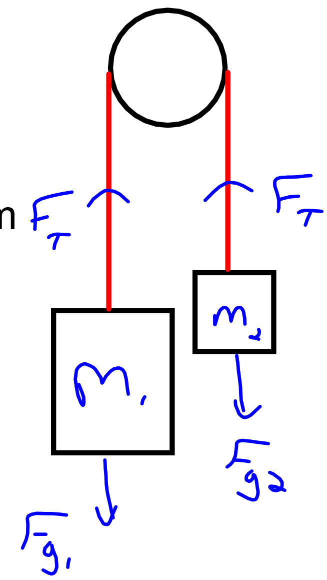
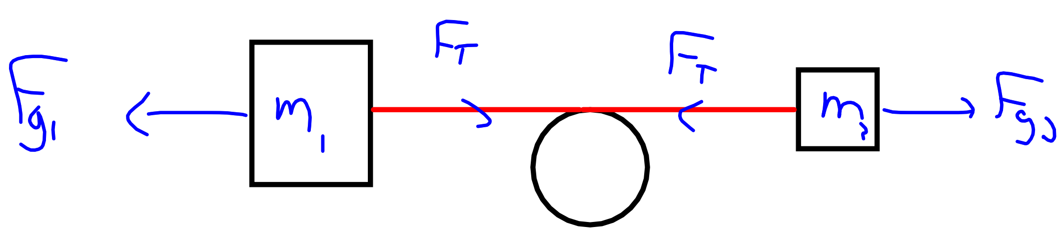
Atwood Machine



- Invented by Rev. George Atwood in 1784.
- It was designed to study the mechanics of uniform acceleration.
- The simple design involves two connected masses accelerating in opposite directions
- Important points to remember:

1. Rope and Pulley massless/frictionless so no Forces used to make them move
2. Pulley only acts to change direction/orientation of Tension in rope
3. Objects are still connected so they have the same magnitude of acceleration - opposite direction though $a_1 = a_2$

- Can re-imagine or model the setup as if it acts in a straight line
  > From this it is more obvious that the internal forces (tension) cancel out and don't contribute to system motion



F_T is Internal

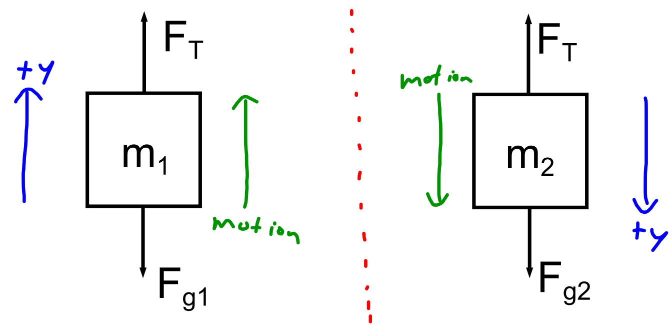
F_g are External

$$F_{Net,sys} = F_{g1} - F_{g2} = m_{sys} a_{sys}$$

a_{sys} has no direction

>

Basic FBD of Atwood Machine (with no extra applied Force)



$$a_1 = a_2 \longrightarrow \vec{a}_1 = -\vec{a}_2$$

$$F_{Net,1} = F_T - F_{g1} = m_1 a_1 \leftarrow \vec{a}_1 \text{ is [up]}, \; F_T > F_{g1}$$

$$F_T = m_1 a_1 + F_{g1} = m_1(a_1 + g) \leftarrow \text{Apparent Weight}$$

$$F_{Net,2} = F_{g2} - F_T = m_2 a_2 \leftarrow \vec{a}_2 \text{ is [down]}$$

$$F_{g2} > F_T > F_{g1}$$

$$m_2 g - m_1(a_1 + g) = m_2 a_2$$

$$m_2 g - m_1 a_1 - m_1 g = m_2 a_2$$

$$\frac{m_2 g - m_1 g}{m_1 + m_2} = m_1 a_1 + m_2 a_2 = \frac{\cancel{(m_1 + m_2)} a_{sys}}{\cancel{m_1 + m_2}}$$

$$\boxed{a_{sys} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g}$$

$= a_1 = a_2$

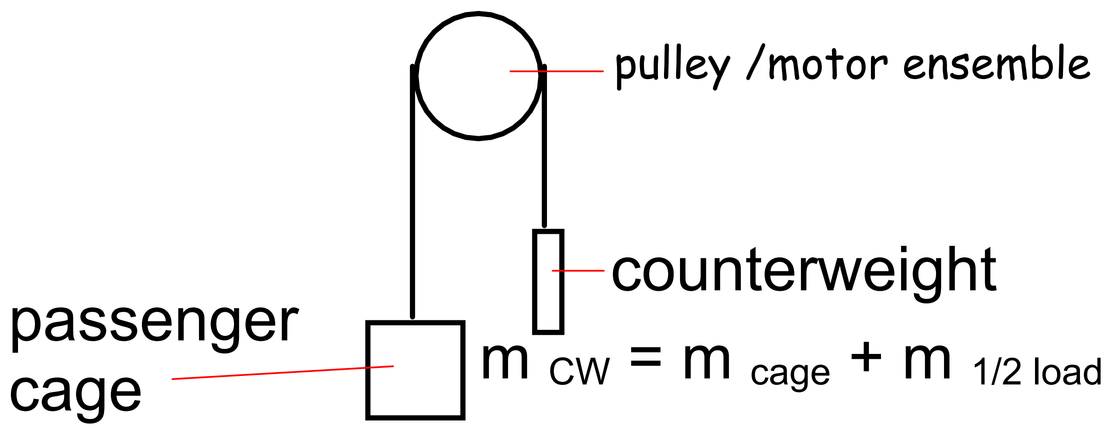
When $m_1 = m_2$
$a_y = 0 \text{ m/s}^2$.

As m_2 get bigger OR m_1 gets smaller a_{sys} will increase

When $m_1 = 0 \text{ kg}$ $a_{sys} = g \longrightarrow$ Free Fall

<u>Application: The Elevator</u> https://www.youtube.com/watch?v=hMdJLXGxynA

pg 479



$$m_{CW} = m_{cage} + m_{1/2\ load}$$

- Elevator wants to accelerate up at a_y - Tension is apparent weight of elevator, so $F_T = M(g+a_y)$
- If no counterweight, Motor must provide all the power to get this Tension in the cable
- The counterweight provides a base level of Tension in the cable at all times - motor just has to provide the extra amount to get it to accelerate
- By setting the counterweight to 1/2 the average load mass means that 1/2 the time the counterweight is enough to provide all the tension required - no motor

Atwood Machine Examples

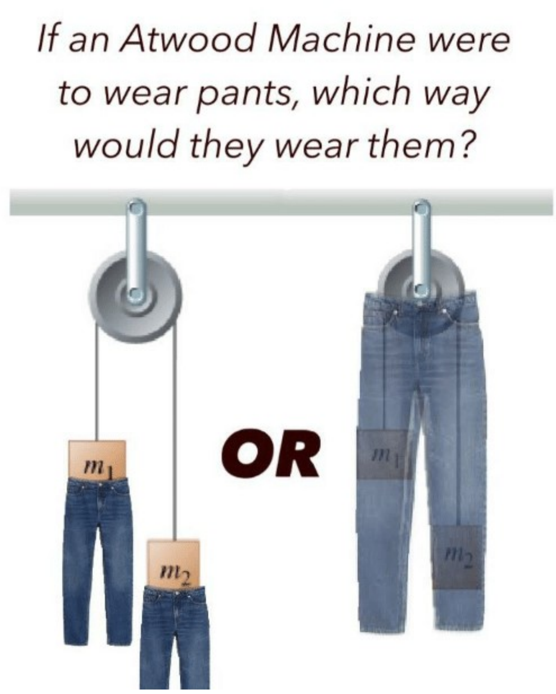
Ex. 1: A 200. g mass and a 100. g mass are suspended on an Atwood's machine. The masses are released simultaneously such that they start to move at the same time.

a) What is the acceleration of the system?
b) What is the size of the tension exerted through the system?
c) What would be the tension acting on each mass in the system?

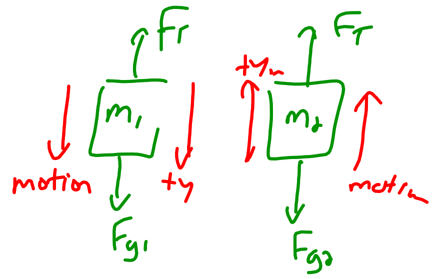
Ex. 2: Two masses are suspended on an Atwood's machine. They are released. The system accelerates at $5.88\ m/s^2$.

a) What is the heavier mass if the lighter mass is 1.50 kg?
b) What is the tension force vector acting on the lighter mass?
c) What is the tension force acting vector on the heavier mass?

Do Practice Problem 19-22 on page 485



Ex 1)



$m_1 = 200.\ g$

$m_2 = 100.\ g$

$\vec{g} = 9.81\ m/s^2\ [down]$

$a_{sys} = ? = a_1 = a_2$

Based on same setup as previous page

$$a_{sys} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{(0.200\ kg - 0.100\ kg)}{0.200\ kg + 0.100\ kg}\right)(9.81\ m/s^2)$$

$$= \boxed{3.27\ m/s^2}$$

b) $F_{Net,1} = F_{g1} - F_T = m_1 a_1$

$$F_T = F_{g1} - m_1 a_1 = m_1 (g - a_1)$$

$$F_T = (0.200\ kg)(9.81\ m/s^2 - 3.27\ m/s^2)$$

$$= 1.308\ N = \boxed{1.31\ N}$$

$$F_T = m_2 (g + a_4)$$

c) $\vec{F}_{T1} = 1.31\ N\ [up]$

$\vec{F}_{T2} = 1.31\ N\ [up]$

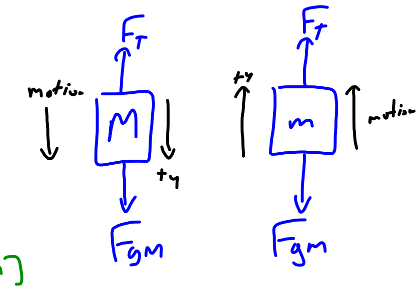
Ex 2)

$a_{sys} = 5.88\ m/s^2$

$m = 1.50\ kg$

$\vec{g} = 9.81\ m/s^2\ [down]$

$M = ?$



$$(M+m)\, a_{sys} = \left(\frac{M-m}{\cancel{M+m}}\right) g \quad \times \cancel{(M+m)}$$

$$(M+m)a_{sys} = (M-m)g$$

$$Ma_{sys} + ma_{sys} = Mg - mg$$

$$ma_{sys} + mg = Mg - Ma_{sys} = M(g - a_{sys})$$

$$M = \frac{ma_{sys} + mg}{g - a_{sys}}$$

$$= \frac{(1.50\ kg)(9.81\ m/s^2) + (1.50\ kg)(5.88\ m/s^2)}{(9.81\ m/s^2 - 5.88\ m/s^2)}$$

$$= 5.9885\ kg$$

$$\boxed{= 5.99\ kg}$$

b) $F_T = ?$

$$F_{Net,1} = F_{gM} - F_T = Ma_1$$

$$F_T = Mg - Ma_1$$

$$= M(g - a_1) = (5.9885\ kg)(9.81\ m/s^2 - 5.88\ m/s^2)$$

$$F_T = 23.535\ N$$

$$\boxed{= 23.5\ N}$$