Section 3.3

Part #3

Prove the rule for deriving a negative exponent.

$$f(x) = x^n, \quad n \in I^-$$

$$f(x) = x^{-m} = \frac{1}{x^m}, \quad m = -n$$

$-m = n$

$$f'(x) = \frac{x^m \cdot \frac{d}{dx}(1) - (1)\frac{d}{dx}x^m}{(x^m)^2}$$

$$= \frac{x^m(0) - (1)\,m x^{m-1}}{x^{2m}} = \frac{-m x^{m-1}}{x^{2m}}$$

$$= \frac{-m x^{m-1}}{x^{2m}} = -m x^{m-1-2m}$$

$$= -m x^{-m-1}$$

$$= n x^{n-1}$$

Find the derivatives of the following

$$f(x) = x^{-3}$$

$$f'(x) = -3x^{-4}$$

$$f(x)=\frac{\sqrt{x}+5}{\sqrt{x}-5} = \frac{u(x)}{v(x)}$$

$$f'(x) = \frac{(\sqrt{x}-5)\frac{d}{dx}(\sqrt{x}+5)-(\sqrt{x}+5)\frac{d}{dx}(\sqrt{x}-5)}{(\sqrt{x}-5)^2}$$

$$= \frac{(\sqrt{x}-5)\frac{d}{dx}(x^{1/2}+5)-(\sqrt{x}+5)\frac{d}{dx}(x^{\frac{1}{2}}-5)}{(\sqrt{x}-5)^2}$$

$$= \frac{(\sqrt{x}-5)\left(\frac{1}{2}x^{-1/2}\right)-(\sqrt{x}+5)\left(\frac{1}{2}x^{-1/2}\right)}{(\sqrt{x}-5)^2}$$

$$= \frac{\frac{1}{2}x^{-\frac{1}{2}}\left[\sqrt{x}-5-(\sqrt{x}+5)\right]}{(\sqrt{x}-5)^2}$$

$$= \frac{\frac{1}{2}x^{-\frac{1}{2}}\left[-10\right]}{(\sqrt{x}-5)^2}$$

$$= \frac{-5x^{-1/2}}{(\sqrt{x}-5)^2} = \frac{-5}{\sqrt{x}(\sqrt{x}-5)^2}$$

Find the first four derivatives.

$$y = -3x^4 + 8x^3 - 7x^2 + 9x - 12$$

$$y' = -12x^3 + 24x^2 - 14x + 9$$

$$y'' = -36x^2 + 48x - 14$$

$$y''' = -72x + 48$$

$$y'''' = y^{(4)} = \frac{d^4y}{dx^4} = -72$$

Find the equation of the line perpendicular to the tangent when x=3 for the equation:

$$f(x) = 4x^2 - 9$$

Point

$$f(3) = 4(3)^2 - 9 = 4(9) - 9$$

$$= 36 - 9 = 27$$

$$(3, 27)$$

Slope

$$f'(x) = 8x$$

$$f'(3) = 8(3) = 24$$

$$m_\perp = \frac{-1}{24}$$

$$y - y_1 = m(x - x_1)$$

$$y - 27 = -\frac{1}{24}(x - 3)$$

$$y - 27 = \frac{-1}{24}x + \frac{1}{8}$$

$$y = \frac{-1}{24}x + 27\tfrac{1}{8}$$

Find the equations of both lines that are tangent to the curve $y=1+x^3$ and have a slope of 12.

$$y' = \frac{3x^2}{3} = \frac{12}{3}$$

$$x^2 = 4$$

$$x = \pm\sqrt{4} = \pm 2$$

$$x = 2$$

$$y = 1 + 2^3 = 1 + 8 = 9$$

$$(2, 9) \quad m = 12$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = 12(x - 2)$$

$$y = 12x - 24 + 9$$

$$y = 12x - 15$$

$$x = -2$$

$$y = 1 + (-2)^3$$

$$y = 1 - 8 = -7$$

$$(-2, -7) \quad m = 12$$

$$y - y_1 = m(x - x_1)$$

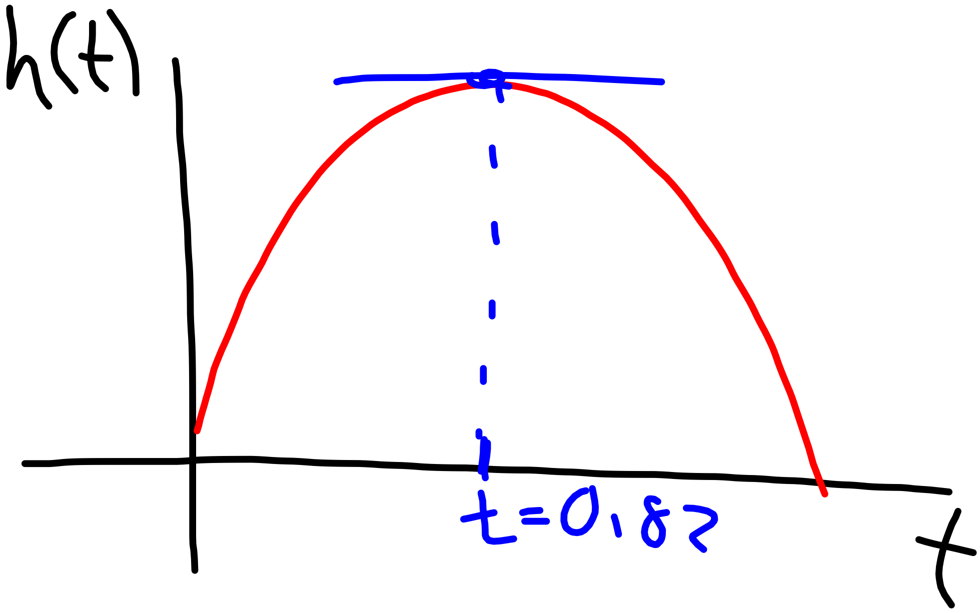
$$y + 7 = 12(x + 2)$$

$$y = 12x + 24 - 7$$

$$y = 12x + 17$$

A projectile's flight is given by the equation below. When does it reach the peak?

$$h(t) = -4.9t^2 + 8t + 15$$



$$h'(t) = -9.8t + 8 = 0$$

$$-9.8t = -8$$

$$t = \frac{-8}{-9.8} = 0.82s$$

p.124-125 #1-47 odd