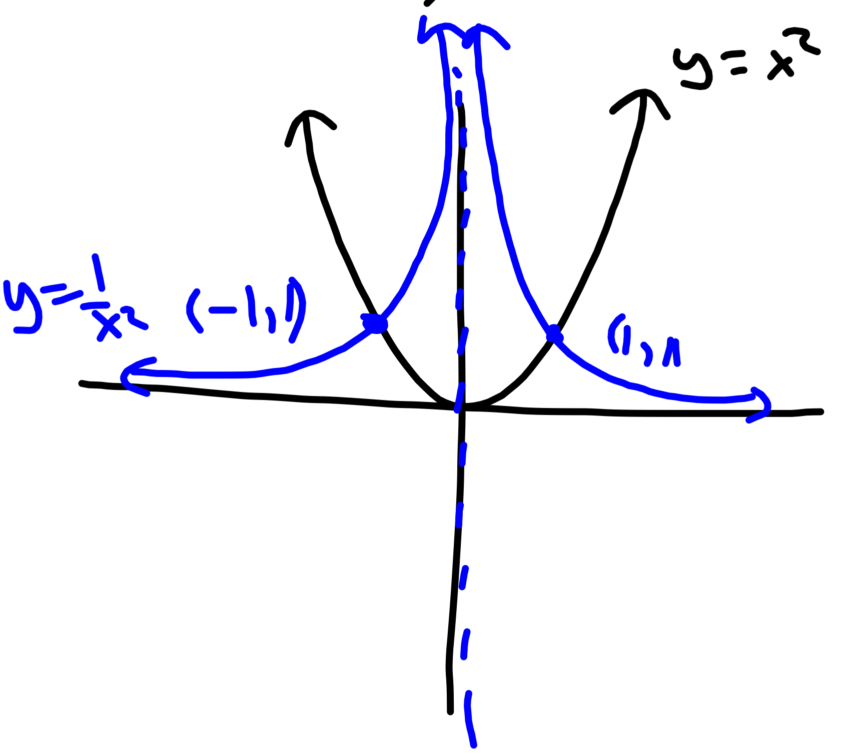
22) $\lim_{x\to 0} \frac{1}{x^2} = \frac{1}{0^2} = \frac{1}{0} \Rightarrow$ DNE

$\lim_{x\to 0^+}$

x	y
0.1	$\frac{1}{0.1^2} = \frac{1}{0.01} = 100$
0.01	$\frac{1}{0.01^2} = \frac{1}{0.0001} = 10{,}000$
0.001	1,000,000

$\lim_{x\to 0^-}$

x	y
-0.1	100
-0.01	10,000
-0.001	1000000



D.N.E.

33

$$\lim_{x \to 0} \frac{\sin^2 x}{x} = \frac{\sin^2 0}{0} = \frac{0^2}{0} = \text{DNE}$$

$$\lim_{x \to 0} \frac{\sin x \cdot \sin x}{x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \cdot \sin x = (1) \cdot (0) = 0$$

22, 33

Yesterday's Fun

Hwk Questions?

28, 30, 34?

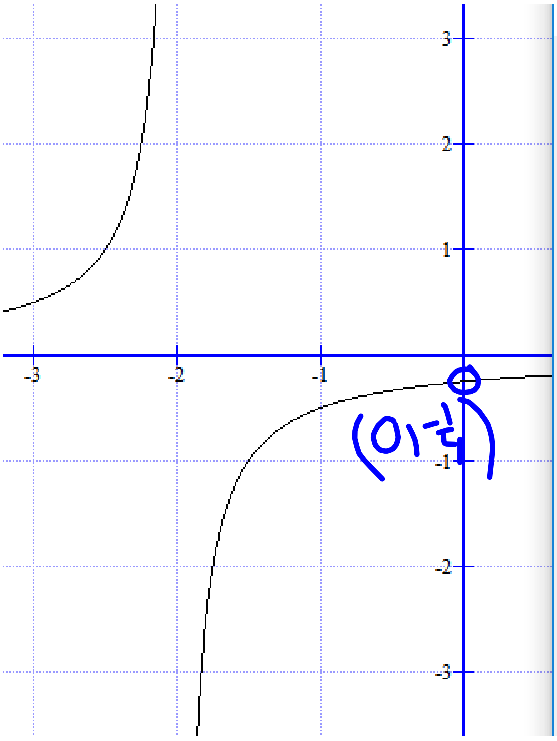
28. $\lim\limits_{x\to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x}$ **30.** $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x}$ **34.** $\lim\limits_{x\to 5} \dfrac{x^3 - 125}{x - 5}$

$$\lim_{x\to 0}\left(\frac{\frac{2(1)}{2(2+x)} - \frac{1(2+x)}{2(2+x)}}{x}\right)$$

$$\lim_{x\to 0}\left(\frac{\frac{2}{2(2+x)} + \frac{-2-x}{2(2+x)}}{x}\right)$$

$$\lim_{x\to 0}\frac{\frac{-\cancel{x}}{2(2+x)}}{\cancel{x}} = \lim_{x\to 0}\frac{-1}{2(2+x)}$$

$$= \frac{-1}{2(2+0)} = -\frac{1}{4}$$



Calculate table

From: -1 To: 1

Δx= 0.05

x	f(x)	f'(x)
−0.55	−0.3448	0.2378
−0.50	−0.3333	0.2222
−0.45	−0.3226	0.2081
−0.40	−0.3125	0.1953
−0.35	−0.303	0.1837
−0.30	−0.2941	0.173
−0.25	−0.2857	0.1633
−0.20	−0.2778	0.1543
−0.15	−0.2703	0.1461
−0.10	−0.2632	0.1385
−0.05	−0.2564	0.1315
0.00		
0.05	−0.2439	0.119
0.10	−0.2381	0.1134
0.15	−0.2326	0.1082
0.20	−0.2273	0.1033
0.25	−0.2222	0.0988
0.30	−0.2174	0.0945

30 $\lim\limits_{x \to 0} \frac{\sin 2x}{x}$

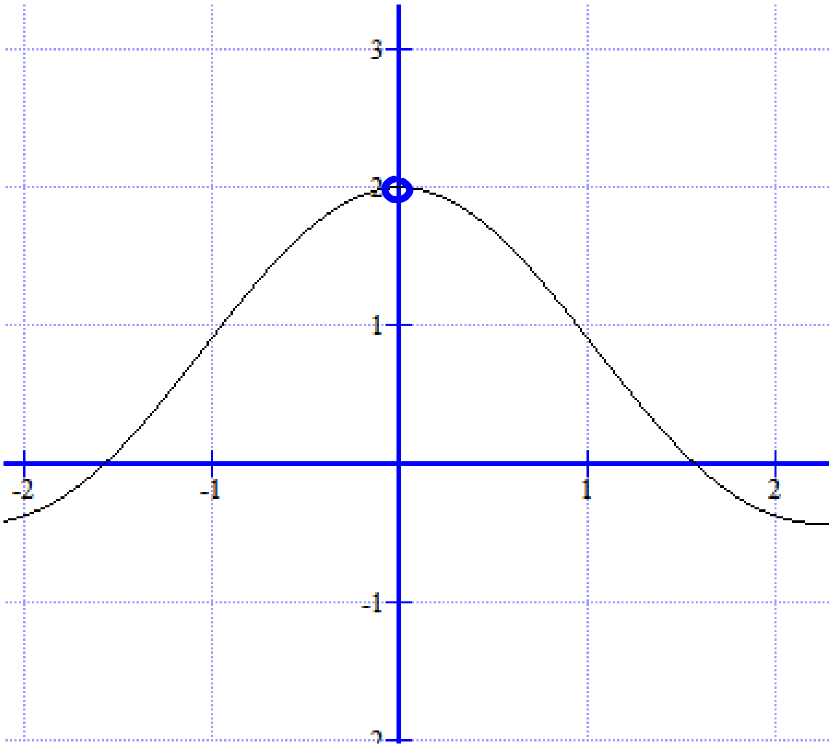
Remember
$\sin 2x = 2\sin x \cos x$

$= \lim\limits_{x \to 0} \frac{2\sin x \cos x}{x}$

$= \lim\limits_{x \to 0} \frac{\sin x}{x} \cdot 2\cos x$

$= \lim\limits_{x \to 0} \frac{\sin x}{x} \cdot \lim\limits_{x \to 0} 2\cos x$

$= (1)(2\cos 0) = (1)(2(1)) = 2$


3
2
1
-1
-2
-2
-1
1
2

$$\lim_{x\to 5} \frac{x^3-125}{x-5}$$

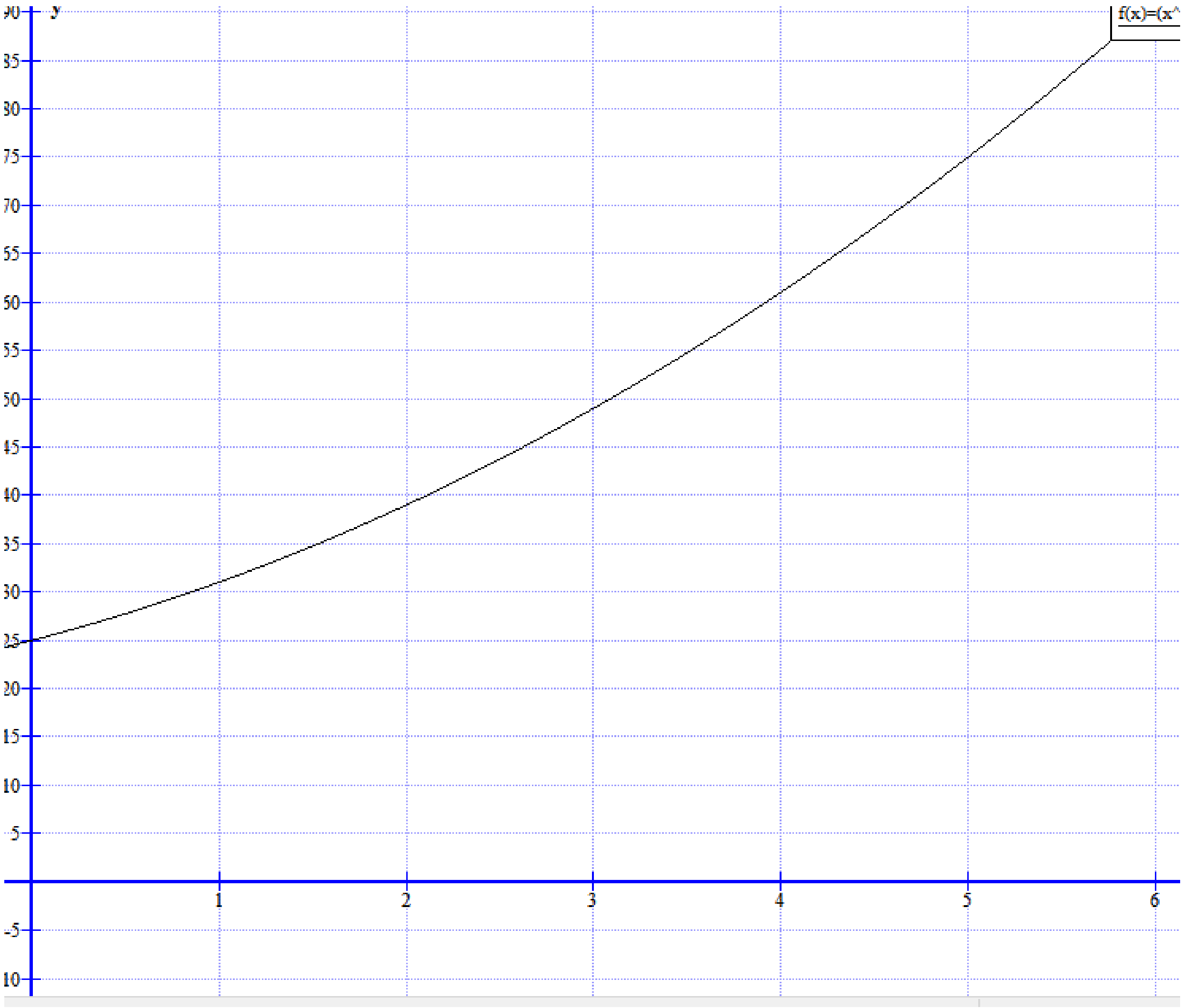
$$= \lim_{x\to 5} \frac{(x-5)(x^2+5x+25)}{(x-5)}$$

$$= \lim_{x\to 5} (x^2+5x+25)$$

$$= 5^2+5(5)+25$$

$$= 25+25+25 = 75$$

$$
\begin{array}{r|l}
 & x^2 + 5x + 25 \\
\hline
x-5 & x^3+0x^2+0x-125 \\
 & -(x^3-5x^2) \\
\hline
 & 5x^2+0x \\
 & -(5x^2-25x) \\
\hline
 & 25x-125 \\
 & -(25x-125) \\
\hline
 & 0
\end{array}
$$


y
f(x)=(x^
1
2
3
4
5
6

2.1 Rate of Change and Limits

One-Sided and Two-Sided Limits:

Sometimes functions will approach different values if you approach from opposite sides. Notation:

right-hand limit: $\lim_{x \to c^+} f(x)$ approach *c* **from** the right

left-hand limit: $\lim_{x \to c-} f(x)$ approach *c* **from** the left

A function only has a defined limit if the left-hand and right-hand limits approach the same value.

(Offer subject to change - does not include **endpoints**. Taxes extra.)

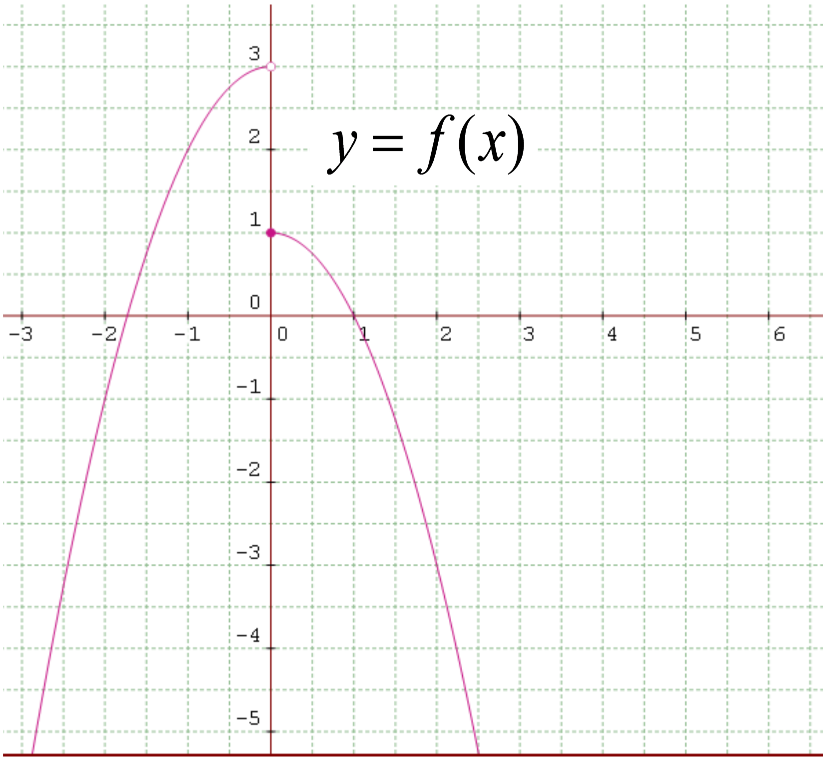
Use the graph to evaluate the following:

$f(0) =$

$\lim_{x \to 0^+} f(x) =$

$\lim_{x \to 0^-} f(x) =$

$\lim_{x \to 0} f(x) =$



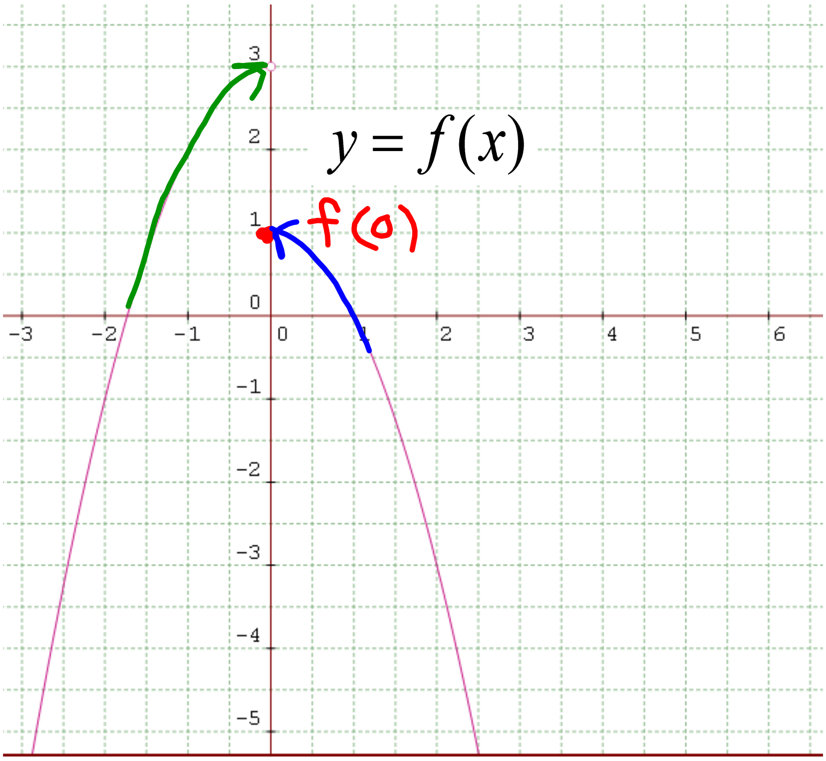
Use the graph to evaluate the following:

$f(0) =$ 1

$\lim_{x\to 0^+} f(x) =$ 1

$\lim_{x\to 0^-} f(x) =$ 3

$\lim_{x\to 0} f(x) =$ DNE



Evaluate the following:

$$\text{int}(4) =$$

$$\lim_{x \to 4^+} \text{int}(x)$$

$$\lim_{x \to 4^-} \text{int}(x)$$

$$\lim_{x \to 4} \text{int}(x)$$

Evaluate the following:

$\text{int}(4) =$ 4

$\lim_{x \to 4^+} \text{int}(x)$

int(4.1) = 4
int(4.01) = 4 } 4

$\lim_{x \to 4^-} \text{int}(x)$

int(3.9) = 3
int(3.99) = 3 } 3

$\lim_{x \to 4} \text{int}(x)$ DNE

$$\lim_{x\to0}\frac{|x|}{x} \Rightarrow \text{DNE}$$

$\lim_{x\to0^+} = 1$

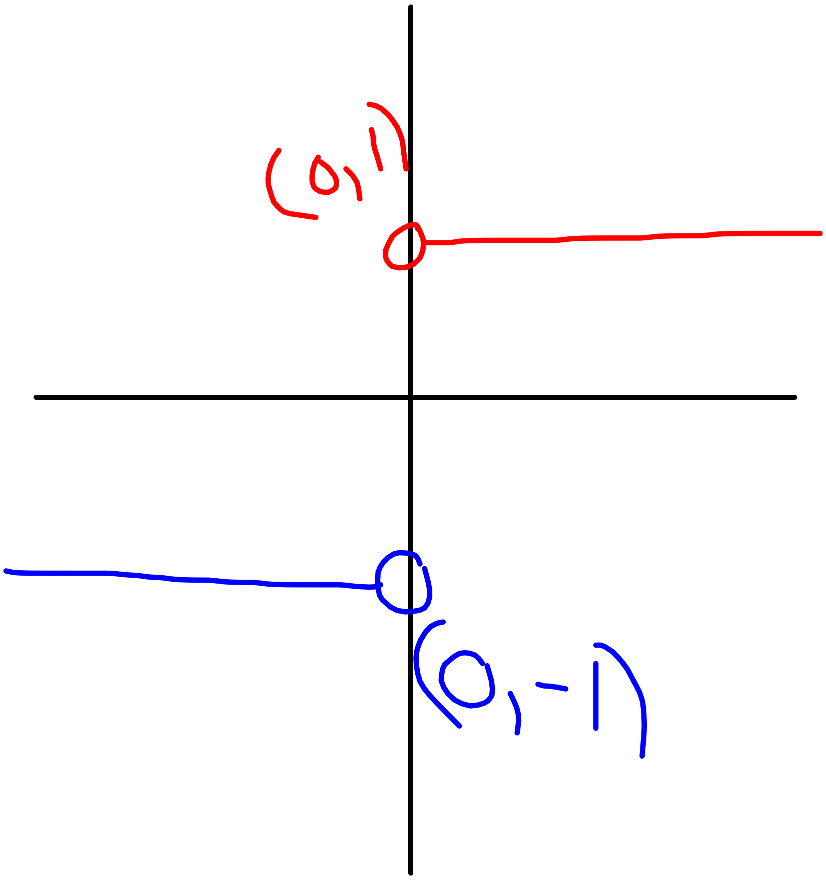
x	y
0.1	1
0.01	1
0.001	1

$\lim_{x\to0^-} = -1$

x	y
-0.1	-1
-0.01	-1
-0.001	-1

Remember $y=|x| \begin{cases} x, & x\geq0 \\ -x, & x<0 \end{cases}$

$\lim_{x\to0^+}\frac{x}{x} = 1$ $\lim_{x\to0^-}\frac{-x}{x} = -1$



(0,1)
(0,-1)

Pages 66-68

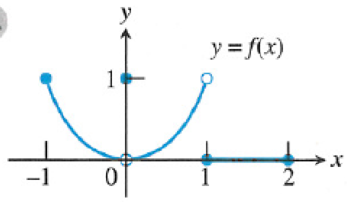
#37-46

(refer to examples 7 and 8)

You will be graphing tomorrow.

In Exercises 43 and 44, which of the statements are true about the function $y = f(x)$ graphed there, and which are false?

3.



(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 0^-} f(x) = 0$

(c) $\lim_{x \to 0^-} f(x) = 1$

(d) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

(e) $\lim_{x \to 0} f(x)$ exists

(f) $\lim_{x \to 0} f(x) = 0$

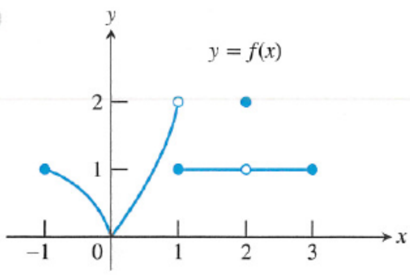
(g) $\lim_{x \to 0} f(x) = 1$

(h) $\lim_{x \to 1} f(x) = 1$

(i) $\lim_{x \to 1} f(x) = 0$

(j) $\lim_{x \to 2^-} f(x) = 2$

44.



(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 2} f(x) = 2$

(d) $\lim_{x \to 1^-} f(x) = 2$

(e) $\lim_{x \to 1^+} f(x) = 1$

(f) $\lim_{x \to 1} f(x)$ does not exist.

(g) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(h) $\lim_{x \to c} f(x)$ exists at every c in $(-1, 1)$.

(i) $\lim_{x \to c} f(x)$ exists at every c in $(1, 3)$.