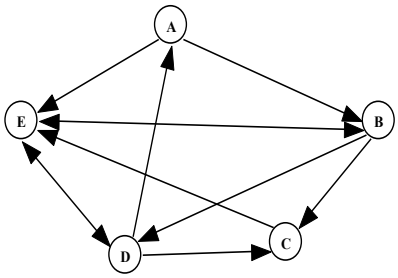
2017-2018 **American Computer Science League** Contest #4

Intermediate Division – Short Problems

1. Graph Theory

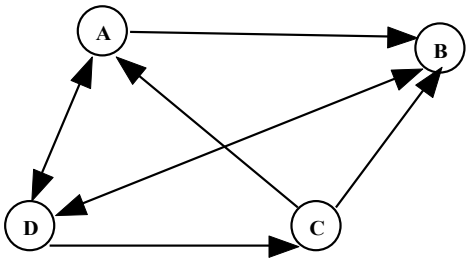
How many paths of length 2 are there in the directed graph at the right?



1.

2. Graph Theory

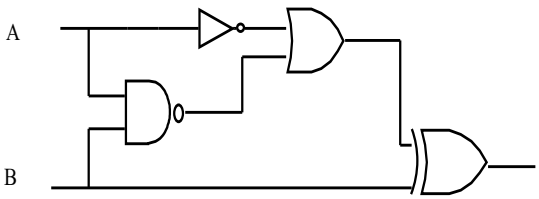
How many cycles exist in the directed graph at the right?



2.

3. Digital Electronics

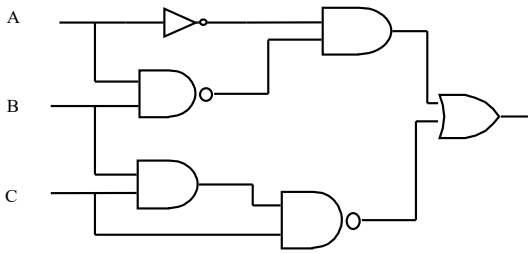
Simplify the Boolean expression represented by the digital circuit at the right.



3.

4. Digital Electronics

Which ordered triple(s) make the digital circuit at the right FALSE?



4.

5. Assembly Language

What is the sum of the outputs of the following assembly program after it is executed?

```
A       DC      4213
B       DC      16
TOP     LOAD    A
        DIV     B
        STORE   C
        BE      DOWN
        LOAD    C
        MULT    B
        STORE   E
        LOAD    A
        SUB     E
        STORE   F
        PRINT   F
        LOAD    C
        STORE   A
        BU      TOP
DOWN    PRINT   A
        END
```

5.

Intermediate Division – Solutions to Short Problems

1. Graph Theory

To find the number of paths of length 2, add the entries in the square of the adjacency matrix. The sum is 24.

$$\begin{vmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}^2 = \begin{vmatrix} 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 2 \\ 1 & 0 & 2 & 1 & 2 \end{vmatrix}$$

1. 24

2. Graph Theory

The cycles are: ABDA, ABDCA, ADCA, ADA, BDB, and BDCB.

2. 6

3. Digital Electronics

The circuit translates to: $(\overline{A} + \overline{AB}) \oplus B$

$(\overline{A} + \overline{AB}) \oplus B = \overline{(\overline{A} + \overline{AB})}B + (\overline{A} + \overline{AB})\overline{B} = (\overline{\overline{A}(\overline{AB})})B + (\overline{A} + (\overline{A} + \overline{B}))\overline{B}$

$= (AAB)B + \overline{A}\overline{B} + \overline{B}\overline{B} = AB + \overline{A}\overline{B} + \overline{B} = AB + \overline{B}(\overline{A} + 1) = AB + \overline{B}$

Note: It would have been fewer steps if the first term had been simplified first.

3. $AB + \overline{B}$ or $A + \overline{B}$

4. Digital Electronics

The circuit translates to: $(\overline{A}(\overline{AB})) + (\overline{(BC)C})$

$(\overline{A}(\overline{AB})) + (\overline{(BC)C}) = (\overline{A}(\overline{A} + \overline{B})) + \overline{BC} = \overline{A} + \overline{A}\overline{B} + \overline{B} + \overline{C}$

$= \overline{A} + \overline{B} + \overline{C}.$

This is FALSE when all three terms are 0, so $\overline{A} = 0 \wedge \overline{B} = 0 \wedge \overline{C} = 0$.

The corresponding ordered triple is (1, 1, 1).

4. (1, 1, 1)

5. Assembly Language

This program converts a base ten number into a base 16 number by repeated division. The integral remainders are outputted.

$4213_{10} = 1075_{16}$ The sum of the digits outputted is 13.

5. 13