Section 3.3

Part #2

Evaluate the following:

$\frac{d}{dx}(x \cdot x) = \frac{d}{dx} x^2$

$2x$

$\frac{d}{dx}(x) \cdot \frac{d}{dx}(x) = (1)(1) = 1$

$2x \neq 1$ So you cannot take the derivative of the factors and then multiply.

The product rule.

Assume you have the product of 2 functions *u* and *v*:

$$\frac{d}{dx}(uv) = \lim_{h\to 0}\rule{3cm}{0.4pt} \qquad f(x) = u(x)\cdot v(x)$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{u(x+h)\cdot v(x+h) - u(x)v(x)}{h}$$

$$= \lim_{h\to 0}\frac{\left(u(x+h)\cdot v(x+h) - u(x+h)v(x)\right) + \left[u(x+h)v(x) - u(x)v(x)\right]}{h}$$

$$= \lim_{h\to 0}\frac{u(x+h)\left[v(x+h)-v(x)\right] + v(x)\left[u(x+h)-u(x)\right]}{h}$$

$$= \lim_{h\to 0}u(x+h)\cdot\lim_{h\to 0}\frac{v(x+h)-v(x)}{h} + \lim_{h\to 0}v(x)\cdot\lim_{h\to 0}\frac{u(x+h)-u(x)}{h}$$

$$= u(x)\,v'(x) + v(x)\,u'(x)$$

The first times the derivative of the second plus the second times the derivative of the first.

The Quotient Rule. $\frac{d}{dx}\left(\frac{u}{v}\right) = \lim_{h\to 0}$ ________

$$f(x) = \frac{u(x)}{v(x)}$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{v(x)}{v(x)}\cdot\frac{u(x+h)}{v(x+h)} - \frac{u(x)}{v(x)}\cdot\frac{v(x+h)}{v(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{u(x+h)v(x) - u(x)v(x+h)}{v(x+h)v(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{[u(x+h)v(x) - u(x)v(x)] + [u(x)v(x) - u(x)v(x+h)]}{v(x+h)v(x)}}{h}$$

$$= \lim \frac{v(x)[u(x+h) - u(x)] - u(x)[v(x+h) - v(x)]}{v(x+h)v(x)\cdot h}$$

$$= \lim_{h\to 0} v(x) \cdot \lim_{h\to 0} \frac{u(x+h)-u(x)}{h} \cdot \frac{1}{v(x+h)v(x)}$$

$$- \lim_{h\to 0} \frac{u(x)}{v(x+h)v(x)} \cdot \lim_{h\to 0} \frac{v(x+h)-v(x)}{h}$$

$$= \frac{v(x)\cdot u'(x)}{(v(x))^2} - \frac{u(x)}{(v(x))^2}\cdot v'(x)$$

$$= \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2}$$

The bottom times the derivative of the top minus the top times the derivative of the bottom over the bottom squared.

The Quotient Rule.

low d hi - hi d low

Product Rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient Rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Bottom times the derivative of the top minus the top times the derivative of the bottom over the bottom squared.

Find the derivative of the following functions:

$$f(x) = \left(x^3 - 4x^2 + 8\right)\left(5x^2 - 12\right)$$

$$f'(x) = (x^3-4x^2+8)\cdot\frac{d}{dx}(5x^2-12) + (5x^2-12)\frac{d}{dx}(x^3-4x^2+8)$$

$$= (x^3-4x^2+8)(10x) + (5x^2-12)(3x^2-8x)$$

$$= 10x^4 - 40x^3 + 80x + 15x^4 - 40x^3 - 36x^2 + 96x$$

$$= 25x^4 - 80x^3 - 36x^2 + 176x$$

$$f(x)=\frac{x^2-12}{x+3}$$

$$f(x)'=\frac{(x+3)\frac{d}{dx}(x^2-12)-(x^2-12)\frac{d}{dx}(x+3)}{(x+3)^2}$$

$$=\frac{(x+3)(2x)-(x^2-12)(1)}{(x+3)^2}$$

$$=\frac{2x^2+6x-x^2+12}{(x+3)^2}=\frac{x^2+6x+12}{(x+3)^2}$$

$$f(x) = \frac{(x+7)(x-5)}{3x^2}$$

Pages 124-125

#13-27 odd

#24