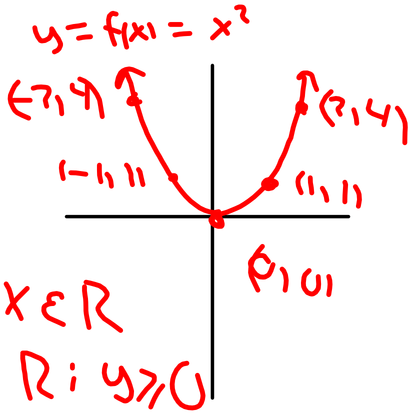
Section 1.6

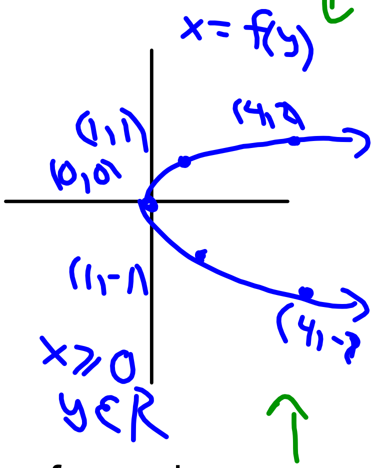
Inverse Trig Functions

Reflects About $y=x$

Review

If y= f(x) is represented by y=x^2 find x=f(y)





State the domain and range for each.

Inverse Relation

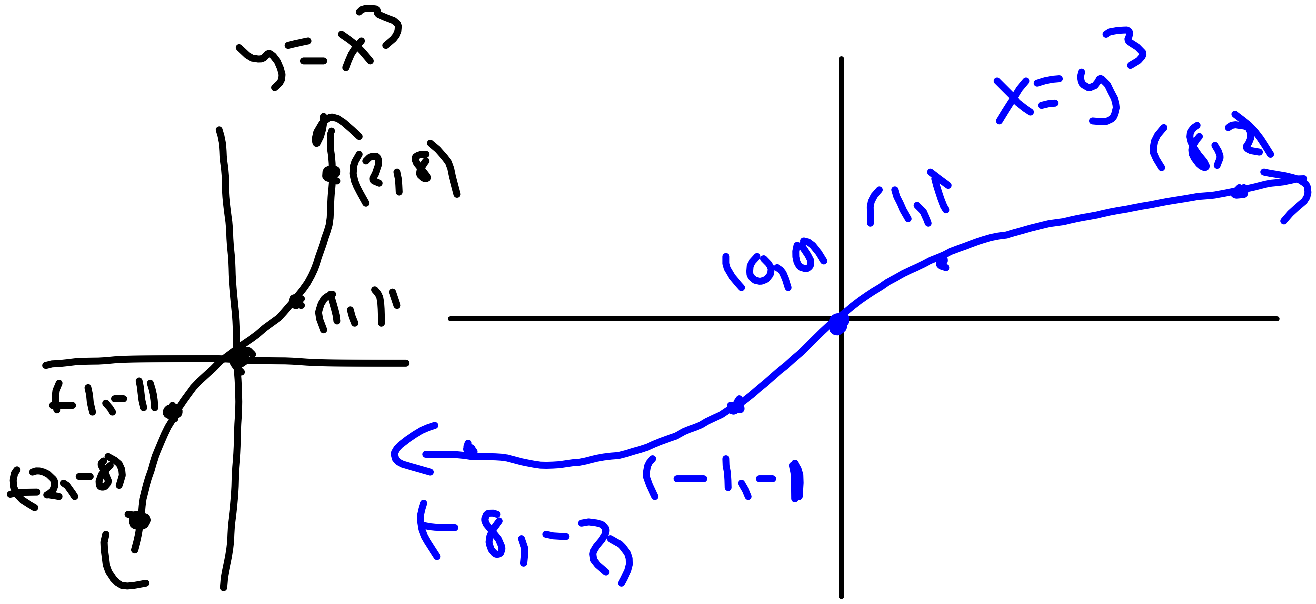
Does $f^{-1}(x)$ exist?

No. $y=f(x)$ does not pass the horizontal line test (or $x=f(y)$ does not pass the vertical line test).

So it is an inverse relation <u>not</u> an inverse function.

$y=f(x)$	$y=x^2$
$x=f(y)$	$x=y^2$

Find and sketch the inverse function of $y=x^3$.



This is an inverse function ($f^{-1}(x)$).

$$x = y^3$$

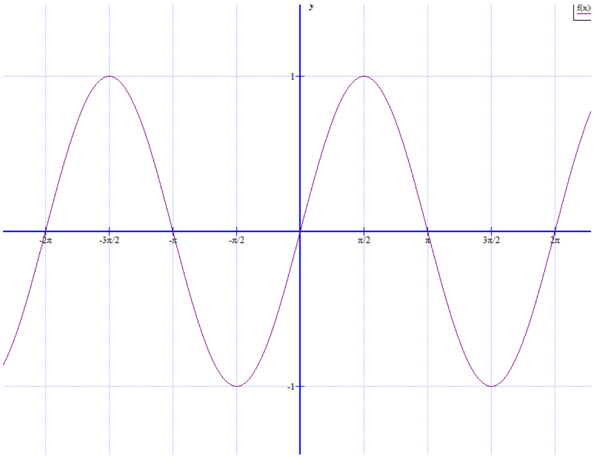
$$\sqrt[3]{x} = \sqrt[3]{y^3}$$

$$y = x^{1/3}$$

$$f^{-1}(x) = x^{1/3}$$

In this section we will learn about **inverse trigonometry functions**.

x	y
$-\frac{\pi}{2}$	-1
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

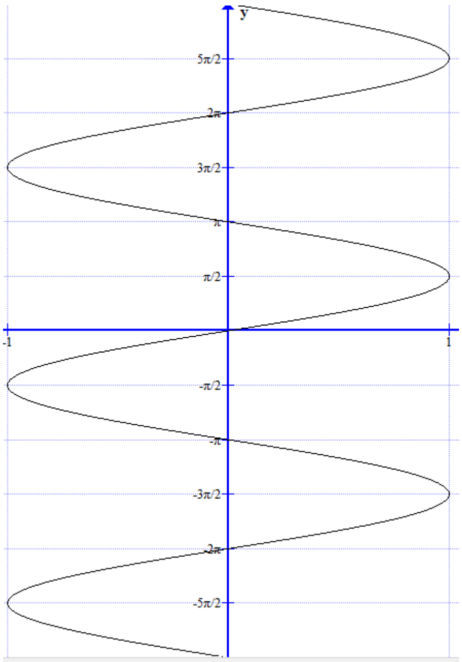


$y=\sin(x)$

$x \in \mathbb{R}$

$-1 \leq y \leq 1$

x	y
-1	$-\pi/2$
0	0
1	$\pi/2$
0	π
-1	$\frac{3\pi}{2}$
0	2π



$x=\sin(y)$

$-1 \leq x \leq 1$

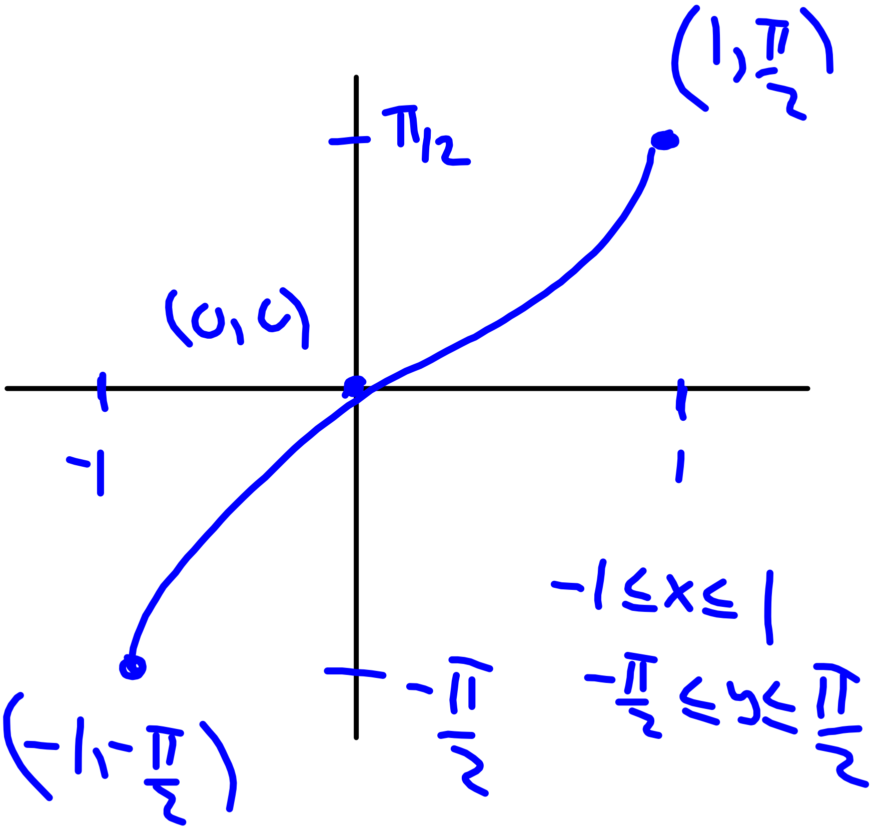
$y \in \mathbb{R}$

Is this an inverse function?

No

Since we want inverse functions we will have to restrict the domain and range.

x	y
-1	$-\pi/2$
0	0
1	$\pi/2$



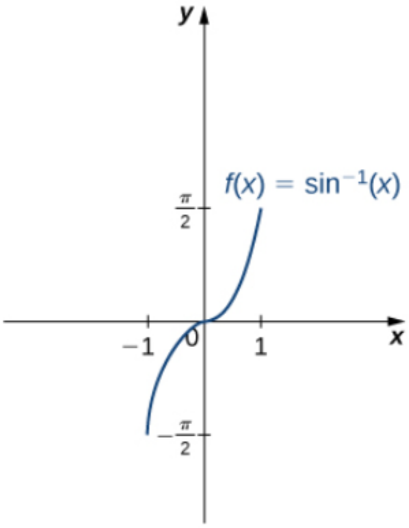
To indicate this is an inverse function $f^{-1}(x)$ we use special notation.

$y=\sin^{-1}(x)=\arcsin(x)$

WARNING!!

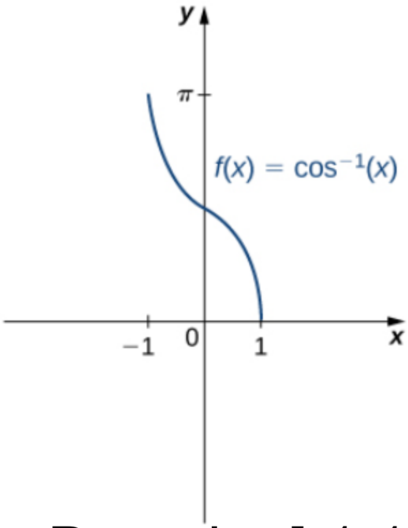
$y=\sin^{-1}(x)$ DOES NOT MEAN $y=\frac{1}{\sin x}=\csc x$

$$y=(\sin x)^{-1}=\frac{1}{\sin x}=\csc x$$



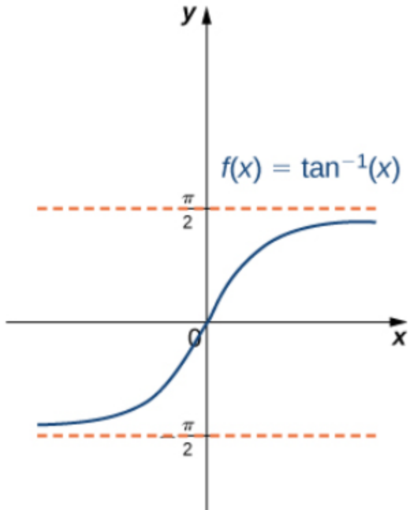
Domain: [-1,1]

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



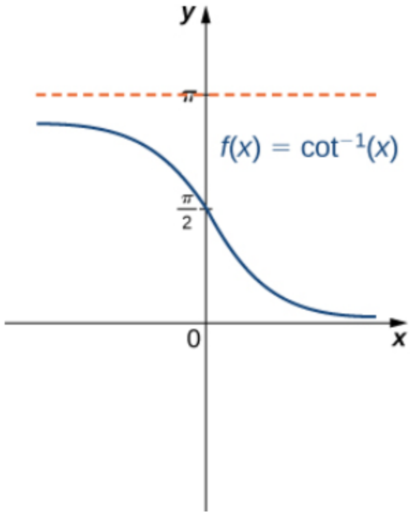
Domain: [-1,1]

Range: $[0, \pi]$



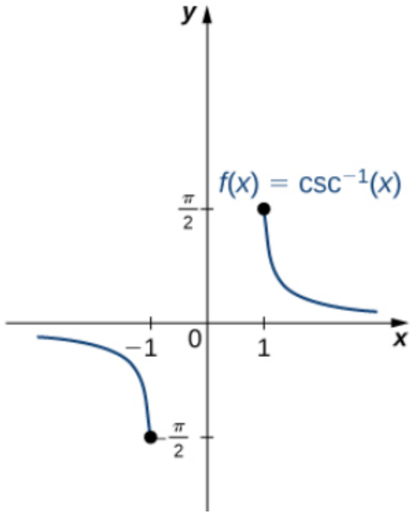
Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Domain: $(-\infty, \infty)$

Range: $(0, \pi)$

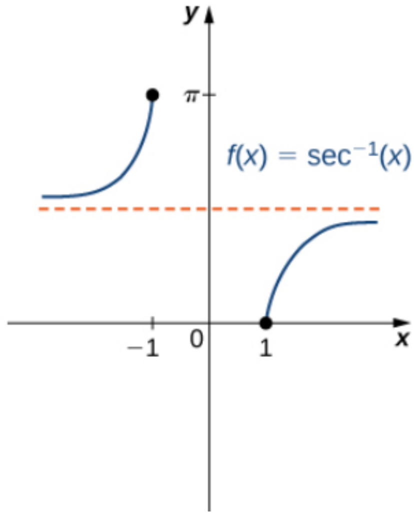


Domain:

$x \leq -1 \text{ or } x \geq 1$

Range:

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y \neq 0$



Domain:

$x \leq -1 \text{ or } x \geq 1$

Range:

$[0, \pi] y \neq \frac{\pi}{2}$

Solve for x on the given interval. Round to 4th decimal if necessary.

a. cos x = 0.4 $[0, \pi)$

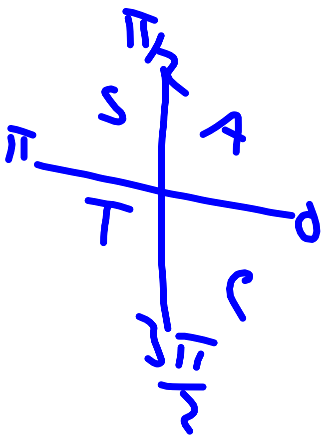
$x = \cos^{-1} 0.4 = 1.1593$

b. $\sin x = \frac{-3}{4}, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x = \sin^{-1} -\frac{3}{4} = -0.8481$

c. $\tan x = 2, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x = \tan^{-1} 2 = 1.1071$



Find the exact value of each expression.

a. $\cos^{-1}(0.5) = \pi/3$

b. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi/4$

c. $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

d. $\cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] = \cos\left[\frac{\pi}{3}\right] = \frac{1}{2}$

e. $\tan\left[\sin^{-1}\left(\frac{2}{5}\right)\right]$

Homework: p51-53

#27 - 41 odd

$x^2+y^2=r^2$

$x^2+y^2=1$ OR $\left(\frac{x}{r}\right)^2+\left(\frac{y}{r}\right)^2=1$

$\sin\theta = \frac{y}{r} = y$

$x^2+y^2=1$

$x^2+\left(\frac{2}{5}\right)^2=1$

$x^2+\frac{4}{25}=1$

$x^2=1-\frac{4}{25}$

$x^2=\frac{25}{25}-\frac{4}{25}=\frac{21}{25}$

$x=\frac{\sqrt{21}}{5}$

$\tan\left(\frac{y}{x}\right)$

$\tan \frac{2/\cancel{5}}{\frac{\sqrt{21}}{\cancel{5}}}$

$\tan \frac{2}{\sqrt{21}}\cdot\frac{\sqrt{21}}{\sqrt{21}}$

$\frac{2\sqrt{21}}{21}$