2.1 Rate of Change and Limits

Use a table of values to state what you believe each limit is. Confirm graphically.

$$\lim_{x\to 0}\frac{\sin(5x)}{x} \quad -\!\!- \quad \frac{\sin(0)}{0} = \frac{0}{0}$$

X	Y
-0.1	4.7942554
-0.01	4.9917917
-0.001	4.9991797
-0.001	4.9999998

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0.1	4.7942554
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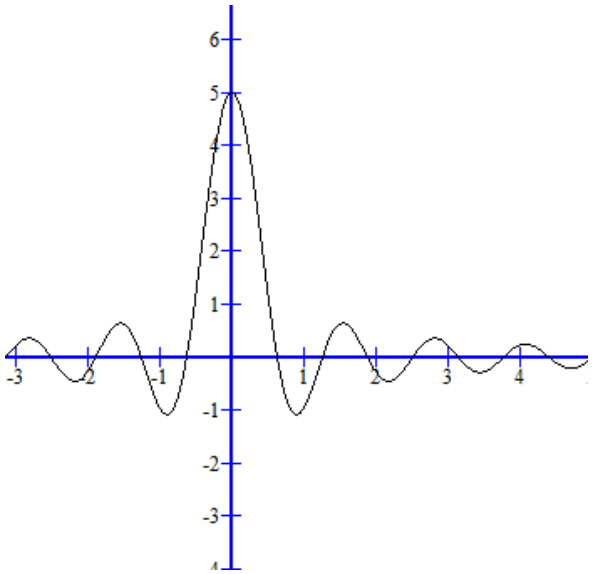


$$\lim_{x\to 0}\frac{\sin(5x)}{x} = \lim_{x\to 0}\frac{\sin(4x+x)}{x}$$

$$\lim_{x\to 0}\frac{\sin 4x\cos x + \cos 4x \sin x}{x}$$

$$\lim_{x\to 0}\frac{\sin 4x\cos x}{x} + \lim_{x\to 0}\frac{\cos 4x \sin x}{x}$$

$$\lim_{x\to 0}\frac{\sin 2(2x)\cos x}{x} + (1)(1)$$

$$\lim_{x\to 0}\frac{2(\sin(2x))\cos(2x)\cos x}{x} + 1$$

$$\lim_{x\to 0}\frac{2(2\sin x\cos x)\cos 2x\cos x}{x} + 1$$

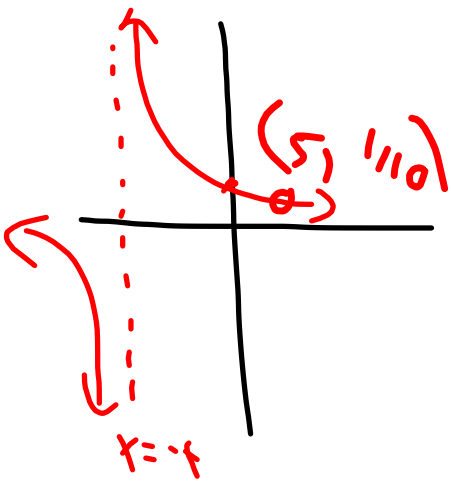
$$(2)(2)(1)(1)(1)(1) + 1$$

$$4 + 1 = 5$$

Evaluate each limit algebraically. Confirm graphically.

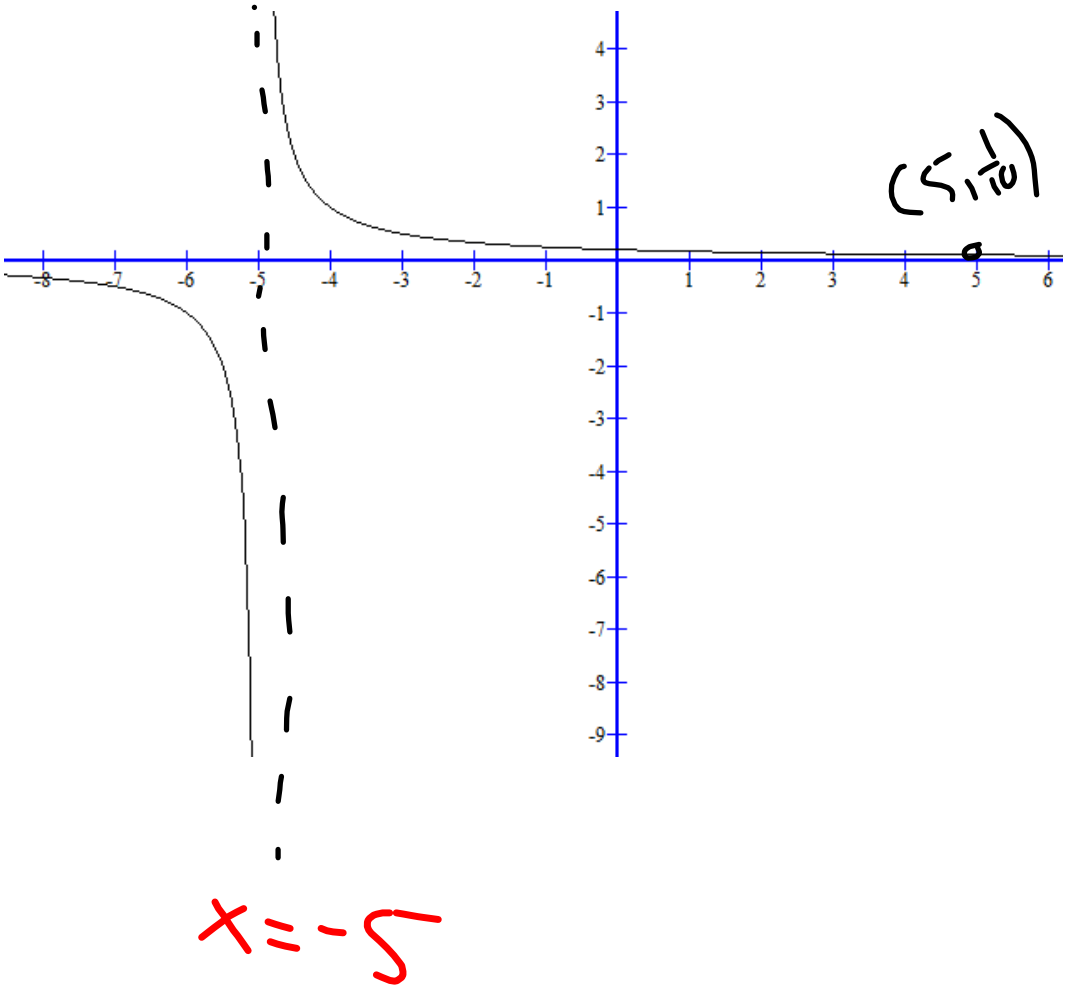
$$\lim_{x\to 5}\frac{x-5}{x^2-25} = \frac{5-5}{5^2-25} = \frac{0}{0}$$

$$\lim_{x\to 5}\frac{x-5}{(x+5)(x-5)} = \lim_{x\to 5}\frac{1}{x+5} = \frac{1}{10}$$



x	y
4.9	0.10010 16
4.99	0.100100 1
4.999	0.10001

x	y
5.1	0.09900
5.01	0.0999
5.001	0.099999


(5, 1/10)
x = -5

$$\lim_{x\to 0}\frac{\sin^3 x+\cos^2 x\sin x}{x} = \lim_{x\to 0}\frac{\sin x(\sin^2 x+\cos^2 x)}{x}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot(\sin^2 x+\cos^2 x)$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot(1) = (1)(1) = 1$$

$$\lim_{x\to 0}\frac{\sin x}{x^3+7x^2+12x}$$

$$\lim_{x\to 0}\frac{\sin x}{x(x^2+7x+12)} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{(x+3)(x+4)}$$

$$= (1)\cdot\frac{1}{(3)(4)} = \frac{1}{12}$$

Homework

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$$\lim_{x\to -14}\frac{(7+x)^2-49}{x+14}$$

$$\lim_{x\to -14}\frac{x^2+14x+49-49}{x+14}$$

$$\lim_{x\to -14}\frac{x^2+14x}{x+14}=\lim_{x\to -14}\frac{x(x+14)}{(x+14)}$$

$$=-14$$

OR

$$\frac{(7+x+7)(7+x-7)}{x+14}=\frac{(x+14)(x)}{(x+14)(1)}$$