

Sum & Difference Identities (6.2) p306 day 3

take up quiz 8 take up asst 3

Quiz 8C

1. An observer sees over the water in a straight line. He starts with a maximum altitude of 40 ft, moves to a low of 10 ft, and back up to 40 ft. He repeats this pattern 20 times in 10 seconds. Draw a graph of the observer's pattern and write an equation representing height as a function of time.

2. Find the general solution for $\sin(2\theta - 3) = 0$

3. Graph the following:
a) $y = 2\sin\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

4. The height of a rider on a Ferris Wheel is modeled by $h(t) = 12\sin\left(\frac{\pi}{30}(t - 30)\right) + 15$ where h is the height in meters and t is the time in seconds.

a) What is the maximum height?

b) How long does it take to go around the wheel?

c) Find the height of a rider after 42 seconds.

5. Find the first two times that the height is 7.6 meters.

6. Divide $2x^2 + 3x - 4x + 15$ by $x + 3$

7. Rewrite $y = \frac{1}{2}$ after a horizontal translation 3 to the left.

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#W: sheet #1 - 7

6. $RS = \frac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$

$= \frac{1}{\frac{1 - \sin x}{\cos x}}$

$= \frac{1}{1} \cdot \frac{\cos x}{1 - \sin x}$

$= \frac{\cos x}{1 - \sin x}$

7. $LS: \cos x \sin x + \cos x - \sin x - 1$

$= \frac{1}{\sin x} \sin x + \frac{1}{\sin x} - \sin x - 1$

$= 1 + \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} - 1$

$= \frac{1 - \sin^2 x}{\sin x}$

$= \frac{\cos^2 x}{\sin x}$

$= \cos x \cdot \frac{\cos x}{\sin x}$

$\cos x \cot x$

$-\sin x$

$-\sin^2 x$

$\frac{-\sin x}{\sin x}$

Proof!

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4. $LS = \frac{\cos \theta}{(\cos \theta - 1)} + \frac{\cos \theta}{(\cos \theta + 1)}$

$= \frac{\cos \theta (\cos \theta + 1) + \cos \theta (\cos \theta - 1)}{(\cos \theta - 1)(\cos \theta + 1)}$

$= \frac{\cos^2 \theta + \cos \theta + \cos^2 \theta - \cos \theta}{\cos^2 \theta - 1}$

$= \frac{2\cos^2 \theta}{\cos^2 \theta - 1}$

$= \frac{2}{\frac{\cos^2 \theta}{\sin^2 \theta} - 1}$

$= \frac{2}{\frac{1 - \sin^2 \theta}{\sin^2 \theta} - 1}$

$= \frac{2}{\frac{1 - \sin^2 \theta - \sin^2 \theta}{\sin^2 \theta}}$

$= \frac{2}{\frac{1 - 2\sin^2 \theta}{\sin^2 \theta}}$

$= \frac{2}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{1 - 2\sin^2 \theta}$

$= \frac{2 \sin^2 \theta}{\cos^2 \theta (1 - 2\sin^2 \theta)}$

$LS = RS$

QED

$\cos(x) = 1$

$(x-1)(x+1)$

$x^2 - 1$

$2x^{-1}$

$= \frac{2}{x}$

$\frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta}$

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ex1: Show that $\tan^2 x + 1 = \sec^2 x$ is true for $x = \frac{\pi}{6}$

$LS = \left(\tan \frac{\pi}{6}\right)^2 + 1$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + 1$

$= \frac{1}{3} + 1$

$= \frac{4}{3}$

$RS = \left(\sec \frac{\pi}{6}\right)^2$

$= \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{4}{3}$

$LS = RS$

\therefore might be an identity

testing a value does not prove anything - it only means it might be

P 298# 12a

12.a) $LS = \frac{\cot \frac{\pi}{4}}{\sec \frac{\pi}{4}} + \sin \frac{\pi}{4}$

$= \frac{1}{\frac{2}{\sqrt{2}}} + \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

$RS = \csc \frac{\pi}{4}$

$= \frac{2}{\frac{\sqrt{2}}{2}}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

might

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ex2: Find the exact value of $\sin 15^\circ$

$$\begin{aligned}
 &= \sin(45^\circ - 30^\circ) \\
 &= \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ \\
 &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\
 &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\
 &= \frac{\sqrt{6} - \sqrt{2}}{4}
 \end{aligned}$$

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ex3: Evaluate $\sin 40^\circ \cos 20^\circ + \cos 40^\circ \sin 20^\circ$

$$\begin{aligned}
 &= \sin(40^\circ + 20^\circ) \\
 &= \sin 60^\circ \\
 &= \frac{\sqrt{3}}{2}
 \end{aligned}$$

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Note:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

and...

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

add to unit circle

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ex4: Find an exact value for

$$\begin{aligned}
 &\cos \frac{7\pi}{12} \cos \frac{\pi}{3} + \sin \frac{7\pi}{12} \sin \frac{\pi}{3} \\
 &= \cos\left(\frac{7\pi}{12} - \frac{\pi}{3}\right) \\
 &= \cos\left(\frac{7\pi}{12} - \frac{4\pi}{12}\right) \\
 &= \cos \frac{3\pi}{12} \\
 &= \cos \frac{\pi}{4} \\
 &= \frac{\sqrt{2}}{2}
 \end{aligned}$$

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ex5: Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$

$$\begin{aligned}
 &= \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) \\
 &= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) \\
 &= \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\
 &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\
 &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \\
 &= \frac{\sqrt{2} - \sqrt{6}}{4}
 \end{aligned}$$

1ab

2bd

8a

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#w: p306 #8e, 9, 20ab

8e is also $\frac{4}{\sqrt{6} - \sqrt{2}}$

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To prove the formula for $\cos(\alpha - \beta)$, place the angles α , β , and $\alpha - \beta$ in standard position and let P , Q , and R be, respectively, the points where their terminal sides intersect the unit circle. The coordinates of the points shown in the diagram are

$P(\cos \alpha, \sin \alpha)$, $Q(\cos \beta, \sin \beta)$,
 $R(\cos(\alpha - \beta), \sin(\alpha - \beta))$, and $A(1, 0)$.

Since \widehat{QP} and \widehat{AR} both have central angles of measure $\alpha - \beta$, they are congruent. Therefore chords QP and AR are congruent and $(QP)^2 = (AR)^2$. Now use the distance formula (page 402) and simplify the result using the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$.

$(QP)^2 = (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2$
 $= \cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta + \sin^2 \alpha - 2 \sin \alpha \sin \beta + \sin^2 \beta$
 $= 2 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$
 $(AR)^2 = [\cos(\alpha - \beta) - 1]^2 + [\sin(\alpha - \beta) - 0]^2$
 $= \cos^2(\alpha - \beta) - 2 \cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta)$
 $= 2 - 2 \cos(\alpha - \beta)$

Since $(QP)^2 = (AR)^2$,
 $2 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) = 2 - 2 \cos(\alpha - \beta)$,
 or $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$.

