Chapter 2: Limits and Continuity

2.1 Rate of Change and Limits

The limit of a function represents the value a function ***approaches*** for a given *x* value.

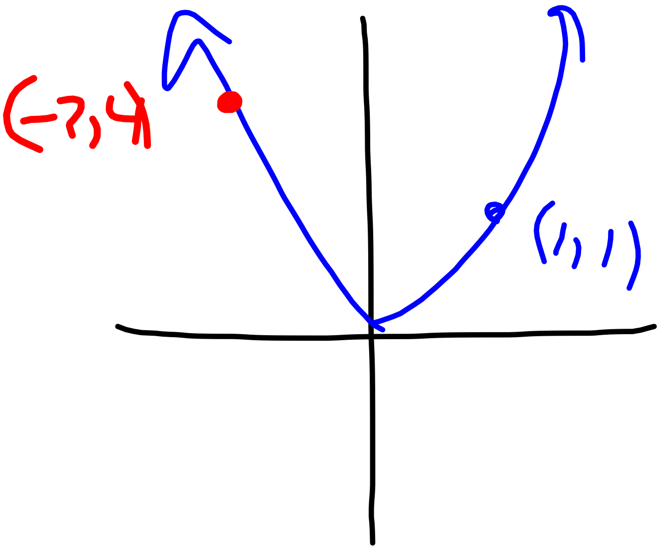
$$\lim_{x \to c} f(x) = L$$

The limit of *f* as *x* **approaches** *c* equals *L*.

This is not always the value of the function at *c*, $f(c)$, but the value the function approaches.

Consider limits for functions you have seen before:

$$\lim_{x \to 1} x^2 = 1$$



$$\lim_{x \to -2} x^2 = 4$$

Consider limits for functions you have seen before:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

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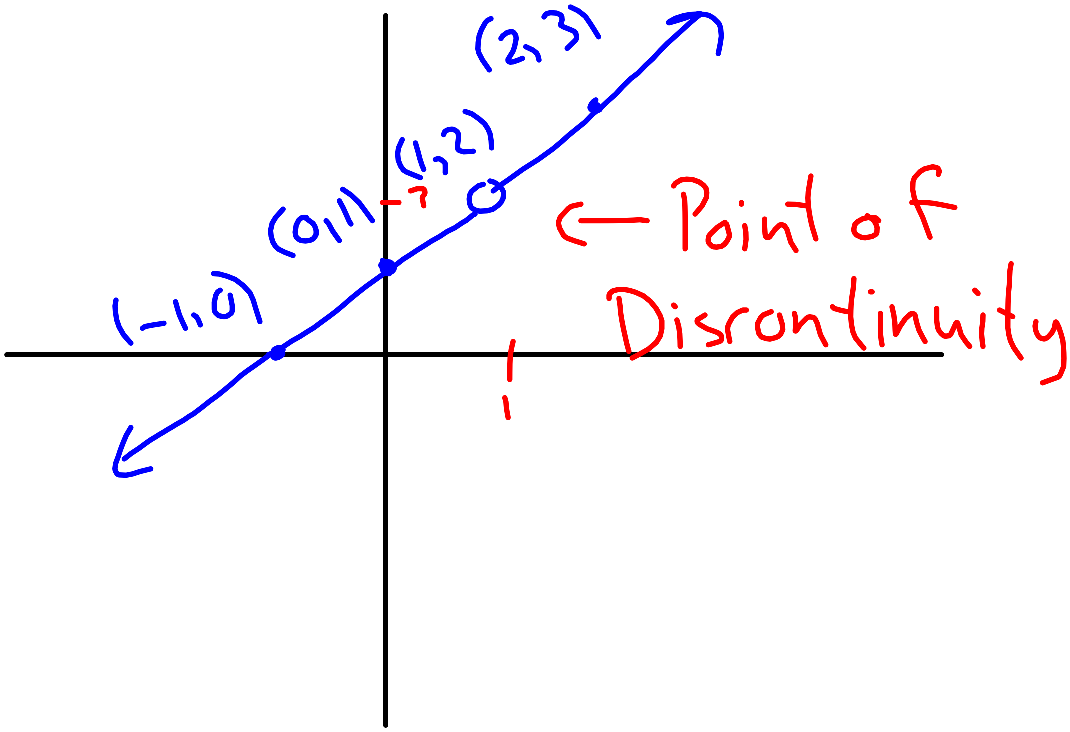
$$\lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$\lim_{x \to 1} (x+1) = (1+1) = 2$$


(2, 3)
(1, 2)
(0, 1)
(-1, 0)
← Point of Discontinuity

Consider limits for functions you have seen before:

$$\lim_{x\to 1}\frac{x+1}{x-1} = \lim_{x\to 1}\left(1+\frac{2}{x-1}\right) = DNE$$

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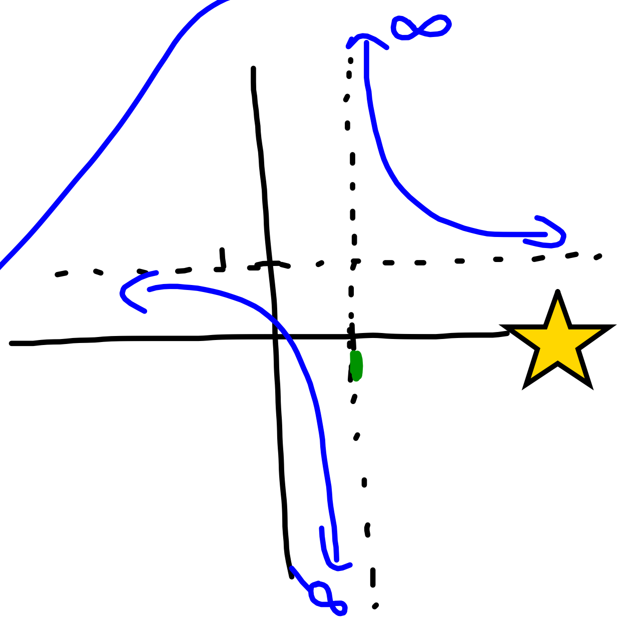
$$\begin{array}{r|l} & \ \ 1 \\ \hline x-1 & \ \ x+1 \\ & -(x-1) \\ \hline & \ \ \ \ 2 \end{array}$$

$$\begin{array}{r|rr} -1 & 1 & 1 \\ \hline & \downarrow & -(-1) \\ \hline & 1 & 2 \end{array}$$



An important limit is given below:

$$\lim_{x\to 0}\frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0} \text{ DNE}$$

Radians $\lim_{x \to 0} \frac{\sin x}{x} = 1$

We know that *x=0* is a discontinuity, but we don't know if it is a hole, or an asymptote. We can use a table of values to see what is happening as *x* approaches *0*.

[x will always be radians for trigonometric functions, unless otherwise stated]

$$y = \frac{\sin x}{x}$$

X	Y
-0.1	0.99833417
-0.01	0.99998333
-0.001	0.99999983
0	DNE

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0.1	0.99833417
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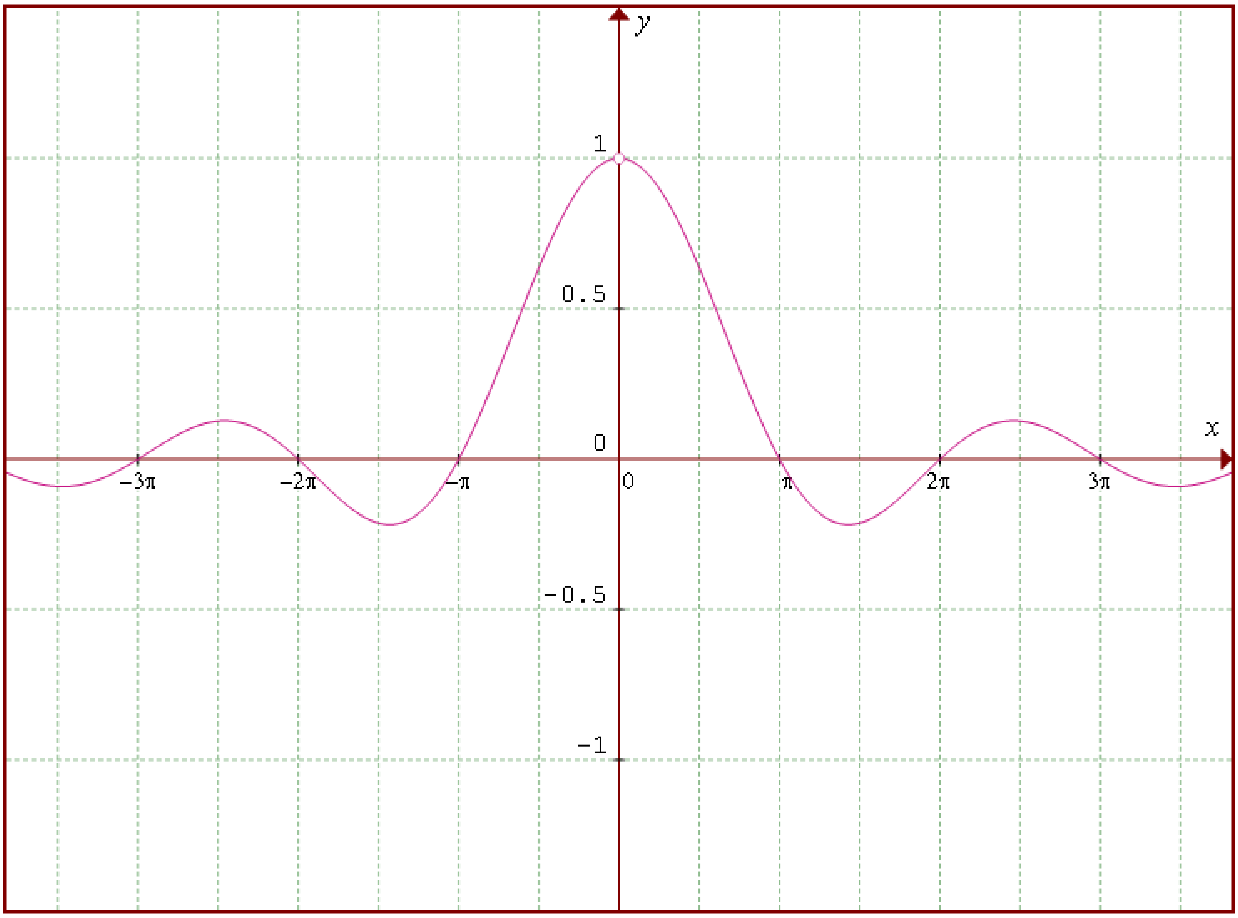
0	DNE

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$



Properties of Limits

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$

where L, M, c, and k are real numbers

Sum $\lim_{x \to c}\left(f(x) + g(x)\right) = L + M$

Difference $\lim_{x \to c}\left(f(x) - g(x)\right) = L - M$

Product $\lim_{x \to c}\left(f(x) \bullet g(x)\right) = L \bullet M$

Quotient $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, M \neq 0$

Constant Multiple $\lim_{x \to c} k \bullet f(x) = k \bullet L$

Power $\lim_{x \to c}\left(f(x)\right)^{r/s} = L^{r/s}$ r and s are integers, and $s \neq 0$

$$\lim_{x \to c} k = k$$

Evaluate the limit:

$$\lim_{x\to 4}\left(x^3-5x^2+2x-5\right) = 4^3-5(4)^2+2(4)-5$$

$$= 64-5(16)+8-5$$

$$= 64-80+3$$

$$= 67-80 = -13$$

$$= \lim_{x\to 4} x^3 + \lim_{x\to 4} -5x^2 + \lim_{x\to 4} 2x + \lim_{x\to 4} -5$$

$$64-80+8-5 = -13$$

Use the product rule to evaluate:

$$\lim_{x\to 0}\frac{\tan x}{x} = \frac{\tan 0}{0} \Rightarrow DNE$$

$$\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\left(\frac{\sin x}{\cos x}\cdot\frac{1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)$$

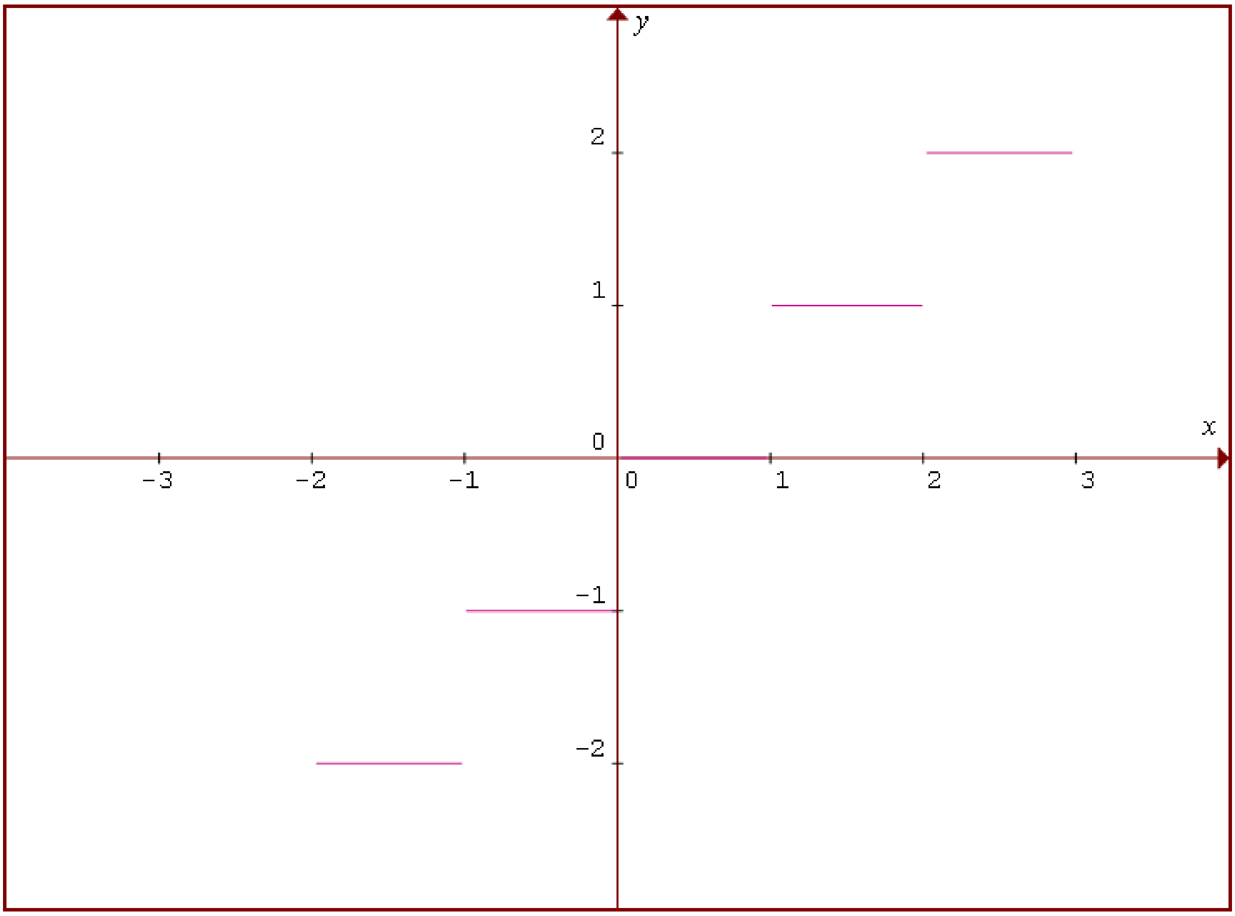
$$= (1)\cdot\left(\frac{1}{\cos 0}\right) = (1)(1) = 1$$

x	y
-0.1	1.0033467
-0.01	1.000033335
-0.001	1.000000333

x	y
0.1	1.0033467
0.01	1.000033334
0.001	1.000000333

The "int" function.

int(x) -> round ***down*** to the nearest integer.



int(4.5)=__4__ int(-4.5)=__-5__

-5 -4.5 -4

int(7.999)=__7__ int(-7.999)=__-8__

int(0.001)=__0__ int(-0.001)=__-1__

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start #15-20 in class