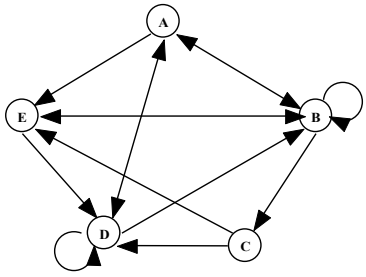
Senior Division – Short Problems

1. Graph Theory

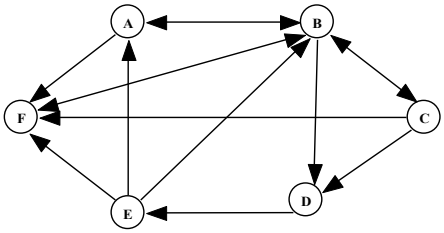
Which starting vertex has the most paths of length 3 in the directed graph at the right and how many are there?



1.

2. Graph Theory

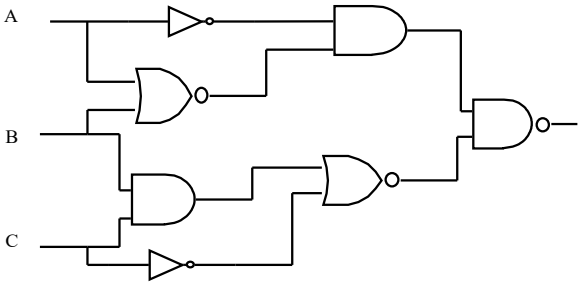
How many cycles exist from B in the directed graph at the right?



2.

3. Digital Electronics

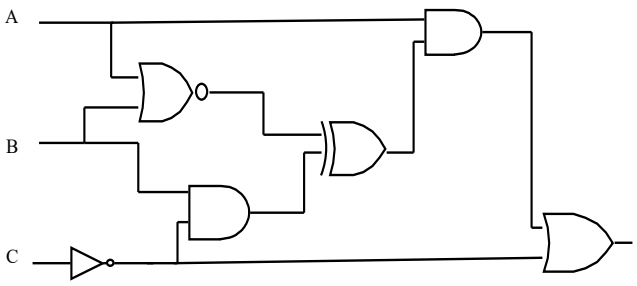
Which ordered triple(s) make the digital circuit at the right FALSE?



3.

4. Digital Electronics

Simplify the Boolean expression represented by the digital circuit at the right.



4.

5. Assembly Language

What is the output of the following assembly program after it is executed?

```
A    DC    7325
B    DC    8
T    DC    0
LOAD    A
DIV     =10
STORE   C
MULT    =10
STORE   D
LOAD    A
SUB     D
STORE   E
LOAD    C
DIV     =10
STORE   D
MULT    =10
STORE   H
LOAD    C
SUB     H
MULT    B
STORE   F
LOAD    D
DIV     =10
STORE   N
MULT    =10
STORE   M
LOAD    D
SUB     M
MULT    B
MULT    B
STORE   X
LOAD    N
MULT    B
MULT    B
MULT    B
STORE   W
LOAD    T
ADD     W
ADD     E
ADD     F
ADD     X
STORE   T
PRINT   T
END
```

(continued to the next column)

5.

Senior Division – Solutions to the Short Problems

1. Graph Theory

To find the number of paths of length 3 from, each vertex, add the entries in the cubed adjacency matrix in each row. B has the most at 32.

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}^3 = \begin{vmatrix} 5 & 8 & 3 & 6 & 6 \\ 6 & 10 & 3 & 8 & 5 \\ 4 & 5 & 2 & 3 & 3 \\ 5 & 9 & 3 & 7 & 6 \\ 3 & 6 & 2 & 5 & 5 \end{vmatrix}$$

1. B 32

2. Graph Theory

The 13 cycles from B are: BAB, BAFB, BCB, BCDEB, BCDEAB, BCDEAFB, BCDEFB, BCFB, BDEB, BDEAB, BDEAFB, BDEFB, BFB

2. 13

3. Digital Electronics

The circuit translates to: $\overline{(\overline{A}(\overline{A+B}))(\overline{BC+\overline{C}})}$

$$\overline{(\overline{A}(\overline{A+B}))(\overline{BC+\overline{C}})} = \overline{\overline{A}(\overline{A+B})} + \overline{\overline{BC+\overline{C}}} = \overline{\overline{A}} + \overline{\overline{A+B}} + BC + \overline{C}$$
$$= A + A + B + BC + \overline{C} = A + B(1+C) + \overline{C} = A + B + \overline{C}$$

So $A + B + \overline{C} = 0$ implies $A = 0 \wedge B = 0 \wedge \overline{C} = 0$. (0, 0, 1) makes it FALSE.

3. (0, 0, 1)

4. Digital Electronics

The circuit translates to: $A(\overline{A+B} \oplus B\overline{C}) + \overline{C}$

$$A(\overline{A+B} \oplus B\overline{C}) + \overline{C} = A(\overline{\overline{A+B}B\overline{C}} + \overline{A+B}\,\overline{B\overline{C}}) + \overline{C}$$
$$= A((A+B)B\overline{C} + \overline{A}\,\overline{B}(\overline{B} + \overline{\overline{C}})) + \overline{C}$$
$$= A(AB\overline{C} + B\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,\overline{B}C) + \overline{C}$$
$$= AB\overline{C} + AB\overline{C} + A\overline{A}\,\overline{B} + A\overline{A}\,\overline{B}C + \overline{C}$$
$$= AB\overline{C} + \overline{C} = \overline{C}(AB+1) = \overline{C}$$

4. $\overline{C}$

5. Assembly Language

This program converts 7325_8 to a base 10 number.

5. 3797