Section 4.2

Part #3

Find $\frac{d^2y}{dx^2}$ or y''

a. $x^3 + y^3 = 8$

$\frac{d}{dx} \quad \frac{d}{dx} \quad \frac{d}{dx}$

$3x^2 + 3y^2 \cdot y' = 0$

$3y^2 y' = -3x^2$

$y' = \frac{-3x^2}{3y^2} = -\frac{x^2}{y^2}$

$y'' = \frac{y^2 \cdot \frac{d}{dx}(-x^2) - (-x^2)\frac{d}{dx}(y^2)}{(y^2)^2}$

$y'' = \frac{y^2(-2x) - (-x^2)(2y \cdot y')}{y^4}$

$= \frac{-2xy^2 + 2x^2 y \cdot y'}{y^4}$

$= \frac{-2xy^2 + 2x^2 \cancel{y}\left(-\frac{x^2}{y^{\cancel{2}}}\right)}{y^4}$

$= \frac{-2xy^2\left(\frac{y}{y}\right) - \frac{2x^4}{y}}{y^4} = \frac{\frac{-2xy^3 - 2x^4}{y}}{y^4}$

$= \frac{-2xy^3 - 2x^4}{y^5} = \frac{-2x(y^3 + x^3)}{y^5}$

$= \frac{-2x(8)}{y^5} = \frac{-16x}{y^5}$

b. $5y^2+6x^2+9y=4$

$$10yy'+12x+9y'=0$$

$$y'(10y+9)=-12x$$

$$y'=\frac{-12x}{10y+9}$$

$$y''=\frac{(10y+9)\frac{d}{dx}(-12x)-(-12x)\frac{d}{dx}(10y+9)}{(10y+9)^2}$$

$$=\frac{(10y+9)(-12)+12x(10y')}{(10y+9)^2}$$

$$=\frac{-120y-108+120xy'}{(10y+9)^2}$$

$$=\frac{-120y-108+120x\left(\frac{-12x}{10y+9}\right)}{(10y+9)^2}$$

$$=\frac{-120y\left(\frac{10y+9}{10y+9}\right)-108\left(\frac{10y+9}{10y+9}\right)+120x\left(\frac{-12x}{10y+9}\right)}{(10y+9)^2}$$

$$=\frac{-120y(10y+9)-108(10y+9)+120x(-12x)}{(10y+9)^3}$$

$$=\frac{-1200y^2-1080y-1080y-972-1440x^2}{(10y+9)^3}$$

$$=\frac{-1200y^2-2160y-972-1440x^2}{(10y+9)^3}$$

$$=\frac{-12(100y^2+180y+81+120x^2)}{(10y+9)^3}$$

$$=\frac{-12((100y^2+180y+120x^2)+81)}{(10y+9)^3}$$

$$=\frac{-12(20(5y^2+9y+6x^2)+81)}{(10y+9)^3}$$

$$=\frac{-12(20(4)+81)}{(10y+9)^3}$$

$$=\frac{-12(161)}{(10y+9)^3}=\frac{-1932}{(10y+9)^3}$$

Prove the power rule applies for rational exponents. $\frac{d}{dx}x^n = nx^{n-1}$

If n<1, then the derivative does not exist at x=0.

Let $n = \frac{p}{q}$, $p, q \in I$, $q \neq 0$

$$y = x^n = x^{p/q}$$

$$(y)^q = (x^{p/q})^q$$

$$\frac{d}{dx} y^q = \frac{d}{dx} x^p$$

$$q y^{q-1} \cdot y' = p x^{p-1}$$

$$y' = \frac{p x^{p-1}}{q y^{q-1}} = \frac{p x^{p-1}}{q (x^{p/q})^{q-1}}$$

$$= \frac{p x^{p-1}}{q x^{p - p/q}}$$

$$= \frac{p}{q} x^{p-1-p+p/q}$$

$$= \frac{p}{q} x^{p/q - 1}$$

$$= n x^{n-1}$$

Find the derivative of the following.

a. $y = x^{\frac{7}{3}}$

$y' = \frac{7}{3} x^{4/3}$

b. $y = (4x^3 + 1)^{\frac{1}{6}}$

$y' = \frac{1}{6}(4x^3+1)^{-\frac{5}{6}}(12x^2)$

$= 2x^2(4x^3+1)^{-\frac{5}{6}}$

Homework:

p. 167 #27-31,

33-41 odd

Answers:

28. $y' = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

$$\frac{d^2y}{dx^2} = \frac{\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right)}{3x^{\frac{4}{3}}y^{\frac{1}{3}}} = \frac{1}{3x^{\frac{4}{3}}y^{\frac{1}{3}}}$$

30. $y' = \frac{1}{y+1}$

$$\frac{d^2y}{dx^2} = -\frac{1}{(y+3)^3}$$