

# Trig Identities

①

$$1. LS = \sec x - \sin x \tan x \quad [\cos x]$$

$$= \frac{1}{\cos x} - \sin x \cdot \frac{\sin x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x}{\cos x}$$

$$= \cos x = RS$$

QED

$$2. LS = \sin^2 x (\csc^2 x + \sec^2 x) \quad [\sec^2 x]$$

$$= \sin^2 x \left( \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \right)$$

$$= \frac{\sin^2 x}{\sin^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

$$= \sec^2 x = RS$$

QED

$$3. LS = \sec^2 x + \csc^2 x \quad [2\sec^2 \theta]$$

$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$$

$$= \frac{1}{\cos^2 x \sin^2 x}$$

$$= \sec^2 x \csc^2 x$$

$$= RS$$

QED

$$4. LS = \frac{\csc \theta}{\csc \theta - 1} + \frac{\csc \theta}{\csc \theta + 1} \quad [2\sec^2 \theta]$$

$$= \frac{\csc \theta (\csc \theta + 1) + \csc \theta (\csc \theta - 1)}{(\csc \theta - 1)(\csc \theta + 1)}$$

$$= \frac{\csc^2 \theta + \csc \theta + \csc^2 \theta - \csc \theta}{\csc^2 \theta - 1}$$

$$4. LS = \frac{2\csc^2 \theta}{\csc^2 \theta - 1}$$

$$= \frac{2}{\frac{1}{\sin^2 \theta} - 1}$$

$$= \frac{2}{\frac{1 - \sin^2 \theta}{\sin^2 \theta}}$$

$$= \frac{2 \sin^2 \theta}{1 - \sin^2 \theta}$$

$$= \frac{2 \sin^2 \theta}{\cos^2 \theta}$$

$$= 2 \sec^2 \theta = RS$$

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$$5. LS = \frac{\sec - \tan}{1 - \sin}$$

$$= \frac{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}{1 - \sin x}$$

$$= \frac{\frac{1 - \sin x}{\cos x}}{1 - \sin x}$$

$$= \frac{1 - \sin x}{\cos x (1 - \sin x)}$$

$$= \frac{1}{\cos x}$$

$$= \frac{1}{\cos x}$$

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$$= \frac{1}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$RS = \sec x$$

$$= \frac{1}{\cos x}$$

$$RS = LS$$

QED

$$6. RS = \frac{1}{\sec x - \tan x}$$

$$= \frac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$$

$$= \frac{1}{\frac{1 - \sin x}{\cos x}}$$

$$= \frac{\cos x}{1 - \sin x} = LS$$

QED

$$7. LS = (\csc x - 1)(\sin x + 1)$$

$$= \csc x \sin x - \sin x + \csc x - 1$$

$$= \frac{1}{\sin x} \sin x - \sin x + \frac{1}{\sin x} - 1$$

$$= \cancel{1} - \frac{\sin^2 x}{\sin x} + \frac{1}{\sin x} - \cancel{1}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \csc x \cot x = RS$$

QED

$$8. RS = \frac{\tan^2 \theta}{\sec \theta - 1} \cdot \frac{\sec \theta + 1}{\sec \theta + 1}$$

$$= \frac{\tan^2 \theta (\sec \theta + 1)}{\sec^2 \theta - 1}$$

$$= \frac{\tan^2 \theta (\sec \theta + 1)}{\tan^2 \theta}$$

$$= \sec \theta + 1$$

$$= 1 + \frac{1}{\cos \theta} = LS$$

QED

$$9. LS = \frac{\sin A + \tan A}{1 + \sec A}$$

$$= \frac{\sin A + \frac{\sin A}{\cos A}}{1 + \frac{1}{\cos A}}$$

$$= \frac{\sin A \cos A + \sin A}{\cos A + 1}$$

$$= \frac{\sin A \cos A + \sin A}{\cos A + 1}$$

$$= \frac{\sin A (\cos A + 1)}{\cos A + 1}$$

$$= \sin A = RS$$

QED

$$10. LS = \frac{1 - \sin^2 \theta}{\csc^2 \theta - 1}$$

$$= \frac{\cos^2 \theta}{\cot^2 \theta}$$

$$= \frac{\cos^2 \theta}{1} \div \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= \frac{\cos^2 \theta}{1} \times \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \sin^2 \theta = RS$$

QED

$$11. RS = \frac{1 + \tan A}{\sec A}$$

$$= \frac{1 + \frac{\sin A}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\cos A + \sin A}{\frac{1}{\cos A}}$$

$$= \cos A + \sin A = LS$$

QED

$$12. LS = \tan^2 x - \sin^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

$$= \frac{\sin^2 x - \sin^2 \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$

$$= \sin^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$$

$$= \sin^2 x \tan^2 x = RS$$

QED

$$13. \quad LS = \cot x + \cos x$$

$$= \frac{\cos x}{\sin x} + \cos x$$

$$= \frac{\cos x + \cos x \sin x}{\sin x}$$

$$= \frac{\cos x (1 + \sin x)}{\sin x}$$

$$= RS$$

it is an identity

14. similar LS to #13  
not an identity

$$15. \quad LS = \cot x + \cos x$$

$$= \frac{\cos x}{\sin x} + \cos x$$

$$= \frac{\cos x + \sin x \cos x}{\sin x}$$

$$RS = \tan x + \sin x$$

$$= \frac{\sin x}{\cos x} + \sin x$$

$$= \frac{\sin x + \sin x \cos x}{\cos x}$$

not an identity

14. test  $30^\circ$

$$LS = \sqrt{3} + \frac{\sqrt{3}}{2}$$

$$= \frac{2\sqrt{3} + \sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2}$$

$$RS = \frac{2 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= 2\sqrt{3}$$

$$LS \neq RS$$

not an identity

15. test  $30^\circ$

$$LS = \frac{3\sqrt{3}}{2}$$

$$RS = \frac{1}{\sqrt{3}} + \frac{1}{2}$$

$$= \frac{2 + \sqrt{3}}{2\sqrt{3}}$$

$$LS \neq RS$$

not an identity