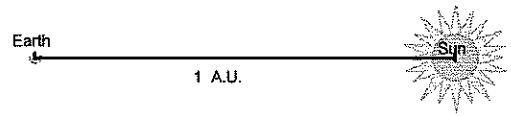
Kepler's Third Law

$$K = \frac{r^3}{T^2}$$

The key to using the third law is to ensure that you use two orbiting bodies that share the same focus mass.

ratio of distances:

$$A.U. = \frac{r_{object}}{r_{Earth}}$$



	body radius	orbital period	mean orbital radius
	m		km
Earth	6.3713×10^{6}	1 y	1.4957×10^{8}
Moon	1.737×10^{6}		3.85×10^{5}
ISS		92.75 min	

conversion factors:	1 y = 365.2422 d	(based on 400 year average)
	1 d = 23 h 56 min 4 s	(solar day based on orbit of Sun)

1. The biggest separation distance between Comet 67P and the Sun is 5.683 AU. Express this measurement in kilometres.

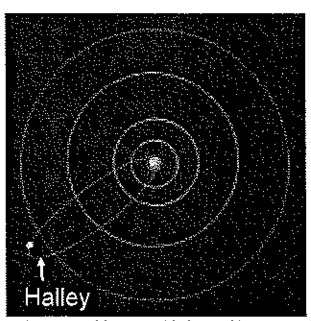
2. By comparison, the biggest separation distance between Halley's Comet and the Sun is 5.25×10^{9} km. Express this measurement in astronomical units.

3. Halley's Comet has an orbital period of 75.3 y around the Sun. Use Kepler's third law to predict the mean orbital radius for Halley's orbit in kilometres.

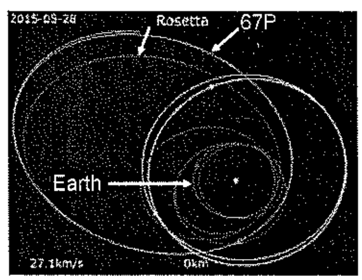
4. Mercury has an orbital period of 87.77 d. Use Kerpler's third law to predict the mean orbital radius of Mercury's orbit using astronomical units.

5. A satellite placed in orbit around the Earth has a mean orbital radius equal to one third of the radius of the Moon's orbit around Earth. Use Kepler's third law to derive an expression for the period of the satellite based on the Moon's period for orbiting the Earth. (Not looking for a numeric answer. Looking for a ratio.)

6. Suppose the ISS orbits at an altitude of 435 km above the Earth. Calculate the period of the Moon, in solar Earth days, orbiting around the Earth. (Numeric answer.)

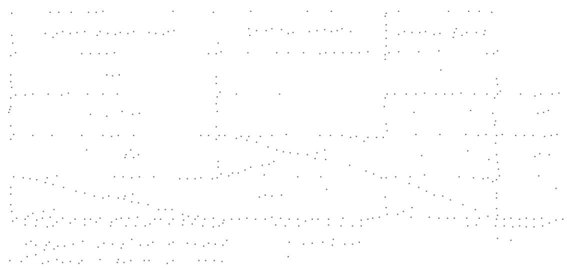


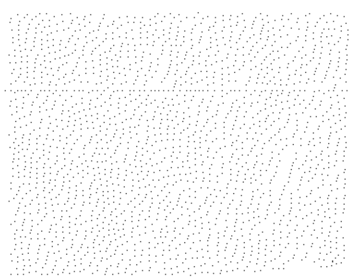
https://en.wikipedia.org/wiki/Halley%27s_Comet

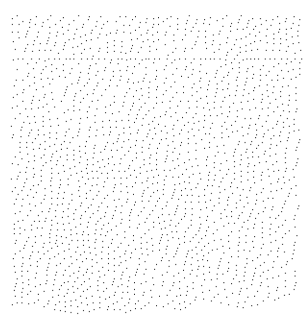


https://en.wikipedia.org/wiki/67P/Churyumov%E2%80%93Gerasimenko

answers: 1. 8.500×10^{8} km
2. 35.1 AU
3. 2.67×10^{9} km
4. 0.3865 AU
4. 5.781×10^{10} m
5. $0.19\, T_{moon}$
6. 25.7 d







Keplers 3rd Law WS

#1) r[AU] = 5.683 AU

r[m] = ?

$$r[m] = 5.683\ A.U. \times \frac{1.4957 \times 10^{11}\ m}{1\ A.U.}$$

$$= 8.500063 \times 10^{11}\ m = \boxed{8.500 \times 10^{8}\ Km}$$

#2) r[m] = 5.25×10^{9} Km

r[A.U.] = ?

$$r[A.U.] = 5.25 \times 10^{12}\ m \times \frac{1\ A.U.}{1.4957 \times 10^{11}\ m}$$

$$= 35.1006\ A.U. = \boxed{35.1\ A.U.}$$

#3) $T_H = 75.3\,y$

$r_H = ?$

$T_E = 1.000\ y$

$r_E = 1.4957 \times 10^{11}\ m$

$$\frac{r_H^3}{T_H^2} = \frac{r_E^3}{T_E^2} \rightarrow r_H = \sqrt[3]{\frac{r_E^3\, T_H^2}{T_E^2}}$$

$$= \sqrt[3]{\frac{(1.4957 \times 10^{8}\ Km)^3 (75.3_y)^2}{(1.000_y)^2}}$$

$$= 2.66711 \times 10^{9}\ Km$$

$$= \boxed{2.67 \times 10^{9}\ Km}$$

#4) $T_M = 87.77\,d$

$r_M\,[A.U.] = ?$

$T_E = 365.2422\,d$

$r_E = 1.000\ A.U.$

$$\frac{r_M^3}{T_M^2} = \frac{r_E^3}{T_E^2}$$

$$r_M = \sqrt[3]{\frac{r_E^3\,T_M^2}{T_E^2}}$$

$$r_M = \sqrt[3]{\frac{(1.000\ A.U)^3\,(87.77\,d)^2}{(365.2422\,d)^2}} = 0.386524\ A.U$$

$$= \boxed{0.3865\ A.U}$$

#5) $r_s = \frac{1}{3} r_m$

$r_m = 3.85 \times 10^5\ m$

$T_s = ?$

$$\frac{r_s^3}{T_s^2} = \frac{r_m^3}{T_m^2}$$

$$\frac{r_s^3}{r_m^3} = \frac{T_s^2}{T_m^2} \longrightarrow \left(\frac{r_s}{r_m}\right)^3 = \left(\frac{T_s}{T_m}\right)^2$$

$$\frac{T_s}{T_M} = \sqrt{\left(\frac{1}{3}\right)^3} = 0.19245 = \boxed{0.19}$$

#6) $r_I = 435\ Km$

$T_I = 92.75\ min$

$r_M = 3.85 \times 10^5\ km$

$T_M = ?$

$R_E = 6.38 \times 10^6\ m$

$$23 + 56\ min \times \frac{1\,h}{60\,min} + 4\,s \times \frac{1\,h}{3600\,s} = 23.9344\ h = 1\,day$$

$$\frac{T_m^2}{r_m^3} = \frac{T_I^2}{r_I^3} \longrightarrow T_M = \sqrt{\frac{r_m^3\,T_I^2}{r_I^3}} = \sqrt{\frac{r_m^3\,T_I^2}{(r_I + R_E)^3}}$$

$$T_M = \sqrt{\frac{(3.85 \times 10^8\,m)^3\,(0.064586\ day)^2}{(6.3713 \times 10^6\,m + 435 \times 10^3\,m)}} = 27.4766\ day$$

$$= \boxed{27.5\ days}$$

$$92.75\ min \times \frac{1\,h}{60\,min} \times \frac{1\,d}{23.9344\,h} = 0.064586\ day$$