Panel 1

Composition of Functions (10.3)   *p507*   *day 2*

$$y = \sin(x^2 - 1)$$ $Y = x^2 - 1$

$$y = \sin^2 x + 2\sin x + 1$$ $x^2 + 2x + 1$ $\sin x$

Panel 2

Composition of Functions (10.3)   *p507*   *day 2*

The Turducken

a chicken stuffed into a duck, stuffed into a turkey

aka $f\big(g(h(x))\big)$

Panel 3

Composition of Functions (10.3)   *p507*   *day 2*

ex1: If $f(x) = x^2 + 1$ and $g(x) = 2x + 4$, find the following:

a) f(2) $= 2^2 + 1$
$= 5$

g(-1) $= 2(-1) + 4$
$= 2$

f(a) $= a^2 + 1$

f(x+h) $= (x+h)^2 + 1$

f(g(x)) $= (2x+4)^2 + 1$

f(f(x)) $= (x^2+1)^2 + 1$

f(g(0)) $= (2(0)+4)^2 + 1$
$= 4^2 + 1$
$\boxed{= 17}$

$g(0) = 2(0)+4 = 4$
$f(4)$

f(f(2)) $= ((2^2+1)^2 + 1$
$= 5^2 + 1$
$\boxed{= 26}$

Panel 4

Composition of Functions (10.3) $f(g(x))$ *p507*   *day 2*

ex2: For each pair of functions, state $f \circ g$ and $g \circ f$

a) $f(x) = \sqrt{x+1}$ $g(x) = x^2$

$(f \circ g)(x) = \sqrt{x^2 + 1}$
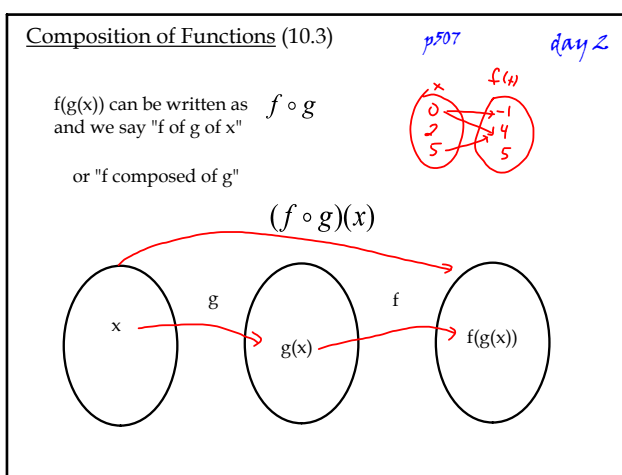 $f(g(x))$

$(g \circ f)(x) = (\sqrt{x+1})^2$
$= x+1$

b) $f(x) = \dfrac{1}{x}$ $g(x) = 3x^2$

$f \circ g = \dfrac{1}{3x^2}$

$g \circ f = 3\left(\dfrac{1}{x}\right)^2 = \dfrac{3}{x^2}$

c) $f(x) = \dfrac{x+1}{2x}$ $g(x) = 3x - 1$

$f \circ g = \dfrac{(3x-1)+1}{2(3x-1)}$

$g \circ f = 3\left(\dfrac{x+1}{2x}\right) - 1$

d) $f(x) = \sin x$ $g(x) = \dfrac{1}{x}$

$f \circ g = \sin\left(\dfrac{1}{x}\right)$

$g \circ f = \dfrac{1}{\sin x}$ $\csc x$

Panel 5

Composition of Functions (10.3)   *p507*   *day 2*

f(g(x)) can be written as $f \circ g$
and we say "f of g of x"

   or "f composed of g"

f(x)
$0 \to -1$
$2 \to 4$
$5 \to 5$

$(f \circ g)(x)$

Panel 6

Composition of Functions (10.3)   *p507*   *day 2*

ex3: If $f(x) = \sqrt{x}$ $g(x) = 2x+3$ $h(x) = \dfrac{1}{x}$

construct

$\dfrac{1}{\sqrt{x}} = h \circ f$ $\sqrt{2x+3} = f \circ g$

$2\sqrt{x} + 3$ $(g \circ f)(x)$ $\dfrac{1}{2x+3} = h \circ g$
$= g \circ f$

Composition of Functions (10.3)   p507   day 2

ex4: State the domain for $f \circ g$ and $g \circ f$ of each pair of functions.

a) $f(x) = \dfrac{1}{x}$

$g(x) = 4x + 3$

$f \circ g = \dfrac{1}{4x+3}$

$D: x \neq -\dfrac{3}{4}$

$g \circ f = 4\left(\dfrac{1}{x}\right) + 3 = \dfrac{4}{x} + 3$

$D: x \neq 0$

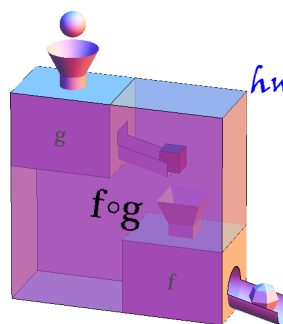b) $f(x) = \sqrt{x}$

$g(x) = x - 5$

$f \circ g = \sqrt{x-5}$

$D: [5, \infty)$

$g \circ f = \sqrt{x} - 5$

$D: [0, \infty)$

Composition of Functions (10.3)   p507   day 2

hw: #4,5,6,7

*p287#14,15,16

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4. If $f(x) = 3x + 4$ and $g(x) = x^2 - 1$, determine each of the following.

a) $f(g(a))$ b) $g(f(a))$
c) $f(g(x))$ d) $g(f(x))$
e) $f(f(x))$ f) $g(g(x))$

a) $3(a^2 - 1) + 4$

$= 3a^2 - 3 + 4$

$\boxed{= 3a^2 + 1}$

b) $g(f(a)) = (3a + 4)^2 - 1$

$= 9a^2 + 24a + 16 - 1$

$\boxed{= 9a^2 + 24a + 15}$

c)   d)

e) $f(f(x)) = 3(3x + 4) + 4$

$= 9x + 12 + 4$

$\boxed{= 9x + 16}$

f) $g(g(x)) = (x^2 - 1)^2 - 1$

$= x^4 - 2x + 1 - 1$

$\boxed{= x^4 - 2x}$

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5. For each pair of functions, $f(x)$ and $g(x)$, determine $f(g(x))$ and $g(f(x))$.

a) $f(x) = x^2 + x$ and $g(x) = x^2 + x$
b) $f(x) = \sqrt{x^2 + 2}$ and $g(x) = x^2$
c) $f(x) = |x|$ and $g(x) = x^2$

a) $(x^2 + x)^2 + x$

$= x^4 + 2x^3 + x^2 + x$

b) $f \circ g = \sqrt{(x^2)^2 + 2}$

$= \sqrt{x^4 + 2}$

$g \circ f = (\sqrt{x^2 + 2})^2$

$= x^2 + 2$

c) $f \circ g = |x^2|$

$= x^2$

$g \circ f = (|x|)^2$

$= x^2$

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6. Given $f(x) = \sqrt{x}$ and $g(x) = x - 1$, sketch the graph of each composite function. Then, determine the domain and range of each composite function.

a) $y = f(g(x))$
b) $y = g(f(x))$

a) $f(g(x)) = \sqrt{x - 1}$

$D: [1, \infty)$
$R: [0, \infty)$

b) $g(f(x)) = (\sqrt{x}) - 1$

$= \sqrt{x} - 1$

$D: [0, \infty)$
$R: [-1, \infty)$

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7. If $h(x) = (f \circ g)(x)$, determine $g(x)$.

a) $h(x) = (2x - 5)^2$ and $f(x) = x^2$
b) $h(x) = (5x + 1)^2 - (5x + 1)$ and $f(x) = x^2 - x$

a) $(2x - 5)^2$ $\therefore g(x) = 2x - 5$

b) $(5x + 1)^2 - (5x + 1)$ $\therefore g(x) = 5x + 1$