More Keplers 3d Law Practice
Physics 621

1. There is a region beyond Neptune known as the Kuiper Belt, which includes lots of small objects (remnants of when the Solar System was first formed about 4.5 billion years ago). The region includes two additional dwarf planets: Haumea and Makemake. Their orbital periods are 283 years and 306 years, respectively. What are their mean orbital radius in both AU and km? **[43.1 AU and 45.4 AU, 6.45x10^9 km and 6.79x10^9 km]**

2. The star HD 4203 has a planet orbiting at a distance of 2.07 AU. The planet takes 432 days to orbit once. What is the Kepler constant for HD 4203 and how many times bigger is it than our Sun's Kepler constant. **[6.34 AU^3/y^2, 6.34 times bigger]**

3. In astronomy, we like to compare everything we see far away in reference to our solar system. Astronomers define a Solar Mass as being Kepler's constant when distance is in AU and period is in years. In this way, our Sun has a Solar Mass of 1 [AU^3/T^2].

   Suppose a planet takes 4.9 Earth years to orbit around a star of mass 2.5 solar masses. How far is the planet from its "Sun"? **[3.9 AU]**

4. Suppose a planet orbits a star of mass 0.80 Solar masses. If the orbit has a mean orbital radius of 0.80 AU how long does it take the planet to make one complete orbit? **[0.8 y]**

5. The moon Io orbits Jupiter in 1.77 days and is 4.2 x 10^8 m from the planet. The other moon, Europa, orbits at a distance of 6.7 x 10^8 m from the planet. How many days does it take Europa to orbit once? **[3.6 days]**

6. Our moon is 3.84 x 10^8 m from the Earth and it takes about 27.3 days to orbit once.
   a. What is the distance in AU? **[0.00256 AU]**
   b. What is the orbital period in years? **[0.0747 y]**
   c. What is the mass of the Earth in solar masses? **[3.03x10^{-6} Solar Masses]**
   d. If 1 solar mass 2.00 x 10^{30} kg, what is the mass of the Earth in kg? **[6.06x10^{24} kg]**

More Keplers 3rd Law

#1)

$T_H = 283\ y$

$T_M = 306\ y$

$T_E = 1.000\ y$

$r_E = 1.000\ AU$

$$\frac{r_H^3}{T_H^2} = \frac{r_E^3}{T_E^2} = 1\ AU^3/y^2$$

$$\therefore\ r_H = \sqrt[3]{T_H^2}$$

$$r_H = \sqrt[3]{(283y)^2} = 43.104\ AU = \boxed{43.1\ AU}$$

$$43.1\ AU \times \frac{1.4957 \times 10^8\ Km}{1\ AU} = 6.44716 \times 10^9\ Km = \boxed{6.45 \times 10^9\ Km}$$

$$r_M = \sqrt[3]{(T_M)^2} = \sqrt[3]{(306y)^2} = 45.4096\ AU = \boxed{45.4\ AU}$$

$$45.4\ AU \times \frac{1.4957 \times 10^8\ km}{1\ AU} = 6.7919 \times 10^9\ km = \boxed{6.79 \times 10^9\ km}$$

#2)

$r_p = 2.07\ AU$

$T_p = 432\ days$

$K_p = ?$

$$T_p = 432\ day \times \frac{1\ year}{365.25\ days} = 1.18275\ y$$

$$K_p = \frac{r_p^3}{T_p^2} = \frac{(2.07\ AU)^3}{(1.18275y)^2} = 6.3405\ AU^3/y^2$$

$$K_p = \boxed{6.34\ AU^3/y^2} \qquad K_{sun} = 1\ AU^3/y^2$$

$$\therefore\ \boxed{K_p = 6.34\ K_{sun}}$$

#3)

$T_p = 4.9\ y$

$K_x = 2.5\ AU^3/y^2$

$r_p = ?$

$$\frac{r_p^3}{T_p^2} = K_x$$

$$r_p = \sqrt[3]{K_x T_p^2}$$

$$= \sqrt[3]{(2.5\ AU^3/y^2)(4.9\ y)^2} = 3.915411\ AU = \boxed{3.9\ AU}$$

#4) $K_x = 0.8\ AU^3/y^2$ $\qquad K_x = \frac{r_p^3}{T_p^2}$

$r_p = 0.8\ AU$

$T_p = ?$

$T_p^2 = r_p^3 / K_x \rightarrow T_p = \sqrt{\frac{r_p^3}{K_x}}$

$T_p = \sqrt{\frac{(0.8 AU)^3}{(0.8 AU^3/y^2)}} = \boxed{0.8\ y}$

#5) $T_I = 1.77\ d$ $\qquad \frac{T_E^2}{r_E^3} = \frac{T_I^2}{r_I^3}$

$r_I = 4.2 \times 10^8\ m$

$T_E = ?$

$r_E = 6.7 \times 10^8\ m$ $\qquad T_E = \sqrt{\frac{T_I^2\ r_E^3}{(r_I)^3}}$

$T_E = \sqrt{\frac{(1.77d)^2 (6.7 \times 10^8 m)^3}{(4.2 \times 10^8 m)^3}} = 3.56625\ \text{days}$

$= \boxed{3.6\ \text{days}}$

#6) $r_m = 3.84 \times 10^8\ m$ $\qquad 1\ AU^3/y^2 \equiv 1$ Solar Mass

$T_M = 27.3$ days

a) $r_m = 3.84 \times 10^8\ m \times \frac{1\ AU}{1.4957 \times 10^{11}\ m} = 0.002567 = \boxed{0.00257\ AU}$

b) $T_M = 27.3\ \text{days} \times \frac{1\ y}{365.25\ \text{day}} = 0.07474\ y = \boxed{0.0747\ y}$

c) $K_{Earth} = \frac{r_M^3}{T_M^2} = \frac{(0.002567\ AU)^3}{(0.07474 y)^2} = 3.029112 \times 10^{-6}\ AU^3/y^2$

$= 3.03 \times 10^{-6}\ AU^3/y^2$

$= \boxed{3.03 \times 10^{-6}\ \text{Solar Mass}}$

d) $M_E = 3.029112 \times 10^{-12}$ Solar Mass $\times \frac{2.00 \times 10^{30}\ Kg}{1\ \text{Solar Mass}} = \boxed{6.06 \times 10^{24}\ Kg}$