Section 4.1

Part #2

The Chain Rule

Review

$$(f \circ g)(x) = f(g(x))$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

OR

$$y = f(u), u = g(x)$$

$$y' = \frac{dy}{dx} = \frac{dy}{du}\left(\frac{du}{dx}\right)$$

Find the derivative of the following.

$$y = \tan(x^3 + 6x)$$

$u = x^3 + 6x$

$\frac{du}{dx} = 3x^2 + 6$

$y = \tan u$

$\frac{dy}{du} = \sec^2 u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \sec^2 u \cdot (3x^2+6)$

$= (3x^2+6)\sec^2(x^3+6x)$

$$y = \cos^3 x$$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$y = u^3$

$\frac{dy}{du} = 3u^2$

$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

$= 3u^2 (-\sin x)$

$= -3\cos^2 x \sin x$

$$y = \sin^4(5x^7 - 9x^2)$$

$u = 5x^7 - 9x^2$

$\frac{du}{dx} = 35x^6 - 18x$

$y = \sin^4 u$

$v = \sin u$

$\frac{dv}{du} = \cos u$

$y = v^4$

$\frac{dy}{dv} = 4v^3$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= 4v^3 \cdot \cos u \, (35x^6 - 18x)$$

$$= 4\sin^3 u \cdot \cos u \, (35x^6 - 18x)$$

$$= 4\sin^3(5x^7 - 9x^2)\cos(5x^7 - 9x^2) \cdot (35x^6 - 18x)$$

$$= (140x^6 - 72x)\sin^3(5x^7 - 9x^2)\cos(5x^7 - 9x^2)$$

$$y = \cos^5(3x^{24} + 8x^3)$$

$y = \cos^5 u$ $\quad y = v^5$

$u = 3x^{24} + 8x^3$ $\quad v = \cos u$ $\quad \frac{dy}{dv} = 5v^4$

$\frac{du}{dx} = 72x^{23} + 24x^2$ $\quad \frac{dv}{du} = -\sin u$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= 5v^4 \cdot (-\sin u)(72x^{23} + 24x^2)$$

$$= 5(\cos^4 u)(-\sin u)(72x^{23} + 24x^2)$$

$$= 5\cos^4(3x^{24} + 8x^3)(-\sin(3x^{24} + 8x^3))(72x^{23} + 24x^2)$$

$$= (-360x^{23} - 120x^2)\sin(3x^{24} + 8x^3)\cos^4(3x^{24} + 8x^3)$$

Power Chain Rule: Given u is a function of x

$$\frac{d}{dx}u^n = nu^{n-1}\cdot\left(\frac{du}{dx}\right)$$

$$f(x) = (5x^4 + 9x^2)^{10} = 10(5x^4+9x^2)^9 \frac{d}{dx}(5x^4+9x^2)$$

$$f'(x) = 10(5x^4+9x^2)^9(20x^3+18x)$$

$$= (200x^3+180x)(5x^4+9x^2)^9$$

$$g(x) = \cos^5 x$$

$$= 5\cos^4 x \cdot \frac{d}{dx}\cos x$$

$$= 5\cos^4 x(-\sin x)$$

$$= -5\cos^4 x \sin x$$

$$f(x)=\left(\frac{\cos x}{\sin x-3}\right)^7$$

$$f'(x)=7\left(\frac{\cos x}{\sin x-3}\right)^6\cdot\frac{d}{dx}\left(\frac{\cos x}{\sin x-3}\right)$$

$$=7\left(\frac{\cos x}{\sin x-3}\right)^6\left(\frac{(\sin x-3)(-\sin x)-\cos x(\cos x)}{(\sin x-3)^2}\right)$$

$$=7\left(\frac{\cos x}{\sin x-3}\right)^6\left(\frac{-\sin^2 x+3\sin x-\cos^2 x}{(\sin x-3)^2}\right)$$

$$=7\left(\frac{\cos x}{\sin x-3}\right)^6\left(\frac{3\sin x-1}{(\sin x-3)^2}\right)$$

$$=\frac{7(3\sin x-1)\cos^6 x}{(\sin x-3)^8}$$

Find the value of $(f \circ g)'$ when x = -1.

$$f(u) = 1 - \frac{1}{u}, \quad u = g(x) = \frac{1}{1-x}$$

$$(f \circ g)(x) = f(g(x))$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

OR

$$f(u) = 1 - \frac{1}{\frac{1}{1-x}}$$

$$= 1 - 1 \div \frac{1}{1-x}$$

$$= 1 - 1 \times \frac{1-x}{1}$$

$$= 1 - (1-x)$$

$$f(g(x)) = x$$

$$f'(g(x)) = (1)$$

p.158 #13-39 odd

$$g(x) = \frac{1}{1-x}$$

$$g'(x) = \frac{(1-x)(0) - 1(-1)}{(1-x)^2}$$

$$g'(x) = \frac{1}{(1-x)^2}$$

$$(f \circ g)'(x) = f'(g(x)) \, g'(x)$$

$$= (1) \frac{1}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$(f \circ g)'(1) = \frac{1}{(1-(-1))^2} = 1/4$$