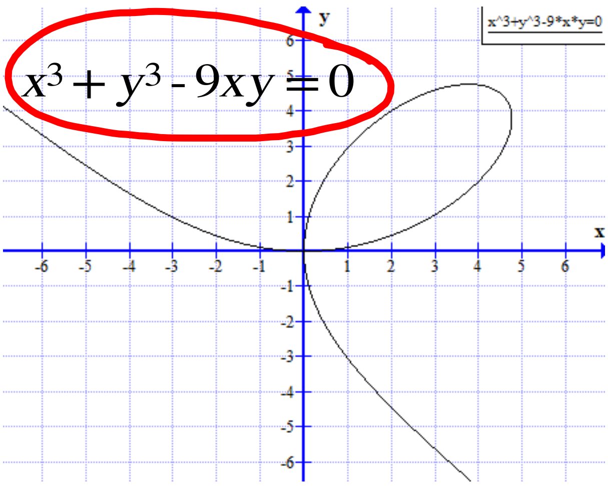
Section 4.2

Implicit Differentiation

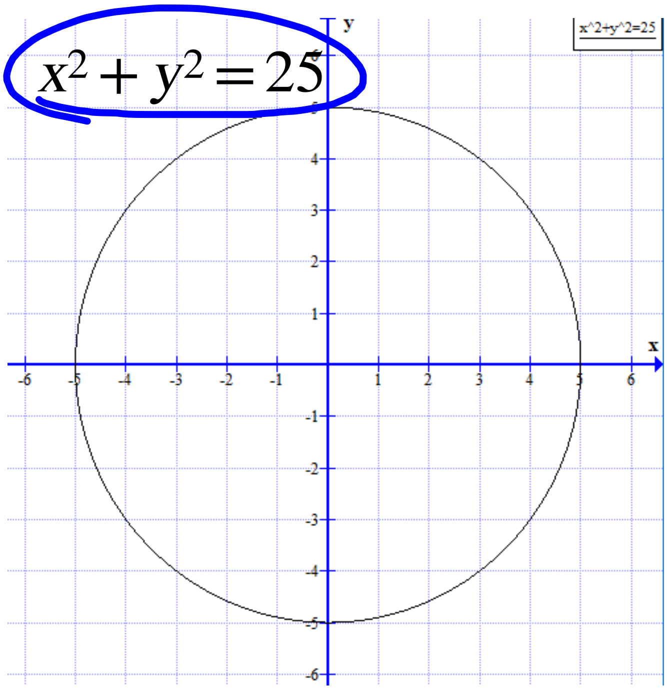
We are used to dealing with functions that are written in y= format such as y=cos(x) or $y = x^2 - 25$.

However, there are lots of relations and equations that exist that we wouldn't want to try and take the derivative of because it would be too tricky.

$x^3 + y^3 - 9xy = 0$



$x^2 + y^2 = 25$



Definitions

$\frac{d}{dx}$ $\longrightarrow$ The derivative of "something" with respect to x.

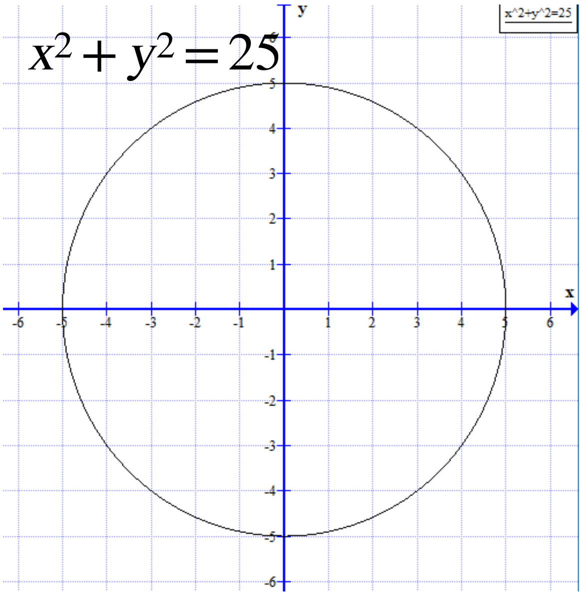
$\frac{dy}{dx}$ $\longrightarrow$ The derivative of y with respect to x.

Explicit: If the dependent variable (y) is a function of the independent variable (x), we express y in terms of x. For example the equation $y = x^2 + 1$, we are defining y ***explicity*** in terms of x.

Implicity: If the function y and the variable x is expressed by an equation where y is not expressed entirely in terms of x, we say that the equation defines y ***implicitly*** in terms of x. For example the equation $y - x^2 = 1$.

https://opentextbc.ca/calculusv1openstax/chapter/implicit-differentiation/

Online Tool: https://www.symbolab.com/solver/implicit-derivative-calculator



Looking at

$$x^2 + y^2 = 25$$

$$y^2 = -x^2 + 25$$

$$y = \pm\sqrt{-x^2+25}$$

So get y' by using the Chain Rule.

$$y = \pm\sqrt{-x^2+25} = \pm(-x^2+25)^{1/2}$$

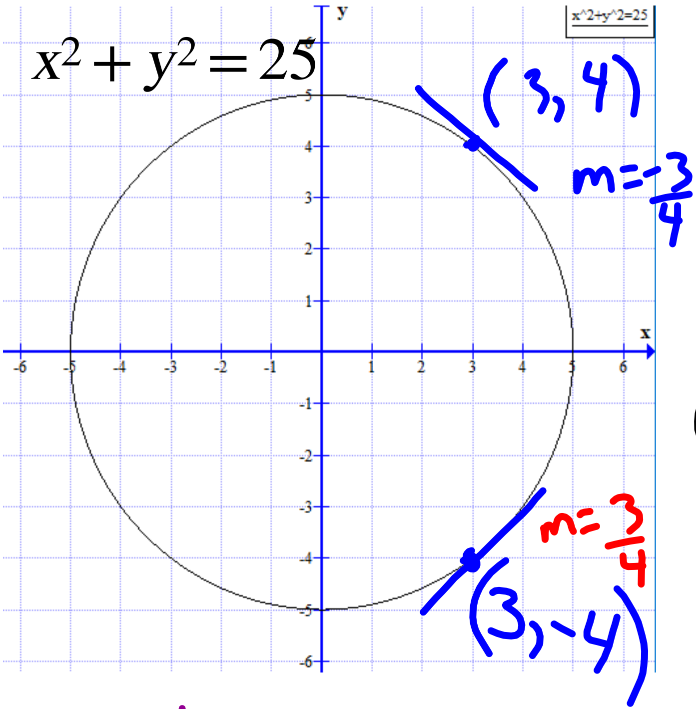
$$y' = \pm\frac{1}{2}(-x^2+25)^{-1/2}\frac{d}{dx}(-x^2+25)$$

$$y' = \pm\frac{1}{\cancel{2}}(-x^2+25)^{-1/2}(-\cancel{2}x)$$

$$y' = \pm(-x^2+25)^{-1/2}(-x)$$

$$y' = \frac{-x}{\pm(-x^2+25)^{1/2}} = \frac{-x}{\pm\sqrt{-x^2+25}} = \frac{-x}{y}$$

$$y' = \frac{-x}{y}$$



Find the slope of the tangent when x=3.

$$x^2 + y^2 = 25$$

$$(3)^2 + y^2 = 25$$

$$9 + y^2 = 25$$

$$y^2 = 25 - 9 = 16$$

$$y = \pm 4$$

$$y' = \frac{-x}{y}$$

(3, 4) $\quad m = y' = \frac{-x}{y} = -\frac{(3)}{4} = -\frac{3}{4}$

(3, -4) $\quad y = \frac{-x}{y} = \frac{-3}{-4} = 3/4$

Review

Find the derivative of $y=(5+2x)^{10}$

$$y' = 10(5+2x)^9 \cdot \frac{d}{dx}(5+2x)$$

$$y' = 10(5+2x)^9(2)$$

$$\frac{dy}{dx} = y' = 20(5+2x)^9$$

Find the derivative of $y^2 = (5+2x)^{10}$

$$\frac{d}{dx}\, y^2 = \frac{d}{dx}\,(5+2x)^{10}$$

$$2y \cdot y' = 20(5+2x)^9$$

Let $z = y^2$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= (2y)\, y'$$

$$\frac{2y \cdot y'}{2y} = \frac{20(5+2x)^9}{2y}$$

$$y' = \frac{10(5+2x)^9}{y}$$

Find the derivative of $x^2 + y^2 = 25$

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = \frac{d}{dx} 25$$

$$2x + 2y \cdot y' = 0$$

$$2y \cdot y' = -2x$$

$$y' = \frac{-2x}{2y}$$

$$y' = -\frac{x}{y}$$

Find the derivative of $x^2y + xy^3 = 10$

$$\frac{d}{dx}\, x^2y + \frac{d}{dx}\, xy^3 = \frac{d}{dx}\, 10$$

$$x^2(1)\cdot y' + y(2x) + (x)(3y^2\cdot y') + y^3(1) = 0$$

$$x^2y' + 2xy + 3xy^2y' + y^3 = 0$$

$$x^2y' + 3xy^2y' = -2xy - y^3$$

$$y'(x^2 + 3xy^2) = -2xy - y^3$$

$$y' = \frac{-2xy - y^3}{x^2 + 3xy^2}$$

Find the slope of the curve at the point (1, 1).

$$x^2 + xy + y^2 = 3$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} xy + \frac{d}{dx} y^2 = \frac{d}{dx} 3$$

$$2x + x(1)y' + y(1) + 2y \cdot y' = 0$$

$$2x + xy' + y + 2yy' = 0$$

$$xy' + 2yy' = -2x - y$$

$$y'(x + 2y) = -2x - y$$

$$y' = \frac{-2x - y}{x + 2y} \qquad (1, 1)$$

$$y' = \frac{-2(1) - (1)}{1 + 2(1)} = \frac{-2-1}{1+2} = \frac{-3}{3} = -1$$

p. 167

#1-4

9-13, 17,19