Confidence Intervals Cheat Sheet

$(1-\alpha)\cdot 100\%$ Confidence interval for θ

$$\boxed{\hat{\theta} \pm (\text{critical value})\, SD[\hat{\theta}]}$$

Population Parameter θ	Sample Statistic $\hat{\theta}$	Standard Error $SD[\hat{\theta}]$	Critical Value to Use	Assumptions that Must Be Met
μ	$\bar{X}$	$\sigma/\sqrt{n}$ → or $s/\sqrt{n}$ →	$Z_{1-\alpha/2}$ $t_{1-\alpha/2}$ (with $df = n-1$)	• Random sample • $n < 10\%$ of population • $n > 30$ or population dist. is normal
p	$\hat{p}$	$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$Z_{1-\alpha/2}$	• Random sample • $n < 10\%$ of population • $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$
$\mu_1 - \mu_2$	$(\bar{X}_1 - \bar{X}_2)$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$Z_{1-\alpha/2}$ $t_{1-\alpha/2}$ (with $df = \min(n_1-1, n_2-1)$)	• Samples 1 and 2 are random • $n_1 < 10\%$ of population 1, $n_2 < 10\%$ of population 2 • $n_1 > 30$ or pop. dist. 1 is normal • $n_2 > 30$ or pop. dist. 2 is normal
$p_1 - p_2$	$(\hat{p}_1 - \hat{p}_2)$	$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	$Z_{1-\alpha/2}$	• Samples 1 and 2 are random • $n_1 < 10\%$ of population 1, $n_2 < 10\%$ of population 2 • $n_1\hat{p}_1 \geq 10$ and $n_1(1-\hat{p}_1) \geq 10$ • $n_2\hat{p}_2 \geq 10$ and $n_2(1-\hat{p}_2) \geq 10$

Two-Tailed Hypothesis Test Cheat Sheet

$H_0: \theta = \theta_0$

$H_A: \theta \neq \theta_0$

Method 1

- If θ_0 is in $(\hat{\theta} - \text{critical value } SD[\hat{\theta}], \hat{\theta} + \text{critical value } SD[\hat{\theta}])$

  $\Rightarrow$ Decision a.

- If not, $\Rightarrow$ decision b.

Method 2

calculate test-statistic $= \dfrac{\hat{\theta} - \theta_0}{SD[\hat{\theta}]}$

- If $|\text{test-statistic}| \leq \text{critical value}$,

  $\Rightarrow$ decision a.

- If not, $\Rightarrow$ decision b.

Population Parameter θ	Sample Statistic $\hat{\theta}$	Standard Error $SD[\hat{\theta}]$	Critical Value to Use	Assumptions that Must Be Met
μ	$\bar{x}$	$\sigma/\sqrt{n}$ → or $s/\sqrt{n}$ →	$z_{1-\alpha/2}$ $t_{1-\alpha/2}$ with df $= n-1$	* Random sample * n<10% of the population * n≥30 or population distribution is normal
p	$\hat{p}$	$\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$	$z_{1-\alpha/2}$	* Random sample * n<10% of the population * $n\hat{p}$≥10 and $n(1-\hat{p})$≥10
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ → or $\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ →	$z_{1-\alpha/2}$ $t_{1-\alpha/2}$ with df $= \min(n_1-1, n_2-1)$	* Samples 1 and 2 are random * n1<10% of population 1 • $n_1 \geq 30$ or pop. dist. 1 is normal * n2<10% of population 2 * [redacted] * n2≥30 or population distribution 2 is normal
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	$z_{1-\alpha/2}$	* Samples 1 and 2 are random * n1<10% of population 1 * n2<10% of population 2 [redacted] • $n_1\hat{p}_1 \geq 10$ and $n_1(1-\hat{p}_1) \geq 10$ [redacted] • $n_2\hat{p}_2 \geq 10$ and $n_2(1-\hat{p}_2) \geq 10$

Method 3

If you're not using an "s" in your standard error

p-value $= 2P(Z < -|\text{test stat}|)$

If you're using an "s" in your standard error

p-value $= 2P(t < -|\text{test stat}|)$

- If p-value $\geq \alpha \Rightarrow$ dec. a
- If p-value $< \alpha \Rightarrow$ dec. b

$\boxed{\text{Decision a}}$

- "Fail to reject Ho. There isn't sufficient evidence to suggest Ha."

$\boxed{\text{Decision b}}$

- Reject Ho, There is sufficient evidence to suggest Ha."

Example of Hypothesis Testing (Two-Tailed Test) for the Difference Between Two Proportions

Using our Pew research results, we would like to estimate if there is a difference in the proportion of all adult Independents living in the US that approve of Trump and the proportion of all adult "that have no political preference" living in the US that approve of Trump. Our Pew research dataset contains a random sample of 530 adult independents living in the US in which 34.5% approve of Trump. Our Pew research dataset contains a random sample of 41 adults with "no political preference" living in the US in which 36.6% approve of Trump.

1. Formulate the null and alternative hypotheses to test this claim.

$H_0: p_{ind} - p_{no\,pref} = 0$

$H_a: p_{ind} - p_{no\,pref} \neq 0$

2. What information do we know about the samples?

$\hat{p}_{ind} = .345$ $\quad \hat{p}_{no\,pref} = .366$

$n_{ind} = 530$ $\quad n_{no\,pref} = 41$

$= 1-\alpha \Rightarrow \alpha = .1$

3. Formulate a 90% confidence interval for the population parameter we are interested in.

$$(\hat{p}_{ind} - \hat{p}_{no\,pref}) \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}_{ind}(1-\hat{p}_{ind})}{n_{ind}} + \frac{\hat{p}_{no\,pref}(1-\hat{p}_{no\,pref})}{n_{no\,pref}}}$$

$$(.345 - .366) \pm z_{.95} \sqrt{\frac{.345(1-.345)}{530} + \frac{.366(1-.366)}{41}}$$

$\boxed{(-.1489, .1077)}$

norm.ppf(.95)
$= 1.645$

4. Interpret this confidence interval.

We are 90% confident $p_{ind} - p_{no\,pref}$ is in

$(-.1489, .1077)$.

5. Use this confidence interval to make a conclusion about our null and alternative hypotheses.

The null value, 0, is in $(-.1489, .1077)$. So we fail to reject Ho. There is not sufficient evidence to suggest Ha.

$H_0: p_{ind} - p_{no\,pref} = 0$

6. **Calculate the z-statistic.**

$$\boxed{z\text{-statistic}} = \frac{(\hat{p}_{ind} - \hat{p}_{no\,pref}) - 0}{\sqrt{\frac{\hat{p}_{ind}(1-\hat{p}_{ind})}{n_{ind}} + \frac{\hat{p}_{no\,pref}(1-\hat{p}_{no\,pref})}{n_{no\,pref}}}}$$

7. **Use this z-statistic to make a conclusion about our null and alternative hypotheses.**

$$= \frac{(.345 - .366) - 0}{\sqrt{\frac{.345(1-.345)}{530} + \frac{.366(1-.366)}{41}}}$$

$$= \boxed{-.2637}$$

$$|z\text{-statistic}| \le \underbrace{z_{1-\alpha/2}}_{1.645} \Rightarrow$$

Fail to reject Ho. There is not sufficient evidence to suggest

8. **Calculate the p-value for this hypothesis test and use it to make a conclusion about our null and alternative hypothesis.**

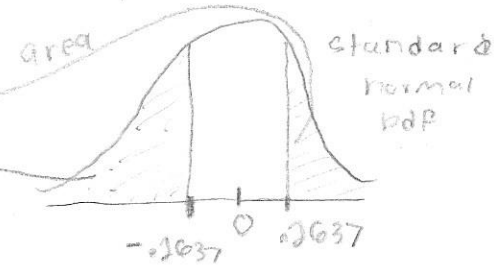
$$\boxed{p\text{-value}} = 2P(Z < -|z\text{-statistic}|)$$

$$= 2P(Z < -|-.2637|)$$

$$= 2\underbrace{P(Z < -.2637)}_{\text{norm.cdf}(-.2637)}$$

$$= 2(.396)$$

$$= \boxed{.792}$$



$p\text{-value} > \alpha = .1$

$\Rightarrow$ Fail to reject Ho. There is not sufficient evidence to suggest Ha.