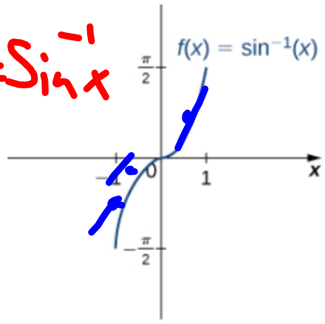
Section 4.3
Part #1
Derivatives of Inverse Trig Functions

$x = \sin y \iff f(x) = \sin^{-1} x$



Domain: [-1,1]

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\frac{d}{dx} x = \frac{d}{dx} \sin y$

$1 = \cos y \cdot y'$

$y' = \frac{1}{\cos y}$

Since $\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \pm\sqrt{1 - \sin^2 y} = \pm\sqrt{1 - x^2}$

$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$, $x \neq \pm 1$

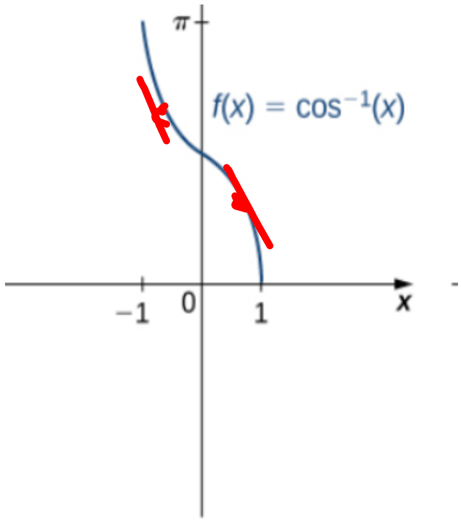
Reject - (Positive Slopes)

$x = \cos y$

$\frac{d}{dx} x = \frac{d}{dx} \cos y$

$(1) = -\sin y \cdot y'$

$y' = \frac{1}{-\sin y} = \frac{-1}{\sin y}$



Domain: [-1,1]

Range: $[0, \pi]$

Using $\sin^2 y + \cos^2 y = 1$

$\sin y = \pm\sqrt{1-\cos^2 y} = \pm\sqrt{1-x^2}$

Reject $-$

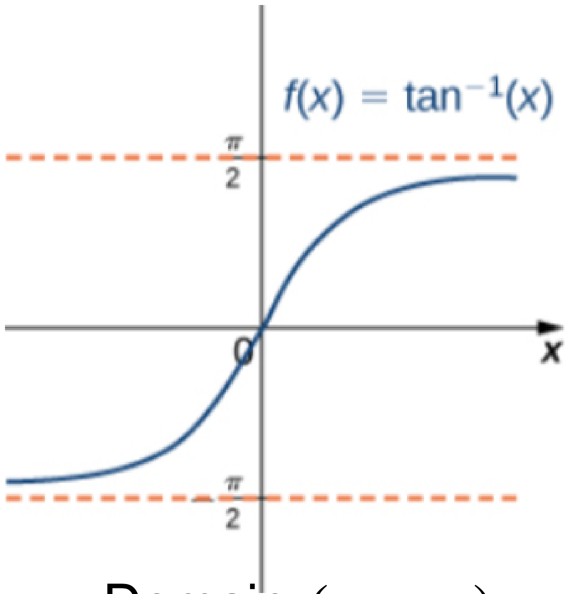
$y' = \frac{-1}{\sqrt{1-x^2}}$

$$x = \tan y$$

$$\frac{d}{dx} x = \frac{d}{dx} \tan y$$

$$(1) = \sec^2 y \cdot y'$$

$$y' = \frac{1}{\sec^2 y}$$



Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Since $\sec^2 y = 1 + \tan^2 y$

$$y' = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}, \quad x \neq \pm 1$$

Formulas:

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}},\ x \neq \pm 1$$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}},\ x \neq \pm 1$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

Find the derivatives of the following.

$$\frac{d}{dx}\, y = \frac{d}{dx}\tan^{-1}(x^2)$$

Let $u = x^2$ $\qquad$ $y = \tan^{-1} u$

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dy}{du} = \frac{1}{1+u^2}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{1+u^2} \cdot (2x) = \frac{1}{1+(x^2)^2} \cdot 2x$$

$$= \frac{2x}{1+x^4}$$

$$y = \tan^{-1}(\cos x)$$

Let $u = \cos x$ $\qquad$ $y = \tan^{-1} u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \frac{du}{dx} = -\sin x \qquad \frac{dy}{du} = \frac{1}{1+u^2}$$

$$= \frac{1}{1+u^2}(-\sin x)$$

$$= \frac{-\sin x}{1+\cos^2 x}$$

$$y = x\sin^{-1}(x) + \sqrt{1 - x^2}$$

$$y = x\sin^{-1}x + (1-x^2)^{1/2}$$

$$\frac{d}{dx}y = \frac{d}{dx}x\sin^{-1}x + \frac{d}{dx}(1-x^2)^{1/2}$$

$$\frac{dy}{dx} = x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x) + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$$

$$= x\left(\frac{1}{\sqrt{1-x^2}}\right) + \sin^{-1}x\,(1) + \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}$$

$$= \cancel{\frac{x}{\sqrt{1-x^2}}} + \sin^{-1}x - \cancel{\frac{x}{\sqrt{1-x^2}}}$$

$$= \sin^{-1}x$$

$$y = \sin^{-1}(2x+1)$$

Let $u = 2x+1$ $\qquad$ $y = \sin^{-1} u$

$$\frac{du}{dx} = 2 \qquad \frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}}(2)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x+1)^2}}(2)$$

$$= \frac{2}{\sqrt{1-(4x^2+4x+1)}}$$

$$= \frac{2}{\sqrt{-4x^2-4x}} = \frac{2}{\sqrt{-4x(x+1)}}$$

$$= \frac{2}{2\sqrt{-x(x+1)}} = \frac{1}{\sqrt{-x(x+1)}}$$

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Answers

2. $$\frac{1}{|x|\sqrt{x^2 - 1}}$$

8. $$\frac{-2}{(\sin^{-1}2x)^2\sqrt{1-4x^2}}$$

4. $$\frac{-1}{\sqrt{2t - t^2}}$$

10. $$\frac{\sqrt{3}}{24}$$

6. $$\frac{-2s^2}{\sqrt{1 - s^2}}$$