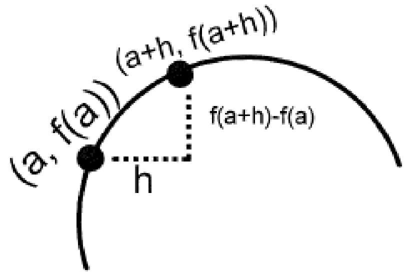
Section 3.1
Part #1
The Derivative

Review: In Chapter 2 we learned that the slope of the tangent of a curve at the point (a, f(a)) is defined by

$$m = \lim_{h \to 0}\left(\frac{f(a+h) - f(a)}{h}\right)$$



https://mathinsight.org/applet/ordinary_derivative_limit_definition

In this chapter we will start to call it the derivative instead of the slope of the tangent and rewrite it slightly different.

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

Notice the new symbol for m and a has changed to x.

Another important note is that the textbook will sometimes use a instead of x. It is nothing to worry about, it's the same thing.

Notation for derivatives:

$$f'(x) \qquad y' \qquad \frac{dy}{dx} \qquad \frac{df}{dx} \qquad \frac{d}{dx}f(x)$$

Using $f'(x) = \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$ find the derivative of

$y = x^2 - 4x$ when x = -2.

$$y' = \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h}$$

$$y' = \lim_{h\to 0} \frac{(-2+h)^2 - 4(-2+h) - \left[(-2)^2 - 4(-2)\right]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4} - 4h + h^2 + \cancel{8} - 4h - \left[\cancel{4 + 8}\right]}{h}$$

$$= \lim_{h\to 0} \frac{h^2 - 8h}{h} = \lim_{h\to 0} \frac{\cancel{h}(h-8)}{\cancel{h}}$$

$$= \lim_{h\to 0} h - 8 = -8$$

Using $f'(a)=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ find the derivative of $y=\dfrac{1}{2x-3}$ when a = -2.

$$\lim_{h\to 0}\frac{\dfrac{1}{2(-2+h)-3}-\left[\dfrac{1}{2(-2)-3}\right]}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{1}{-4+2h-3}-\left[\dfrac{1}{-4-3}\right]}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{7}{7}\cdot\dfrac{1}{2h-7}+\dfrac{1}{7}\cdot\dfrac{2h-7}{2h-7}}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{\cancel{7}}{7(2h-7)}+\dfrac{2h-\cancel{7}}{7(2h-7)}}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{2h}{7(2h-7)}}{h}=\lim_{h\to 0}\frac{2\cancel{h}}{7(2h-7)}\times\frac{1}{\cancel{h}}$$

$$=\lim_{h\to 0}\frac{2}{7(2h-7)}=\frac{2}{7(2(0)-7)}=\frac{-2}{49}$$

Find the derivative of each of the following functions at the indicated point

a) ~~$y = x^2 - 3x$;~~ $x = 10$

$$f(x) = x^3 + 3x - 8$$

$$\lim_{h\to 0} \frac{(10+h)^3 + 3(10+h) - 8 - [10^3 + 3(10) - 8]}{h}$$

$$\lim_{h\to 0} \frac{1000 + 300h + 30h^2 + h^3 + 30 + 3h - 8 - [1000 + 30 - 8]}{h}$$

$$\lim_{h\to 0} \frac{h^3 + 30h^2 + 303h + \cancel{1022} - \cancel{1022}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(h^2 + 30h + 303)}{\cancel{h}} = 0 + 0 + 303$$

$$= 303$$

b) $y = \sqrt{x}$; x = 5

$$y' = \lim_{h\to 0} \frac{\sqrt{5+h} - \sqrt{5}}{h} \cdot \frac{\sqrt{5+h} + \sqrt{5}}{\sqrt{5+h} + \sqrt{5}}$$

$$= \lim_{h\to 0} \frac{\cancel{5} + \cancel{h} - \cancel{5}}{\cancel{h}(\sqrt{5+h} + \sqrt{5})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{5+h} + \sqrt{5}} = \frac{1}{\sqrt{5} + \sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{10}$$

Alternate definition of a derivative at a point:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Find the derivative of $y = x^2 - 4x$ when a = -2.

$$f'(-2) = \lim_{x \to -2} \frac{f(x) - f(-2)}{x - (-2)}$$

$$= \lim_{x \to -2} \frac{x^2 - 4x - [(-2)^2 - 4(-2)]}{x+2}$$

$$= \lim_{x \to -2} \frac{x^2 - 4x - [4+8]}{x+2}$$

$$= \lim_{x \to -2} \frac{x^2 - 4x - 12}{x+2} = \lim_{x \to -2} \frac{(x-6)\cancel{(x+2)}}{\cancel{(x+2)}}$$

$$= \lim_{x \to -2} x - 6 = -8$$

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$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} =$$

$$\lim_{x \to a} \frac{x^2 - 4x - [a^2 - 4a]}{x - a}$$

$$\lim_{x \to a} \frac{x^2 - 4x - a^2 + 4a}{x - a}$$

$$\lim_{x \to a} \frac{(x^2 - a^2) + (-4x + 4a)}{x - a}$$

$$\lim_{x \to a} \frac{(x+a)(x-a) + (-4)(x-a)}{x - a}$$

$$\lim_{x \to a} \frac{\cancel{(x-a)}[x + a - 4]}{\cancel{x-a}}$$

$$\lim_{x \to a} x + a - 4 = a + a - 4 = 2a - 4$$

Since $a = -2$: $2(-2) - 4 = -8$

Find the derivative of $y = \frac{1}{2x-3}$ when a = -2.

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{x \to -2} \frac{\frac{1}{2x-3} - \frac{1}{2(-2)-3}}{x-(-2)}$$

$$= \lim_{x \to -2} \frac{\frac{1}{2x-3} - \left(-\frac{1}{7}\right)}{x+2} = \lim_{x \to -2} \frac{\frac{1}{2x-3} + \frac{1}{7}}{x+2}$$

$$= \lim_{x \to -2} \frac{\frac{7+2x-3}{7(2x-3)}}{x+2} = \lim_{x \to -2} \frac{2x+4}{7(2x-3)} \cdot \frac{1}{x+2}$$

$$= \lim_{x \to -2} \frac{2(x+\cancel{2})}{7(2x-3)} \cdot \frac{1}{x+\cancel{2}} = \frac{2}{7(2(-2)-3)}$$

$$= -\frac{2}{49}$$

$$\lim_{x \to a} \frac{\frac{1}{2x-3} - \frac{1}{2a-3}}{x-a}$$

$$\lim_{x \to a} \frac{\frac{2a-3-(2x-3)}{(2x-3)(2a-3)}}{x-a}$$

$$\lim_{x \to a} \frac{2a-2x}{(2x-3)(2a-3)} \cdot \frac{1}{(x-a)}$$

$$\lim_{x \to a} \frac{-2(x-\cancel{a})}{(2x-3)(2a-3)} \cdot \frac{1}{x-\cancel{a}}$$

$$\lim_{x \to a} \frac{-2}{(2x-3)(2a-3)} = \frac{-2}{(2a-3)^2}$$

$$\frac{-2}{(2(-2)-3)^2} = \frac{-2}{49}$$

b) $y=\sqrt{x}$; $x=5$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=$$

$$\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{x\to a}\cdot\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$$

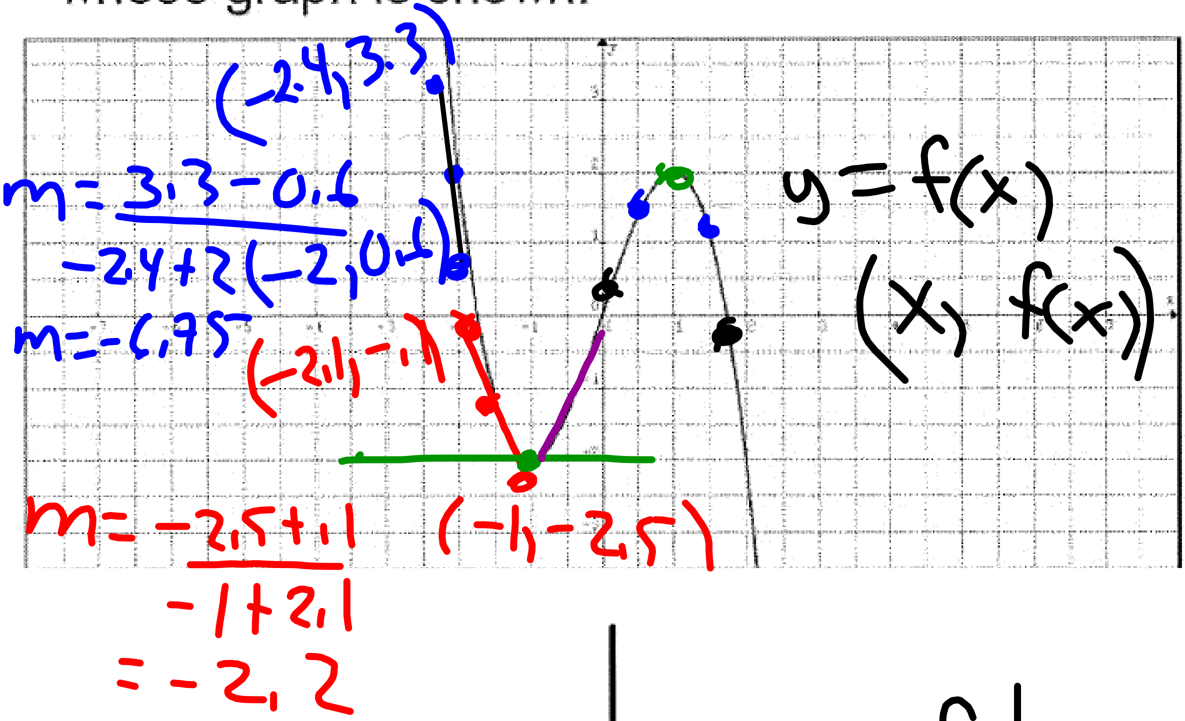
$$\lim_{x\to a}\frac{\cancel{x-a}}{\cancel{(x-a)}(\sqrt{x}+\sqrt{a})}$$

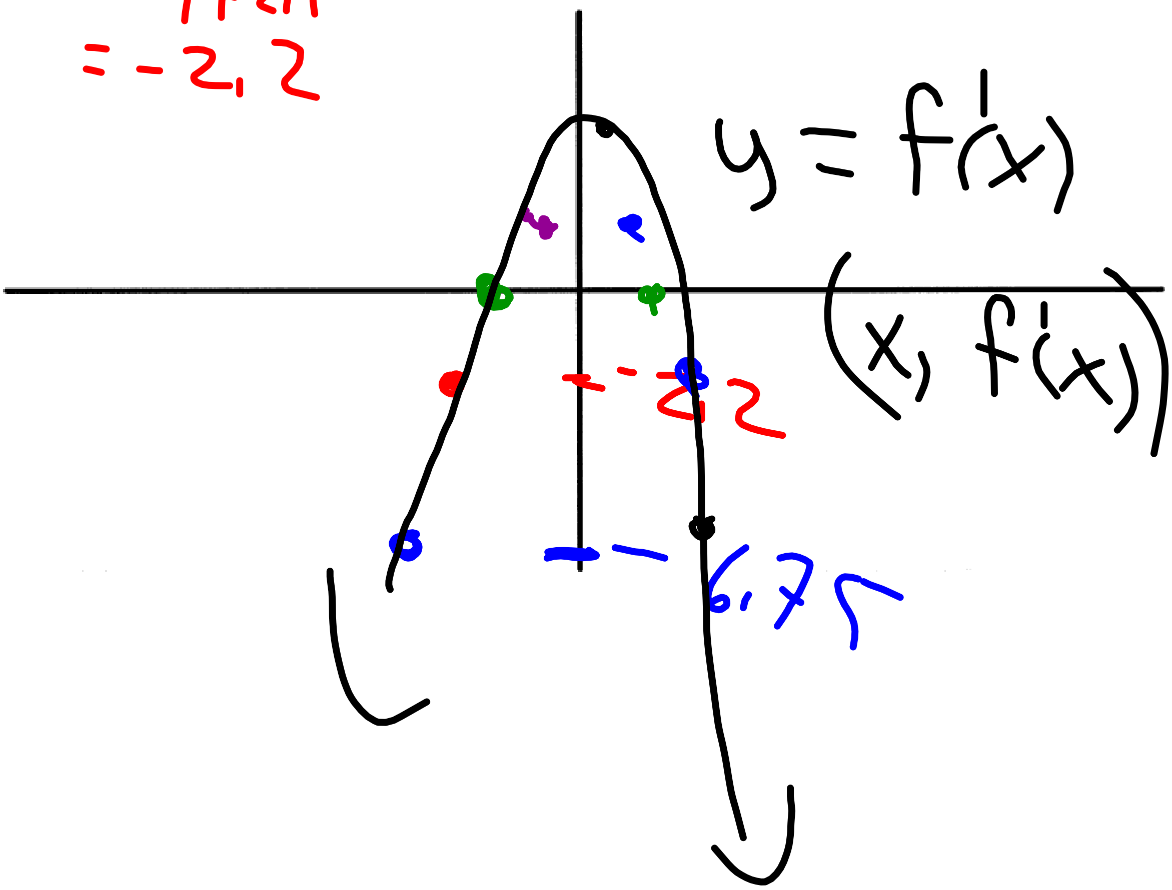
$$\lim_{x\to a}\frac{1}{\sqrt{x}+\sqrt{a}}=\frac{1}{\sqrt{a}+\sqrt{a}}=\frac{1}{2\sqrt{a}}\cdot\frac{\sqrt{a}}{\sqrt{a}}$$

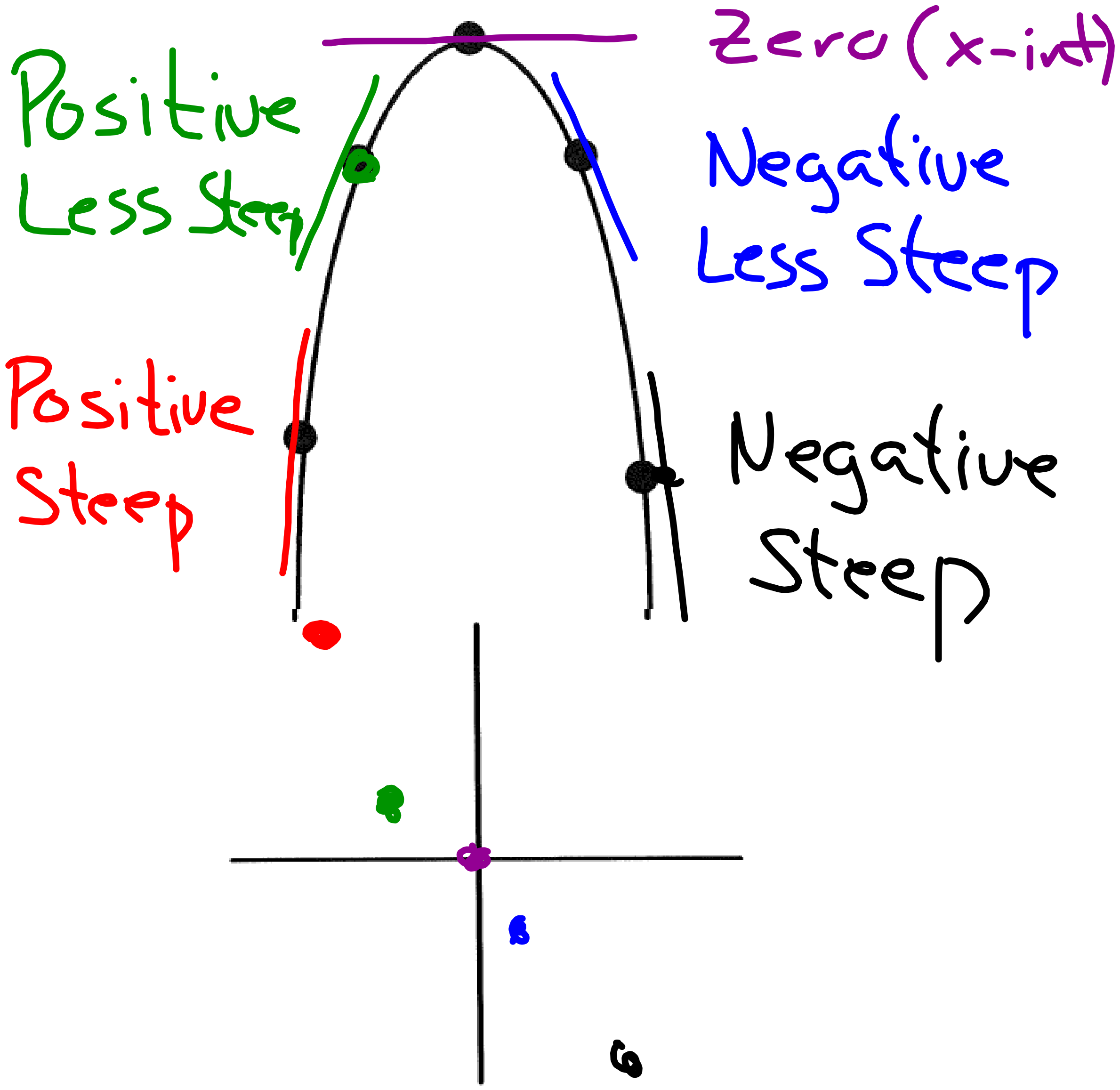
$$=\frac{\sqrt{a}}{2a}=\frac{\sqrt{5}}{2(5)}=\frac{\sqrt{5}}{10}$$

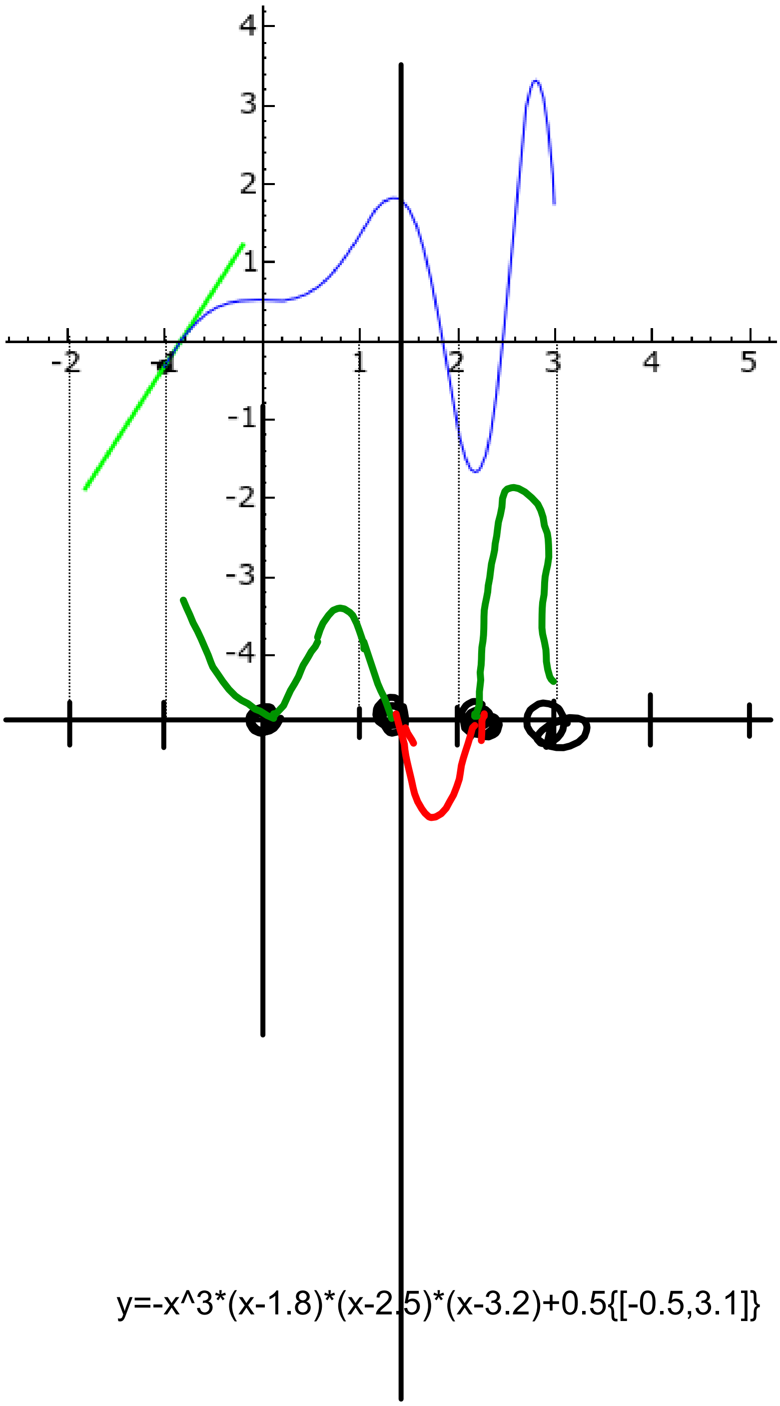
Section 3.1
Part #2
Graph of f vs. f'

https://www.desmos.com/calculator/codaiowpo9 whose graph is shown.





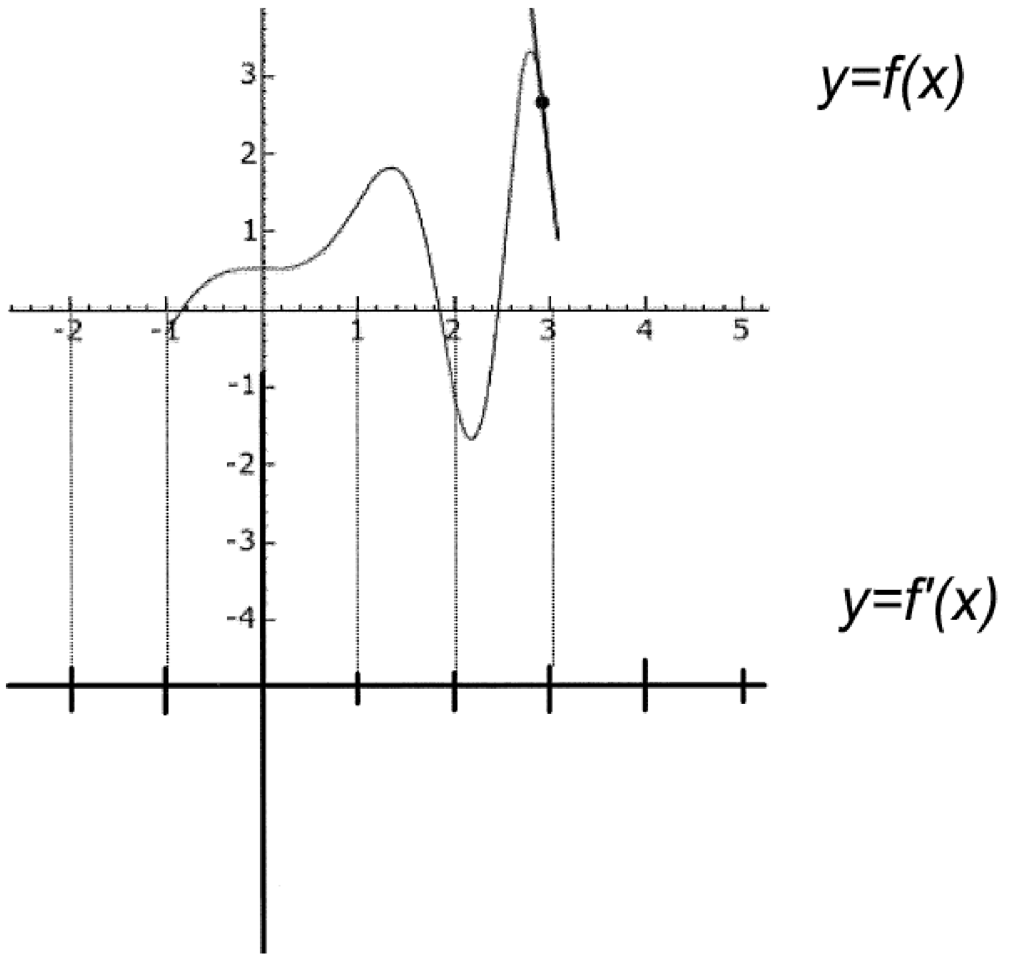

Zero (x-int)
Positive Less Steep
Negative Less Steep
Positive Steep
Negative Steep

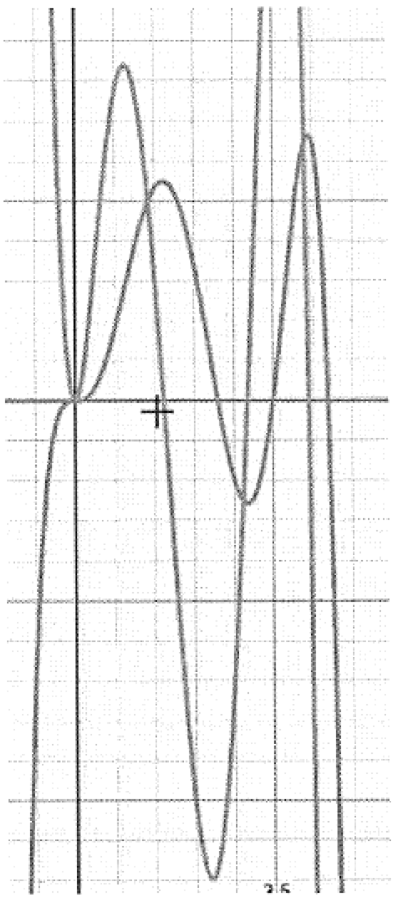


y=f(x)

y=f′(x)

y=-x^3*(x-1.8)*(x-2.5)*(x-3.2)+0.5{[-0.5,3.1]}


y=f(x)
y=f'(x)
3
2
1
-1
-2
-3
-4
-2
-1
1
2
3
4
5



$$f'(x) = -\frac{x^2\left(300x^3 - 1875x^2 + 3652x - 2160\right)}{50}$$

How do I find the derivative of a linear equation?

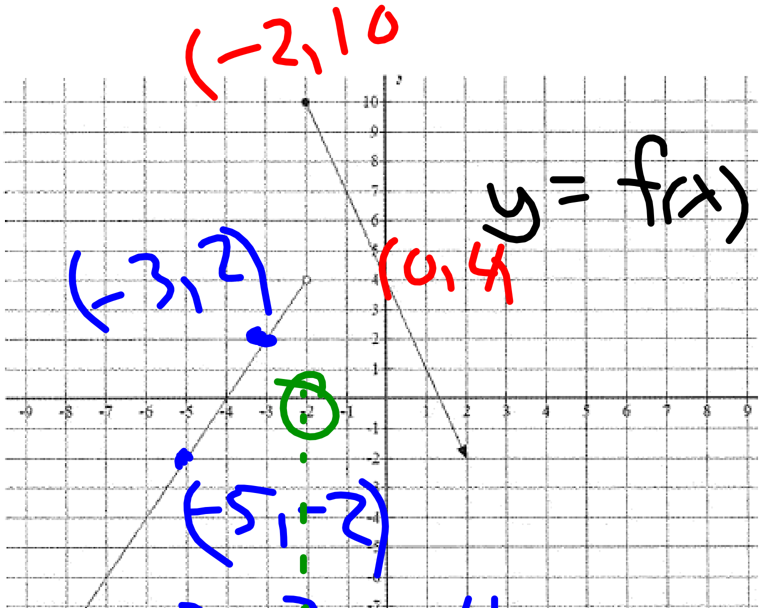
$$y = mx + b$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{m(x+h) + b - [mx + b]}{h}$$

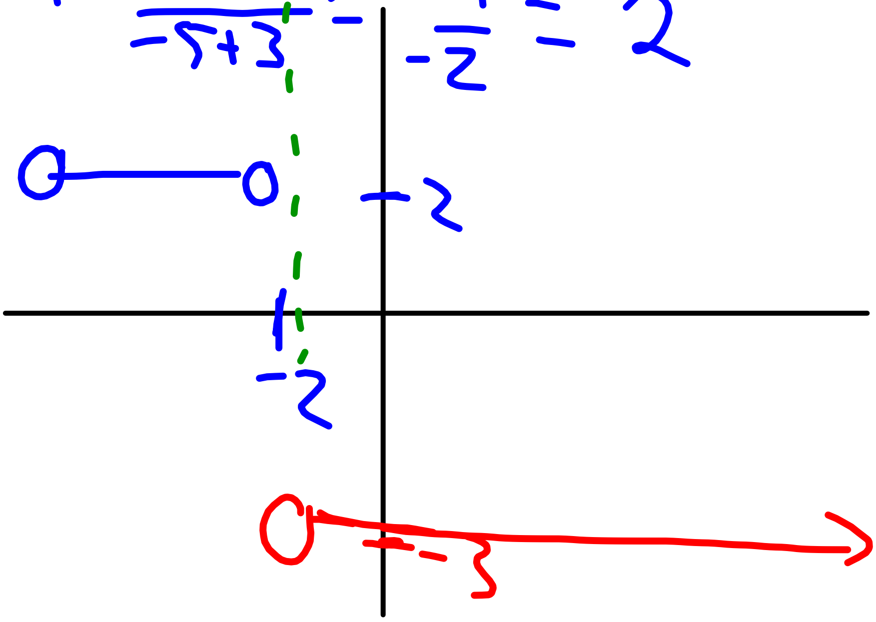
$$\lim_{h \to 0} \frac{m\cancel{x} + mh + \cancel{b} - m\cancel{x} - \cancel{b}}{h}$$

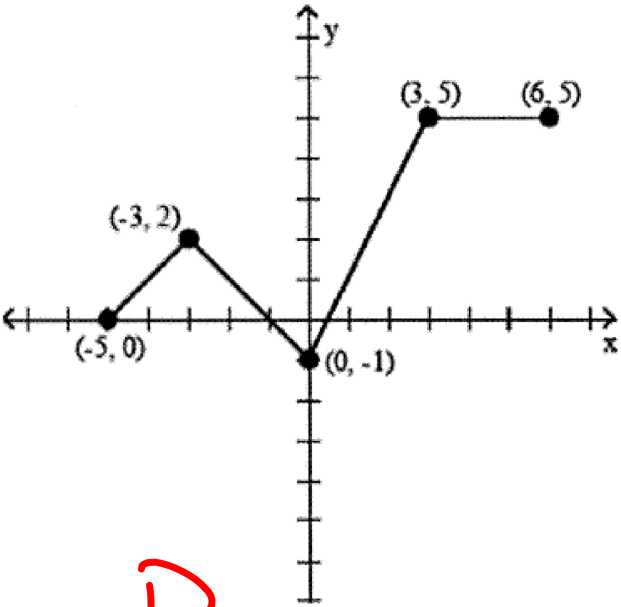
$$\lim_{h \to 0} \frac{m\cancel{h}}{\cancel{h}} = m$$



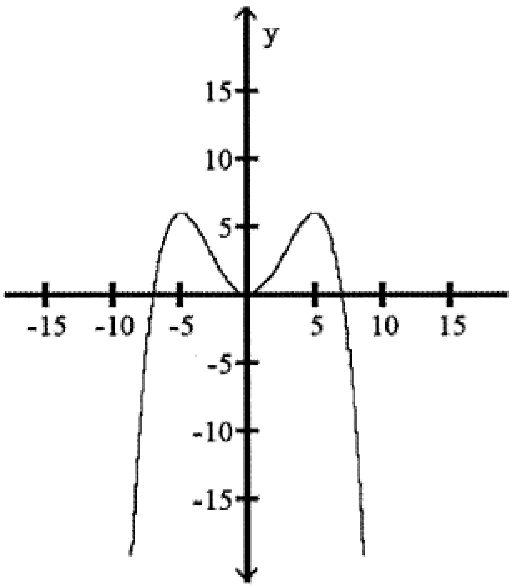
$$m = \frac{4-10}{0+2} = \frac{-6}{2} = -3$$

$$m = \frac{-2-2}{-5+3} = \frac{-4}{-2} = 2$$





B



D

Homework

Finish 3^{rd} Booklet

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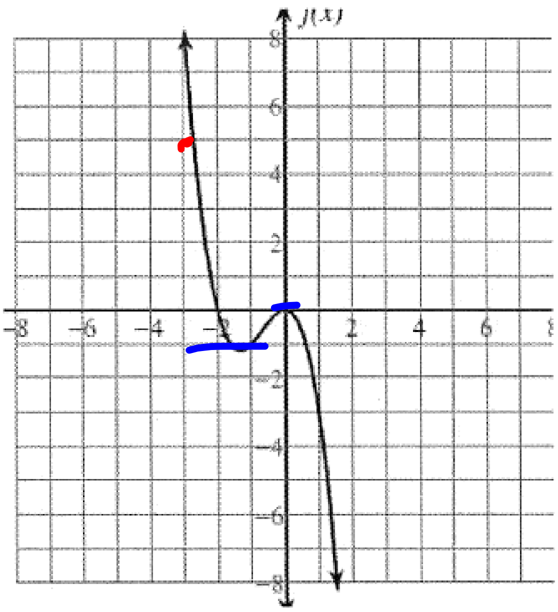
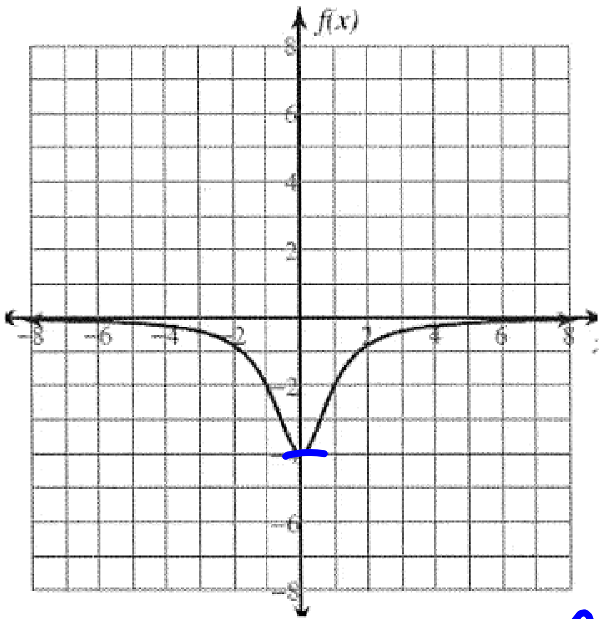
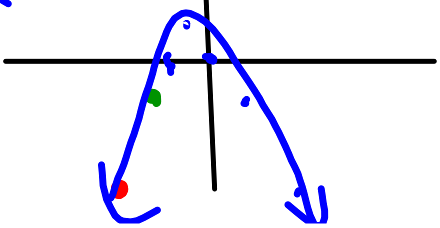
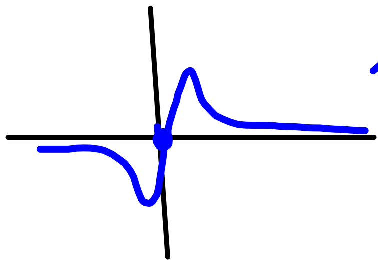




C

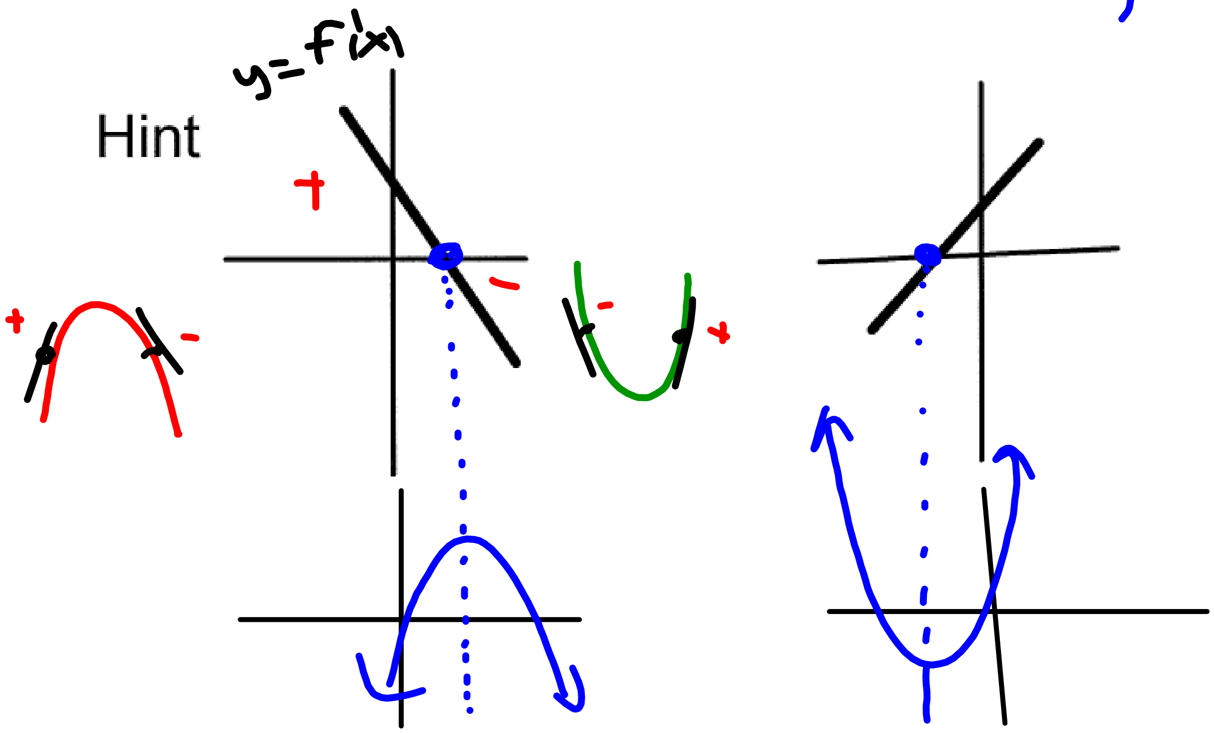


A



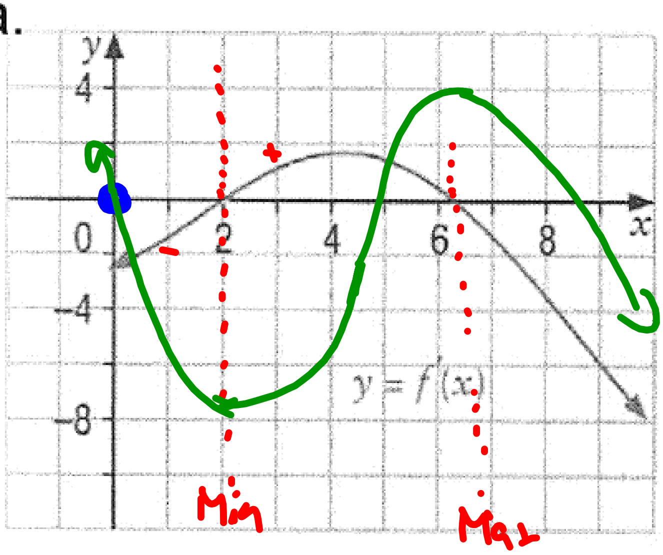
Sketching f(x) given f'(x).

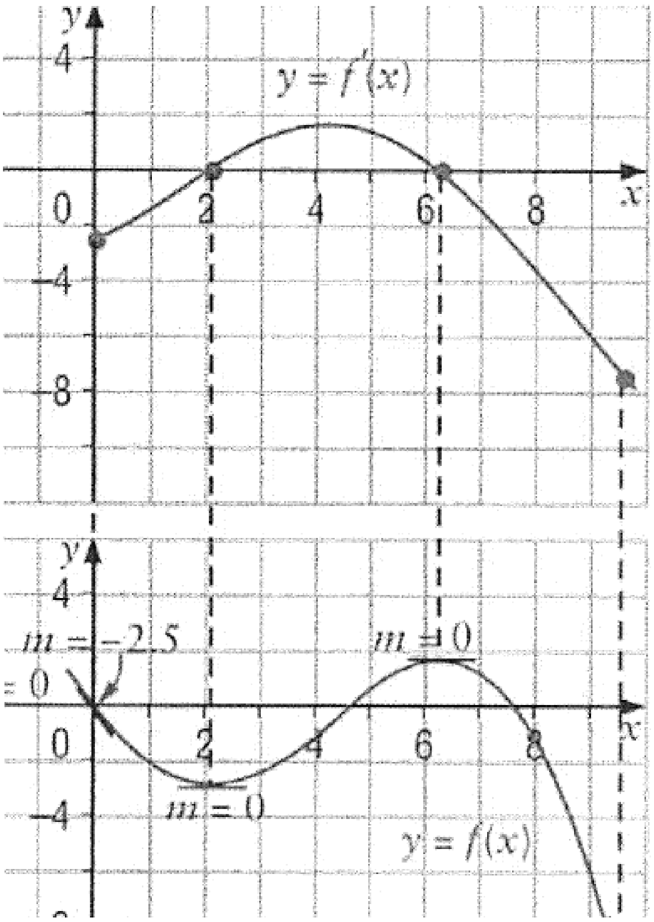
If $f'(x) > 0$, the graph of the function curves upward to the right.
If $f'(x) < 0$, the graph of the function curves downward to the right.
If $f'(x) = 0$, the graph of the function is horizontal. (Usually Max/Min)

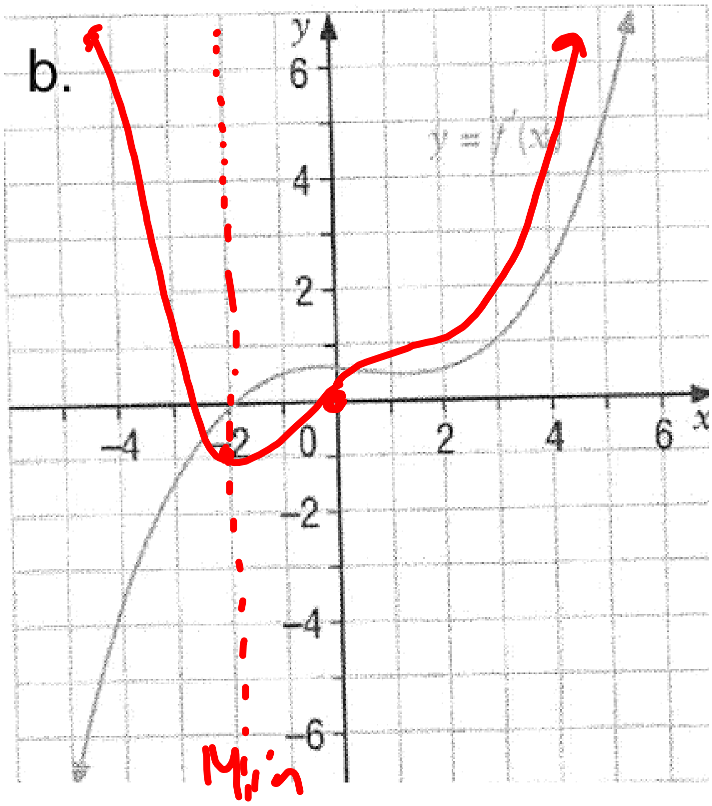


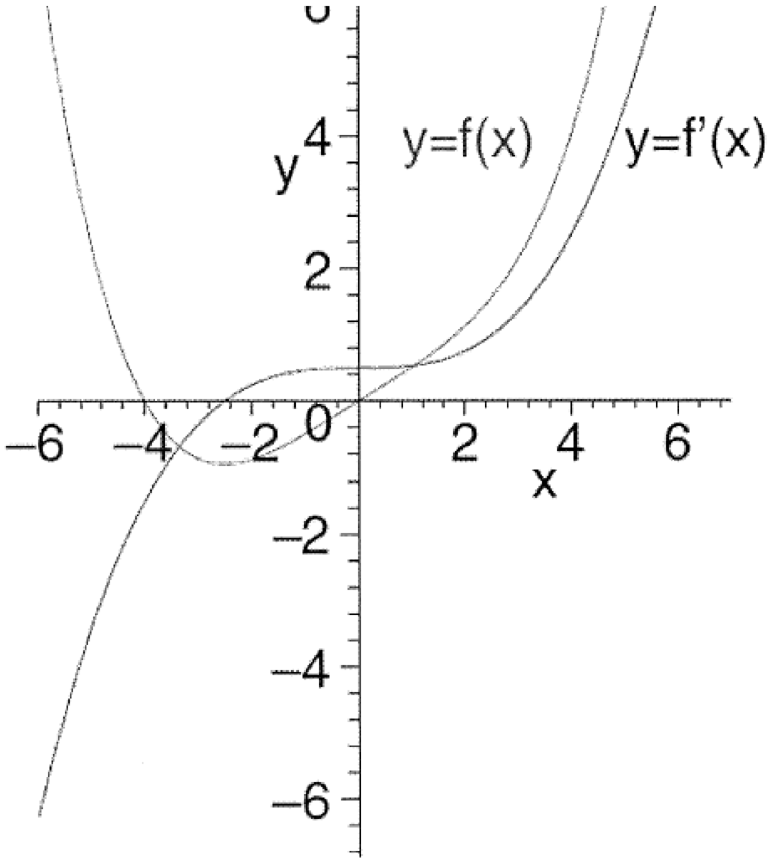
Given the graph of f'(x) below, sketch a possible graph for f(x) if f(0)=0.

a.




y
4
y = f′(x)
0
2
4
6
8
x
−4
−8
y
4
m = −2.5
= 0
0
2
4
6
8
x
m = 0
m = 0
−4
y = f(x)


b.
y
6
4
2
-4
-2
0
2
4
6
x
-2
-4
-6
Min


y=f(x)
y=f'(x)
y
x
4
2
0
−2
−4
−6
−6
−4
−2
2
4
6

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