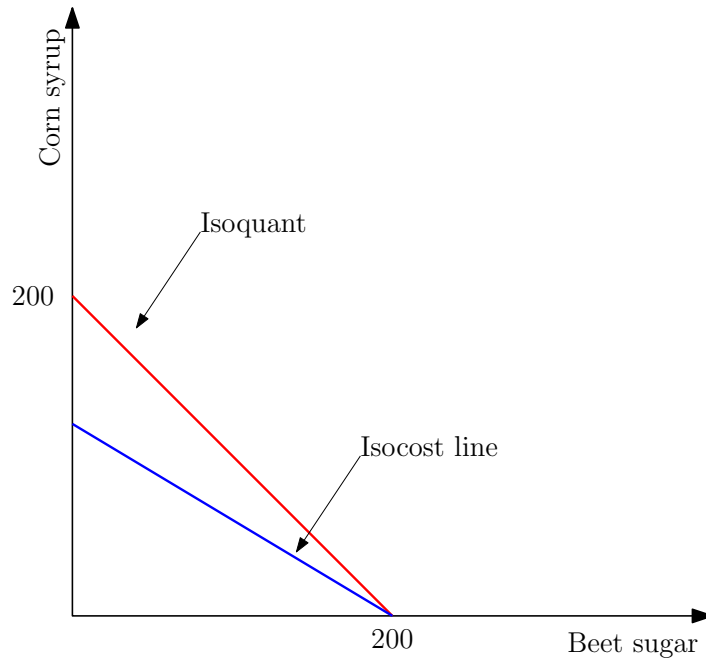
(iii) (*0.75 points*) The slope of an isocost line is now: $-\dfrac{2}{1.5} = -1.33$, which is more (in absolute value) than the slope of the isoquant. The isocost lines are then steeper than the isoquant and the lowest attainable isocost line has only the vertical intercept in common with the isoquant, as shown in the graph below. This means that the optimal input mix will include only corn syrup (the cheaper input in this case), 200 pounds of it, and no beet sugar.

Corn syrup

200

Isocost line

Isoquant

200   Beet sugar