Section 3.3

Part #1

Proof Time!

The derivative of a constant... $f(x) = c$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \underline{\qquad\qquad}$$

$$\lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h} = 0$$

Power Rule for Positive Integers: $f(x) = x^n$, $n \in \mathbb{N}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \underline{\qquad\qquad}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\qquad}{}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}$$

$$\lim_{h\to 0}\frac{{}_nC_0\,x^n h^0+{}_nC_1\,x^{n-1}h^1+{}_nC_2\,x^{n-2}h^2+\dots\dots+{}_nC_n\,x^0h^n-x^n}{h}$$

$$\lim_{h\to 0}\frac{\cancel{x^n}+{}_nC_1\,x^{n-1}h+{}_nC_2\,x^{n-2}h^2+\dots+{}_nC_n\,h^n-\cancel{x^n}}{h}$$

$$\lim_{h\to 0}\ {}_nC_1\,x^{n-1}+{}_nC_2\,x^{n-2}h+\dots\dots+h^{n-1}$$

$${}_nC_1\,x^{n-1}=nx^{n-1}$$

$$\boxed{\frac{d}{dx}x^n=nx^{n-1}}$$

$$\frac{d}{dx}x^{10}=10x^9$$

Constant Multiple $f(x) = c \cdot u(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \underline{\qquad\qquad}$$

$$\lim_{h \to 0} \frac{c\,u(x+h) - c\,u(x)}{h} =$$

$$= \lim_{h \to 0} c \cdot \left(\frac{u(x+h) - u(x)}{h} \right)$$

$$= \lim_{h \to 0} c \cdot \lim_{h \to 0} \frac{u(x+h) - u(x)}{h} = c\,u'(x)$$

Sum and Difference $f(x) = u(x) \pm v(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \underline{\qquad\qquad}$$

Sum $f(x) = u(x) + v(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{u(x+h) + v(x+h) - [u(x) + v(x)]}{h}$$

$$= \lim_{h \to 0} \frac{u(x+h) + v(x+h) - u(x) - v(x)}{h}$$

$$= \lim_{h \to 0} \frac{u(x+h) - u(x)}{h} + \lim_{h \to 0} \frac{v(x+h) - v(x)}{h}$$

$$= u'(x) + v'(x)$$

Difference

$$f(x) = u(x) - v(x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{u(x+h) - v(x+h) - [u(x) - v(x)]}{h}$$

$$\lim_{h \to 0} \frac{u(x+h) - v(x+h) - u(x) + v(x)}{h}$$

$$\lim_{h \to 0} \frac{u(x+h) - u(x)}{h} + \lim_{h \to 0} \frac{-v(x+h) + v(x)}{h}$$

$$= u'(x) + (-1) \lim_{h \to 0} \frac{v(x+h) - v(x)}{h}$$

$$= u'(x) - v'(x)$$

NAME	LAW
Constant Rule	$\frac{d}{dx}(c) = 0$
Power Rule	$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$
Constant Multiple Rule	$\frac{d}{dx}(cu) = c\frac{du}{dx}$
Sum Rule	$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
Difference Rule	$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

Find the derivative of the following functions:

a) $f(x) = x^5 - 3x^4 + 7x^2 - 8$

$$f'(x) = 5x^4 - (3)(4)x^3 + 7(2)x^1 + 0$$

$$= 5x^4 - 12x^3 + 14x$$

b) $f(x) = x^3 + 2x^2 - \frac{8x}{5} + 3$

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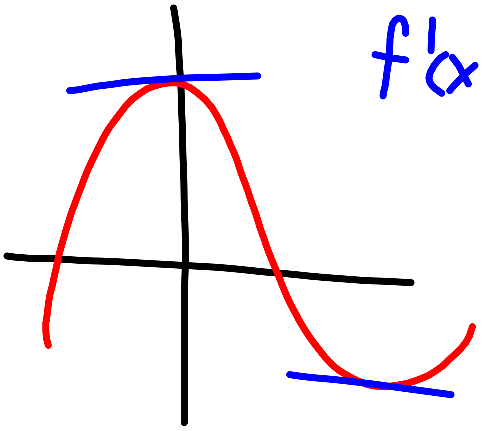
(odd)

$f'(x) = 3x^2 + 4x - \frac{8}{5}$

Where does $y = x^3 - 2x^2 - 10x + 5$ have horizontal tangents?



$f'(x) = 0$

$y' = 3x^2 - 4x - 10 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-10)}}{2(3)}$

$x = \frac{4 \pm \sqrt{136}}{6} = \frac{4 \pm \sqrt{4}\sqrt{34}}{6}$

$x = \frac{4 \pm 2\sqrt{34}}{6} = \frac{2 \pm \sqrt{34}}{3}$