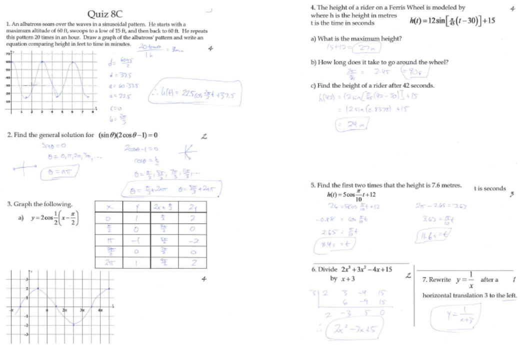
Sum & Difference Identities (6.2) p306 day 3

take up quiz 8

take up asst 3



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Sum & Difference Identities (6.2) p306 day 3

HW: sheet #1 - 7

6. $RS = \dfrac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$

$= \dfrac{1}{\frac{1-\sin x}{\cos x}}$

$= \dfrac{1}{1} \cdot \dfrac{\cos x}{1-\sin x}$

$= \dfrac{\cancel{X}}{\frac{1-\sin x}{\cos x}}$

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7. $LS: \csc x \sin x + \csc x - \sin x - 1$

$= \frac{1}{\cancel{\sin x}} \cdot \cancel{\sin x} + \frac{1}{\sin x} - \sin x - 1$

$= \cancel{1} + \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \cancel{-1}$

$= \dfrac{1-\sin^2 x}{\sin x}$

$= \dfrac{\cos^2 x}{\sin x}$

$= \cos x \cdot \dfrac{\cos x}{\sin x}$

$\cos x \cot x$

$-\frac{\sin x}{1} \cdot \frac{\sin x}{\sin x} = \frac{-\sin^2 x}{\sin x}$

Poof!

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$LS = \dfrac{\csc\theta}{(\csc\theta - 1)} + \dfrac{\csc\theta}{(\csc\theta+1)}$

$= \dfrac{\csc\theta(\csc\theta+1)}{(\csc\theta-1)(\csc\theta+1)} + \dfrac{\csc\theta(\csc\theta-1)}{(\csc\theta-1)(\csc\theta+1)}$

$= \dfrac{\csc^2\theta + \cancel{\csc\theta} + \csc^2\theta - \cancel{\csc\theta}}{\csc^2\theta - 1}$

$= \dfrac{2\csc^2\theta}{\csc^2\theta - 1}$

$= \dfrac{\frac{2}{\sin^2\theta}}{\frac{1}{\sin^2\theta} - 1}$

$= \dfrac{\frac{2}{\sin^2\theta}}{\frac{1-\sin^2\theta}{\sin^2\theta}}$

$\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}$

$= \frac{2}{\cancel{\sin^2\theta}} \cdot \frac{\cancel{\sin^2\theta}}{\cos^2\theta}$

$= \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

LS = RS

QED

$\cos(2x) = 1$

$(x-1)(x+1)$

$x^2 - 1$

$2x^{-1}$

$= \frac{2}{x}$

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Sum & Difference Identities (6.2) p306 day 3

ex1: Show that $\tan^2 x + 1 = \sec^2 x$ is true for $x = \dfrac{\pi}{6}$

$LS = \left(\tan\frac{\pi}{6}\right)^2 + 1$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + 1$

$= \frac{1}{3} + 1$

$= \frac{4}{3}$

$RS: \left(\sec\frac{\pi}{6}\right)^2$

$= \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{4}{3}$

LS = RS

∴ might be an identity

's docha!

testing a value does not prove anything - it only means it might be

P 298# 12a

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12.a) $LS = \dfrac{\cot\frac{\pi}{4}}{\sec\frac{\pi}{4}} + \sin\frac{\pi}{4}$

$= \dfrac{1}{\frac{2}{\sqrt{2}}} + \dfrac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

$RS = \csc\frac{\pi}{4}$

$= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{2}$

might

Sum & Difference Identities (6.2) p306 day 3

ex2: Find the exact value of $\sin 15°$

$= \sin(45° - 30°)$

$= \sin 45° \cos 30° - \sin 30° \cos 45°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\boxed{= \frac{\sqrt{6} - \sqrt{2}}{4}}$

Sum & Difference Identities (6.2) p306 day 3

ex3: Evaluate $\sin 40° \cos 20° + \cos 40° \sin 20°$

$= \sin(40° + 20°)$

$= \sin 60°$

$\boxed{= \frac{\sqrt{3}}{2}}$

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Note:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

and...

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

add to unit circle

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ex4: Find an exact value for

$$\cos\frac{7\pi}{12}\cos\frac{\pi}{3} + \sin\frac{7\pi}{12}\sin\frac{\pi}{3}$$

$= \cos\left(\frac{7\pi}{12} - \frac{\pi}{3}\right)$

$= \cos\left(\frac{7\pi}{12} - \frac{4\pi}{12}\right)$

$= \cos\frac{3\pi}{12}$

$= \cos\frac{\pi}{4}$

$\boxed{= \frac{\sqrt{2}}{2}}$

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ex5: Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

1ab

2bd

8a

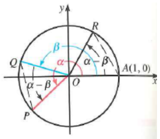
Sum & Difference Identities (6.2) p306 day 3

HW: p306#8e, 9, 20ab

8e is also $\frac{4}{\sqrt{6} - \sqrt{2}}$

identities 10, 11, 12

To prove the formula for $\cos(\alpha - \beta)$, place the angles $\alpha$, $\beta$, and $\alpha - \beta$ in standard position and let $P$, $Q$, and $R$ be, respectively, the points where their terminal sides intersect the unit circle. The coordinates of the points shown in the diagram are




$P(\cos\alpha, \sin\alpha)$, $Q(\cos\beta, \sin\beta)$,
$R(\cos(\alpha - \beta), \sin(\alpha - \beta))$, and $A(1, 0)$.

Since $\widehat{QP}$ and $\widehat{AR}$ both have central angles of measure $\alpha - \beta$, they are congruent. Therefore chords $\overline{QP}$ and $\overline{AR}$ are congruent and $(QP)^2 = (AR)^2$. Now use the distance formula (page 402) and simplify the result using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

$$\begin{aligned}(QP)^2 &= (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2\\ &= \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta\\ &= 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)\\ (AR)^2 &= [\cos(\alpha - \beta) - 1]^2 + [\sin(\alpha - \beta) - 0]^2\\ &= \cos^2(\alpha - \beta) - 2\cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta)\\ &= 2 - 2\cos(\alpha - \beta)\end{aligned}$$

Since $(QP)^2 = (AR)^2$,

$$2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 2 - 2\cos(\alpha - \beta),$$

or

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$