2.2 Limits Involving Infinity

Infinity Exists!

We will look at limits as x approaches infinity:

$$\lim_{x \to \infty} f(x)$$

The limit of *f* as *x* approaches infinity

DEFINITION **Horizontal Asymptote**

The line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if either

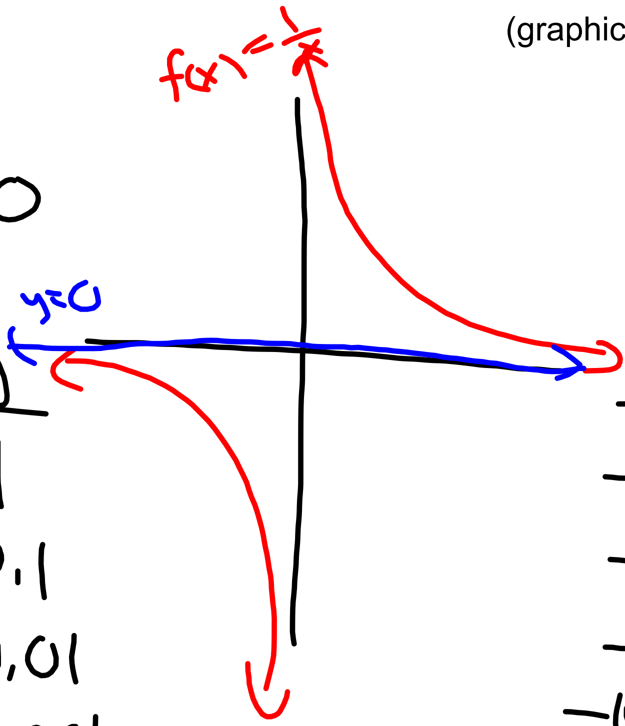
$$\lim_{x \to \infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b.$$

Determine the following limits:

(graphically and table)

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$



x	y
1	1
10	0.1
100	0.01
1000	0.001
10,000	0.0001
1,000,000	0.000001

x	y
−1	−1
−10	−0.1
−100	−0.01
−1000	−0.001
−10,000	−0.0001
−1000000	−0.000001

$$y = \frac{\sin x}{x}$$ Revisited

Determine:

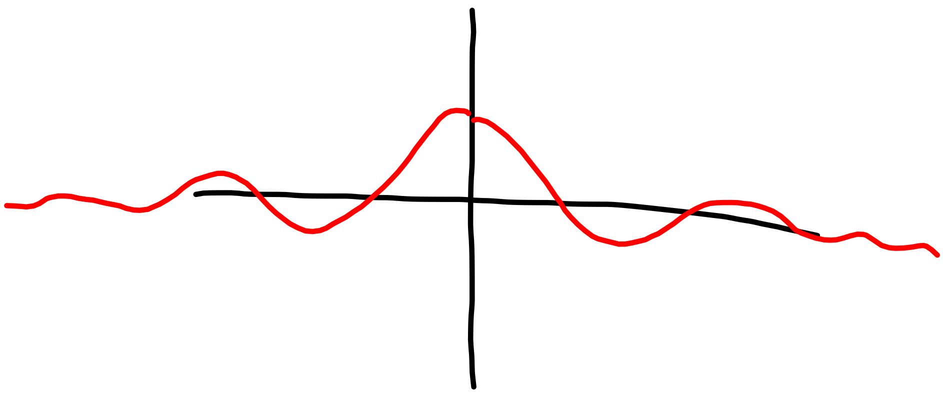
(logic, table, graph, sandwich theorem)

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

x	y
1	0.84
10	-0.05
100	-0.005
1000	0.00083
10,000	-0.0000031
1000000	-0.00000035

$$\lim_{x\to-\infty} \frac{\sin x}{x} = 0$$

x	y
-1	0.84
-10	-0.05
-100	-0.005
-1000	0.00083
-10,000	-0.000031
-1,000,000	-0.00000035



All the limit rules from 2.1 apply when dealing with ±∞.

If $\lim\limits_{x\to c} f(x) = L$ $\lim\limits_{x\to c} g(x) = M$

where L, M, c, and k are real numbers

Sum $\lim\limits_{x\to c}\left(f(x)+g(x)\right) = L+M$

Difference $\lim\limits_{x\to c}\left(f(x)-g(x)\right) = L-M$

Product $\lim\limits_{x\to c}\left(f(x)\bullet g(x)\right) = L\bullet M$

Quotient $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{L}{M}, M \neq 0$

Constant Multiple $\lim\limits_{x\to c} k\bullet f(x) = k\bullet L$

Power $\lim\limits_{x\to c}\left(f(x)\right)^{r/s} = L^{r/s}$ r and s are integers, and $s \neq 0$

Determine $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to -\infty} f(x)$ for each of the following:

(table and graph --> logic --> algebra tomorrow)

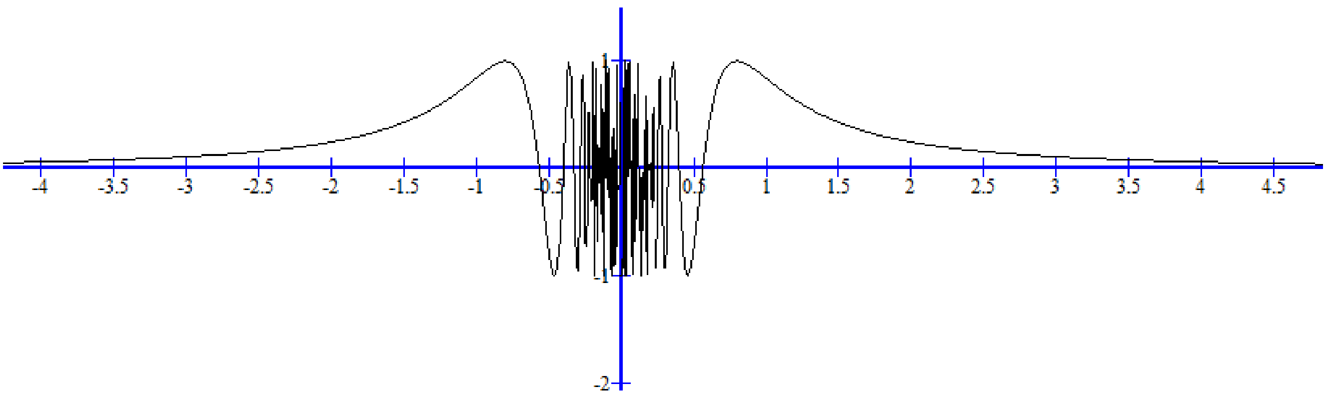
$$f(x)=\sin\left(\frac{1}{x^2}\right)$$

x	y
1	0.84
10	0.09
100	0.000099
1000	0.000 001
10,000	0.000 000 01
1,000,000	0.000 000 000 001

x	y
−1	0.84
−10	0.09
−100	0.000099
−1000	0.000001
−10,000	0.00000001
−1,000,000	0.000000000001

$$\lim_{x\to\infty} \sin\left(\frac{1}{x^2}\right)=0$$

$$\lim_{x\to-\infty} \sin\left(\frac{1}{x^2}\right)=0$$



$$f(x) = \sin\left(\frac{1}{2x-8}\right)$$

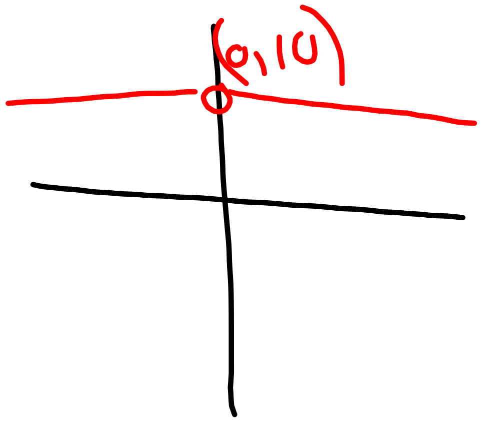
$$f(x) = \frac{10x}{x}$$

x	y
1	$10\frac{(1)}{(1)} = 10$
1000	$10\left(\frac{1000}{1000}\right) = 10$
1000000	10

$$f(x) = \frac{10\cancel{x}}{\cancel{x}} = 10$$

$$\lim_{x \to \infty} f(x) = 10$$

$$\lim_{x \to -\infty} f(x) = 10$$



$$f(x) = \frac{10x - 7}{x + 5}$$

$$\begin{array}{r} 10 \\ x+5 \overline{)\,10x - 7} \\ -(10x + 50) \\ \hline -57 \end{array}$$

$$f(x) = 10 - \frac{57}{x+5}$$

$$\lim_{x \to \infty} \left(10 - \frac{57}{x+5}\right)$$

$$= \lim_{x \to \infty} 10 + \lim_{x \to \infty} \frac{-57}{x+5}$$

$$= 10 + 0 = 10$$

$$\lim_{x \to -\infty} \left(10 + \frac{-57}{x+5}\right)$$

$$= 10 + 0 = 0$$

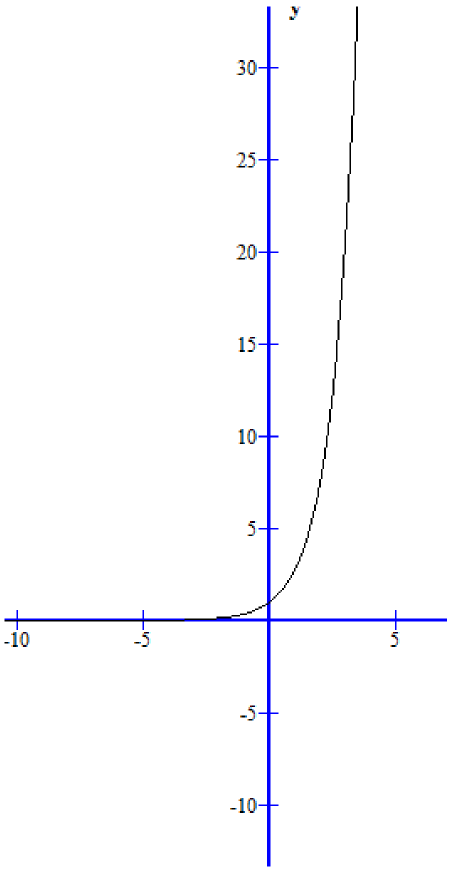
$$f(x) = e^x$$

x	y
1	2.72
10	22026
100	2.69×10^{43}
150	1.39×10^{65}

$$\lim_{x \to \infty} e^x = \infty$$

x	y
-1	0.368
-10	4.5×10^{-5}
-100	3.72×10^{-44}
-1000	0
-10000	0
-100000	

$$\lim_{x \to -\infty} e^x = 0$$



Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for each of the following algebraically:

22. $y = \left(\frac{2}{x} + 1\right)\left(\frac{5x^2 - 1}{x^2}\right)$

24. $y = \frac{2x + \sin x}{x}$

26. $y = \frac{x \sin x + 2 \sin x}{2x^2}$

$$\frac{5x^2}{x^2} - \frac{1}{x^2}$$

$$5 - \frac{1}{x^2}$$

$$\lim_{x\to\infty} \left(\frac{2}{x}+1\right)\left(\frac{5x^2-1}{x^2}\right)$$

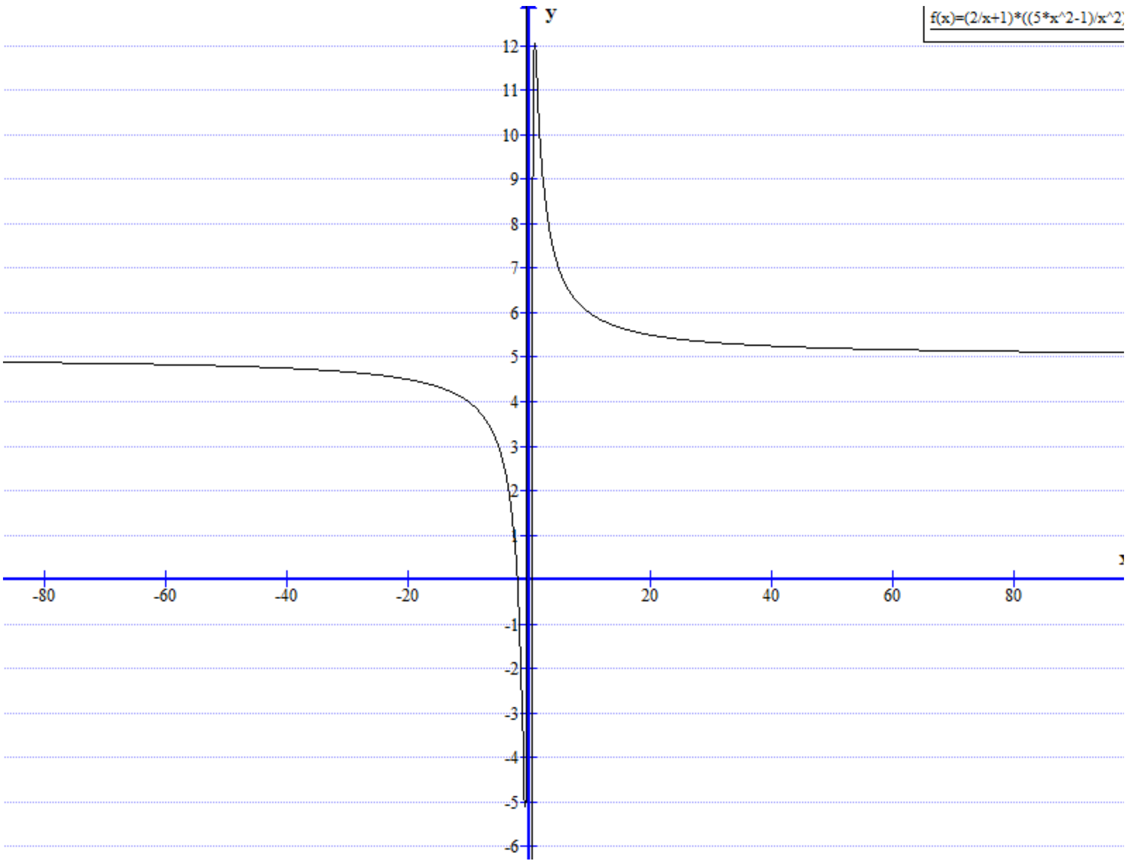
$$\lim_{x\to\infty} \left(\frac{2}{x}+1\right)\left(5-\frac{1}{x^2}\right)$$

$$= (1)(5) = 5$$

$$\lim_{x\to-\infty} \left(\frac{2}{x}+1\right)\left(5-\frac{1}{x^2}\right)$$

$$= (1)(5)$$

$$= 5$$



$$y = \frac{2x + \sin x}{x}$$

$$y = \frac{2x}{x} + \frac{\sin x}{x}$$

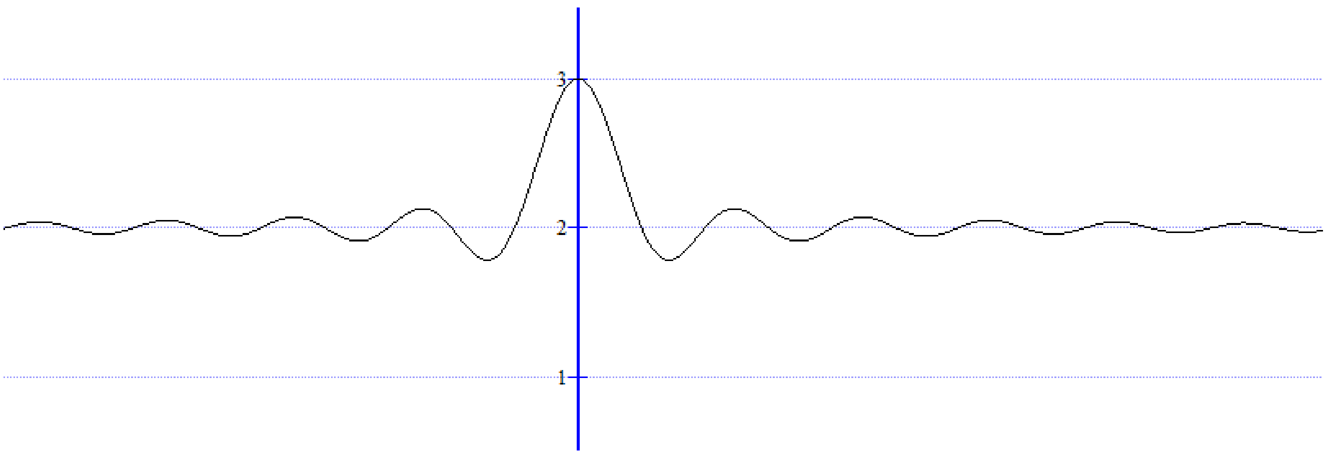
$$y = 2 + \frac{\sin x}{x}$$

$$\lim_{x \to \infty} \left(2 + \frac{\sin x}{x}\right)$$

$$= 2 + 0 = 2$$

$$\lim_{x \to -\infty} \left(2 + \frac{\sin x}{x}\right)$$

$$= 2 + 0 = 2$$



$$y = \frac{x\sin x + 2\sin x}{2x^2}$$

$$y = \frac{x\sin x}{2x^2} + \frac{2\sin x}{2x^2}$$

$$y = \frac{\sin x}{2x} + \frac{\sin x}{x^2}$$

$$\lim_{x\to\infty}\left(\frac{\sin x}{2x} + \frac{\sin x}{x^2}\right)$$

$$\lim_{x\to\infty}\left(\frac{1}{2}\,\overset{\nearrow 0}{\frac{\sin x}{x}} + \overset{\nearrow 0}{\frac{1}{x}}\cdot\overset{\nearrow 0}{\frac{\sin x}{x}}\right)$$

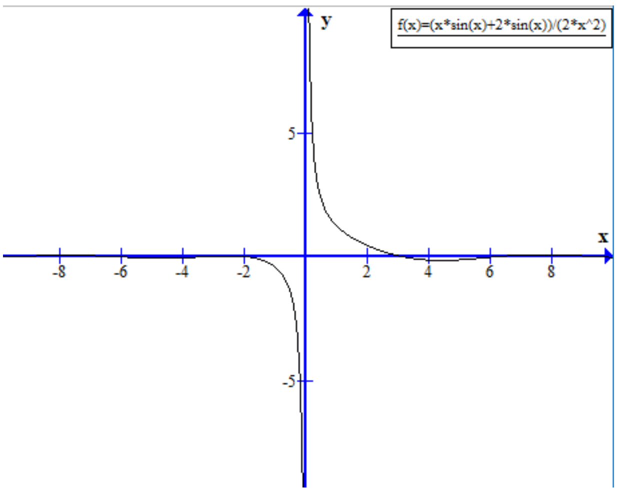
$$0$$

$$\frac{x}{2x} + \frac{2}{2x}$$

$$\frac{1}{2} + \frac{1}{x}$$

$$\lim_{x\to-\infty}\left(\frac{\sin x}{x}\cdot\frac{x+2}{2x}\right)$$

$$\lim_{x\to-\infty}\left(\overset{\nearrow 0}{\frac{\sin x}{x}}\cdot\left(\frac{1}{2}+\overset{\nearrow 0}{\frac{1}{x}}\right)\right)$$



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#1-7 odd

#21-25 all

logic out #9-12