Partial Differential Equations Study Guide

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1 Energy Methods to Prove Uniqueness

We can consider the following problem:

1. $E_x = u_{tt} + 2u_{xt} - 3u_{xx} = 0, u(x,0) = 0$

Note that the above equation is equivalent to the following:

$$E = \frac{1}{2}\int_{-\infty}^{\infty} (1)u_t^2 + (-3)u_x^2 \; dx$$

If it is not clear at first, note how we integrate with respect to x. Then, once we rewrite the problem in the form of an integral with respect to x, we can derive the energy equation with respect to t. This gives us the following:

$$\frac{dE(t)}{dt} = \frac{1}{2}\int_{-\infty}^{\infty} (2)u_t u_{tt} + (-3)(2)u_{xt}u_x \; dx$$

We can then use a method from calculus: integration by parts! Recall:

$$\int u^{'} v \; dx = uv - \int uv^{'}$$

We can set u, $u^{'}$, v, $v^{'}$ to be the following:

$$u = u_x \qquad v^{'} = u_{xt}$$

$$u^{'} = u_{xx} \qquad v = u_t$$

Integration by parts thus gives us:

$$\frac{dE(t)}{dt} = \int_{-\infty}^{\infty} u_t(3u_{xx} - 2u_{xt}) - 3u_{xt}(u_x) \; dx$$

$$\implies u \equiv C$$

$$u(x,0) = 0 \implies u \equiv 0$$