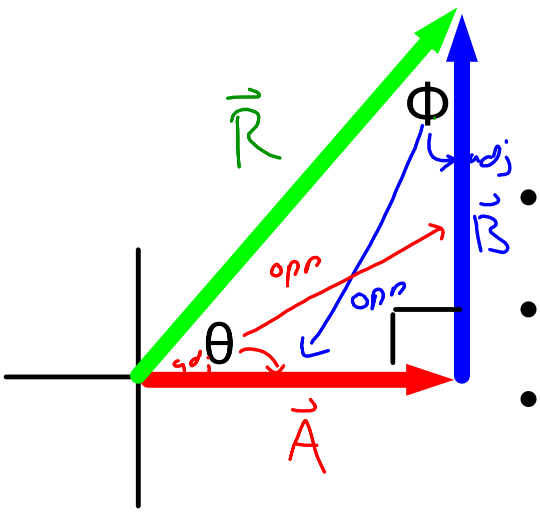
Vector Mathematics

- To begin, let's consider adding 2 perpendicular vectors
- Remember: vector subtraction is just addition with the vector opposite --> A - B = A + (-B)

- Perpendicular vectors add Head-to-Tail creating a right angle triangle



R = A + B

- The resultant is the Hypotenuse
- θ + Φ = 90^0
- $|\vec{R}|$ mean magnitude of vector

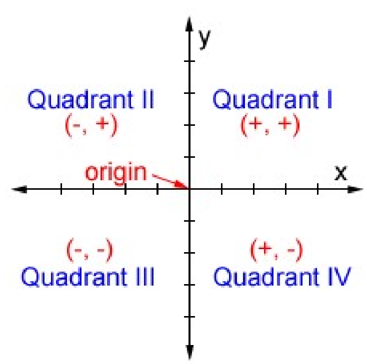
size

- According to the Pythagorean Theorem

$$|\vec{R}|^2 = |\vec{A}|^2 + |\vec{B}|^2$$
$$|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2}$$

- Using trigonometry (SOHCAHTOA)
  > tan = opposite / adjacent

$$tan(\theta) = \frac{|\vec{B}|}{|\vec{A}|} \rightarrow \theta = tan^{-1}\left(\frac{|\vec{B}|}{|\vec{A}|}\right)$$
$$tan(\phi) = \frac{|\vec{A}|}{|\vec{B}|} \rightarrow \phi = tan^{-1}\left(\frac{|\vec{A}|}{|\vec{B}|}\right)$$



Adding Perpendicular Vectors

1. F_1 = 3.95 N [S], F_2 = 6.32 N [W], F_R = ?

2. An abandoned motor boat is drifting at sea. The wind is blowing it north with a force of 5.8 N. The tide is pulling it east with a force of 2.5 N. What is the resultant force exerted on the motor boat?

3. When Ryan started to run he was 5.5 hm east of the school's front door. When he finished the run, he was 8.4 hm south of the school's front door. The run took him 15 minutes. What was Ryan's resultant displacement during the run?

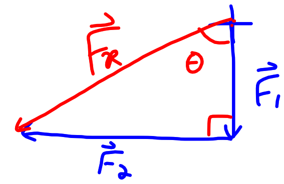
Checklist for answer (full marks):

- givens listed
- sketch of vector addition-labelled
- angle to be calculated shown on sketch
- equations (using vector symbols)
- substitutions into equations, carrying units
- 1 additional intermediate step (so that you can count sig. figs when rules multiplication to addition)
- raw answers
- final answer

#1 $\vec{F}_1 = 3.95\,N\,[S]$

$\vec{F}_2 = 6.32\,N\,[W]$

$\vec{F}_R = ?$



$$|\vec{F}_R|^2 = |\vec{F}_1|^2 + |\vec{F}_2|^2$$

$$|\vec{F}_R| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2}$$

$$= \sqrt{(3.95\,N)^2 + (6.32\,N)^2}$$

$$= \sqrt{15.6025\,N^2 + 39.9424\,N^2}$$

$$= \sqrt{55.5449\,N^2}$$

$$= 7.45285\,N = 7.45\,N$$

$$\theta = \tan^{-1}\left(\frac{|\vec{F}_2|}{|\vec{F}_1|}\right) = \tan^{-1}\left(\frac{6.32\,N}{3.95\,N}\right) = 57.9946^\circ$$

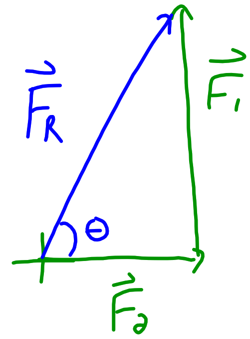
$$\doteq 58.0^\circ$$

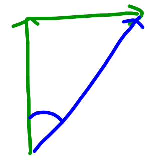
$$\boxed{\vec{F}_R = 7.45\,N\,[S\,58.0^\circ\,W]}$$

#2 $\vec{F}_1 = 5.8N\ [N]$

$\vec{F}_2 = 2.5N\ [E]$

$\vec{F}_R = ?$





$$|\vec{F}_R|^2 = |\vec{F}_1|^2 + |\vec{F}_2|^2$$

$$|\vec{F}_R| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2}$$

$$= \sqrt{(5.8N)^2 + (2.5N)^2}$$

$$= \sqrt{33.64N^2 + 6.25N^2}$$

$$= 6.316N = 6.3N$$

$$\theta = \tan^{-1}\left(\frac{|\vec{F}_1|}{|\vec{F}_2|}\right) = \tan^{-1}\left(\frac{5.8\cancel{N}}{2.5\cancel{N}}\right) = 66.682^\circ$$

$$= 67^\circ$$

$$\boxed{\vec{F}_R = 6.3N\ [E\ 67^\circ\ N]}$$

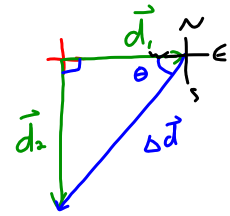
#3 $\vec{d}_1 = 5.5\text{ km [E]}$ $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

$\vec{d}_2 = 8.4\text{ km [S]}$

$\Delta t = 15\text{ min}$

$\vec{v} = ? = \vec{v}_2 - \vec{v}_1$

$= \frac{\Delta\vec{d}}{\Delta t}$



$$|\Delta\vec{d}|^2 = |\vec{d}_1|^2 + |\vec{d}_2|^2$$

$$|\Delta\vec{d}| = \sqrt{|\vec{d}_1|^2 + |\vec{d}_2|^2}$$

$$= \sqrt{(5.5\text{ km})^2 + (8.4\text{ km})^2}$$

$$= \sqrt{30.25\text{ km}^2 + 70.56\text{ km}^2}$$ 100.81 km^2

$$= 10.0404\text{ km} = 10.0\text{ km}$$

$$\theta = \tan^{-1}\left(\frac{|\vec{d}_2|}{|\vec{d}_1|}\right) = \tan^{-1}\left(\frac{8.4\cancel{\text{km}}}{5.5\cancel{\text{km}}}\right) = 56.7847°$$

$$= 57°$$

$\Delta\vec{d} = 10.0\text{ km [W 57° S]}$

$$\Delta t = 15\text{ min} \times \frac{60\text{ s}}{1\text{ min}} = 900\text{ s}$$

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t} = \frac{10.0404\times10^3\text{ m}}{900\text{ s}} = 1.1156\text{ m/s}$$

$$\boxed{\vec{v} = 1.1\text{ m/s [W 57° S]}}$$