Derivatives of Trigometric Functions

Section 3.5 Part #1

Review

sin(A+B) = sinAcosB + cosAsinB

cos(A+B) = cosAcosB - sinAsinB

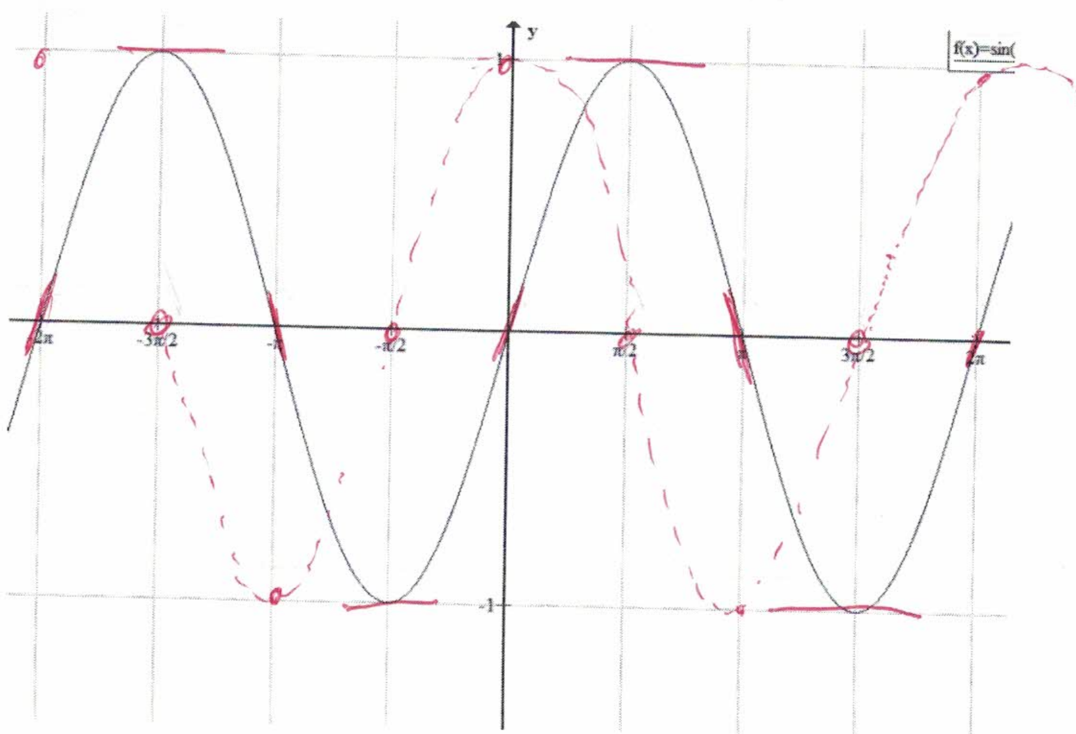
$$y = \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right) = 1$$

$$y = \lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) = 0$$

h	y
-0.1	0.04996
-0.01	0.00499996
-0.001	0.000499
-0.0001	0.00005
-1×10^{-10}	0

h	y
0.1	-0.05
0.01	-0.005
0.001	-0.0005
0.0001	-0.00005
1×10^{-10}	0

What is the derivative of f(x)=sin(x) ?



How to prove that if f(x)=sin(x) then f'(x)=cos(x).

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} = \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

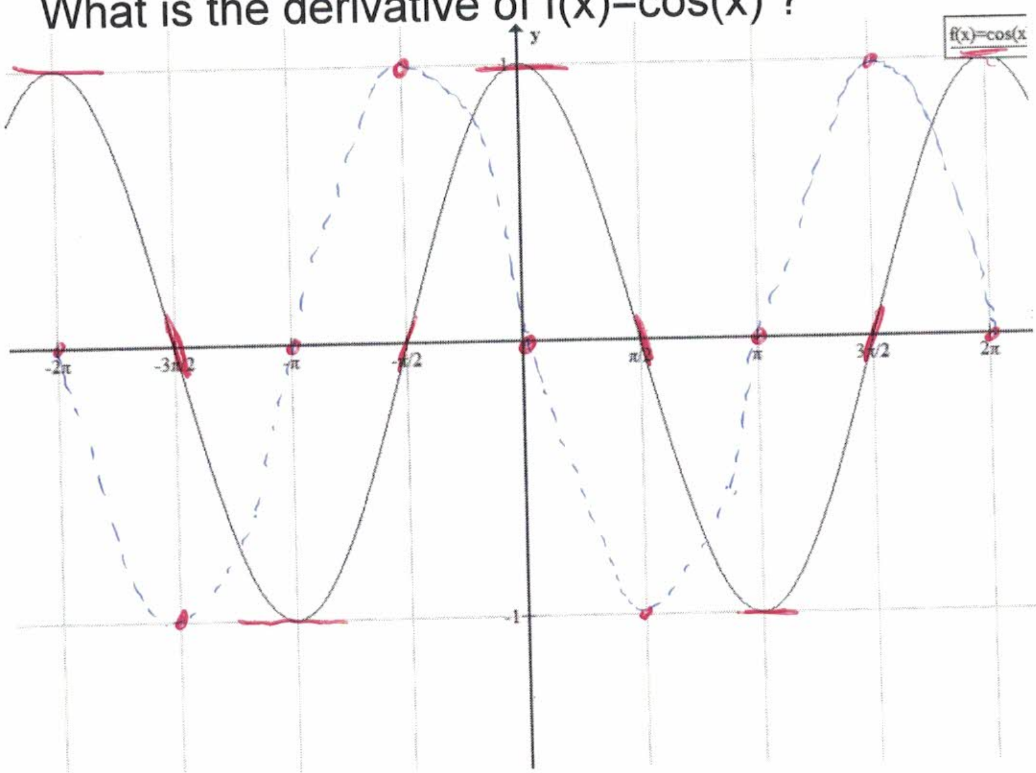
$$= \lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h} + \lim_{h\to 0} \frac{\cos x \sin h}{h}$$

$$= \lim_{h\to 0} \frac{\sin x (\cos h - 1)}{h} + \lim_{h\to 0} \cos x \left(\frac{\sin h}{h}\right)$$

$$= \lim_{h\to 0} \sin x \left(\frac{\cos h - 1}{h}\right) + \lim_{h\to 0} \cos x \left(\frac{\sin h}{h}\right)$$

$$= \sin x (0) + \cos x (1) = \cos x$$

What is the derivative of f(x)=cos(x) ?



How to prove that if f(x)=cos(x) then f'(x)=-sin(x).

$$f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h\to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x)\cos(h) - \cos(x)}{h} + \lim_{h\to 0} \frac{-\sin(x)\sin(h)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x)(\cos(h)-1)}{h} + \lim_{h\to 0} -\sin(x)\left(\frac{\sin(h)}{h}\right)$$

$$= \lim_{h\to 0} \cos(x)\left(\frac{\cos(h)-1}{h}\right) + \lim_{h\to 0} -\sin(x)\left(\frac{\sin(h)}{h}\right)$$

$$= \cos(x)(0) + (-\sin(x))(1)$$

$$= -\sin(x)$$

Given f(x)=tan(x) find f'(x).

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

$$f'(x) = \frac{\cos(x)\frac{d}{dx}\sin(x) - \sin(x)\,{}^{d}/_{dx}\cos(x)}{\cos^2(x)}$$

$$f'(x) = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\therefore f'(x) = \sec^2(x)$$

Given f(x)=csc(x) find f'(x).

$$f(x) = \csc(x) = \frac{1}{\sin(x)}$$

$$f'(x) = \frac{\sin(x)\frac{d}{dx}(1) - (1)\frac{d}{dx}\sin(x)}{\sin^2(x)} = \frac{\sin(x)(0) - (1)\cos(x)}{\sin^2(x)}$$

$$= \frac{-\cos(x)}{\sin^2(x)} = -\frac{\cos(x)}{\sin(x)} \cdot \frac{1}{\sin(x)} = -\cot(x) \cdot \csc(x)$$

Given f(x)=sec(x) find f'(x).

$$f(x) = \sec(x) = \frac{1}{\cos(x)}$$

$$f'(x) = \frac{\cos(x)\frac{d}{dx}(1) - (1)\,{}^{d}/_{dx}\cos(x)}{\cos^2(x)} = \frac{\cos(x)(0) - (1)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} = \tan(x)\sec(x)$$

Given f(x)=cot(x) find f'(x).

$$f(x) = \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$f'(x) = \frac{\sin(x)\cdot\frac{d}{dx}\cos(x) - \cos(x)\,\frac{d}{dx}\sin(x)}{\sin^2(x)}$$

$$= \frac{\sin(x)(-\sin(x)) - \cos(x)(\cos(x))}{\sin^2(x)}$$

$$= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-(\sin^2(x) + \cos^2(x))}{\sin^2 x} = \frac{-1}{\sin^2(x)}$$

$$= -\csc^2(x)$$

Know that

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\csc x = -\csc x\cot x$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\sec x = \sec x\tan x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

Given f(x)=4cos(x)-5tan(x) find f'(x).

$f'(x) = 4(-\sin(x)) - 5(\sec^2(x))$

$f'(x) = -4\sin(x) - 5\sec^2(x)$

Given f(x)= $\dfrac{8}{\sin x}$ find f'(x).

$f'(x) = \dfrac{\sin(x)\frac{d}{dx}(8) - 8\frac{d}{dx}\sin(x)}{\sin^2 x} = \dfrac{\sin(x)(0) - 8\cos(x)}{\sin^2(x)}$

$= \dfrac{-8\cos(x)}{\sin^2(x)} = -8\dfrac{\cos(x)}{\sin(x)} \cdot \dfrac{1}{\sin(x)} = -8\cot(x)\csc(x)$

OR

$f(x) = \dfrac{8}{\sin(x)} = 8\csc(x)$

$f'(x) = 8(-\csc(x)\cot(x))$

$f'(x) = -8\csc(x)\cot(x)$

Hemework:

p.146

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