Section 4.3

Part #2

Derivatives of Inverse Trig Functions

Simplify

$|6| = 6$

$|6x| = 6|x|$

$|6x^2| = 6x^2$

$|6x^3| = 6|x^3|$ or $6x^2|x|$

$|x^2 - 1| = |x^2 - 1|$

$|x^2 + 1| = x^2 + 1$

$|(x-9)^2| = (x-9)^2$

$|(x-9)^3| = |(x-9)^3|$ OR $(x-9)^2|x-9|$

-Even Root Index

-Even Power Inside

-Odd Power Outside

-Absolute Value | |

$\sqrt{25} = 5$

$\sqrt{25x^2} = 5|x|$

$\sqrt{25x^3} = \sqrt{25x^2}\cdot\sqrt{x}$

$= 5|x|\sqrt{x}$

$\sqrt{25x^4} = 5x^2$

$\sqrt{x^6} = |x^3|$

$\sqrt{x^{12}} = x^6$

$\sqrt{9x^{14}y^{24}}$

$3|x^7|y^{12}$

$(a^9b^3)^{\frac{1}{3}}$

a^3b

$\sqrt{(x+5)^2} = |x+5|$

$\sqrt{x^6 - x^4}$ $\sqrt{x^4(x^2-1)}$

$x^2\sqrt{x^2-1}$

$\sqrt{(x+5)^4}$ $(x+5)^2$

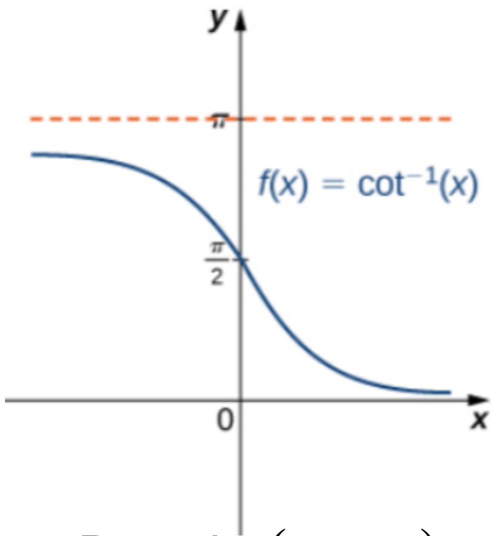
$\sqrt{x^{10}y^{18}}$

$|x^5y^9|$

$\sqrt{t}$

$\sqrt{t}$

$$x = \cot y$$



Domain: $(-\infty, \infty)$

Range: $(0, \pi)$

$$\frac{d}{dx} x = \frac{d}{dx} \cot y$$

$$1 = -\csc^2 y \cdot y'$$

$$y' = \frac{-1}{\csc^2 y}$$

Since $1 + \cot^2 y = \csc^2 y$

$$y' = \frac{-1}{1 + \cot^2 y}$$

$$y' = \frac{-1}{1 + x^2}$$

Find the derivative

$$y = \cot^{-1}(x^3)$$

Let $u = x^3$

$$\frac{du}{dx} = 3x^2$$

$$y = \cot^{-1} u$$

$$\frac{dy}{du} = \frac{-1}{1+u^2}$$

$$= \frac{-1}{1+(x^3)^2}$$

$$= \frac{-1}{1+x^6}$$

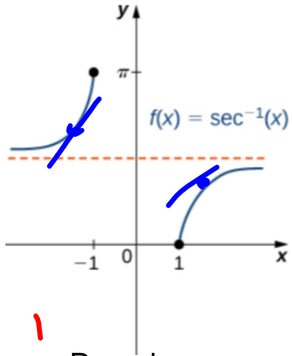
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{-1}{1+x^6} \cdot 3x^2$$

$$= \frac{-3x^2}{1+x^6}$$

$$x = \sec y$$

$$\frac{d}{dx} x = \frac{d}{dx} \sec y$$



Domain:
$x \leq -1$ or $x \geq 1$

Range:
$[0, \pi]$ $y \neq \frac{\pi}{2}$

$$1 = \sec y \cdot \tan y \cdot y'$$

$$y' = \frac{1}{\sec y \cdot \tan y}$$

$$y' = \frac{1}{x\left(\pm\sqrt{x^2-1}\right)}$$

Which do we use??

Since $\tan^2 y + 1 = \sec^2 y$

$\tan^2 y = \sec^2 y - 1$

$\tan y = \pm\sqrt{\sec^2 y - 1}$

$= \pm\sqrt{x^2-1}$

Need positive slopes

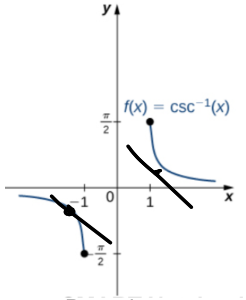
$x \leq -1$ $\qquad$ $x \geq 1$

$$y' = \frac{1}{x\left(-\sqrt{x^2-1}\right)} \qquad y' = \frac{1}{x\left(+\sqrt{x^2-1}\right)}$$

Neg $\qquad$ Positive

$$y' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$x = \csc y$$



Domain:
$x \leq -1$ or $x \geq 1$

Range:
$[-\frac{\pi}{2}, \frac{\pi}{2}], y \neq 0$

$$\frac{d}{dx} x = \frac{d}{dx} \csc y$$

$$1 = -\csc y \cot y \cdot y'$$

$$y' = \frac{-1}{\csc y \cot y}$$

$$1 + \cot^2 y = \csc^2 y$$

$$\cot y = \pm\sqrt{\csc^2 y - 1}$$

$$= \pm\sqrt{x^2 - 1}$$

$$y' = \frac{-1}{x(\pm\sqrt{x^2-1})}$$

which do we use?

Need negative slopes.

$x \leq -1$ $\qquad$ $x \geq 1$

Neg $\rightarrow$ $y' = \frac{-1}{x(-\sqrt{x^2-1})}$ $\qquad$ Pos $\rightarrow$ $y' = \frac{-1}{x(+\sqrt{x^2-1})}$

$y' = \frac{1}{x\sqrt{x^2-1}}$ $\qquad$ $y' = \frac{-1}{x\sqrt{x^2-1}}$

$$y' = \frac{-1}{|x|\sqrt{x^2-1}}$$

Formulas:

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}},\ x \neq \pm 1$$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}},\ x \neq \pm 1$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}},\ x \neq \pm 1, 0$$

$$\frac{d}{dx}(\csc^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}},\ x \neq \pm 1, 0$$

Find the derivative

$$y = \csc^{-1}\sqrt{t}$$

Let $u = \sqrt{t}$

$u = t^{1/2}$

$$\frac{du}{dt} = \frac{1}{2}t^{-1/2}$$

$y = \csc^{-1} u$

$$\frac{dy}{du} = \frac{-1}{|u|\sqrt{u^2-1}}$$

$$= \frac{-1}{|\sqrt{t}|\sqrt{(\sqrt{t})^2-1}}$$

$$= \frac{-1}{\sqrt{t}\sqrt{t-1}}$$

$$\frac{dy}{dt} = \frac{du}{dt} \cdot \frac{dy}{du}$$

$$= \left(\frac{1}{2}t^{-1/2}\right)\left(\frac{-1}{\sqrt{t}\sqrt{t-1}}\right)$$

$$= \left(\frac{1}{2\sqrt{t}}\right)\left(\frac{-1}{\sqrt{t}\sqrt{t-1}}\right)$$

$$= \frac{-1}{2t\sqrt{t-1}}$$

$$y = \sec^{-1}(x^2)$$

Let $u = x^2$ $\qquad y = \sec^{-1} u$

$\frac{du}{dx} = 2x \qquad \frac{dy}{du} = \frac{1}{|u|\sqrt{u^2-1}}$

$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du} \qquad = \frac{1}{|x^2|\sqrt{(x^2)^2-1}}$

$= (2x)\left(\frac{1}{x^2\sqrt{x^4-1}}\right) \qquad = \frac{1}{x^2\sqrt{x^4-1}}$

$= \frac{2}{x\sqrt{x^4-1}}$

$$y = \csc^{-1}\left(\frac{x}{4}\right)$$

$$y = \csc^{-1}\left(\frac{1}{x^2}\right)$$

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Answers

14. $\frac{1}{|s|\sqrt{25s^2-1}}$

16. $-\frac{2}{|x|\sqrt{x^2-4}}$

18. $-\frac{1}{2\sqrt{t}(t+1)}$

20. $\frac{s|s|-1}{|s|\sqrt{s^2-1}}$

22. Answer = 0

$x \neq 0$