2.1 Rate of Change and Limits

Interval Notation:

Brackets - inequality *without* equal to

Square Brackets - inequality *with* equal to

$-5\pi \leq x \leq \pi$	$[-5\pi, \pi]$
$-100 < x \leq 8$	$(-100, 8]$
$0 \leq x < 10$	$[0, 10)$
$-20 < x < 5$	$(-20, 5)$

Piecewise Functions:

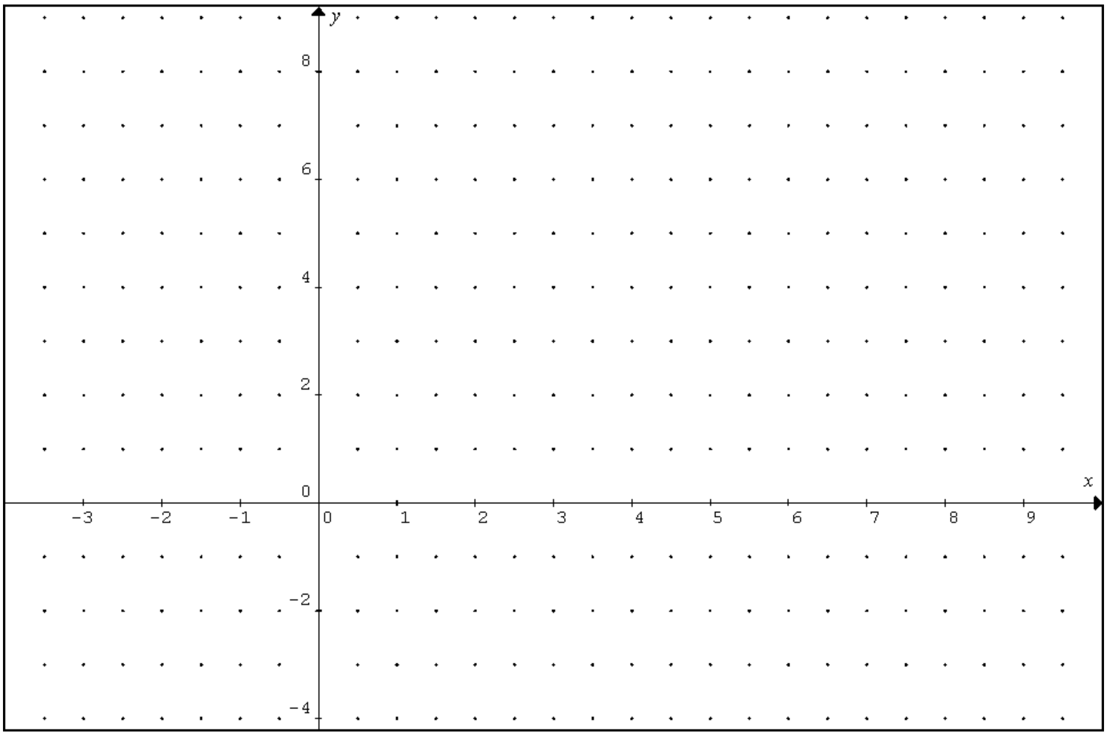
a) Draw the graph of *f*.

b) At what points does only the left-hand limit of $\lim_{x \to c} f(x)$ exist.

c) At what points does only the right-hand limit exist?

d) At what points does the limit exist?

$$f(x) = \begin{cases} x^2, & -3 \le x \le 0 \\ 2x - 4, & 0 < x \le 4 \\ \sqrt{x}, & 4 < x \le 9 \end{cases}$$



Piecewise Functions:

a) Draw the graph of *f*.

b) At what points does only the left-hand limit of $\lim_{x \to c} f(x)$ exist. $x = 9$

c) At what points does only the right-hand limit exist? $x = -3$

d) At what points does the limit exist?

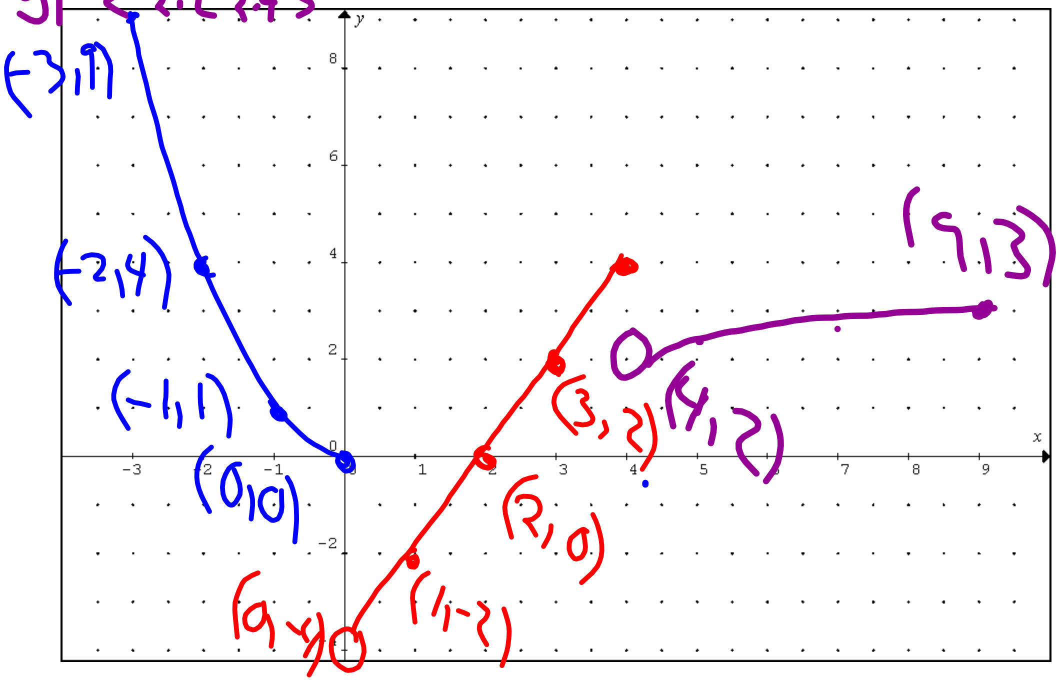
$-3 \le x \le 9 \mid x \ne 0, 4$ $[-3,0)(0,4)(4,9]$

$$f(x) = \begin{cases} x^2, & -3 \le x \le 0 \\ 2x - 4, & 0 < x \le 4 \\ \sqrt{x}, & 4 < x \le 9 \end{cases}$$

x	-3	-2	-1	0
y	9	4	1	0

x	0	1	2	3	4
y	-4	-2	0	2	4

x	4	5	7	9
y	2	2.2	2.4	3



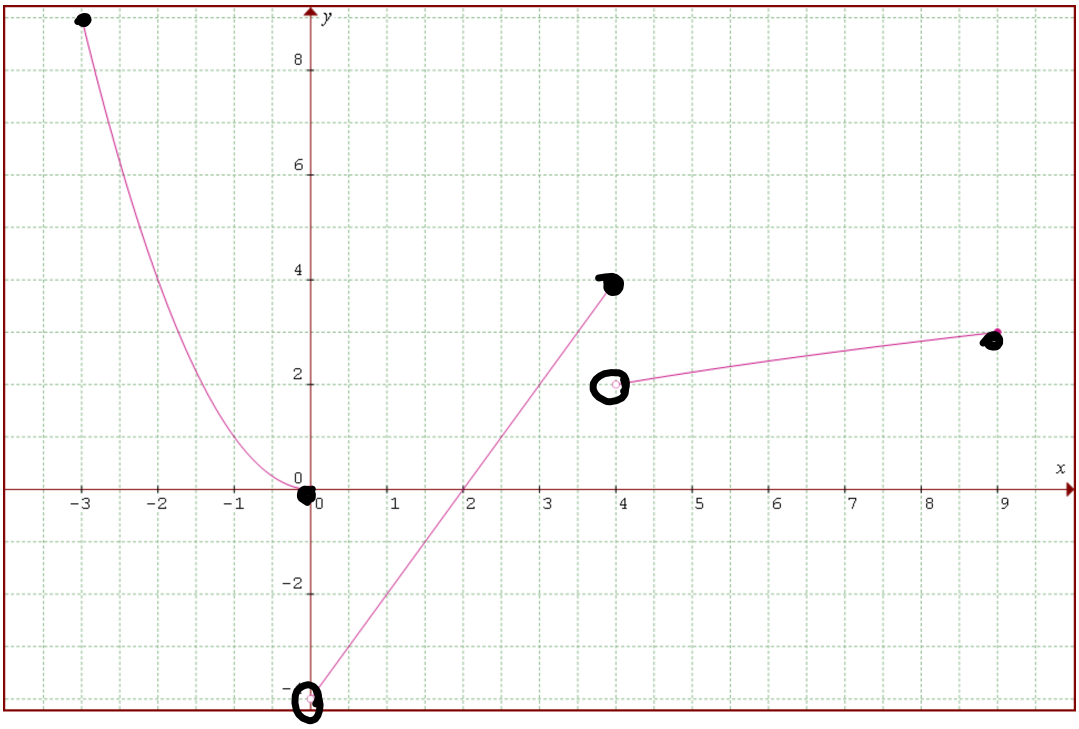
Where does a left-hand limit exist? $-3 < x \le 9$ $(-3, 9]$

Where does a right-hand limit exist?

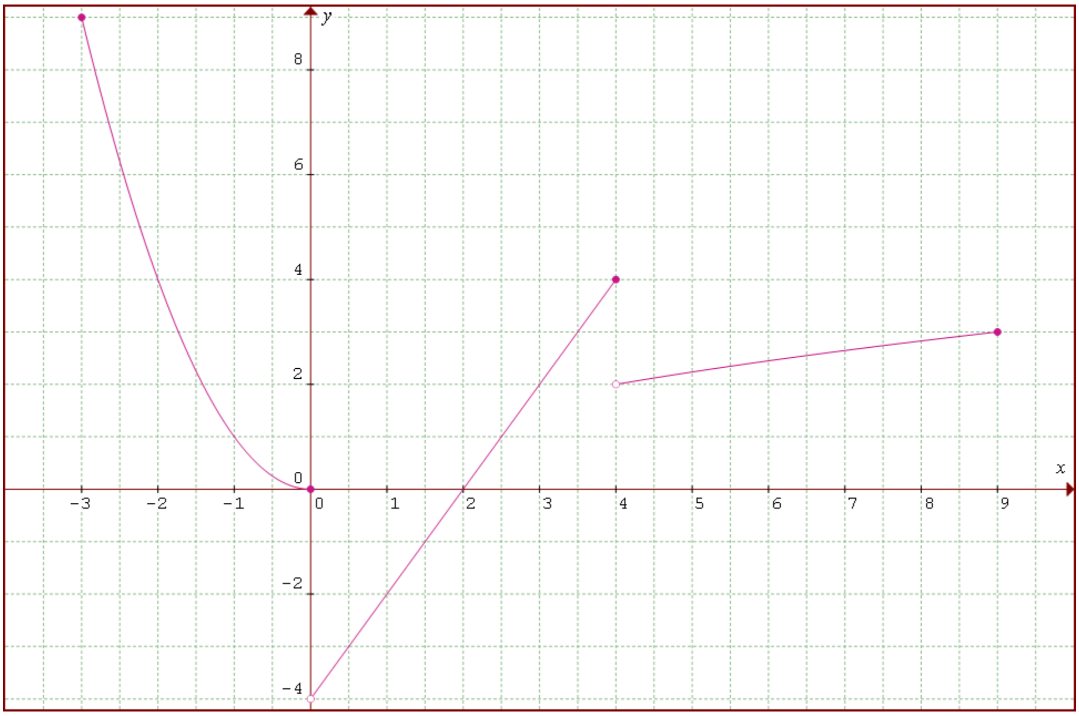
$-3 \le x < 9$

$[-3, 9)$

Where does the limit not exist? $x = 0, 4$

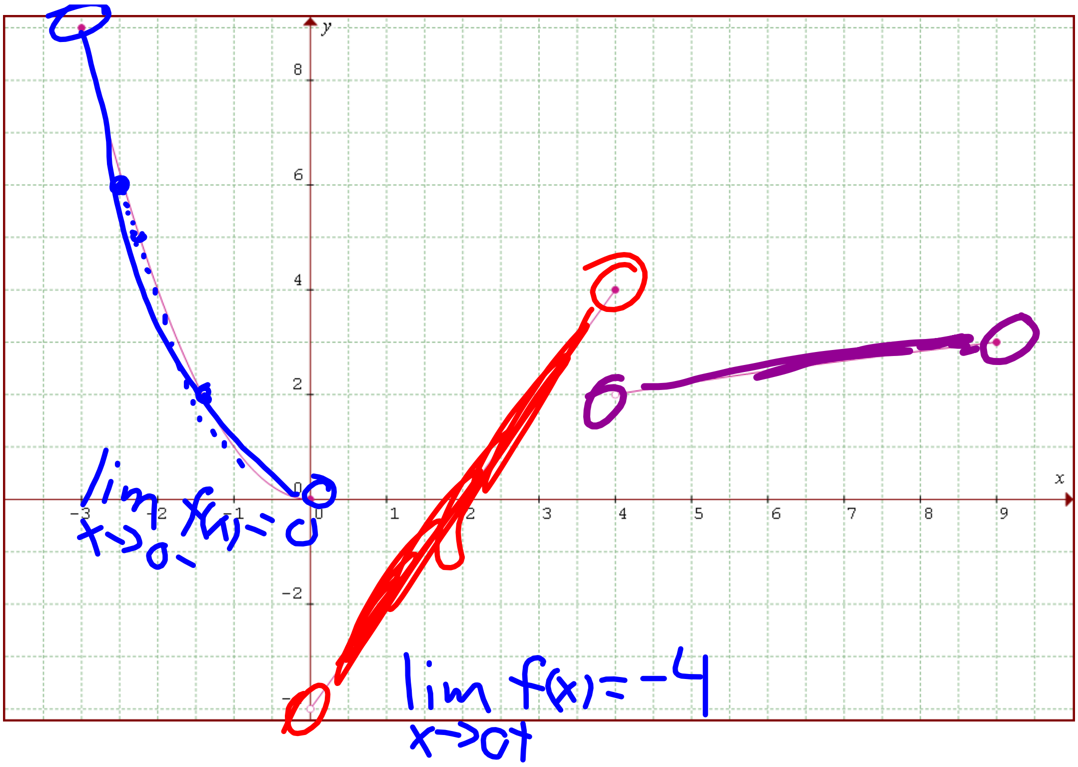


b) At what points does only the left-hand limit of $\lim_{x \to c} f(x)$ exist.



c) At what points does only the right-hand limit exist?

$\lim_{x \to c}$ $c = -3$



d) At what points does the limit exist?

$-3 < x < 0$, $0 < x < 4$, $4 < x < 9$

$(-3, 0)$ $(0, 4)$ $(4, 9)$

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#57-61, 63, 64, 71-76

I'll show 62 if time permits

Integer values within the domain of the graphs.

Get enough points for a visual confirmation of the answers.

--> Graph all now, with the aim of being able to answer the problems without the graph.

$$\lim_{x \to -4} \frac{\frac{x}{x}\cdot\frac{1}{4} + \frac{1}{x}\cdot\frac{4}{4}}{4+x}$$

$$\lim_{x \to -4} \frac{\frac{x}{4x} + \frac{4}{4x}}{4+x}$$

$$\lim_{x \to -4} \frac{\frac{x+4}{4x}}{x+4} = \lim_{x \to 4} \frac{\cancel{x+4}}{4x} \cdot \frac{1}{\cancel{x+4}}$$

$$= \lim_{x \to -4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$$

$$
\begin{array}{r|l}
 & x^2 - 15x + 7 \\
\hline
x-3 & x^3 - 18x^2 + 52x - 21 \\
 & -(x^3 - 3x^2) \\
\hline
 & -15x^2 + 52x \\
 & -(-15x^2 + 45x) \\
\hline
 & 7x - 21 \\
 & -(7x - 21) \\
\hline
 & 0
\end{array}
$$

$$\lim_{x \to 3} \frac{x^3 - 18x^2 + 52x - 21}{x-3}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x^2 - 15x + 7)}{\cancel{(x-3)}}$$

$$= \lim_{x \to 3} (x^2 - 15x + 7) = 3^2 - 15(3) + 7$$

$$= -29$$

c. $$\lim_{x\to 0} \frac{\sin^2 x \cos x}{1-\cos x}$$

$$= \lim_{x\to 0} \frac{(1-\cos^2 x)(\cos x)}{(1-\cos x)}$$

$$= \lim_{x\to 0} \frac{(1+\cos x)\cancel{(1-\cos x)}(\cos x)}{\cancel{(1-\cos x)}}$$

$$= \lim_{x\to 0} (1+\cos x)(\cos x)$$

$$= (1+\cos(0))(\cos(0)) = (1+1)(1) = 2$$

$$\lim_{x \to 0} \frac{1-\cos 2x}{x^2}$$

$$= \lim_{x \to 0} \frac{2\sin^2 x}{x^2}$$

$$= 2\lim_{x \to c} \left(\frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$$