Section 4.4

Part #2

Derivative of ln(x)

Review

$$\frac{d}{dx}e^x = e^x$$

$$y = e^u \longrightarrow \frac{dy}{dx} = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x} \longrightarrow \frac{d}{dx}\ln(u) = \frac{1}{u} \cdot \left(\frac{du}{dx}\right)$$

$$y = a^x \longrightarrow \frac{dy}{dx} = a^x \ln(a)$$

$$y = a^u \longrightarrow \frac{dy}{dx} = a^u \ln(a) \cdot \left(\frac{du}{dx}\right)$$

Review: Find the derivatives of the following.

a. $y = e^{x^4 + 5x}$

Let $u = x^4 + 5x$ $\frac{du}{dx} = 4x^3 + 5$

$y = e^u$ $\frac{dy}{du} = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (e^u) \cdot (4x^3 + 5)$$

$$\frac{dy}{dx} = (4x^3 + 5)e^{x^4 + 5x}$$

b. $y = 8^{x^3 + 5x^2}$

$u = x^3 + 5x^2$

$\frac{du}{dx} = 3x^2 + 10x$

$y = 8^u$

$\frac{dy}{du} = 8^u \ln 8$

$\frac{dy}{du} = 8^{x^3+5x^2} \ln 8$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$= \left(8^{x^3+5x^2} \ln 8\right)\left(3x^2 + 10x\right)$$

$$= \ln 8 \left(3x^2 + 10x\right) 8^{x^3+5x^2}$$

Proof of the derivative of y = ln (x).

$$y=\ln(x)$$

$$e^y = x$$

$$\frac{d}{dx}e^y = \frac{d}{dx}x$$

$$e^y \cdot y' = 1$$

$$y' = \frac{1}{e^y}$$

From above $e^y = x$

So $$y' = \frac{1}{e^y} = \frac{1}{x}$$

Therefore $$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

Find the derivatives of the following. and state the domain.

a. $y = \ln(x^6)$ $x^6 > 0$

Let $u = x^6$ $\frac{du}{dx} = 6x^5$ $x > 0$ $x < 0$ OR $x \neq 0$

$y = \ln(u)$ $\frac{dy}{du} = \frac{1}{u}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u} \cdot (6x^5) = \frac{6x^5}{x^6} = \frac{6}{x}$$

OR

$y = \ln(x^6)$

$\frac{d}{dx} y = \frac{d}{dx} 6 \ln x$

$y' = 6 \frac{1}{x} = \frac{6}{x}$

b. $y = (\ln(x))^6 = \ln^6(x)$

Let $u = \ln(x)$ $\frac{du}{dx} = \frac{1}{x}$

$y = u^6$ $\frac{dy}{du} = 6u^5$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (6u^5) \cdot \left(\frac{1}{x}\right) = \frac{6(\ln(x))^5}{x}$$

$= \frac{6 \ln^5 x}{x}, x > 0$

c. $y=\ln\left(\frac{15}{x}\right)$

Let $u=\frac{15}{x}$ $\frac{du}{dx}=\frac{x\cdot(0)-15(1)}{x^2}=-\frac{15}{x^2}$

$y=\ln(u)$ $\frac{dy}{du}=\frac{1}{u}$

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

$\frac{dy}{dx}=\frac{1}{u}\cdot\left(\frac{-15}{x^2}\right)=\frac{1}{\frac{15}{x}}\cdot\left(\frac{-15}{x^2}\right)=\frac{x}{15}\cdot\left(\frac{-15}{x^2}\right)=-\frac{1}{x}$

$\frac{15}{x}>0$

$x>0$

d. $y=\ln(2x+3)$

Let $u=2x+3$ $\frac{du}{dx}=2$

$y=\ln(u)$ $\frac{dy}{du}=\frac{1}{u}$

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

$\frac{dy}{dx}=\frac{1}{u}\cdot(2)=\frac{2}{2x+3}$

$2x+3>0$

$x>-\frac{3}{2}$

Proof: Derivative of $y = \log_a x$

$$y = \log_a x$$

$$a^y = x$$

$$\ln(a^y) = \ln(x)$$

$$y\ln(a) = \ln(x)$$

$$y = \frac{\ln(x)}{\ln(a)}$$

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{\ln(x)}{\ln(a)}\right)$$

$$\frac{dy}{dx} = \frac{\ln(a)\cdot\left(\frac{d}{dx}\ln(x)\right) - \ln(x)\cdot\left(\frac{d}{dx}\ln(a)\right)}{(\ln(a))^2}$$

$$\frac{dy}{dx} = \frac{\ln(a)\cdot\left(\frac{1}{x}\right) - \ln(x)\cdot(0)}{(\ln(a))^2} = \frac{\ln(a)}{x(\ln a)^2} = \frac{1}{x\ln(a)}$$

$$\frac{dy}{dx} = \frac{1}{x\ln(a)}$$

If $y = \log_a x \longrightarrow \frac{dy}{dx} = \frac{1}{x\ln(a)}$

If $y = \log_a u \longrightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u\ln(a)} \cdot \frac{du}{dx}$

$u = f(x)$

Find the derivative of

a. $y = \log_5(x^3)$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$y = \log_5 u$

$\frac{dy}{du} = \frac{1}{u \ln 5} = \frac{1}{x^3 \ln 5}$

$\frac{dy}{dx} = \frac{du}{dx} \frac{dy}{du}$

$3x^2 \cdot \frac{1}{x^3 \ln 5}$

$= \frac{3}{x \ln 5}$

$= \frac{3}{\ln 5\, x}$

b. $y = \log_7(\sin(x) + x^2)$

$u = \sin x + x^2$

$\frac{du}{dx} = \cos x + 2x$

$y = \log_7 u$

$\frac{dy}{du} = \frac{1}{u \ln 7}$

$= \frac{1}{(\sin x + x^2) \ln 7}$

$\frac{dy}{dx} = (\cos x + 2x) \frac{1}{(\sin x + x^2) \ln 7}$

$= \frac{\cos x + 2x}{\ln 7 \cdot (\sin x + x^2)}$

Homework

p.183 #15-27 odd

37-42

Bonus: If time allows I recommend that you do #16-28 even as well before the test.

Even Answers:

16. y'=2ln(x)/x

18. y'=-1/x, x>0

20. y'=lnx

22. y=1/(2xln5), x>0

24. $-\frac{1}{x(\ln 2)(\log_2 x)^2}$ or $-\frac{\ln 2}{x(\ln x)^2}$

26. $\frac{1}{1+x\ln 3}$, $x>-\frac{1}{\ln 3}$

28. ln10

38. y'=1/(x+1), x>-1

40. $\frac{2x}{x^2+1}$ All reals

42. $=\frac{1}{2(x+1)\ln 10}$

Domain of f: $x+1>0$

$x>-1$

Domain of f': $x\neq -1$ and $x>-1$, so $x>-1$.