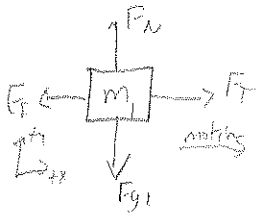
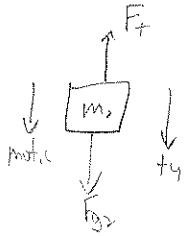
Advanced Connected Systems

#1) $m_1 = 10.0\ kg$
$m_2 = 15.0\ kg$
$\mu_K = 0.20$
$a_{sys} = ?$
$F_T = ?$





$\vec{g} = 9.81\ m/s^2$ [down]
$\vec{a}_{1y} = 0\ m/s^2$

$F_{Net,sys} = F_{g2} - F_f = m_{sys}\, a_{sys}$

$F_{Net,1y} = F_N - F_{g1} = m a_{1y} = 0\ N \rightarrow F_N = F_{g1}$

$F_{g2} - \mu F_N = m_{sys}\, a_{sys}$

$$a_{sys} = \frac{m_2 g - \mu m_1 g}{m_{sys}} = \left(\frac{m_2 - \mu_K m_1}{m_1 + m_2}\right) g$$

$$a_{sys} = \frac{(15.0\ kg - (0.20)(10.0\ kg))}{(15.0\ kg + 10.0\ kg)} (9.81\ m/s^2)$$

$= 5.1012\ m/s^2 = \boxed{5.10\ m/s^2}$

$F_{Net,2} = F_{g2} - F_T = m_2 a_2 \rightarrow F_T = F_{g2} - m_2 a_2 = m_2 (g - a_2)$

$F_T = (15.0\ kg)(9.81\ m/s^2 - 5.1012\ m/s^2) = 70.632\ N = \boxed{70.6\ N}$

#2 $m_1 = 3.0\,kg$

$m_2 = 5.0\,kg$

$\mu_k = 0.18$

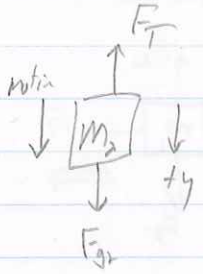
$g = 9.81\,m/s^2$

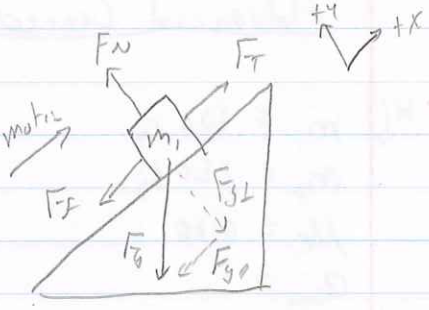
$\vec{a}_{1y} = 0\,m/s^2$

$\theta = 35°$

$a_{sys} = ?$

$F_T = ?$





$F_{net,sys} = F_{g2} - F_F - F_{g\parallel} = m_{sys}\, a_{sys}$

$F_{net,1y} = F_N - F_{g\perp} = ma_{1y} = 0\,N \rightarrow F_N = F_{g\perp}$

$\therefore m_{sys}\, a_{sys} = F_{g2} - \mu F_N - F_{g\parallel}$

$= m_2 g - \mu F_{g\perp} - F_{g\parallel}$

$$a_{sys} = \frac{m_2 g - \mu m_1 g \cos\theta - m_1 g \sin\theta}{m_1 + m_2}$$

$$= \frac{(5.0\,kg)(9.81\,m/s^2) - (0.18)(3.0\,kg)(9.81\,m/s^2)\cos(35°) - (3.0\,kg)(9.81\,m/s^2)\sin(35°)}{(3.0\,kg + 5.0\,kg)}$$

$= 3.4788\,m/s^2 = \boxed{3.5\,m/s^2}$

$F_{net,2} = F_{g2} - F_T = m_2 a_2 \rightarrow F_T = F_{g2} - m_2 a_2$

$F_T = m_2 (g - a_{sys}) = (5.0\,kg)(9.81\,m/s^2 - 3.4788\,m/s^2)$

$= 31.656\,N = \boxed{32\,N}$

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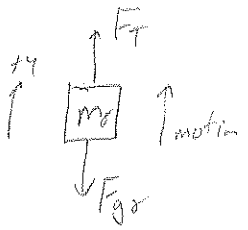
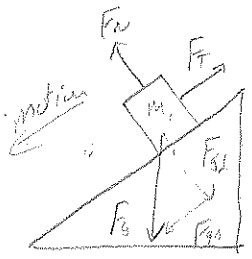
#3

$m_1 = 10.0\ kg$
$m_2 = 5.0\ kg$
$\vec{g} = 9.81\ m/s^2\ [down]$
$\vec{a}_{1y} = 0\ m/s^2$
$\theta = 40°$
$a_{sys} = ?$
$F_T = ?$





$F_{net,sys} = F_{g//} - F_{g2} = m_{sys}\, a_{sys}$

$m_{sys}\, a_{sys} = m_1 g \sin\theta - m_2 g$

$a_{sys} = \dfrac{(m_1 \sin\theta - m_2)\, g}{m_1 + m_2}$

$= \dfrac{((10.0\ kg)\sin(40°) - (5.0\ kg))\,(9.81\ m/s^2)}{(10.0\ kg + 5.0\ kg)}$

$= 0.93383\ m/s^2 = \boxed{0.93\ m/s^2}$

$F_{net,2} = F_T - F_{g2} = m_2 a_2 \rightarrow F_T = F_{g2} + m_2 a_2$

$\therefore F_T = m_2 (g + a_{sys}) = (50\ kg)(9.81\ m/s^2 + 0.93383\ m/s^2)$

$= 53.719\ N = \boxed{54\ N}$

#4 $m_1 = 189.3\ g$

$\mu_K = 0.208$

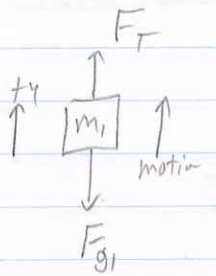
$\vec{g} = 9.81\ m/s^2$ [down]

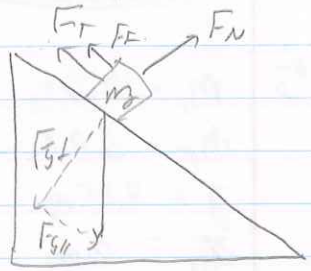
$\vec{a}_{2y} = 0\ m/s^2$

$\theta = 35.0°$

$a_{sys} = 0.175\ m/s^2$

$m_2 = ?$





$F_{Net,sys} = F_{g\parallel} - F_F - F_{g1} = m_{sys}\, a_{sys}$

$F_{Net,2y} = F_N - F_{g\perp} = m_2 a_{2y} = 0N \rightarrow F_N = F_{g\perp}\ (F_F = \mu F_N = \mu F_{g\perp})$

$m_1 a_{sys} + m_2 a_{sys} = m_2 g \sin\theta - \mu F_{g\perp} - m_1 g$

$m_1 a_{sys} + m_1 g = m_2 g \sin\theta - \mu m_2 g \cos\theta - m_2 a_{sys}$

$m_1 (a_{sys} + g) = m_2 (g \sin\theta - \mu g \cos\theta - a_{sys})$

$\therefore m_2 = \dfrac{(a_{sys} + g)\, m_1}{(g \sin\theta - \mu g \cos\theta - a_{sys})}$

$= \dfrac{(0.175\ m/s^2 + 9.81\ m/s^2)(189.3\ g)}{((9.81\ m/s^2)\sin 35° - (0.208)(9.81\ m/s^2)\cos(35°) - 0.175\ m/s^2)}$

$= 499.99994\ g = \boxed{500.\ g}$

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#5)

$m_2 = 250\ g$

$\mu_K = 0.164$

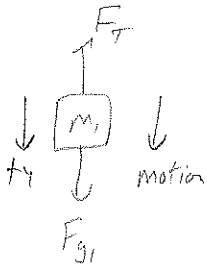
$a_{sys} = 0.0968\ m/s^2$

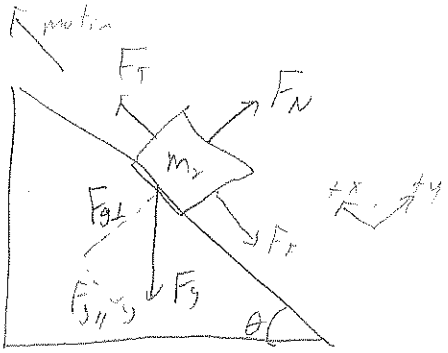
$\vec{g} = 9.81\ m/s^2$ [down]

$\theta = 35.0°$

$m_1 = ?$

$\vec{a}_{2y} = 0\ m/s^2$





$F_{net,sys} = F_{g1} - F_{g\parallel} - F_F = m_{sys}\, a_{sys}$

$F_{net,2y} = F_N - F_{g\perp} = m_2 a_{2y} = 0N \rightarrow F_N = F_{g\perp}$

$(\therefore F_F = \mu_K F_N = \mu_K F_{g\perp})$

$m_{sys}\, a_{sys} = m_1 g - m_2 g \sin\theta - \mu_K m_2 g \cos\theta$

$m_2 a_{sys} + m_2 g \sin\theta + \mu_K m_2 g \cos\theta = m_1 g - m_1 a_{sys}$

$m_2 (a_{sys} + g\sin\theta + \mu g \cos\theta) = m_1 (g - a_{sys})$

$\therefore m_1 = m_2 \dfrac{(a_{sys} + g\sin\theta + \mu_K g \cos\theta)}{g - a_{sys}}$

$= (250\ g)\dfrac{(0.0968\ m/s^2 + (9.81\ m/s^2)\sin 35° + (0.164)(9.81\ m/s^2)\cos(35°))}{(9.81\ m/s^2 - 0.0968\ m/s^2)}$

$= 181.23\ g = \boxed{181\ g}$

#6) $m_1 = 20.\ kg$

$m_2 = 30.\ kg$

$\mu_{1K} = 0.20$

$\mu_{K2} = 0.30$

$g = 9.81\ m/s^2$ [down]

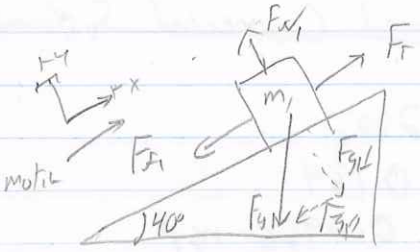
$a_y = 0\ m/s^2$

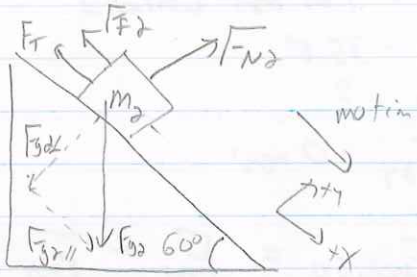
$\theta_1 = 40°$

$\theta_2 = 60°$

$a_{sys} = ?$

$F_T = ?$





$F_{Net,sys} = F_{g2//} - F_{F2} - F_{F1} - F_{g1//} = m_{sys}\, a_{sys}$

$F_{Net,1,y} = F_{N1} - F_{g1\perp} = m_1 a_y = 0N \rightarrow F_{N1} = F_{g1\perp} \rightarrow F_{F1} = \mu_{K1} F_{g1\perp}$

$F_{Net,2,y} = F_{N2} - F_{g2\perp} = m_2 a_y = 0N \rightarrow F_{N2} = F_{g2\perp} \rightarrow F_{F2} = \mu_{K1} F_{g2\perp}$

$\therefore m_{sys}\, a_{sys} = m_2 g \sin\theta_2 - \mu_{K2} m_2 g \cos\theta_2 - \mu_{K1} m_1 g \cos\theta_1 - m_1 g \sin\theta_1$

$= m_2 g (\sin\theta_2 - \mu_{K2} \cos\theta_2) - m_1 g (\sin\theta_1 + \mu_{K1} \cos\theta_1)$

$\therefore a_{sys} = \dfrac{[m_2 (\sin\theta_2 - \mu_{K2} \cos\theta_2) - m_1 (\sin\theta_1 + \mu_{K1} \cos\theta_1)]\, g}{m_1 + m_2}$

$= \dfrac{[(30.\ kg)(\sin(60°) - (0.30)\cos(60°)) - (20\ kg)(\sin(40°) + (0.20)\cos(40°))](9.81\ m/s^2)}{(20.\ kg + 30.\ kg)}$

$= 1.0910\ m/s^2 = \boxed{1\ m/s^2}$

$F_{Net,2} = F_{g2//} - F_T - F_{F2} = m_2 a_2$

$\therefore F_T = m_2 g \sin\theta - \mu_K m_2 g \cos\theta - m_2 a_2 = m_2 (g \sin\theta - \mu_K g \cos\theta - a_2)$

$= (30.\ kg)((9.81\ m/s^2)\sin(60°) - (0.30)(9.81\ m/s^2)\cos(60°) - 1.0910\ m/s^2) = 177.995\ N$

$= \boxed{180\ N}$