Acceleration Due to Gravity

- On Earth, g=9.81 m/s^2
  > But why?
  > And what is it on other Planets / Moons?

- Fg = mg is just an approximation used near the surface of the Earth. It depends on both the mass and the radius of the Earth.
- Obviously being on a different planet (or a moon) then the acceleration felt would be different

Table 4.4 Free-Fall Accelerations Due to Gravity in the Solar System page 133

Location	Acceleration due to gravity (m/s^2)
Earth	9.81
Moon	1.64
Mars	3.72
Jupiter	25.9

- In a sense, the pull of gravity is then related to the density of the planet

- Even on/near the surface of a planet, Newtons Law of Gravity is still true, so we equate the two equations for the

$$\boxed{F_g = mg} = \frac{GmM_E}{R_E^2} = m\frac{GM_E}{R_E^2}$$

$$\therefore g_E = \frac{GM_E}{R_E^2}$$

surface

$$\boxed{\therefore g_{Planet} = \frac{GM_{Planet}}{R_{Planet}^2}}$$

Acceleration Due to Gravity

- What elevation is it when $r = R_E$?
  > Sea Level
- What if you are at an elevation/height above sea level?
  > $r = R_E + h$

$$g_{new} = \frac{GM_E}{(R_E + h)^2} \neq \frac{GM_E}{R_E^2 + h^2}$$

- How does this new g value compare to the g value at the surface?

$$g_{new} = \frac{GM_E}{(R_E + h)^2} \Longrightarrow GM_E = g_{new}(R_E + h)^2$$

$$g_{surface} = \frac{GM_E}{R_E^2} \Longrightarrow GM_E = g_{surface}R_E^2$$

$$\therefore \boxed{g_{new} = \left(\frac{R_E^2}{(R_E + h)^2}\right) g_{surface}}$$

9.81

- g changes depending on not only the elevation but even the latitude of your position on Earth. Also changes because the Earth's mass distribution is not constant not is its shape perfectly spherical

Table 4.3 Free-Fall Accelerations Due to Gravity on Earth

Location	Acceleration due to gravity (m/s^2)	Altitude (m)	Distance from Earth's centre (km)
North Pole	9.8322	0 (sea level)	6357
equator	9.7805	0 (sea level)	6378
Mt. Everest (peak)	9.7647	8850	6387
Mariana Ocean Trench* (bottom)	9.8331	11 034 (below sea level)	6367
International Space Station*	9.0795	250 000	6628

Acceleration From Gravity Examples

Ex 1: Calculate the acceleration due to gravity on the surface of Venus.

Ex 2: A woman is flying in a plane 9.65 km above the surface of the Earth. She stands on a scale and it reads 508 N.

a) What is the acceleration due to gravity she is experiencing?
b) What is her mass?

Ex 3: Assuming you could stand on the Sun, what would be the acceleration due to gravity on the surface? How far would you have to be off the surface to experience a g of $9.81 m/s^2$?

Ex 1

$M_v = 4.88 \times 10^{24}\ kg$

$R_v = 6.073 \times 10^{6}\ m$

$$g_v = \frac{GM_v}{R_v^2}$$

$$= \frac{(6.673 \times 10^{-11}\ Nm^2/kg^2)(4.88 \times 10^{24}\ kg)}{(6.073 \times 10^{6}\ m)^2}$$

$$= 8.82946\ m/s^2$$

$$= \boxed{8.83\ m/s^2}$$

Ex 2)

$h = 9.65\,km$

$F_g = 508\,N$

$R_E = 6.38 \times 10^6\,m$

$M_E = 5.98 \times 10^{24}\,kg$

$g = 9.81\,m/s^2$

a) $g_{new} = \frac{R_E^2}{(R_E + h)^2} g = \left(\frac{R_E}{R_E + h}\right)^2 g$

$= \left(\frac{6.38 \times 10^6\,m}{6.38 \times 10^6\,m + 9.65 \times 10^3\,m}\right)^2 (9.81\,m/s^2)$

$= 9.78039\,m/s^2 = \boxed{9.78\,m/s^2}$

b) $F_g = mg_{new}$

$m = \frac{F_g}{g_{new}} = \frac{508\,N}{9.78039\,m/s^2} = 51.94\,kg$

$\boxed{= 51.9\,kg}$

Ex 3

$M_s = 1.99 \times 10^{30}\ kg$ ~ 1 million times bigger

$R_s = 6.96 \times 10^{8}\ m$ ~(100 times bigger)^2

$$g_s = \frac{GM_s}{R_s^2}$$

$$= \frac{(6.673 \times 10^{-11}\ Nm^2/kg^2)(1.99 \times 10^{30}\ kg)}{(6.96 \times 10^{8}\ m)^2}$$

$$= 274.13\ m/s^2$$

$$= \boxed{274\ m/s^2} \div 9.81 = 27\times$$

$h = ?$

$g = 9.81\ m/s^2$

$$g = \frac{GM_s}{(R_s + h)^2}$$

$$\sqrt{(R_s+h)^2} = \sqrt{\frac{GM_s}{g}}$$

$$h = \sqrt{\frac{GM_s}{g}} - R_s$$

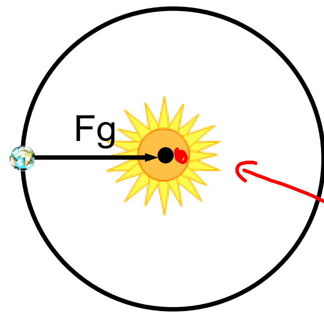
$$= \sqrt{\frac{(6.673 \times 10^{-11}\ Nm^2/kg^2)(1.99 \times 10^{30}\ kg)}{9.81\ m/s^2}} - 6.96 \times 10^{8}\ m$$

$$= \boxed{2.98 \times 10^{9}\ m} \approx 0.02\ AU$$

Orbital speeds, acceleration, and periods:

You can approximate an orbit to be close to that of a circle allowing you to use the equations derived for uniform circular motion.

orbiting body



focus, cause of rotation

Moon is a natural satellite of Earth.
Planets are natural satellites of the Sun.

Fc = ma_c Fc = $\frac{mv^2}{r}$ Fc = $\frac{m4\pi^2 r}{T^2}$ Fc= $m4\pi^2 r f^2$

orbiting body mass

Mean orbital radius

mean orbital Period

The centripetal force acting on an orbiting object is supplied by the force of gravity pulling the object to the "focus" mass.

So, many problems can be answered by letting

orbit body mass

$$F_c = F_G.$$

$$F_g = G\frac{mM}{r^2}$$

focus Mass

mean orbital radius

Connection between Kepler and Newton

- Keplers Laws are about the periodic (circular(ish)) nature of the motion
- Newtons Gravity describes the force that causes this motion
- What is the connection?

$$F_c=F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$\frac{\cancel{m}\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{G\cancel{m}M}{r^2}$$

$$\frac{4\pi^2 r^{\cancel{2}}}{T^2\,\cancel{r}} = \frac{GM}{r^2}$$

$$\boxed{K = \frac{r^3}{T^2} = \frac{GM}{4\pi^2}}$$

$$K \propto M$$

- This is why the Kepler constant is referred to as Solar Mass since it depends almost entirely on the mass of the object (Sun for us) that causes the rotational orbit

Calculating Kepler's Constant

- Calculate K for the Sun using
  - > $G = 6.673 \times 10^{-11}\ Nm^2/kg^2$
  - > $M_S = 1.988 \times 10^{30}$ kg
- Do unit analysis to see that units are m^3/s^2
- Convert it to AU^3/y^2 using
  - > 1 AU = 1.4957×10^{11} m
  - > 1 y = 3.15576×10^7 s

$$K = \frac{GM_s}{4\pi^2} = \frac{(6.673 \times 10^{-11}\ Nm^2/kg^2)(1.988 \times 10^{30}\ kg)}{4\pi^2}$$

$$= 3.360298 \times 10^{18}\ m^3/s^2$$

$$\left[\frac{Nm^2}{\cancel{kg^2}}\right] \times [\cancel{kg}] = \left[\frac{\left(\frac{\cancel{kg}\,m}{s^2}\right)m^2}{\cancel{kg}}\right] = \left[\frac{m^3}{s^2}\right]$$

$$3.360298 \times 10^{18}\ \frac{\cancel{m^3}}{\cancel{s^2}} \times \left(\frac{1\ AU}{1.4957 \times 10^{11}\ \cancel{m}}\right)^3 \times \left(\frac{3.15576 \times 10^7\ \cancel{s}}{1\ y}\right)^2$$

$$= 1.00012\ AU^3/y^2$$