Uniform Circular Motion

very special case

Frequency (f) is the number of revolutions per second.
It is measured in Hertz, Hz
where 1 Hz = 1 revolution/second

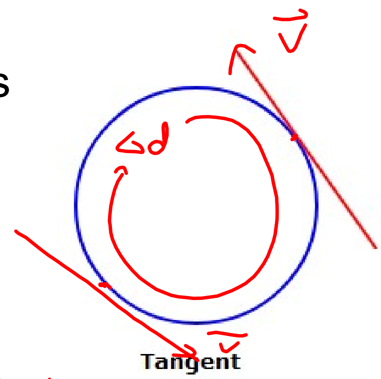
$$f = \frac{n}{\Delta t} = \frac{1}{T}$$

Period (T) is the time needed to make one complete revolution.
It is measured in seconds.

$$T = \frac{\Delta t}{n} = \frac{1}{f}$$

The circumference of a circle is found with the equation C = 2πr , where r is the radius of the circle.

the speed to travel one circle is calculated as



constant

$$v = \frac{\Delta d}{\Delta t} = \frac{C}{T} = \frac{2\pi r}{T} = 2\pi r f$$

Move in circular path in constant time

This speed, v, is the speed an object has as it travels in a circular path

The velocity vector is always pointing tangentially to the circle

As the object moves around the circle

- Speed remains constant

- Velocity always changing

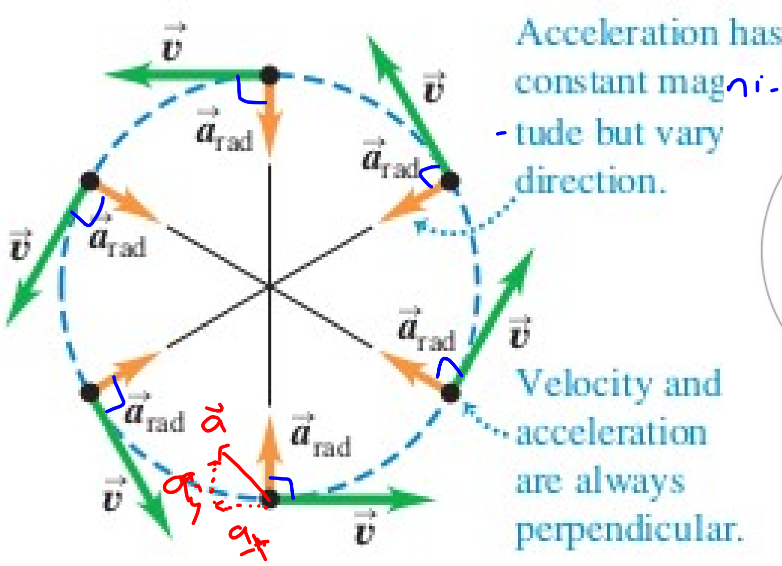
Centripetal Acceleration

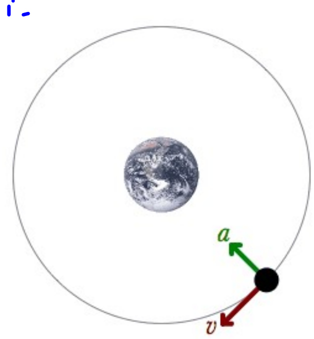
- An acceleration changes velocity - a vector quantity

- Acceleration can change
  > Velocity magnitude ← speed up, slow down
  > Velocity direction ← same speed
  > Both ← not interesting

- Centripetal acceleration (a_c) is the acceleration experienced by a body undergoing circular motion. It is a vector quantity that is directed towards the centre of the circle. It does not change the magnitude of the velocity, only its direction

- Centripetal means "Centre Seeking"





During Uniform Circular motion

1. Tangential speed will remain constant

2. Centripetal acceleration always points radially inward

3. Velocity is always changing as a result of the centripetal acceleration and is always perpendicular to it

Centripetal Acceleration, a_c

- For a detailed derivation of the ac equation, see page 552

$$a_c = \frac{v^2}{r}$$ ← not a vector eq'n

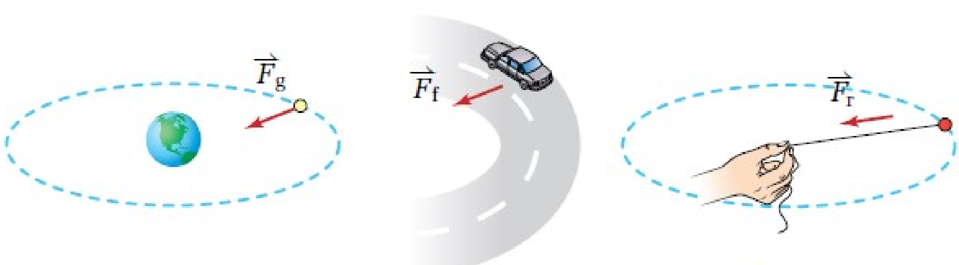
- v is the speed of the object
- r is called the "radius of curvature" - radius of the circle
- when written as a vector, it is implied as being directed towards the centre of the circle, [inwards]
  - > usually just calculate magnitude $\vec{a}_c$

N, W, S, E, up, down, ±x, ±y

Centripetal Force, F_c

- By Newton's 2nd Law, $F_c = ma_c$
- The Net result of other forces acting on an object
- Can not apply a Centripetal force on its own (not like F_g or F_f)
  - > All forces acting on an object allow it to move in a circle
  - > We then characterize the Net result as a single Centripetal Force

$$F_{Net} = \sum F = ma_c = F_c$$



$v = \frac{2\pi r}{T} = 2\pi r f$

- Think of object moving in a 2D plane
- Force acts within this plane as well
- Force always points perpendicularly to velocity, thus accelerating it inward
-

$$F_c = ma_c = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2} = m4\pi^2 r f^2$$

Centripetal Force Examples

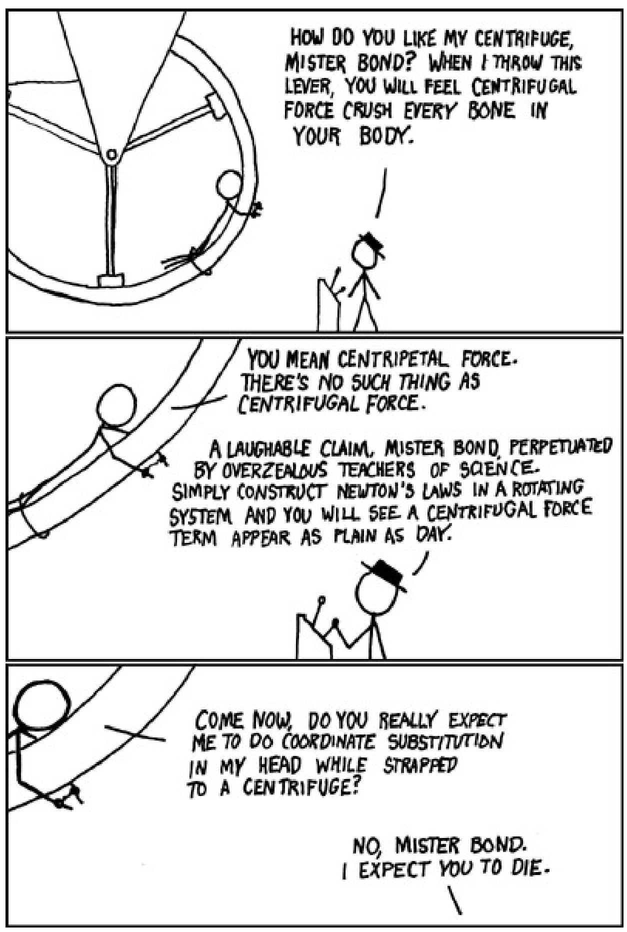
Ex. 1: While driving on a level surface, a car (2200 kg) tries to turn a corner that has a radius of curvature of 27 m. The driver does not apply any brakes during the turn so it's speed remains constant. If the coefficient of friction between the car and the road is 0.81, what is the maximum speed he could have been going at without skidding off the road?

Ex. 2: Goliath was killed when David used a sling to spin a rock around in a circle really fast until he could release it an propel it towards his target. If he was able to spin the rock using 105 N of tension and release it at 165 km/h. If the sling was only 1.20 m long, what was the mass of the rock?

#16 answer incorrect

Do Practice Problems 15-19 on Page 559

Will do more examples that will help



Ex 1)

$m = 2200\ kg$

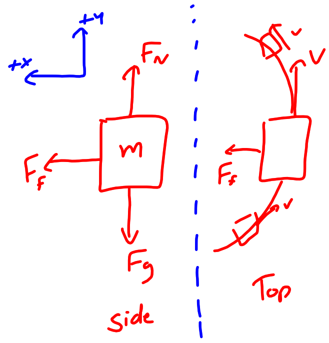
$r = 27\ m$

$\mu_k = 0.81$

$\vec{a}_y = 0\ m/s^2$

$\vec{g} = 9.81\ m/s^2$ (down)

$v = ?$



$$F_{Net,x} = F_F = ma_c$$

$$F_{Net,y} = F_N - F_g = ma_y = 0\ N$$

$$\therefore F_N = F_g$$

$$ma_c = F_F = \mu_k F_N = \mu_k F_g$$

$$\cancel{m}a_c = \mu_k \cancel{m}g$$

$$\frac{v^2}{r} = \mu_k g$$

$$v = \sqrt{\mu_k g r}$$

$$= \sqrt{(0.81)(9.81\ m/s^2)(27\ m)}$$

$$= 14.647\ m/s \doteq \boxed{15\ m/s}$$

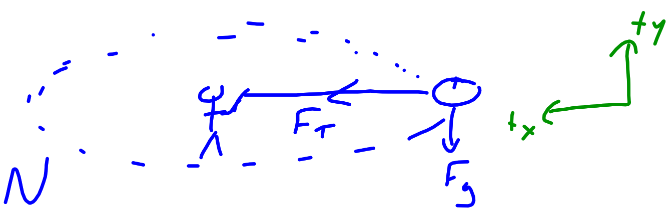
$$F_f = \frac{mv^2}{r}$$

if speed increased → start skidding so $r\uparrow$

if Friction lower either slow down ($v\downarrow$)

or r goes up (larger turn)

Ex 2



$F_T = 105\ N$

$V = 165\ km/h$

$r = 1.20\ m$

$\vec{g} = 9.81\ m/s^2$

$\vec{a}_y = 0\ m/s^2$

$$165\ km/h \times \frac{1000\ m}{1\ km} \times \frac{1\ h}{3600\ s}$$

$$= 45.8333\ m/s$$

$$F_{Net,x} = F_T = ma_c$$

$$F_T = \frac{mv^2}{r}$$

$$m = \frac{F_T\, r}{V^2} = \frac{(105\ N)(1.20\ m)}{(45.8333\ m/s)^2}$$

$$m = 0\ 05998\ kg$$

$$\boxed{= 0.0600\ kg}$$ $$\boxed{6.00 \times 10^{-2}\ kg}$$

$$= \boxed{60.0\ g}$$