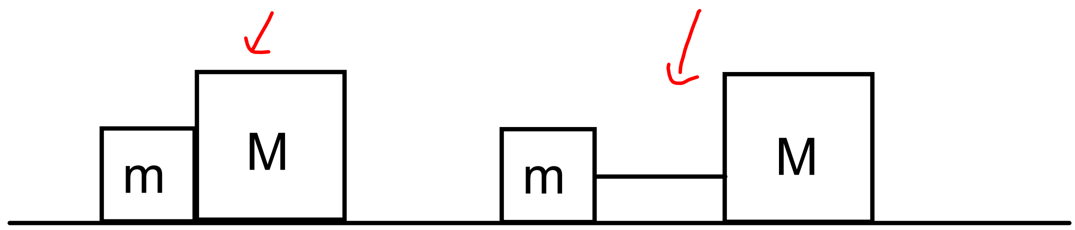
Connected Systems

- A system is a collection of any number (including 1) of objects
- Objects can be connected through various means
  > E.g. physically touching or connected by a rope



- Even within a system, you can look at another subsystem
  > System within a system (Systemption?)
- The system can be modelled as 1 object of the total mass

$$m_{sys} = m_1 + m_2 + \cdots$$

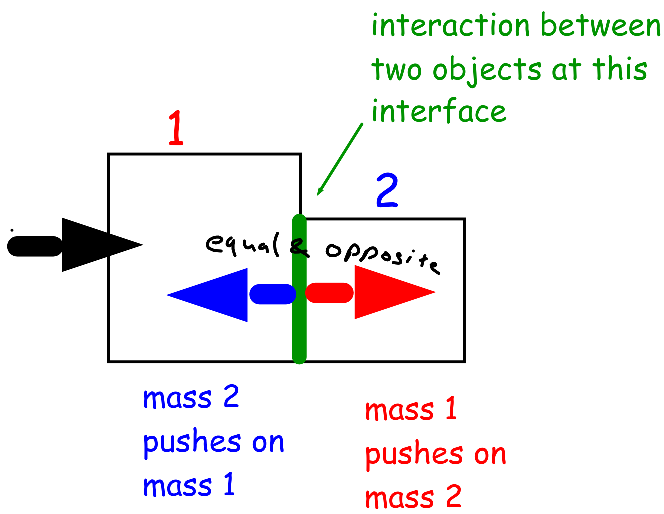
- All objects in a system move with the same acceleration

$$F_{net,sys} = m_{sys} a_{sys}$$ $a_{sys} = a_1 = a_2 = \cdots$

- Forces can act on the objects within the system and can be classified in two ways
- Internal Forces
  > Originate from within the system by other parts of the system
  > Do not affect motion how it's connected
  > E.g. Tension in rope connecting objects
- External Forces
  > Originate from outside the system
  > Affects the motion
  > E.g. Gravity, Normal, and Friction

- Newton's 3rd Law vital in these situations
  > At every connection (interface) between objects A and B, there will be a pair of equal and opposite forces

$$\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A}$$



system: 2 boxes touching

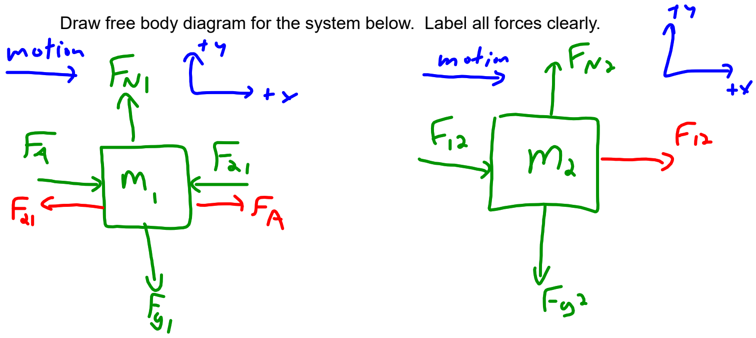
Forces Exerted on Box 1

Symbol	Word Description	Classification
F_{g1}	Gravity pulls box 1 down	Ext
F_{N1}	Ground pushing up on box 1	Ext
F_A	Hand pushing box 1 right	Ext
F_{21}	Box 2 pushes on Box 1 to left	Int

Forces Exerted on Box 2

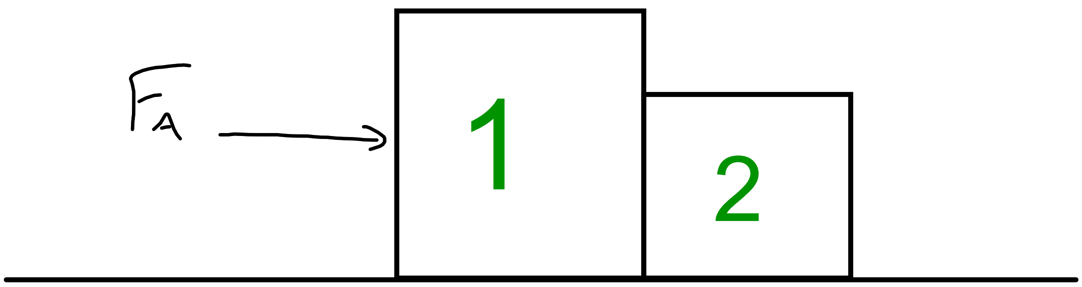
Symbol	Word Description	Classification
F_{g2}	Gravity pulls box 2 down	Ext
F_{N2}	Ground pushes box 2 up	Ext
F_{12}	Box 1 pushes on box 2 to the right	Int

Draw free body diagram for the system below. Label all forces clearly.



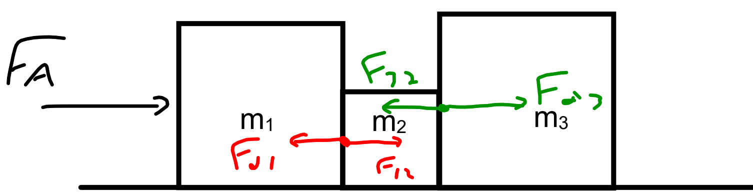
Applying Newton's Laws to Connected Objects

Ex 1: A horizontal force of 10.N is applied eastward to M_1. M_1 and M_2 are in contact with each other supported by a frictionless surface.
M_1 is 2.0 kg. M_2 is 1.0 kg.



a) What is the horizontal acceleration of the system?
b) What force does M_1 exert on M_2?
c) What force does M_2 exert on M_1?

Ex. 2: Masses one, two, and three are 2.0 kg, 1.0 kg, and 3.0 kg, respectively. They are in contact with each other throughout the motion.



frictionless surface

A student pushes mass 1 with a force of 6.0 N [E].

a) What is the acceleration of the system of masses? $1.0\ m/s^2$ [E]

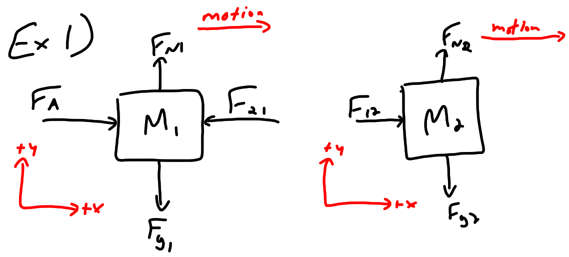
b) What is the magnitude of Net Force on each mass? $F_{net,1} = 2.0\ N$, $F_{net,2} = 1.0\ N$, $F_{net,3} = 3.0\ N$

c) What force does mass 1 apply to mass 2?
$F_{12} = 4.0\ N$ [E]

d) What force does mass 2 apply to mass 3?
$F_{23} = 3.0\ N$ [E]

e) What relationship does this system illustrate with respect to Newtons 2nd law.

Ex 1)



$M_1 = 2.0\,kg$ $\quad \vec{F}_A = 10.\,N\ [E]$

$M_2 = 1.0\,kg$ $\quad \vec{a}_{y,sys} = 0\ m/s^2$

$M_{sys} = M_1 + M_2 = 3.0\,kg$ $\quad \vec{g} = 9.81\ m/s^2\ [down]$

$\vec{a}_{sys} = ?$

$$|\vec{F}_{12}| = |-\vec{F}_{21}|$$

a) $F_{Net,sys} = \Sigma F = F_A + \cancel{F_{12}} - \cancel{F_{21}} = M_{sys}a_{sys}$

$\therefore\ F_A = M_{sys}\,a_{sys}$

$$a_{sys} = \frac{F_A}{M_{sys}} = \frac{10.\,N}{3.0\,kg} = 3.3\ m/s^2$$

$\vec{a}_{sys} = 3.3\ m/s^2\ [E]$ ← both have same accel.

b) $\vec{F}_{12} = ?$

$F_{Net,2} = F_{12} = M_2 a_2$

$F_{12} = (1.0\,kg)(3.333\ m/s^2) = 3.3\,N$

$\vec{F}_{12} = 3.3\,N\ [E]$

c) $\vec{F}_{21} = ?$

Easy way → 3rd Law → $\vec{F}_{21} = 3.3\,N\ [W]$

Robust way → 1st principles

$F_{Net,1} = F_A - F_{21} = M_1 a_1$

$F_{21} = F_A - M_1 a_1$

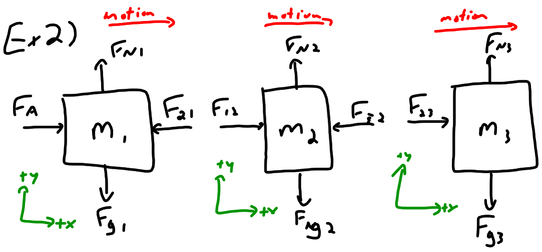
$= (10.\,N) - (2.0\,kg)(3.333\ m/s^2)$

$= 10.\,N - 6.6667\,N$

$= 3.333\,N$

$\vec{F}_{21} = 3.3\,N\ [W]$

Ex2)



$m_1 = 2.0\ kg$ $\qquad \vec{a}_{y_{sys}} = 0\ m/s^2$

$m_2 = 1.0\ kg$ $\qquad \vec{g} = 9.81\ m/s^2$ [down]

$m_3 = 3.0\ kg$ $\qquad \vec{F}_A = 6.0\ N$ [E]

$m_{sys} = m_1 + m_2 + m_3 = 6.0\ kg$ $\qquad \vec{a}_{sys} = ?$

a) $F_{Net,sys} = F_A + \cancel{F_{12}} - \cancel{F_{21}} + \cancel{F_{23}} - \cancel{F_{32}} = m_{sys}a_{sys}$

$$F_A = m_{sys}a_{sys}$$

$$a_{sys} = \frac{F_A}{m_{sys}} = \frac{6.0\ N}{6.0\ kg} = 1.0\ m/s^2$$

$$\boxed{\vec{a}_{sys} = 1.0\ m/s^2\ [E]}$$

b) $F_{Net,1} = m_1a_1 = (2.0\ kg)(1.0\ m/s^2) = \boxed{2.0\ N}$

$F_{Net,2} = m_2a_2 = (1.0\ kg)(1.0\ m/s^2) = \boxed{1.0\ N}$

$F_{Net,3} = m_3a_3 = (3.0\ kg)(1.0\ m/s^2) = \boxed{3.0\ N}$

c) $\vec{F}_{12} = ?$ $\qquad F_{12} = F_{21}$

$$F_{Net,1} = F_A - F_{21} = m_1a_1$$

$$F_{21} = F_A - m_1a_1 = 6.0\ N - 2.0\ N = 4.0\ N$$

$$\therefore \boxed{\vec{F}_{12} = 4.0\ N\ [E]}$$

d) $\vec{F}_{23} = ?$

$$F_{Net,3} = F_{23} = m_3a_3 = 3.0\ N$$

OR (b/c $F_{23} = F_{32}$)

$$F_{Net,2} = F_{12} - F_{32} = m_2a_2$$

$$F_{32} = F_{12} - m_2a_2 = 4.0\ N - 1.0\ N = 3.0\ N$$

$$\vec{F}_{32} = 3.0\ N\ [W]$$

$$\boxed{\therefore \vec{F}_{23} = 3.0\ N\ [E]}$$

Ex 2 e) Newton's second law states that the net force acting on an object is the product of the objects's mass and the object's acceleration. All of the boxes in the system accelerate at the same rate. Since the boxes have different masses, the net force acting on each box will be different. The net force required to accelerate a box or system of boxes is directly proportional to the mass being accelerated. The net force to accelerate the entire system will be the biggest force because it must accelerate all of the masses in the system.