Section 4.4

Part #3

Derivative of $y = \log_a u, \quad y = u^n$

Proof of the derivative of $y = x^n$

$$y = x^n$$

$$\ln(y) = \ln(x^n)$$

$$\ln(y) = n \cdot \ln x$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(n \cdot \ln x)$$

$$\frac{1}{y} \cdot y' = n \cdot \frac{1}{x}$$

$$y' = n \cdot \frac{y}{x} = n \cdot \frac{x^n}{x} = n \cdot x^{n-1}$$

If $y = x^n \longrightarrow y' = n \cdot x^{n-1}$

If $y = u^n \longrightarrow y' = \frac{dy}{du} \cdot \left(\frac{du}{dx}\right) = (n \cdot u^{n-1}) \cdot \left(\frac{du}{dx}\right)$

Find the derivative of $y = \sin^4(x)$.

Chain Rule

$u = \sin x$

$\frac{du}{dx} = \cos x$

$y = u^4$

$\frac{dy}{du} = 4u^3 = 4\sin^3 x$

$\frac{dy}{dx} = 4\sin^3 x \cos x$

Using Logs

$y = \sin^4 x$

$\ln y = \ln \sin^4 x$

$\ln y = 4 \ln \sin x$

$\frac{d}{dx} \ln y = \frac{d}{dx} 4 \ln \sin x$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{y'}{y} = \frac{4}{\sin x} \cdot \cos x$

$y' = \frac{4}{\sin x} \cos x \, y$

$y = \frac{4 \cos x (\sin^4 x)}{\sin x}$

$y = 4 \sin^3 x \cos x$

Using logarithms can make finding the derivative easier. It is especially helpful if you understand the proofs we did throughout Section 4.4.

Find the derivative of

a. $y=(\cos(x))^x$

$$\ln y = \ln \cos x^{x}$$

$$\ln y = x \ln \cos x$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} x \ln \cos x$$

$$\frac{1}{y} \cdot y' = x \cdot \frac{d}{dx} \ln \cos x + \ln \cos x \cdot \frac{d}{dx} x$$

$$\frac{y'}{y} = x\left(\frac{-\sin x}{\cos x}\right) + \ln \cos x(1)$$

$$y' = \left(\frac{-x \sin x}{\cos x} + \ln \cos x\right) \cdot y$$

$$y' = \left(\frac{-x \sin x}{\cos x} + \ln \cos x\right) \cos^{x} x$$

$$y' = \frac{-x \sin x \cos^{x} x}{\cos x} + \ln \cos x \cdot \cos^{x} x$$

$$y' = -x \sin x \cos^{x-1} x + \ln \cos x \cdot \cos^{x} x$$

b. $y=\frac{(3x+1)^4\cdot(x-1)^6}{\sqrt{6x-5}}$

$$\ln y = \ln \frac{(3x+1)^4(x-1)^6}{\sqrt{6x-5}}$$

$$\ln y = \ln(3x+1)^4 + \ln(x-1)^6 - \ln\sqrt{6x-5}$$

$$\ln y = 4\ln(3x+1) + 6\ln(x-1) - \frac{1}{2}\ln(6x-5)$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}4\ln(3x+1) + \frac{d}{dx}6\ln(x-1) + \frac{d}{dx}-\frac{1}{2}\ln(6x-5)$$

$$\frac{1}{y}y' = \frac{4}{3x+1}\cdot(3) + 6\left(\frac{1}{x-1}\right)\cdot(1) - \frac{1}{2}\frac{1}{6x-5}(6)$$

$$\frac{y'}{y} = \frac{12}{3x+1} + \frac{6}{x-1} - \frac{3}{6x-5}$$

$$y' = \frac{12}{3x+1}y + \frac{6}{x-1}y - \frac{3}{6x-5}y$$

$$= \frac{12}{3x+1}\,\frac{(3x+1)^4(x-1)^6}{(6x-5)^{1/2}} +$$

$$+ \frac{6}{x-1}\,\frac{(3x+1)^4(x-1)^6}{(6x-5)^{1/2}}$$

$$- \frac{3}{(6x-5)}\cdot\frac{(3x+1)^4(x-1)^6}{(6x-5)^{1/2}}$$

$$= \frac{12(3x+1)^3(x-1)^6}{(6x-5)^{1/2}} + \frac{6(3x+1)^4(x-1)^5}{(6x-5)^{1/2}} - \frac{3(3x+1)^4(x-1)^6}{(6x-5)^{3/2}}$$

$$= \frac{3(3x+1)^3(x-1)^5}{(6x-5)^{1/2}}\left[4(x-1)+2(3x+1) - \frac{(3x+1)(x-1)}{(6x-5)}\right]$$

$$= \frac{3(3x+1)^3(x-1)^5}{(6x-5)^{1/2}}\left[4x-4+6x+2 - \frac{(3x^2-2x-1)}{6x-5}\right]$$

$$= \frac{3(3x+1)^3(x-1)^5}{(6x-5)^{1/2}}\left[\frac{(6x-5)(10x-2)}{6x-5} + \frac{-3x^2+2x+1}{6x-5}\right]$$

$$= \frac{3(3x+1)^3(x-1)^5}{(6x-5)^{1/2}}\left[\frac{60x^2-12x-50x+10-3x^2+2x+1}{6x-5}\right]$$

$$= \frac{3(3x+1)^3(x-1)^5}{(6x-5)^{1/2}}\left[\frac{57x^2-60x+11}{6x-5}\right]$$

$$= \frac{3(3x+1)^3(x-1)^5\left[57x^2-60x+11\right]}{(6x-5)^{3/2}}$$

52. *Spread of Flu* The spread of flu in a certain school is modeled by the equation

$$P(t) = \frac{200}{1 + e^{5-t}},$$

where $P(t)$ is the total number of students infected t days after the flu first started to spread.

(a) Estimate the initial number of students infected with this flu.

(b) How fast is the flu spreading after 4 days?

(c) When will the flu spread at its maximum rate? What is that rate?

(A) $P(0) = \frac{200}{1+e^{5-0}} = 1.3 \approx 1$ person

(B) $\frac{dP}{dt} = \frac{d}{dt} \frac{200}{1+e^{5-t}}$

$$= \frac{(1+e^{5-t})\frac{d}{dt}(200) - 200\frac{d}{dt}(1+e^{5-t})}{(1+e^{5-t})^2}$$

$$= \frac{(1+e^{5-t})(0) - 200(0+e^{5-t}(-1))}{(1+e^{5-t})^2}$$

$$\frac{dP}{dt} = \frac{+200e^{5-t}}{(1+e^{5-t})^2} = P'(t)$$

$$P'(4) = \frac{200e^{5-4}}{(1+e^{5-4})^2} = \frac{200e}{(1+e)^2}$$

$= 39.32 \approx 39$ people/day

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Test

$\frac{d}{dx} \ln x$

$\frac{d}{dx} a^x$

$\frac{d}{dx} \log_a x$

p.183-184

#33-36, 43-48,

51, 53

Even Answers:

34. $=(1+\sqrt{2})x^{\sqrt{2}}$

46. $\frac{dy}{dx}=\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\left(\frac{1}{x}+\frac{x}{x^2+1}-\frac{2}{3(x+1)}\right)$

36. $(1-e)x^{-e}$

48. y'=0, x>0

44.

$$\frac{dy}{dx}=x^{\tan x}\left[\frac{\tan x}{x}+(\ln x)(\sec^2 x)\right]$$

Review Questions For Test

p. 186-188

#1-52

57-60

70, 78, 79