Section 3.4

Given $f(x)=9x^3-5x^2+7x-13$, find the

a. average rate of change between x=1 and x=4.

$$m = \frac{f(4)-f(1)}{4-1} = \frac{511-(-2)}{3} = \frac{513}{3}$$

$$= 171$$

b. instantaneous rate when x=-4.

$$f'(x) = 27x^2-10x+7$$

$$f'(-4) = 27(-4)^2-10(-4)+7$$

$$= 479$$

The volume of a sphere is $V(r) = \frac{4\pi}{3} r^3$

a. Find the average rate of change when between r=1 cm and r=3 cm.

b. Find the instantaneous rate of change of volume with respect to r.

c. If volume is measured in cm^3 and the radius is measured in cm, what is the unit measurement of the instantaneous rate of change of volume with respect to r?

d. Evaluate the rate of change when r=3 cm.

a. $\frac{V(3) - V(1)}{3-1} \; \frac{cm^3}{cm} = \frac{\frac{4\pi}{3}(3)^3 - \frac{4\pi}{3}(1)^3}{3-1}$

$= \frac{4\pi}{3}\frac{(27-1)}{2} = \frac{4\pi}{3}\left(\frac{26}{2}\right)$

$= \frac{52\pi}{3} \; \frac{cm^3}{cm}$

$= 54.45 \; \frac{cm^3}{cm}$

(b.) $V(r) = \frac{4\pi}{3} r^3$

$V'(r) = 3 \frac{4\pi}{3} r^2$

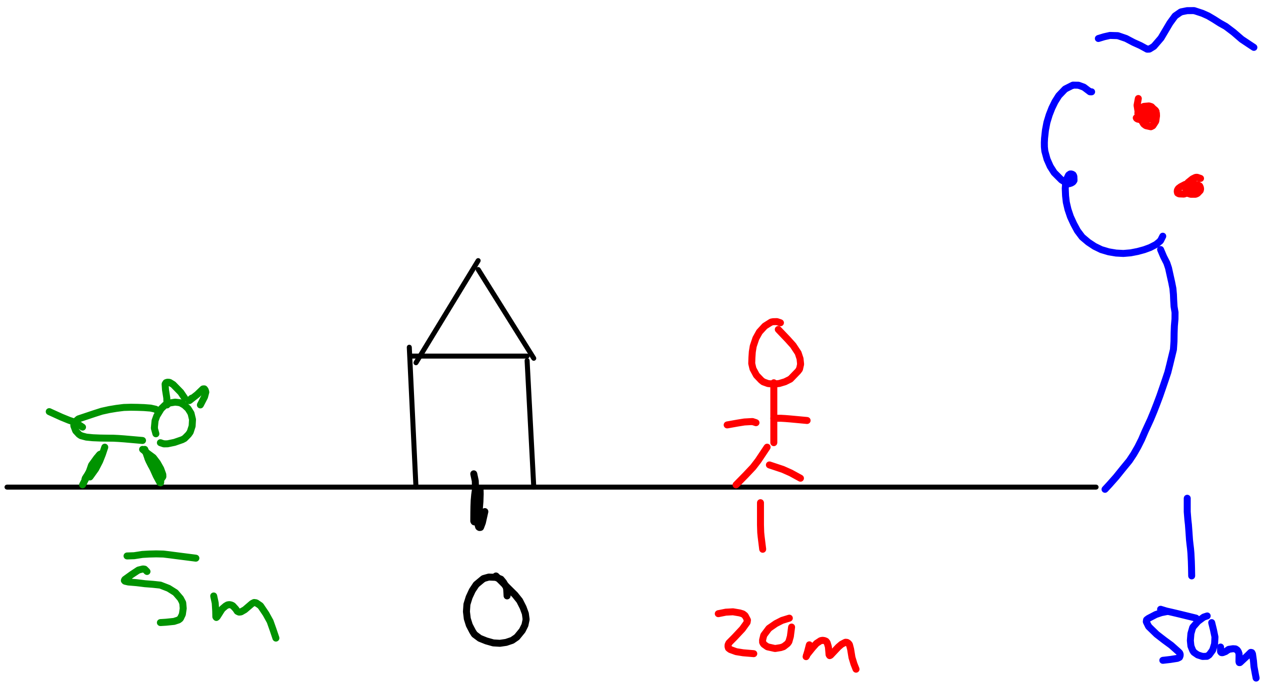
$V'(r) = 4\pi r^2$

c. $\frac{cm^3}{cm}$

d. $V'(3) = 4\pi(3)^2$

$= 36\pi \frac{cm^3}{cm}$

$= 113.10 \frac{cm^3}{cm}$



dist = 5m

positio = -5m

20m

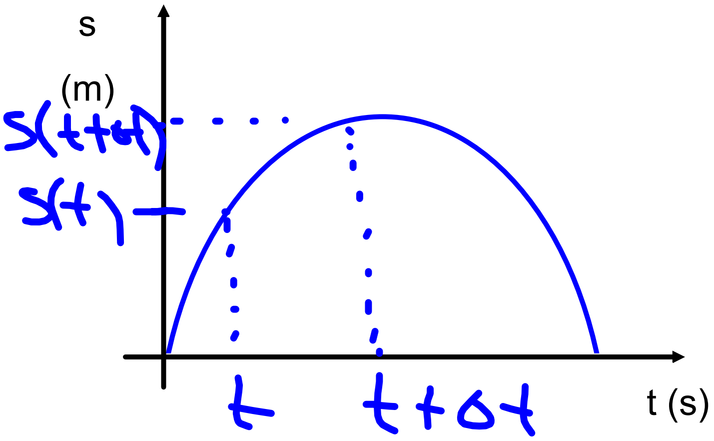
+20m

50m

+50m

Position, Speed, and Acceleration:

Consider a position (s) - time (t) graph of an object:



$$s = f(t)$$

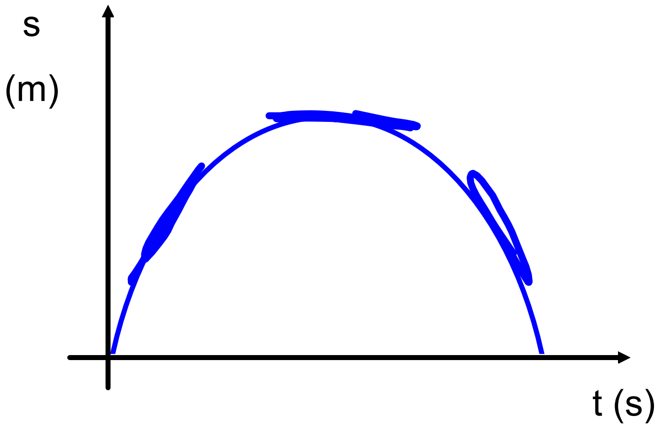
The displacement of the object between time *t* and time *t+Δt* :

$$s(t+\Delta t) - s(t)$$

So the average speed of the object between these times would be:

$$\frac{s(t+\Delta t) - s(t)}{t+\Delta t - t}$$

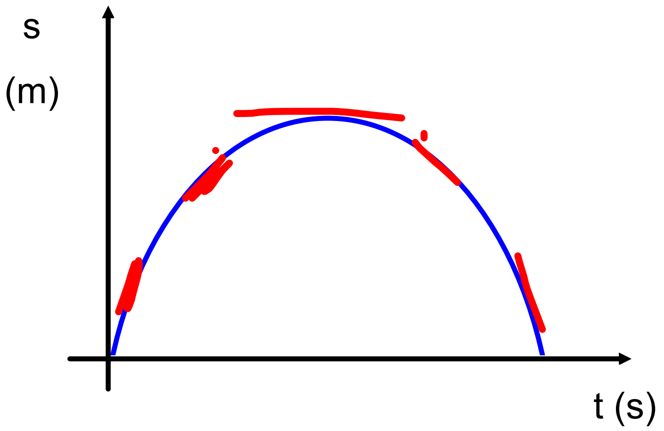
$$\frac{s(t+\Delta t) - s(t)}{\Delta t}$$



$$s = f(t)$$

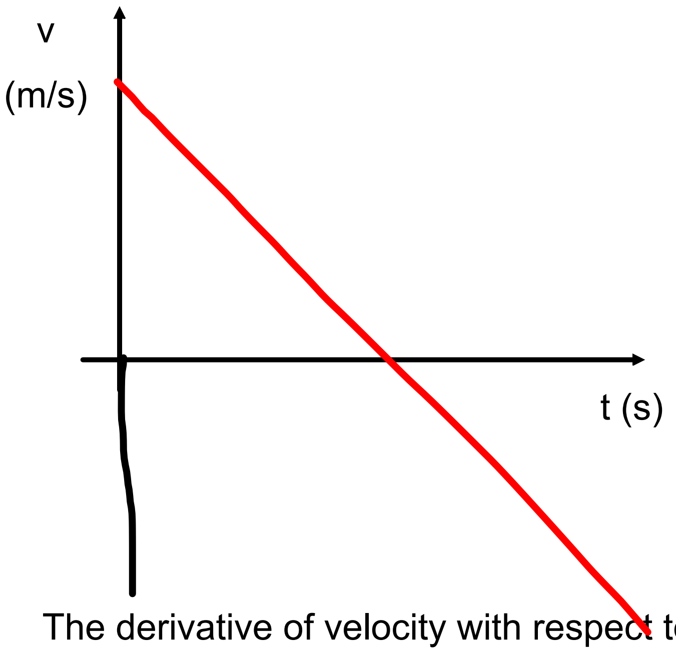
We can calculate the instantaneous velocity of an object using the equation:

$$v(t) = s'(t)$$



$$s = f(t)$$

Sketch a graph of the velocity (v) - time (t) graph for this object:



The derivative of velocity with respect to time is:

$$a(t) = v'(t) = s''(t)$$

$$a(t) = \frac{v(t_2) - v(t_1)}{t_2 - t_1} \quad \frac{m/s}{s}$$

The height, in metres, of a projectile shot vertically upward from ground level with an initial velocity of 24.5 m/s is $s(t) = 24.5t - 4.9t^2$ after t seconds.

a. Find the velocity after 2 s and after 4 s.
b. When does the projectile reach its maximum height?
c. What is the maximum height?
d. Are you thinking about what you are writing?
e. When does it hit the ground?
f. With what velocity does it hit the ground?

a. $v(t) = s'(t)$
$= 24.5 - 9.8t$

$v(2) = 24.5 - 9.8(2) = +4.9 \text{ m/s}$

$v(4) = 24.5 - 9.8(4) = -14.7 \frac{m}{s}$

(b) $0 = 24.5 - 9.8t$

$t = \frac{24.5}{9.8} = 2.5 \text{ s}$

(c)

$s(2.5) = 24.5(2.5) - 4.9(2.5)^2$

$= 30.625 \text{ m}$

(e)

$s(t) = 24.5t - 4.9t^2 = 0$

$t(24.5 - 4.9t) = 0$

$t = 0 \text{ s}$ $\quad 24.5 - 4.9t = 0$

$t = \frac{24.5}{4.9} = 5 \text{ s}$

(f) $v(t) = 24.5 - 9.8t$

$v(5) = 24.5 - 9.8(5)$

$= -24.5 \text{ m/s}$

The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and s in metres.

a. Find the velocity at time t.

b. What is the velocity after 2 s? after 4 s?

c. When is the particle at rest?

d. When is the particle moving in the positive direction? in the negative direction?

e. Find the acceleration at time t and after 4 s.

f. When is the particle speeding up? When is the particle slowing down?

a. $v(t) = 3t^2 - 12t + 9$

b. $v(2) = 3(2)^2 - 12(2) + 9$
$= -3 \text{ m/s}$

$v(4) = 9 \text{ m/s}$

c. $v(t) = 3t^2 - 12t + 9 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(9)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{36}}{6} = \frac{12 \pm 6}{6}$$

$t_1 = \frac{18}{6} = 3s$ $\quad t_2 = \frac{6}{6} = 1 \text{sec}$

	$t<1$	$1<t<3$	$t>3$
$3t^2-12t+9$	+	−	+

Positive Direction $t<1$, $t>3$

Negative Direction $1<t<3$

$3t^2-12t+9$	$t<1$	$1<t<3$	$t>3$
$3(t^2-4t+3)$			
$3(t-3)(t-1)$			
3	+	+	+
$t-3$	−	−	+
$t-1$	−	+	+
$v(t)$	+	−	+

Positive Direction $t<1$, $t>3$

Negative Direction $1<t<3$

e.

$v(t) = 3t^2 - 12t + 9$

$a(t) = v'(t) = 6t - 12$

$a(4) = 6(4) - 12 = 12 \, m/s^2$

$= 12 \, m/s/s$

f. When is $v(t)$ inc/dec?

$6t - 12 = 0$

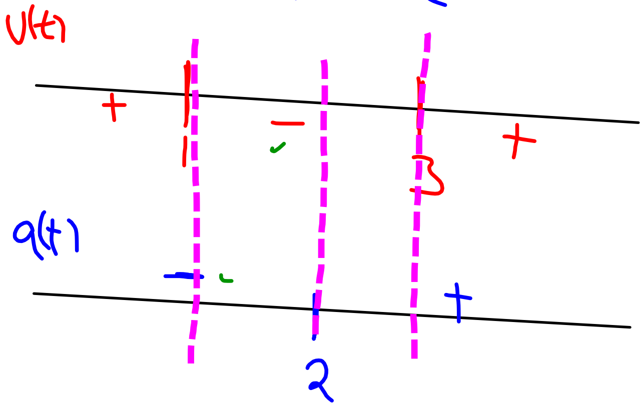
$6t = 12$

$t = 2 \, sec$

	$t<2$	$t>2$
$6t-12$	$-$	$+$

Positive Acceleration $t > 2$

Negative Acceleration $t < 2$



Speeding Up
Both signs the same
$1 < t < 2$ $\quad t > 3$

Slowing Down
Opposite Signs
$t < 1$ $\quad 2 < t < 3$

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14 (4.46 s, 0.726 s)

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If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume of water remaining in the tank after t minutes as

$$V = 5000\left(1 - \frac{1}{40}t\right)^2, \text{ where } 0 \le t \le 40.$$

a. Find the rate at which the water is draining from the tank after 5 min, 10 min, 20 min, and 40 min.

b. At what time is the water flowing out the fastest? the slowest?

$$V(t) = 5000\left(1 - \frac{1}{40}t\right)^2$$

$$V(t) = 5000\left(1 - \frac{1}{20}t + \frac{1}{1600}t^2\right)$$

$$V(t) = 5000 - 250t + \frac{25}{8}t^2$$

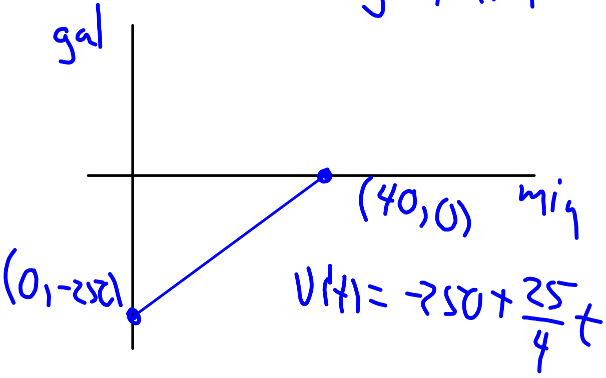
$$V'(t) = -250 + \frac{25}{4}t$$

$V'(5) = -250 + \frac{25}{4}(5) = -218.75$ gal/min

$V'(10) = -187.5$ gal/min

$V'(20) = -125$ gal/min

$V'(40) = 0$ gal/min



Most $t = 0$ min

Least $t = 40$ min

What Is Marginal Analysis?

Marginal analysis is an examination of the additional benefits of an activity compared to the additional costs incurred by that same activity. Companies use marginal analysis as a decision-making tool to help them maximize their potential profits. Marginal refers to the focus on the cost or benefit of the next unit or individual, for example, the cost to produce one more widget or the profit earned by adding one more worker.

For example, consider a hat manufacturer. Each hat produced requires seventy-five cents of plastic and fabric. Your hat factory incurs $100 dollars of fixed costs per month. If you make 50 hats per month, then each hat incurs $2 of fixed costs. In this simple example, the total cost per hat, including the plastic and fabric, would be $2.75 ($2.75 = $0.75 + ($100/50)). But, if you cranked up production volume and produced 100 hats per month, then each hat would incur $1 dollar of fixed costs because fixed costs are spread out across units of output. The total cost per hat would then drop to $1.75 ($1.75 = $0.75 + ($100/100)). In this situation, increasing production volume causes marginal costs to go down.

Marginal Cost: The instantaneous rate of change of cost with respect to the number of items produced. The marginal cost is the derivative of the cost function.

Marginal Revenue: The instantaneous rate of change of revenue with respect to the number of items produced. The marginal revenue is the derivative of the revenue function.

A bicycle tire manufacturer's cost for producing mountain bike tires is given by the formula where c(x) is the cost and x is the number of tires produce. $c(x)=4930+8.4x-0.0006x^2$. [0,2000]

a. Find the average cost of producing 500 tires.

b. Find the average rate of cost per tire.

c. Find the marginal cost of producing 500 tires.

d. Find the actual cost of producing the 501st tire.

(a) $C(x) = 4930 + 8.4x - 0.0006x^2$

$C(500) = 4930 + 8.4(500) - 0.0006(500)^2$

$= \$8{,}980$

$\text{Avg} = \frac{\$8{,}980}{500 \text{ tires}} = \$17.96/\text{tire}$

(b) $\begin{matrix}\text{Avg}\\ \text{Rate}\end{matrix} = \frac{C(500) - C(0)}{500 - 0}$

$= \frac{\$8{,}980 - \$4930}{500}$

$= \frac{4050}{500} = \$8.10/\text{tire}$

c.

$C'(x) = 8.4 - 0.0012x$

$C'(500) = 8.4 - 0.0012(500)$

$= \$7.80/\text{tire}$

d.

$C(501) = 4930 + 8.4(501) - 0.0006(501)^2$

$= \$8987.80$

$\$8987.80 - 8980 = \$7.80$

$C(501) - C(500)$

The revenue function for producing milkshakes is given where r(x) is the revenue and x is the number of milkshakes produced.

$$r(x)=\frac{500{,}000x-x^2}{100{,}000}$$

Find the marginal revenue at a production of 200,000 milkshakes per year.

$$r'(x)=\frac{100{,}000\frac{d}{dx}(500{,}000x-x^2)-(500{,}000x-x^2)\frac{d}{dx}100{,}000}{(100{,}000)^2}$$

$$=\frac{(100{,}0\cancel{00})(500{,}000-2x)-0}{(100{,}000)^{\cancel{2}}}$$

$$r'(x)=\frac{500{,}000-2x}{100{,}000}$$

$$r'(200{,}000)=\frac{500{,}000-2(200{,}000)}{100{,}000}$$

$$=\$1/\text{milkshake}$$

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