CFA® Program
Level I

FORMULA SHEET (2023) Version 1.0

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FINANCE | RISK | SUSTAINABILITY

FOR REFERENCE ONLY

(Note: Formula Sheet is not provided in the CFA exam)

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CFA Level 1 – Formula Sheet (2023)

Setting Up the Texas BA II Plus Financial Calculator

Video: https://youtu.be/0MS8d8QOFmc

Using Texas BA II Plus Financial Calculator

Video: https://youtu.be/LWmTTiZz8BU

Video (Requires Login to Facebook): https://fb.watch/nci5V7Dwtj/

QUANTITATIVE METHODS

Learning Module 1: The Time Value of Money

Interest Rates

Interest rate, r = Real risk-free rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

Nominal risk-free rate = Real risk-free rate + Inflation premium

Maturity premium = Interest rate on longer-maturity, liquid Treasury debt – Interest rate on short-term Treasury debt

Single Cash Flow

Annual Compounding

$$FV_N = PV(1+r)^N$$

$$PV = \frac{FV_N}{(1+r)^N} \qquad or \qquad PV = FV_N \times (1+r)^{-N}$$

where: FV_N = Future value of the investment N periods from today
PV = Present value of the investment (i.e., principal)
r = Interest rate per period

Video (FV of Single Cash Flow): https://youtu.be/DqnbyYLU1e4

Video (PV of Single Cash Flow): https://youtu.be/nVjPtUWTab4

Non-Annual Compounding

$$FV_N = PV\left(1 + \frac{r_s}{m}\right)^{m \times N}$$

$$PV = FV_N\left(1 + \frac{r_s}{m}\right)^{-m \times N}$$

where: r_s = Stated annual interest rate (or quoted interest rate)
m = Number of compounding periods per year
N = Number of years

Continuous Compounding

$$FV_N = PV \times e^{r_s \times N}$$

$$PV = FV_N \times e^{-r_s \times N}$$

Effective Annual Rate (EAR)

Non-Annual Compounding

$$EAR = \left(1 + \frac{r_s}{m}\right)^{m} - 1$$

Continuous Compounding

$$EAR = e^{r_s} - 1$$

Video 1: https://youtu.be/ubmSCZZKWJE

Video 2: https://youtu.be/a3PcCbjneU8

Annuities

Ordinary Annuity **(Mode: END)**

$$FV_N = A\left[\frac{(1+r)^N - 1}{r}\right]$$

$$PV = A\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right]$$

Video (FV of Ordinary Annuity): https://youtu.be/ogSu7t0iWr8

Video (PV of Ordinary Annuity): https://youtu.be/VgFQOltNRK0

Annuity Due **(Mode: BGN)**

$$FV_N = A\left[\frac{(1+r)^N - 1}{r}\right] \times (1+r)$$

$$PV = A\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right] \times (1+r)$$

Perpetuity

If first payment starts one period from now:

$$PV = \frac{A}{r}$$

If first payment starts immediately:

$$PV = \frac{A}{r}(1+r)$$

Video: https://youtu.be/bBiVRtsHgUs

Learning Module 2: Organizing, Visualizing, and Describing Data

Frequency Distribution

Range of data = Maximum value – Minimum value

$$Bin\ width = \frac{Range}{Number\ of\ bins}$$

$$First\ bin = Minimum\ value + Bin\ width$$

Measures of Central Tendency

$$Sample\ Mean, \qquad \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$Weighted\ mean, \qquad \bar{X}_w = \sum_{i=1}^{n} w_i X_i$$

where: X_i = Observation i ($i = 1, 2, 3, \dots, n$)
w_i = Weight of each observation ($i = 1, 2, 3, \dots, n$)

Geometric Mean

$$\bar{X}_G = \sqrt[n]{X_1 X_2 X_3 \dots X_n} \qquad X_i \geq 0 \; for \; i = 1, 2, 3, \dots, n$$

Geometric Mean Return

$$R_G = [(1 + R_1)(1 + R_2) \dots (1 + R_T)]^{1/T} - 1$$

where: R_t = Holding period returns, where $t = 1, 2, 3, \dots, T$

Harmonic Mean

$$\bar{X}_H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}}$$

Median

Position of median $= \frac{n+1}{2}$

Quantiles

Interquartile range $= Q_3 - Q_1$

where: Q_1 = First quartile
Q_3 = Third quartile

Position of y^{th} percentile, $L_y = (n + 1)\frac{y}{100}$

Measures of Dispersion

Range = Maximum value – Minimum value

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n} |X_i - \bar{X}|}{n}$$

Sample Variance

$$s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}$$

Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}}$$

Relationship Between Arithmetic and Geometric Mean

$$\bar{X}_G = \bar{X} - \frac{s^2}{2}$$

Sample Target Semideviation

$$s_{Target} = \sqrt{\frac{\sum_{X_i \le B}^{n} (X_i - B)^2}{n-1}}$$

where: $B =$ target

$n =$ total number of sample observations

Coefficient of Variation

$$CV = \frac{s}{\bar{x}}$$

Sample Skewness

$$Skewness \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^{n} (X_i - \bar{X})^3}{s^3}$$

Sample excess kurtosis

$$K_E \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^{n} (X_i - \bar{X})^4}{s^4} - 3$$

Sample covariance

$$s_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})$$

Sample correlation coefficient

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

Learning Module 3: Probability Concepts

Odds

Given a probability $P(E)$,

$$Odds\ for\ E = \frac{P(E)}{1 - P(E)}$$

$$Odds\ against\ E = \frac{1 - P(E)}{P(E)}$$

Conditional Probability

$$P(A|B) = \frac{P(AB)}{P(B)}$$

Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Multiplication Rule

$$P(AB) = P(A|B)P(B)$$

If two events A and B are **independent**, then $P(AB) = P(A)P(B)$

Total Probability Rule

$$\begin{aligned} P(A) &= P(AS_1) + P(AS_2) + \cdots + P(AS_n) \\ &= P(A|S_1)P(S_1) + P(A|S_2)P(S_2) + \cdots + P(A|S_n)P(S_n) \end{aligned}$$

Expected Value

$$E(X) = \sum_{i=1}^{n} P(X_i)X_i$$

Variance

$$\begin{aligned} \sigma^2(X) &= E\{[X - E(X)]^2\} \\ &= \sum_{i=1}^{n} P(X_i)[X_i - E(X)]^2 \end{aligned}$$

Conditional Expected Value

$$E(X|S) = P(X_1|S)X_1 + P(X_2|S)X_2 + \cdots + P(X_n|S)X_n$$

Total Probability Rule for Expected Value

$$E(X) = E(X|S_1)P(S_1) + E(X|S_2)P(S_2) + \cdots + E(X|S_n)P(S_n)$$

Portfolio Expected Return

$$E(R_P) = w_1E(R_1) + w_2E(R_2) + \cdots + w_nE(R_n)$$

Covariance

$$Cov(R_i, R_j) = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

Sample Covariance

$$Cov(R_i, R_j) = \frac{1}{n-1}\sum_{t=1}^{n}(R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)$$

Portfolio Variance

$$\begin{aligned}\sigma^2(R_P) &= E\{[R_P - E(R_P)]^2\}\\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(R_i, R_j)\end{aligned}$$

Requires n variances and $\frac{n(n-1)}{2}$ distinct covariances to estimate portfolio variance.

For a two-asset ($n = 2$) portfolio:

$$\sigma^2(R_P) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1, R_2)$$

where: $Cov(R_1, R_2) = \rho(R_1, R_2) \times \sigma(R_1) \times \sigma(R_2)$

Video: https://youtu.be/lUwulZ9ONC0

For a three-asset ($n = 3$) portfolio:

$$\begin{aligned}\sigma^2(R_P) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2Cov(R_1, R_2)\\ +2w_1w_3Cov(R_1, R_3) + 2w_2w_3Cov(R_2, R_3)\end{aligned}$$

Covariance Given a Joint Probability Function

$$Cov(R_A, R_B) = \sum_{i=1}\sum_{j=1} P(R_{A,i}, R_{B,j}) \times [R_{A,i} - E(R_A)] \times [R_{B,j} - E(R_B)]$$

If X and Y are uncorrelated, then $E(XY) = E(X)E(Y)$

If X and Y are independent, then $P(X, Y) = P(X)P(Y)$

Bayes' Formula

$$P(A|B) = \frac{P(B|A)}{P(B)} \times P(A)$$

$$P(Event|Information) = \frac{P(Information|Event)}{P(Information)} \times P(Event)$$

Video (Bayes' Formula and Total Probability Rule): https://youtu.be/9_h0EzssPZ4

Multiplication Rule for Counting

If Task 1 can be done in n_1 ways, Task 2 can be done in n_2 ways (given Task 1 is done), Task 3 can be done in n_3 ways (given Task 1 and 2 are done), and so on for k tasks.

The number of ways the k tasks can be done is $(n_1 \times n_2 \times \ldots \times n_k)$.

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

Multinomial Formula for Labeling Problems

There are n objects to be labelled with k different labels, with n_1 of the first type, n_2 of the second type, and so on, with $n_1 + n_2 + \cdots + n_k = 1$.

The number of ways to perform this is:

$$\frac{n!}{n_1!\,n_2!\ldots n_k!}$$

Combination Formula

The number of ways to choose r objects from a total of n objects (where order does not matter).

$$_nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Permutation Formula

The number of ways to choose r objects from a total of n objects (where order does matter).

$$_nP_r = \frac{n!}{(n-r)!}$$

Learning Module 4: Common Probability Distributions

Cumulative distribution function (CDF)

$$F(x) = P(X \leq x)$$

Continuous Uniform Distribution

$$f(x) = \begin{cases} \dfrac{1}{b-a} & for\ a \leq x \leq b \\ 0 & otherwise \end{cases}$$

$$F(x) = \begin{cases} 0 & for\ x < a \\ \dfrac{x-a}{b-a} & for\ a \leq x \leq b \\ 1 & for\ x > b \end{cases}$$

$$E(X) = \frac{a+b}{2}$$

$$\sigma^2(X) = \frac{(b-a)^2}{12}$$

Binomial Distribution

$$X \sim B(n, p)$$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$E(X) = np$$

$$\sigma^2(X) = np(1-p)$$

Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \qquad for\ \ -\infty < x < +\infty$$

Standardized Normal Random Variable

$$Z = \frac{X-\mu}{\sigma}$$

Safety-First Ratio

$$SFRatio = \frac{E(R_P) - R_L}{\sigma_P}$$

$$P(R_P < R_L) = N(-SFRatio)$$

where: R_L = Minimum acceptable return

Video: https://youtu.be/S3x5JrGIOUA

The Lognormal Distribution

Mean of a lognormal random variable, Y

$$\mu_L = \exp(\mu + 0.50\sigma^2)$$

Variance of a lognormal random variable

$$\sigma_L^2 = \exp(2\mu + \sigma^2) \times [\exp(\sigma^2) - 1]$$

where:
$Y = \exp(X)$, where X is normal
μ = Mean of the normal distribution
σ^2 = Variance of the normal distribution

Continuously Compounded Rates of Return

$$S_T = S_0 \exp(r_{0,T})$$

$$r_{0,T} = \ln\left(\frac{S_T}{S_0}\right)$$

In general,

$$r_{t,t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right)$$

$$r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$$

$$E(r_{0,T}) = E(r_{T-1,T}) + E(r_{T-2,T-1}) + \cdots + E(r_{0,1}) = \mu T$$

$$\sigma^2(r_{0,T}) = \sigma^2 T$$

Student's *t*-Distribution

Degrees of freedom, $df = n - 1$

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

where: $n =$ Sample size

Chi-Square Distribution

Degrees of freedom, $df = k$

where $k =$ Number of independent standard normal random variables

F-Distribution

$$F = \frac{(\chi_1^2/m)}{(\chi_2^2/n)}$$

χ_1^2 is a chi-square random variable with m degrees of freedom.
χ_2^2 is a chi-square random variable with n degrees of freedom.

Learning Module 5: Sampling and Estimation

Mean of sampling distribution $= \mu$

Variance of sampling distribution $= \frac{\sigma^2}{n}$

Standard error of the sample mean, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ $s_{\bar{X}} = \frac{s}{\sqrt{n}}$

where: $\mu =$ Population mean
$\sigma^2 =$ Population variance
$n =$ Sample size
Absolute Error = Difference between sample mean and population mean

Confidence Intervals

A $100(1-\alpha)\%$ confidence interval for a parameter has the following structure:

$$Point\ estimate \pm Reliability\ factor \times Standard\ error$$

where: $Precision\ of\ the\ estimator = Reliability\ factor \times Standard\ error$

A $100(1-\alpha)\%$ confidence interval for the Population Mean, μ (Normally distributed with known variance)

$$\bar{X} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

A $100(1-\alpha)\%$ confidence interval for the Population Mean, μ (Large sample, population variance unknown)

$$\bar{X} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

OR

$$\bar{X} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

where degrees of freedom for $t_{\alpha/2} = n-1$

A $100(1-\alpha)\%$ confidence interval for the Population Mean, μ (Population variance unknown; Sample is small but population is approximately normally distributed)

$$\bar{X} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

Selection of Sample Size

$$Sample\ size, \qquad n = \left(\frac{t_{\alpha/2} \times s}{E}\right)^2$$

where $E = t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

Learning Module 6: Hypothesis Testing

Test of a single mean ($df = n - 1$):

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

where: $\bar{X}$ = Sample mean

μ_0 = Hypothesized population mean

$\frac{s}{\sqrt{n}}$ = Standard error of the mean

Test of the difference in means ($df = n_1 + n_2 - 2$):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Test of the mean of differences ($df = n - 1$):

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$$

Test of a single variance ($df = n - 1$):

$$\chi^2 = \frac{s^2(n-1)}{\sigma_0^2}$$

Test of the differences in variances ($df_1 = n_1 - 1; df_2 = n_2 - 1$):

$$F^2 = \frac{s_1^2}{s_2^2}$$

Test of a correlation ($df = n - 2$)

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

where: r = Correlation

Test of independence (categorical data) $df = (r-1)(c-1)$

$$\chi^2 = \sum_{i=1}^{m} \frac{(O - ijE)ij^2}{Eij}$$

where: O_{ij} = Observed frequencies

E_{ij} = Expected frequencies

r = Number of rows in the contingency table

c = Number of columns in the contingency table

Type I and II Error

Significance level (Type I error) $= \alpha$

Confidence level = $1 - \alpha$

Type II error $= \beta$

Power of the test $= 1 - \beta$

Confidence Intervals

A 95% confidence interval for the population mean, μ,

$$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

Significance Interpretation

$$p(k = 1) \leq \alpha \times \frac{Rank\ of\ i}{Number\ of\ tests}$$

Learning Module 7: Introduction to Linear Regression

$$Y_i = b_0 + b_1 X_1 + \cdots + b_n X_n + \varepsilon_i, \quad i = 1, 2, \ldots, n$$

where: Y = Dependent variable
X = Independent variable
b_0 = Intercept
b_i = Slope coefficient, $i = 1, 2, \ldots, n$
ε_i = Error term
$b_0, b_1, \ldots, b_n$ = Regression coefficients

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i + e_i$$

where: $\hat{Y}_i$ = Estimated value on the regression line for the *i*th observation
$\hat{b}_0$ = Intercept
$\hat{b}_1$ = Slope
e_i = Residual for the *i*th observation

$$\hat{b}_1 = \frac{Covariance\ of\ X\ and\ Y}{Variance\ of\ X} = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

$$\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

Sum of Squares Total, $SST = \sum_{i=1}^{n}(Y_i - \bar{Y})^2 = SSR + SSE$

Sum of Squares Regression, $SSR = \sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2$

Sum of Squares Error, $SSE = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n} e_i^2$

Coefficient of Determination,

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

Correlation coefficient,

$$r = \frac{Covariance\ of\ X\ and\ Y}{(Standard\ deviation\ of\ X)(Standard\ deviation\ of\ Y)}$$

Note: Correlation coefficient $= \sqrt{Coefficient\ of\ determination}$

Sample standard deviation of X

$$S_X = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

Sample standard deviation of Y

$$S_Y = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n-1}}$$

Homoskedasticity

$$E(\varepsilon_i^2) = \sigma_\varepsilon^2, \quad i = 1, 2, \dots, n$$

ANOVA F-Test

Mean square regression (MSR)

$$MSR = \frac{SSR}{k}$$

Mean square error (MSE)

$$MSE = \frac{SSE}{n-k-1}$$

F-distributed test statistic

$$F = \frac{MSR}{MSE}$$

where: n = Number of observations

k = Number of independent variables

Standard error of estimate

$$s_e = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n-k-1}}$$

Hypothesis Test of the Slope Coefficient

$$t = \frac{\hat{b}_1 - B_1}{s_{\hat{b}_1}}$$

Degrees of freedom, $df = n - k - 1$

where: B_1 = Hypothesized population slope

$s_{\hat{b}_1}$ = Standard error of the slope coefficient

$$= \frac{s_e}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

Hypothesis Test of the Intercept

$$t_{intercept} = \frac{\hat{b}_0 - B_0}{s_{\hat{b}_0}}$$

Standard error of the intercept, $s_{\hat{b}_0}$

$$s_{\hat{b}_0} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

Prediction Intervals

$$\hat{Y}_f \pm t_{\alpha/2} \times s_f$$

where: $\hat{Y}_f = \hat{b}_0 + \hat{b}_1 X_f$

Variance of the prediction error of Y, given X

$$s_f^2 = s_e^2\left[1 + \frac{1}{n} + \frac{\left(X_f - \bar{X}\right)^2}{(n-1)s_X^2}\right]$$

Standard error of the forecast

$$s_f = s_e\sqrt{1 + \frac{1}{n} + \frac{\left(X_f - \bar{X}\right)^2}{(n-1)s_X^2}}$$

The Log-Lin Model

$$\ln Y_i = b_0 + b_1 X_i$$

The Lin-Log Model

$$Y_i = b_0 + b_1 \ln X_i$$

The Log-Log Model

$$\ln Y_i = b_0 + b_1 \ln X_i$$

ECONOMICS

Learning Module 1: Topics in Demand and Supply Analysis

Inverse demand function (Demand curve), $P_x = a + bQ_x$

Price elasticity of demand

$$E_{P_x}^d = \frac{\%\Delta Q_x^d}{\%\Delta P_x} = \frac{\Delta Q_x^d}{\Delta P_x}\left(\frac{P_x}{Q_x^d}\right)$$

Income elasticity of demand

$$E_I^d = \frac{\%\Delta Q_x^d}{\%\Delta I} = \frac{\Delta Q_x^d}{\Delta I}\left(\frac{I}{Q_x^d}\right)$$

Cross-Price elasticity of demand

$$E_{P_y}^d = \frac{\%\Delta Q_x^d}{\%\Delta P_y} = \frac{\Delta Q_x^d}{\Delta P_y}\left(\frac{P_y}{Q_x^d}\right)$$

Supply Analysis

Total cost of production

$$TC = w \times L + r \times K$$

where: w = Wage rate
L = Labor hours
K = Hours of capital
r = Rental rate of capital

Total product = Q

$$Average\ product, AP = \frac{Q}{L}$$

$$Marginal\ product, MP = \frac{\Delta Q}{\Delta L}$$

Economic profit = Total revenue – Total **economic** costs

Accounting profit = Total revenue – Total **accounting** costs

$$Total\ revenue = Price \times Quantity = P \times Q$$

$$Average\ revenue = \frac{Total\ revenue}{Quantity}$$

$$Marginal\ revenue = \frac{\Delta TR}{\Delta Q} = P + Q\left(\frac{\Delta P}{\Delta Q}\right)$$

$$Marginal\ cost = \frac{\Delta TC}{\Delta Q}$$

Short-run marginal cost

$$SMC = \frac{Wage\ rate}{Marginal\ product\ of\ labor}$$

$$Average\ variable\ cost = \frac{Total\ variable\ cost}{Quantity}$$

$$Average\ fixed\ cost = \frac{Total\ fixed\ cost}{Quantity}$$

Total cost = Total fixed cost + Total variable cost

Average total cost = Average fixed cost + Average variable cost

Total profit = Total revenue – Total cost

Profit maximization occurs when $MR = MC$ and MC is not falling.

Learning Module 2: The Firm and Market Structures

Marginal revenue

$$MR = P\left[1 - \frac{1}{E_P^d}\right]$$

where: P = Price

E_P^d = Price elasticity of demand

Concentration Ratio

$$Concentration\ ratio = \sum_{i=1}^{n} (Market\ share)_i$$

Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^{n} (Market\ share)_i^2$$

Learning Module 3: Aggregate Output, Prices, and Economic Growth

$$Per\ capita\ real\ GDP = \frac{Real\ GDP}{Size\ of\ population}$$

$$Nominal\ GDP_t = Prices\ in\ year\ t \times Quantity\ produced\ in\ year\ t$$

$$Real\ GDP_t = Prices\ in\ Base\ year \times Quantity\ produced\ in\ year\ t$$

$$GDP\ deflator = \frac{Value\ of\ current\ year\ output\ at\ current\ year\ prices}{Value\ of\ current\ year\ output\ at\ base\ year\ prices} \times 100$$
$$= \frac{Nominal\ GDP}{Real\ GDP} \times 100$$

Components of GDP

$$GDP = C + I + G + (X - M)$$

where:
GDP = Gross domestic product
C = Consumer spending
I = Gross private domestic investment
G = Government spending
X = Exports
M = Imports

Gross domestic income
GDI = Net domestic income + Consumption of fixed capital + Statistical discrepancy

Gross domestic income = Compensation of employees + Gross operating surplus
+ Gross mixed income + Taxes less subsidies on production
+ Taxes less subsidies on products and imports

Household primary income = Compensation of employees + Net mixed income
+ Net property income

Household saving = Personal disposable income – Household final consumption expenditure
+ Change in pension entitlements

Household net saving = Personal disposable income – Net current transfers paid
– Household final consumption expenditure
+ Change in pension entitlements

$$Savings\ rate = \frac{Household\ net\ saving}{Compensation\ of\ employees}$$

$$S = I + (G - T) + (X - M)$$

where S = Domestic savings
T = Net taxes

Marginal propensity to save = 1 – Marginal propensity to consume

$$MPS = 1 - MPC$$

$$Marginal\ propensity\ to\ consume, MPC = \frac{C}{Disposable\ income}$$

$$Average\ propensity\ to\ consume, APC = \frac{C}{Y}$$

$$\begin{matrix}\%\ Change\ in\ unit \\ labor\ cost\end{matrix} = \begin{matrix}\%\ Change\ in \\ nominal\ wages\end{matrix} - \begin{matrix}\%\ Change\ in \\ productivity\end{matrix}$$

<u>The Production Function</u>

$$Y = A\boldsymbol{F}(L, K)$$

where:

Y = Level of aggregate output in the economy

L = Quantity of labor (or number of workers in the economy)

K = Capital stock

A = Total factor productivity (TFP)

Growth Accounting Equation

$$\begin{matrix}Growth\ in \\ potential\ GDP\end{matrix} = \begin{matrix}Growth\ in \\ TFP\end{matrix} + W_L\begin{pmatrix}Growth\ in \\ labor\end{pmatrix} + W_C\begin{pmatrix}Growth\ in \\ capital\end{pmatrix}$$

W_L = Relative share of labor in national income

W_C = Relative share of capital in national income

$$= \frac{Corporate\ profits + Net\ interest\ income + Net\ rental\ income + Depreciation}{GDP}$$

$$\begin{matrix}Growth\ in\ per\ capita \\ potential\ GDP\end{matrix} = \begin{matrix}Growth\ in \\ TFP\end{matrix} + W_C\begin{pmatrix}Growth\ in\ capital \\ to\ labor\ labor\end{pmatrix}$$

Total hours worked = Labor force × Average hours worked per worker

$$Labor\ productivity = \frac{Real\ GDP}{Aggregate\ hours} = \frac{Y}{L}$$

$$Potential\ GDP = Aggregate\ hours\ worked \times Labor\ productivity$$

$$\begin{matrix} Potential\ growth \\ rate \end{matrix} = \begin{matrix} Long\ term\ growth\ rate \\ of\ aggregate\ hours\ worked \end{matrix} + \begin{matrix} Long\ term\ labor\ productivity \\ growth\ rate \end{matrix}$$

Learning Module 4: Understanding Business Cycles

Measuring Inflation

$$Laspeyres\ price\ index = \frac{Value\ of\ current\ period\ output\ at\ base\ prices}{Value\ of\ base\ period\ output\ at\ base\ prices} \times 100$$

$$Paasche\ price\ index = \frac{Value\ of\ current\ period\ output\ at\ current\ prices}{Value\ of\ base\ period\ output\ at\ current\ prices} \times 100$$

$$Fisher\ price\ index = \sqrt{Laspeyres\ price\ index \times Paasche\ price\ index}$$

$$Unit\ labor\ cost, ULC = \frac{Wage\ per\ hour\ per\ worker}{Output\ per\ hour\ per\ worker}$$

Learning Module 5: Monetary and Fiscal Policy

$$Money\ multiplier = \frac{1}{Reserve\ requirement}$$

$$Money\ created\ from\ deposit = \frac{New\ deposit}{Reserve\ requirement}$$

Quantity equation of exchange

$$M \times V = P \times Y$$

where: M = Quantity of money
V = Velocity of circulation of money
P = Average price level
Y = Real output

Fisher effect

$$R_{nominal} = R_{real} + Expected\ inflation\ rate$$

The Fiscal Multiplier

$$Fiscal\ multiplier = \frac{1}{1 - c(1-t)}$$

where: c = Marginal propensity to consume
t = Tax rate

Learning Module 6: Introduction to Geopolitics

No formula

Learning Module 7: International Trade and Capital Flows

Current account balance

$$CA = X - M = S_p + S_g - I$$

where: S_p = Private savings
S_g = Government surplus = $T - G$
I = Private investments

Learning Module 8: Currency Exchange Rates

$$Real\ exchange\ rate_{d/f} = S_{d/f} \times \frac{P_f}{P_d}$$

$$\%\ Change\ in\ real\ exchange\ rate \approx \%\Delta S_{d/f} + \%\Delta P_f - \%\Delta P_d$$

Percentage change in currency f (vs currency d)

$$\frac{E(S_{d/f}) - S_{d/f}}{S_{d/f}}$$

Cross-Rate

$$\frac{A}{B} = \frac{A}{C} \times \frac{C}{D}$$

Forward Rate

$$F_{A/B} = S_{A/B} \times \left[\frac{1 + r_A \times T}{1 + r_B \times T}\right]$$

$$F_{A/B} = S_{A/B} + Forward\ points\ (in\ decimal)$$

$$F_{A/B} - S_{A/B} = S_{A/B}\left(\frac{r_A - r_B}{1 + r_B}\right)T$$

Marshall-Lerner Condition

$$\omega_X \varepsilon_X + \omega_M(\varepsilon_M - 1) > 0$$

where: ω_X = Share of exports in total trade
ω_M = Share of imports in total trade
ε_X = Price elasticity of foreign demand for domestic country exports
ε_M = Price elasticity of domestic demand for imports

$\omega_X + \omega_M = 1$

FINANCIAL STATEMENT ANALYSIS

Learning Module 1: Introduction to Financial Statement Analysis

$$Assets = Liabilities + Equity$$

$$Net\ income = Revenue - Expenses$$

$$Total\ comprehensive\ income = Net\ income + Other\ comprehensive\ income$$

Learning Module 2: Financial Reporting Standards

No formula

Learning Module 3: Understanding Income Statements

$$Gross\ profit = Revenue - Cost\ of\ sales$$

$$Operating\ profit = Gross\ profit - Operating\ expenses$$

Net book value of asset = Asset cost – Accumulated depreciation

$$Basic\ EPS = \frac{Net\ income - Preferred\ dividends}{Weighted\ average\ number\ of\ shares\ outstanding}$$

Diluted EPS (for convertible preferred stock)

$$Diluted\ EPS = \frac{Net\ income}{\begin{matrix} Weighted\ average\ number \\ of\ shares\ outstanding \end{matrix} + \begin{matrix} New\ common\ shares\ that\ would \\ have\ been\ issued\ at\ conversion \end{matrix}}$$

Diluted EPS (for convertible debt)

$$Diluted\ EPS = \frac{Net\ income - Preferred\ dividends + \begin{matrix} After\ tax\ interest\ expense \\ on\ convertible\ debt \end{matrix}}{\begin{matrix} Weighted\ average\ number \\ of\ shares\ outstanding \end{matrix} + \begin{matrix} New\ common\ shares\ that\ would \\ have\ been\ issued\ at\ conversion \end{matrix}}$$

Diluted EPS (for options)

$$Diluted\ EPS = \frac{Net\ income - Preferred\ dividends}{\begin{matrix} Weighted\ average\ number \\ of\ shares\ outstanding \end{matrix} + \begin{matrix} Additional\ common\ shares\ issued \\ upon\ conversion \end{matrix}}$$

Using Treasury stock method,

$$\begin{matrix}\text{Additional common}\\ \text{shares issued upon}\\ \text{conversion}\end{matrix} = \begin{matrix}\text{New shares that would}\\ \text{have been issued at}\\ \text{option exercise}\end{matrix} - \begin{matrix}\text{Shares that could have}\\ \text{been purchased with}\\ \text{cash received option exercise}\end{matrix}$$

Video (Basic & Diluted EPS): https://youtu.be/2C-mwVqO2SQ

Learning Module 4: Understanding Balance Sheets

For balance sheet ratios, refer to Learning Module 6: Financial Analysis Techniques.

$$Working\ capital = Current\ assets - Current\ liabilities$$

Learning Module 5: Understanding Cash Flow Statements

$$\begin{matrix}\text{Change in cash}\\ \text{and cash equivalents}\end{matrix} = \begin{matrix}\text{Cash flow}\\ \text{from operating}\\ \text{activities}\end{matrix} + \begin{matrix}\text{Cash flow}\\ \text{from investing}\\ \text{activities}\end{matrix} + \begin{matrix}\text{Cash flow}\\ \text{from financing}\\ \text{activities}\end{matrix}$$

$$\begin{matrix}\text{Ending accounts}\\ \text{receivable}\end{matrix} = \begin{matrix}\text{Beginning accounts}\\ \text{receivable}\end{matrix} + Revenue - \begin{matrix}\text{Cash collected}\\ \text{from customers}\end{matrix}$$

$$\begin{matrix}\text{Ending}\\ \text{inventory}\end{matrix} = \begin{matrix}\text{Beginning}\\ \text{inventory}\end{matrix} + Purchases - \begin{matrix}\text{Cost of}\\ \text{goods sold}\end{matrix}$$

$$\begin{matrix}\text{Ending accounts}\\ \text{payable}\end{matrix} = \begin{matrix}\text{Beginning accounts}\\ \text{payable}\end{matrix} + Purchases - \begin{matrix}\text{Cash paid}\\ \text{to suppliers}\end{matrix}$$

$$\begin{matrix}\text{Ending wages}\\ \text{payable}\end{matrix} = \begin{matrix}\text{Beginning wages}\\ \text{payable}\end{matrix} + \begin{matrix}\text{Wages}\\ \text{expense}\end{matrix} - \begin{matrix}\text{Cash paid}\\ \text{to employees}\end{matrix}$$

$$\begin{matrix}\text{Ending interest}\\ \text{payable}\end{matrix} = \begin{matrix}\text{Beginning interest}\\ \text{payable}\end{matrix} + \begin{matrix}\text{Interest}\\ \text{expense}\end{matrix} - \begin{matrix}\text{Cash paid}\\ \text{for interest}\end{matrix}$$

$$\begin{matrix}\text{Ending income}\\ \text{tax payable}\end{matrix} = \begin{matrix}\text{Beginning income}\\ \text{tax payable}\end{matrix} + \begin{matrix}\text{Income tax}\\ \text{expense}\end{matrix} - \begin{matrix}\text{Cash paid}\\ \text{for income taxes}\end{matrix}$$

$$Ending\ PP\&E = Beginning\ PP\&E + \begin{matrix}\text{Equipment}\\ \text{purchased}\end{matrix} - \begin{matrix}\text{Equipment}\\ \text{sold}\end{matrix}$$

$$\begin{matrix}\text{Ending accumulated}\\ \text{depreciation}\end{matrix} = \begin{matrix}\text{Beginning accumulated}\\ \text{depreciation}\end{matrix} + \begin{matrix}\text{Depreciation}\\ \text{expense}\end{matrix} - \begin{matrix}\text{Accumulated}\\ \text{depreciation on}\\ \text{equipment sold}\end{matrix}$$

Note:

$$\begin{matrix}\text{Gain on sale}\\\text{of equipment}\end{matrix} = \begin{matrix}\text{Cash received from}\\\text{sale of equipment}\end{matrix} - \begin{matrix}\text{Book value of}\\\text{equipment sold}\end{matrix}$$

$$\begin{matrix}\text{Ending retained}\\\text{earnings}\end{matrix} = \begin{matrix}\text{Beginning retained}\\\text{earnings}\end{matrix} + \begin{matrix}\text{Net}\\\text{income}\end{matrix} - \text{Dividends}$$

Free Cash Flow To Firm (FCFF)

$$FCFF = NI + NCC + Int(1 - Tax\ rate) - FCInv - WCInv$$

where: NI = Net income

NCC = Non-cash charges (e.g., depreciation and amortization)

Int = Interest expense

$FCInv$ = Capital expenditures

$WCInv$ = Working capital expenditures

$$FCFF = CFO + Int(1 - Tax\ rate) - FCInv$$

$$CFO = NI + NCC - WCInv$$

Free Cash Flow to Equity (FCFE)

$$FCFE = CFO - FCInv + Net\ Borrowing$$

where: $Net\ Borrowing = Debt\ issued - Debt\ repaid$

Performance Ratios

$$Cash\ flow\ to\ revenue = \frac{CFO}{Revenue}$$

$$Cash\ return\ on\ assets = \frac{CFO}{Average\ total\ assets}$$

$$Cash\ return\ on\ equity = \frac{CFO}{Average\ shareholders\ equity}$$

$$Cash\ to\ income = \frac{CFO}{Operating\ income}$$

$$Cash\ flow\ per\ share = \frac{CFO - Preferred\ dividends}{Number\ of\ common\ shares\ outstanding}$$

Coverage Ratios

$$Debt\ coverage\ ratio = \frac{CFO}{Total\ debt}$$

$$Interest\ coverage\ ratio = \frac{CFO + Interest\ paid + Taxes\ paid}{Interest\ paid}$$

$$Reinvestment\ ratio = \frac{CFO}{Cash\ paid\ for\ long\ term\ assets}$$

$$Debt\ payment\ ratio = \frac{CFO}{Cash\ paid\ for\ long\ term\ debt\ repayment}$$

$$Dividend\ payment\ ratio = \frac{CFO}{Dividends\ paid}$$

$$Investing\ and\ financing\ ratio = \frac{CFO}{\substack{Cash\ flow\ for\ investing\ and \\ financing\ activities}}$$

Learning Module 6: Financial Analysis Techniques

Activity Ratios

$$Inventory\ turnover = \frac{Cost\ of\ sales}{Average\ inventory}$$

$$Days\ of\ inventory\ on\ hand = \frac{Number\ of\ days\ in\ the\ period}{Inventory\ turnover}$$

$$Receivables\ turnover = \frac{Revenue}{Average\ receivables}$$

$$Days\ of\ sales\ outstanding = \frac{Number\ of\ days\ in\ the\ period}{Receivables\ turnover}$$

$$Payables\ turnover = \frac{Purchases}{Average\ payables}$$

$$Number\ of\ days\ of\ payables = \frac{Number\ of\ days\ in\ the\ period}{Payables\ turnover}$$

$$Working\ capital\ turnover = \frac{Revenue}{Average\ working\ capital}$$

$$Fixed\ asset\ turnover = \frac{Revenue}{Average\ net\ fixed\ assets}$$

$$Total\ asset\ turnover = \frac{Revenue}{Average\ total\ assets}$$

Liquidity Ratios

$$Current\ ratio = \frac{Current\ assets}{Current\ liabilities}$$

$$Quick\ ratio = \frac{Cash + Short\ term\ marketable\ investments + Receivables}{Current\ liabilities}$$

$$Cash\ ratio = \frac{Cash + Short\ term\ marketable\ investments}{Current\ liabilities}$$

$$\begin{matrix} Defensive\ interval \\ ratio \end{matrix} = \frac{Cash + Short\ term\ marketable\ investments + Receivables}{Daily\ cash\ expenditures}$$

$$\begin{matrix} Cash\ conversion \\ cycle \end{matrix} = \begin{matrix} Days\ of\ inventory \\ on\ hand \end{matrix} + \begin{matrix} Days\ of\ sales \\ outstanding \end{matrix} - \begin{matrix} Number\ of\ days \\ of\ payables \end{matrix}$$

Video (Cash Conversion Cycle): https://youtu.be/IFsI9c4wUD4

Solvency Ratios

$$Long\ term\ debt\ to\ equity\ ratio = \frac{Total\ long\ term\ debt}{Total\ equity}$$

$$Total\ debt\ ratio = \frac{Total\ debt}{Total\ assets}$$

$$Debt\ to\ capital\ ratio = \frac{Total\ debt}{Total\ debt + Total\ equity}$$

$$Debt\ to\ equity\ ratio = \frac{Total\ debt}{Total\ equity}$$

$$Financial\ leverage\ ratio = \frac{Average\ total\ assets}{Average\ total\ equity}$$

$$Debt\ to\ EBITDA\ ratio = \frac{Total\ debt}{EBITDA}$$

Coverage Ratios

$$Interest\ coverage\ ratio = \frac{EBIT}{Interest\ payments}$$

$$Fixed\ charge\ coverage\ ratio = \frac{EBIT + Lease\ payments}{Interest\ payments + Lease\ payments}$$

Profitability Ratios

$$Gross\ profit\ margin = \frac{Gross\ profit}{Revenue}$$

$$Operating\ profit\ margin = \frac{Operating\ income}{Revenue}$$

$$Pretax\ margin = \frac{EBT}{Revenue}$$

$$Net\ profit\ margin = \frac{Net\ income}{Revenue}$$

$$Operating\ ROA = \frac{Operating\ income}{Average\ total\ assets}$$

$$ROA = \frac{Net\ income}{Average\ total\ assets}$$

$$Return\ on\ total\ capital = \frac{EBIT}{Average\ total\ debt\ and\ equity}$$

$$ROE = \frac{Net\ income}{Average\ total\ equity}$$

$$Return\ on\ common\ equity = \frac{Net\ income - Preferred\ dividends}{Average\ common\ equity}$$

DuPont Analysis

$$ROE = ROA \times Financial\ Leverage$$

$$ROE = Net\ profit\ margin \times Total\ asset\ turnover \times Financial\ leverage$$

$$ROE = \begin{matrix} Tax \\ burden \end{matrix} \times \begin{matrix} Interest \\ burden \end{matrix} \times \begin{matrix} EBIT \\ margin \end{matrix} \times \begin{matrix} Total\ asset \\ turnover \end{matrix} \times \begin{matrix} Financial \\ leverage \end{matrix}$$

where:

$$Tax\ burden = \frac{Net\ income}{EBT}$$

$$Interest\ burden = \frac{EBT}{EBIT}$$

Valuation Ratios

$$P/E = \frac{Price\ per\ share}{Earnings\ per\ share}$$

$$P/CF = \frac{Price\ per\ share}{Cash\ flow\ per\ share}$$

$$P/S = \frac{Price\ per\ share}{Sales\ per\ share}$$

$$P/BV = \frac{Price\ per\ share}{Book\ value\ per\ share}$$

$$Cash\ flow\ per\ share = \frac{Cash\ flow\ from\ operations}{Weighted\ average\ number\ of\ shares\ outstanding}$$

$$Dividend\ payout\ ratio = \frac{Common\ share\ dividends}{Net\ income\ attributable\ to\ common\ shares}$$

$$Retention\ rate = 1 - Dividend\ payout\ ratio$$

$$Sustainable\ growth\ rate = Retention\ rate \times ROE$$

Business Risk

$$\begin{matrix} Coefficient\ of\ variation \\ of\ operating\ income \end{matrix} = \frac{Standard\ deviation\ of\ operating\ income}{Average\ operating\ income}$$

$$\begin{matrix} Coefficient\ of\ variation \\ of\ net\ income \end{matrix} = \frac{Standard\ deviation\ of\ net\ income}{Average\ net\ income}$$

$$\begin{matrix} Coefficient\ of\ variation \\ of\ revenue \end{matrix} = \frac{Standard\ deviation\ of\ revenue}{Average\ revenue}$$

Financial Sector Ratios

$$Net\ interest\ margin = \frac{Net\ interest\ income}{Total\ interest\ earning\ assets}$$

Credit Ratios

$$EBITDA\ interest\ coverage\ ratio = \frac{EBITDA}{Interest\ expense}$$

$$FFO\ to\ debt = \frac{Funds\ from\ operations}{Total\ debt}$$

$$Free\ operating\ cash\ flow\ to\ debt = \frac{CFO - Capital\ expenditures}{Total\ debt}$$

$$Debt\ to\ EBITDA = \frac{Total\ debt}{EBITDA}$$

$$Return\ on\ capital = \frac{EBIT}{Average\ capital}$$

Segment Ratios

$$Segment\ margin\ ratio = \frac{Segment\ profit}{Segment\ revenue}$$

$$Segment\ turnover\ ratio = \frac{Segment\ revenue}{Segment\ assets}$$

$$Segment\ ROA\ ratio = \frac{Segment\ profit}{Segment\ assets}$$

$$Segment\ debt\ ratio = \frac{Segment\ liabilities}{Segment\ assets}$$

Learning Module 7: Inventories

Inventory cost (capitalized) = Raw materials + Direct labour + Production overhead + Transportation for raw materials

Inventory cost (expensed) = Abnormal waste + Storage of finished goods inventory

$$FIFO\ Inventory\ value = LIFO\ Inventory\ value + LIFO\ Reserve$$

$$FIFO\ Cost\ of\ goods\ sold = LIFO\ Cost\ of\ goods\ sold - Change\ in\ LIFO\ Reserve$$

IFRS

$$Inventories = Lower\ of\ Cost\ and\ Net\ Realizable\ Value\ (NRV)$$

NRV = Estimated selling price less estimated costs of completion and costs necessary to complete the sale

US GAAP

$$Inventories = Lower\ of\ Cost\ and\ NRV$$

For last-in, first-out (LIFO) method or retail inventory methods

$$Inventories = Lower\ of\ Cost\ and\ Market\ Value$$

$$Market\ value = Current\ replacement\ cost\ (subject\ to\ lower\ and\ upper\ limits)$$

Lower limit = $NRV - Normal\ profit\ margin$

Upper limit = NRV

Video: https://youtu.be/V8C31msIBzs

Learning Module 8: Long-Lived Assets

Video (Capitalizing vs. Expensing): https://youtu.be/c-3D1yuH4xU

Property, Plant, and Equipment (PP&E)

Cost Model

Carrying amount of PP&E = Historical cost – Accumulated depreciation – Impairment loss

Historical cost of PP&E = Purchase price + Delivery cost + Cost to make the asset operable

Goodwill

$$Goodwill = \begin{matrix} Purchase \\ price \end{matrix} - \begin{matrix} Acquirer's\ interest\ in\ the \\ fair\ value\ of\ the\ identifiable \\ assets\ and\ liabilities\ acquired \end{matrix}$$

Straight-line method of depreciation

$$Depreciation\ expense = \frac{Cost - Salvage\ value}{Useful\ life}$$

Double-Declining Balance method

$$Depreciaton\ expense = \frac{2}{Useful\ life} \times (Cost - Accumulated\ depreciation)$$

Video: https://youtu.be/6RskYAxdAFk

Units-of-Production method

$$Depreciation\ expense = \frac{Units\ produced}{Total\ units\ over\ useful\ life} \times (Cost - Salvage\ value)$$

Impairment of PP&E (IFRS)

$$Impairment\ loss = Carrying\ amount - Recoverable\ amount$$

Recoverable amount = HIGHER of its Fair value less costs to sell AND its value in use

Value in use = Present value of expected future cash flows

Impairment of PP&E (US GAAP)

If carrying amount > Undiscounted expected future cash flows

Impairment loss = Carrying amount of PP&E – Fair value of PP&E

Disclosures

$$Estimated\ total\ useful\ life = \begin{matrix} Estimated\ age \\ of\ equipment \end{matrix} + \begin{matrix} Estimated \\ remaining\ life \end{matrix}$$

$$Estimated\ total\ useful\ life = \frac{Gross\ PP\&E}{Annual\ depreciation\ expense}$$

$$Estimated\ age\ of\ equipment = \frac{Accumulated\ depreciation}{Annual\ depreciation\ expense}$$

$$Estimated\ remaining\ life = \frac{Net\ PP\&E}{Annual\ depreciation\ expense}$$

Learning Module 9: Income Taxes

$$Income\ tax\ expense = Income\ tax\ payable + \begin{matrix} Changes\ in\ deferred\ tax \\ assets\ and\ liabilities \end{matrix}$$

$$\begin{matrix} Tax\ base \\ of\ liability \end{matrix} = \begin{matrix} Carrying\ amount \\ of\ liability \end{matrix} - \begin{matrix} Amounts\ that\ will\ be\ deductible \\ for\ tax\ purposes\ in\ the\ future \end{matrix}$$

$$Effective\ tax\ rate = \frac{Income\ tax\ expense}{Earnings\ before\ tax}$$

Learning Module 10: Non-Current (Long Term) Liabilities

$$\begin{matrix} Interest\ payment \\ on\ bonds \end{matrix} = Coupon\ rate \times Face\ value$$

Effective Interest Rate method

$$Interest\ expense = \begin{matrix} Beginning\ bond \\ liability \end{matrix} \times \begin{matrix} Market\ interest \\ rate \end{matrix}$$

$$\begin{matrix} Ending\ bond \\ liability \end{matrix} = \begin{matrix} Beginning\ bond \\ liability \end{matrix} + \begin{matrix} Interest \\ expense \end{matrix} - \begin{matrix} Interest \\ payment \end{matrix}$$

Video: https://youtu.be/wv0gFrbJQu8

Derecognition of Debt

$$\begin{matrix} Gain\ on\ extinguishment \\ of\ debt \end{matrix} = \begin{matrix} Gain\ on\ extinguishment \\ of\ debt \end{matrix} - \begin{matrix} Cash\ required\ to \\ redeem\ the\ debt \end{matrix}$$

Learning Module 11: Financial Reporting Quality

Nothing new

Learning Module 12: Applications of Financial Statement Analysis

$$\begin{array}{c}\textit{Percentage of asset base being renewed}\\ \textit{through new capital investment}\end{array} = \frac{Capex}{PP\&E + Capex}$$

CORPORATE ISSUERS

Learning Module 1: Corporate Structures and Ownership

No formula.

Learning Module 2: Introduction to Corporate Governance and Other ESG Considerations

No formula.

Learning Module 3: Business Models and Risks

$$Breakeven\ point\ (unit) = \frac{Fixed\ costs}{Contribution\ margin}$$

where: Contribution margin = Selling price per unit – Variable cost per unit

$$Total\ leverage = Operating\ leverage \times Financial\ leverage$$

$$Operating\ leverage = \frac{Contribution}{EBIT}$$

$$Financial\ leverage = \frac{EBIT}{EBT}$$

Learning Module 4: Capital Investments

Net Present Value (NPV)

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - Outlay$$

where: CF_t = After-tax cash flow at time t

r = Required rate of return for the investment

$Outlay$ = Investment cash flow at time zero

Internal Rate of Return (IRR)

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} - Outlay = 0$$

Video: https://youtu.be/bzck7QLhICw

Return on Invested Capital (ROIC)

$$ROIC = \frac{After\ tax\ Operating\ Profit}{Average\ Book\ Value\ of\ Invested\ Capital}$$

Real Options

$$Project\ NPV = \begin{matrix} NPV\ based\ on \\ DCF\ alone \end{matrix} - \begin{matrix} Cost\ of \\ options \end{matrix} + \begin{matrix} Value\ of \\ options \end{matrix}$$

Learning Module 5: Working Capital and Liquidity

$$Net\ working\ capital = Current\ assets - Current\ liabilities$$

$$Operating\ cash\ flows = \begin{matrix} After\ tax\ operating \\ cash\ flows \end{matrix} - \begin{matrix} Interest\ and\ dividend\ payments \\ (adjusted\ for\ taxes) \end{matrix}$$

$$\begin{matrix} Effective\ annual\ rate\ on \\ foregone\ trade\ credit \end{matrix} = \left(1 + \frac{\%Trade\ discount}{100 - \%Trade\ discount}\right)^{\frac{365}{Days\ past\ discount}}$$

Learning Module 6: Cost of Capital – Foundational Topics

Weighted Average Cost of Capital

$$WACC = w_d r_d (1 - t) + w_p r_p + w_e r_e$$

where:

w_d = Target weight of debt in capital structure

w_p = Target weight of preferred stock in capital structure

w_e = Target weight of common stock in capital structure

r_d = Before-tax marginal cost of debt

t = Marginal tax rate

$r_d(1 - t)$ = After-tax marginal cost of debt

r_p = Marginal cost of preferred stock = $\frac{Preferred\ dividend\ per\ share}{Preferred\ stock\ price\ per\ share}$

r_e = Marginal cost of common stock

Capital Asset Pricing Model (CAPM)

$$E(R_i) = R_F + \beta_i[E(R_M) - R_F]$$

where: R_F = Risk-free rate

β_i = Beta of stock i

$E(R_M)$ = Expected return on market

$E(R_M) - R_F$ = Expected market risk premium

Historical Equity Risk Premium

$$ERP = \bar{R}_M - \bar{R}_F$$

Bond Yield Plus Risk Premium Approach

$$r_e = r_d + Risk\ premium$$

where: r_d = Before-tax cost of debt

Estimating Beta for Public Companies

$$Adjusted\ beta = \frac{2}{3} \times Unadjusted\ beta + \frac{1}{3}$$

Estimating Beta for Thinly Traded and Nonpublic Companies

$$\beta_U = \beta_E \left[\frac{1}{1 + (1 - t)\frac{D}{E}}\right]$$

where: β_E = Equity beta of peer company
β_U = Unlevered beta (or asset beta)
$\frac{D}{E}$ = Debt-to-equity ratio of peer company
t = Marginal tax rate of peer company

$$\beta_E = \beta_U \left[1 + (1 - t)\frac{D}{E}\right]$$

where: β_E = Equity beta of nonpublic company
$\frac{D}{E}$ = Debt-to-equity ratio of nonpublic company
t = Marginal tax rate of nonpublic company

Flotation Costs (Video: https://youtu.be/T8pJ4bwPo48)

$$r_e = \frac{D_1}{P_0 - F} + g$$

$$r_e = \frac{D_1}{P_0(1 - f)} + g$$

where: r_e = Cost of equity
D_1 = Expected dividend = $D_0(1 + g)$
P_0 = Current stock price
F = Flotation cost per share (in monetary terms)
f = Flotation cost as a percentage of the issue price
g = Growth rate

Learning Module 7: Capital Structure

$$Net\ debt = Interest\ bearing\ debt - Cash\ and\ cash\ equivalents$$

$$\begin{aligned} Operating\ income &= Operating\ revenue - Operating\ expenses \\ &= EBIT - Nonoperating\ profit + Nonoperating\ expenses \end{aligned}$$

$$\begin{aligned} EBIT &= Net\ income + interest\ expense + taxes \\ &= EBITDA - Depreciation\ and\ amortization\ expense \end{aligned}$$

Modigliani-Miller Proposition I

$$V_L = V_U + t \times D$$

where: V_L = Value of levered firm

V_U = Value of unlevered firm

t = Marginal tax rate

D = Debt

$t \times D$ = Debt tax shield

$$V_U = \frac{CF_e(1-t)}{r_{WACC}}$$

$$V_L = \frac{(CF_e - r_D \times D)(1-t)}{r_{WACC}}$$

CF_e = Annual cash flow to equityholders

Modigliani-Miller Proposition II

$$r_e = r_0 + (r_0 - r_d)(1-t)\ \frac{D}{E}$$

$$\beta_e = \beta_a + (\beta_a - \beta_d)\left(\frac{D}{E}\right)(1-t)$$

$$r_{WACC} = \left(\frac{D}{D+E}\right) r_d(1-t) + \left(\frac{E}{D+E}\right) r_e$$

where: r_e = Cost of equity

r_0 = Cost of equity of an unlevered firm

β_e = Equity beta

β_a = Asset beta

β_d = Debt beta

Static Trade-Off Theory of Capital Structure

$$V_L = V_U + t \times D - PV(Costs\ of\ financial\ distress)$$

Learning Module 8: Measures of Leverage

Degree of Operating Leverage

$$DOL = \frac{\%\ Change\ in\ operating\ income}{\%\ Change\ in\ units\ sold}$$

$$DOL = \frac{Contribution}{Operating\ income} = \frac{Q(P - V)}{Q(P - V) - F}$$

where: Q = Quantity of units sold
P = Selling price per unit
V = Variable cost per unit
F = Fixed operating costs

Degree of Financial Leverage

$$DFL = \frac{\%\ Change\ in\ net\ income}{\%\ Change\ in\ operating\ income}$$

$$DFL = \frac{Operating\ income}{Net\ income} = \frac{Q(P - V) - F}{Q(P - V) - F - C}$$

where: C = Fixed financial costs

Degree of Total Leverage

$$DTL = \frac{\%\ Change\ in\ net\ income}{\%\ Change\ in\ number\ of\ units\ sold}$$

$$DTL = \frac{Contribution}{Net\ income} = \frac{Q(P - V)}{Q(P - V) - F - C}$$

$$DTL = DOL \times DFL$$

Breakeven Number of Units

$$Q_{BE} = \frac{F + C}{P - V}$$

Operating Breakeven Number of Units

$$Q_{OBE} = \frac{F}{P - V}$$

EQUITY INVESTMENTS

Learning Module 1: Market Organization and Structure

$$Maximum\ leverage\ ratio = \frac{1}{\text{Minimum margin requirement}}$$

Total return on leveraged stock investment:

$$Total\ Return = \frac{\begin{matrix} Sales \\ proceeds \end{matrix} + Dividends - Loan - \begin{matrix} Margin \\ interest \end{matrix} - \begin{matrix} Sales \\ commission \end{matrix}}{\begin{matrix} Initial \\ equity \end{matrix} + \begin{matrix} Purchase \\ commission \end{matrix}} - 1$$

$$Initial\ equity = \begin{matrix} Minimum\ margin \\ requirement \end{matrix} \times Total\ purchase\ price$$

Video (Return on Leveraged Stock Position): https://youtu.be/tZd4Xtvjjll

$$Margin\ Call\ Price = \frac{P_0(1 - Initial\ Margin)}{(1 - Maintenance\ Margin)}$$

Learning Module 2: Security Market Indexes

$$\text{Price Return Index,} \qquad V_{PRI} = \frac{\sum_{i=1}^{N} n_i P_i}{D}$$

where: n_i = the number of units of constituent security *i* held in the index portfolio
N = the number of constituent securities in the index
P_i = the unit price of constituent security *i*
D = value of the divisor

$$\text{Price return of an index,} \qquad PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

$$\text{Total Return Index,} \qquad TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where: V_{PRI1} = value of the price return index at the **end** of the period
V_{PRI0} = value of the price return index at the **beginning** of the period
Inc_I = total income (dividends and/or interest) from all securities in the index held over the period

Weighting Methods

$$\text{Price weighting,} \qquad w_i^P = \frac{P_i}{\sum_{j=1}^{N} P_j}$$

Video (Recalculating the divisor of a price weighted index): https://youtu.be/eYiZNK-ETrg

$$\text{Equal weighting,} \qquad w_i^E = \frac{1}{N}$$

$$\text{Market-capitalization weighting,} \qquad w_i^M = \frac{Q_i P_i}{\sum_{j=1}^{N} Q_j P_j}$$

$$\text{Float-adjusted market capitalization weighting,} \qquad w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^{N} f_j Q_j P_j}$$

where: f_i = fraction of shares outstanding in the market float
Q_i = number of shares outstanding of security *i*
P_i = share price of security *i*
N = number of securities in the index

$$\text{Fundamental weighting,} \qquad w_i^F = \frac{F_i}{\sum_{j=1}^{N} F_j}$$

where: F_i denotes a fundamental size measure of company *i*

Learning Module 3: Market Efficiency

No formula

Learning Module 4: Overview of Equity Securities

Return on Equity (using **average** total book value of equity)

$$ROE_t = \frac{NI_t}{(BVE_t + BVE_{t-1})/2}$$

Return on Equity (using **beginning** book value of equity)

$$ROE_t = \frac{NI_t}{BVE_{t-1}}$$

where BVE = book value (Assets – Liabilities)

Learning Module 5: Introduction to Industry and Company Analysis

No formula.

Learning Module 6: Equity Valuation: Concepts and Basic Tools

Intrinsic value of a share (at $t = 0$):

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

where: D_t = expected dividend in year t
r = required rate of return on stock
P_n = expected price per share at $t = n$ (terminal value)

Value of **preferred stock** (non-callable, non-convertible, perpetual)

$$V_0 = \frac{D_0}{r}$$

Value of **preferred stock** (non-callable, non-convertible, maturity at time n)

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{Par\ value}{(1+r)^n}$$

Gordon Growth Model

$$P_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$

Sustainable growth rate

$$g = b \times ROE$$

where b = earnings retention rate (1 – Dividend payout ratio)

Video: https://youtu.be/MnfRRRhuGpA

Two-Stage Dividend Discount Model

$$V_0 = \sum_{t=1}^{n} \frac{D_0(1+g_s)^t}{(1+r)^t} + \frac{V_n}{(1+r)^t}$$

where: g_L = Long-term stable growth rate
g_s = Short-term growth rate

$$V_n = \frac{D_{n+1}}{r-g_L} = \frac{D_0(1+g_s)^t(1+g_L)}{r-g_L}$$

Justified forward P/E

$$\frac{P_0}{E_1} = \frac{Dividend\ payout\ ratio}{r - g}$$

Enterprise Value

$$EV = \begin{matrix} Market\ value \\ of\ equity \end{matrix} + \begin{matrix} Market\ value \\ of\ preferred\ stock \end{matrix} + \begin{matrix} Market\ value \\ of\ debt \end{matrix} - \begin{matrix} Cash\ and \\ short\ term \\ investments \end{matrix}$$

Asset-based Valuation

$$\begin{matrix} Adjusted \\ book\ value \end{matrix} = \begin{matrix} Market\ value \\ of\ assets \end{matrix} - \begin{matrix} Market\ value \\ of\ liabilities \end{matrix}$$

FIXED INCOME

Learning Module 1: Fixed-Income Securities: Defining Elements

Deferred Coupon Bond

Video: https://youtu.be/erRbAUOGIyM

Convertible Bonds

$$\begin{matrix} Conversion \\ ratio \end{matrix} = \frac{Par\ value}{Conversion\ price}$$

$$\begin{matrix} Conversion \\ value \end{matrix} = \begin{matrix} Current\ share \\ price \end{matrix} \times \begin{matrix} Conversion \\ ratio \end{matrix}$$

$$\begin{matrix} Conversion \\ premium \end{matrix} = \begin{matrix} Convertible\ bond \\ price \end{matrix} - \begin{matrix} Conversion \\ value \end{matrix}$$

Learning Module 2: Fixed-Income Markets: Issuance, Trading, and Funding

No formula.

Learning Module 3: Introduction to Fixed-Income Valuation

$$Bond\ Price = \sum_{t=1}^{n} \frac{Coupon}{(1+Z_t)^t} + \frac{Face\ Value}{(1+Z_n)^n}$$

where: Z_t = spot rate or zero rate for period *t*
n = remaining term to maturity

Full Price, Flat Price, and Accrued Interest (Video: https://youtu.be/l7G075JAu5w)

$PV^{Full} = PV^{Flat}$ + Accrued Interest
$= PV_{BOP} \times (1+r)^{t/T}$

where: Accrued Interest $= \frac{t}{T} \times Coupon$
t = number of days from the last coupon payment to the settlement date
T = number of days in the coupon period
t/T = fraction of the coupon period that has gone by since the last payment
PV_{BOP} = price on the previous coupon date (before the settlement date)

Value of Floating Rate Note (FRN)

$$PV = \frac{\left(\frac{Index + QM}{m}\right) \times FV}{\left(1 + \frac{Index + DM}{m}\right)^1} + \frac{\left(\frac{Index + QM}{m}\right) \times FV}{\left(1 + \frac{Index + DM}{m}\right)^2} + \cdots + \frac{\left(\frac{Index + QM}{m}\right) \times FV + FV}{\left(1 + \frac{Index + DM}{m}\right)^n}$$

where QM = Quoted Margin;
DM = Discount Margin
m = periodicity
FV = Face Value of FRN

Video: https://youtu.be/zqYOtVLkYR8

Periodicity Conversion

$$\left(1 + \frac{APR_m}{m}\right)^m = \left(1 + \frac{APR_n}{n}\right)^n$$

$$Current\ yield = \frac{Annual\ Coupon\ payment}{Flat\ price}$$

$$Simple\ yield = \frac{Coupon\ + \left(\frac{FV - PV}{N}\right)}{Flat\ price}$$

$$\begin{matrix} Price\ of \\ callable\ bond \end{matrix} = \begin{matrix} Price\ of \\ option\ free\ bond \end{matrix} - \begin{matrix} Value\ of\ embedded \\ call\ option \end{matrix}$$

Yield Measures for Money Market Instruments

Discount Rate Basis

$$PV = FV \times \left(1 - \frac{Days}{Year} \times DR\right)$$

where:
PV = present value of money market instrument
FV = future value paid at maturity
$Days$ = number of days between settlement and maturity
$Year$ = number of days in the year
DR = discount rate (stated as annual percentage rate)

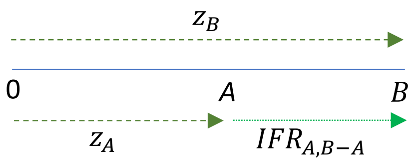
Add-on Rate Basis

$$PV = \frac{FV}{\left(1 + \frac{Days}{Year} \times AOR\right)}$$

Forward Rates, *IFR*

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$$

Note: If YTM is stated as semiannual bond basis, remember to divide YTM by 2



__Yield Spreads__

$$G\ Spread = \begin{matrix} YTM\ of \\ corporate\ bond \end{matrix} - \begin{matrix} YTM\ of \\ government\ bond \end{matrix}$$

$$I\ Spread = \begin{matrix} YTM\ of \\ corporate\ bond \end{matrix} - \begin{matrix} Swap \\ rate \end{matrix}$$

OAS = Z-spread – Option value (in bps)

__Learning Module 4: Introduction to Asset-Backed Securities__

Single Monthly Mortality Rate (SMM)

$$SMM = \frac{Prepayment\ for\ the\ month}{\begin{matrix} Beginning\ outstanding \\ mortgage\ balance \\ for\ the\ month \end{matrix} - \begin{matrix} Scheduled\ principal \\ repayment\ for \\ for\ the\ month \end{matrix}}$$

__Learning Module 5: Understanding Fixed-Income Risk and Return__

__Horizon Yield__

$$\begin{matrix} Bond\ purchase \\ price \end{matrix} = \frac{\begin{matrix} Sum\ of\ reinvested \\ coupon\ payments \end{matrix} + \begin{matrix} Sale\ price\ or \\ redemption\ amount \end{matrix}}{(1 + Horizon\ yield)^{Holding\ period}}$$

Video: https://youtu.be/Bq1FG1jQV2M

Duration Measures

$$Macaulay\ duration = \frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r} - \frac{t}{T}$$

$$Modified\ Duration = \frac{Macaulay\ Duration}{1+r}$$

where r = yield per period
c = coupon rate
N = remaining term to maturity

Video: https://youtu.be/USgjcdCk7Fs

$$\%\Delta PV^{Full} \approx -AnnModDur \times \Delta Yield$$

$$ModDur \approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta Yield) \times (PV_0)}$$

Effective Duration

$$EffDur = \frac{(PV_-) - (PV_+)}{2 \times (\Delta Curve) \times (PV_0)}$$

Portfolio Duration

$$\text{Portfolio duration} = \sum_{i=1}^{n} w_i \times Duration_i$$

where w_i is the weight of the market value of Bond *i*

Key Rate Duration

$$KeyRateDur_k = -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r_k}$$

$$\sum_{k=1}^{n} KeyRateDur_k = EffDur$$

Money Duration

$$\text{Money duration} = AnnModDur \times PV^{full}$$

$$\Delta PV^{Full} \approx -MoneyDur \times \Delta Yield$$

$$\%\Delta PV^{Full} \approx -AnnModDur \times \Delta Yield + \frac{1}{2} \times AnnConvexity \times (\Delta Yield)^2$$

Convexity

$$Convexity \approx \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta Yield)^2 \times (PV_0)}$$

$$\Delta PV^{Full} \approx -MoneyDur \times \Delta Yield + \frac{1}{2} \times MoneyConvexity \times (\Delta Yield)^2$$

$$Effective\ Convexity = \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta Curve)^2 \times (PV_0)}$$

Duration Gap

Duration gap = Macaulay duration – Investment horizon

Learning Module 6: Fundamentals of Credit Analysis

$$Yield\ spread = Liquidity\ premium + Credit\ spread$$

$$\%\Delta PV^{Full} \approx -AnnModDur \times \Delta Spread + \frac{1}{2} \times AnnConvexity \times (\Delta Spread)^2$$

DERIVATIVES

Learning Module 1: Derivative Instrument and Derivatives Market Features

No formula.

Learning Module 2: Forward Commitments and Contingent Claim Features and Instruments

Forward Contract

Long Forward payoff $= S_T - F_0(T)$

Short Forward payoff $= -[S_T - F_0(T)]$

Futures Contract

Long Futures daily mark-to-market $= f_t(T) - f_{t-1}(T)$

Short Futures daily mark-to-market $= -[f_t(T) - f_{t-1}(T)]$

Options Contract

LONG Call option

$$Payoff\ or\ Value\ at\ expiration, c_T = \max(0, S_T - X)$$

$$Profit\ at\ expiration, \Pi = \max(0, S_T - X) - c_0$$

where: c_0 = Call premium
X = Exercise/Strike price
S_T = Spot price at expiration

SHORT Call option

$$Payoff\ or\ Value\ at\ expiration, c_T = -\max(0, S_T - X)$$

$$Profit\ at\ expiration, \Pi = -[\max(0, S_T - X) - c_0]$$

LONG Put option

$$Payoff\ or\ Value\ at\ expiration, p_T = \max(0, X - S_T)$$

$$Profit\ at\ expiration, \Pi = \max(0, X - S_T) - p_0$$

SHORT Put option

$$Payoff\ or\ Value\ at\ expiration, p_T = -\max(0, X - S_T)$$

$$Profit\ at\ expiration, \Pi = -[\max(0, X - S_T) - p_0]$$

Credit Default Swap (CDS)

$$CDS\ MTM\ Change = \Delta CDS\ Spread \times CDS\ Notional \times EffDur_{CDS}$$

In a credit event, payment from CDS seller to CDS buyer ≈ $LGD\ (\%) \times Notional$

Learning Module 3: Derivative Benefits, Risks, and Issuer and Investor Uses

No formula.

Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

If there are no underlying costs or benefits:

$$\text{Forward price, } F_0(T) = S_0(1+r)^T$$

If there are underlying costs or benefits in present value terms:

$$\text{Forward price, } F_0(T) = [S_0 - PV_0(Income) + PV_0(Cost)](1+r)^T$$

where: S_0 = Current spot price
r = Risk-free rate
T = Tenor of forward contract

Under continuous compounding, $F_0(T) = S_0 e^{rT}$
Under continuous compounding, with income (i) and cost (c) expressed in %:

$$F_0(T) = S_0 e^{(r+c-i)T}$$

Foreign Exchange Forward Contract

$$F_{0,f/d}(T) = S_{0,f/d}(T)e^{(r_f - r_d)T}$$

Learning Module 5: Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Value of LONG Forward Prior to Expiration

$$V_0(T) = 0$$

$$V_t(T) = S_t - \frac{F_0(T)}{(1+r)^{T-t}} = S_t - F_0(T) \times (1+r)^{-(T-t)}$$

$$V_T(T) = S_0 - F_0(T)$$

If the asset incurs cost or generates income from time t through maturity,

$$V_t(T) = [S_t - PV_t(Income) + PV_t(Cost)] - F_0(T) \times (1+r)^{-(T-t)}$$

For foreign exchange forward contract,

$$V_t(T) = S_{t,f/d} - F_{0,f/d}(T) \times e^{-(r_f - r_d)(T-t)}$$

Value of SHORT Forward Prior to Expiration

$$V_0(T) = 0$$

$$V_t(T) = -\left[S_t - \frac{F_0(T)}{(1+r)^{T-t}}\right]$$

$$V_T(T) = -[S_0 - F_0(T)]$$

Interest Rate Forward Contracts (Forward Rate Agreements (FRA))

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$$

where: z_A = Spot rate for A periods
z_B = Spot rate for B periods
$IFR_{A,B-A}$ = Implied forward rate for $(B-A)$ periods, starting in A periods

Payoff for a Long FRA = $\left(MRR_{B-A} - IFR_{A,B-A}\right) \times Notional\ principal \times Period$

Payoff for a Short FRA = $-\left(MRR_{B-A} - IFR_{A,B-A}\right) \times Notional\ principal \times Period$

Learning Module 6: Pricing and Valuation of Futures Contracts

If there are no underlying costs or benefits:
Futures price, $f_0(T) = S_0(1+r)^T$

If there are underlying costs or benefits in present value terms:
$$f_0(T) = [S_0 - PV_0(Income) + PV_0(Cost)](1+r)^T$$

Under continuous compounding, $f_0(T) = S_0 e^{rT}$

Under continuous compounding, with income (i) and cost (c) expressed in %:
$$f_0(T) = S_0 e^{(r+c-i)T}$$

Foreign Exchange Forward Contract

$$f_{0,f/d}(T) = S_{0,f/d}(T)e^{(r_f - r_d)T}$$

Interest Rate Futures Contract

$$f_{A,B-A} = 100 - \left(100 \times MRR_{A,B-A}\right)$$

$f_{A,B-A}$ = Futures price for a market reference rate for $(B-A)$ periods that begins in A periods

Futures contract basis point value, $BPV = Notional\ principal \times 0.01\% \times Period$

Learning Module 7: Pricing and Valuation of Interest Rates and Other Swaps

For a **fixed-rate payer** in an interest rate swap:

$$Periodic\ settlement\ value = (MRR - s_N) \times Swap\ Notional \times Period$$

For a **fixed-rate receiver** in an interest rate swap:

$$Periodic\ settlement\ value = (s_N - MRR) \times Swap\ Notional \times Period$$

where: s_N = Fixed swap rate
MRR = Market reference rate

Value of a **pay-fixed** interest rate swap on a settlement date after inception

$$= \begin{matrix} Current\ settlement \\ value \end{matrix} + \Sigma(Floating\ payments) - \Sigma(Fixed\ payments)$$

Value of a **receive-fixed** interest rate swap on a settlement date after inception

$$= \begin{matrix} Current\ settlement \\ value \end{matrix} + \Sigma(Fixed\ payments) - \Sigma(Floating\ payments)$$

Learning Module 8: Pricing and Valuation of Options

Option value = Exercise value + Time value

$$Call\ option\ exercise\ value = Max\left[0, S_t - X(1+r)^{-(T-t)}\right]$$

$$Call\ option\ time\ value = c_t - Max\left[0, S_t - X(1+r)^{-(T-t)}\right]$$

$$Put\ option\ exercise\ value = Max\left[0, X(1+r)^{-(T-t)} - S_t\right]$$

$$Put\ option\ time\ value = p_t - Max\left[0, X(1+r)^{-(T-t)} - S_t\right]$$

Lower bound of call option value $= Max\left[0, S_t - X(1+r)^{-(T-t)}\right]$

Upper bound of call option value $= S_t$

Lower bound of put option value $= Max\left[0, X(1+r)^{-(T-t)} - S_t\right]$

Upper bound of put option value $= X$

Learning Module 9: Option Replication Using Put-Call Parity

Put-Call Parity

$$S_0 + p_0 = c_0 + X(1+r)^{-T}$$

Put-Call Forward Parity

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

Value of the Firm

$$V_0 = c_0 + PV(Debt) - p_0$$

Value of debt $= PV(Debt) - p_0$

Value of equity $= c_0$

Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model

Risk-neutral probability of a price increase in underlying

$$\pi = \frac{1 + r - R^d}{R^u - R^d}$$

where: R^u = Up factor $= \frac{S_1^u}{S_0} > 1$

R^d = Down factor $= \frac{S_1^d}{S_0} < 1$

Video: https://youtu.be/ymUlKgz-rAw

Hedge ratio

$$h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

where: $c_1^u = \max(0, S_1^u - X)$

$c_1^d = \max\left(0, S_1^d - X\right)$

Riskless portfolio with a Call: h of the underlying, S, and short call position, c

$V_0 = hS_0 - c_0$
$V_1^u = hS_1^u - c_1^u$
$V_1^d = hS_1^d - c_1^d$

Riskless portfolio with a Put: h of the underlying, S, and long put position, p

$V_0 = hS_0 + p_0$
$V_1^u = hS_1^u + p_1^u$
$V_1^d = hS_1^d + p_1^d$

Value of a one-period call option

$$c_0 = \frac{\pi c_1^u + (1-\pi)c_1^d}{1+r}$$

Value of a one-period put option

$$p_0 = \frac{\pi p_1^u + (1-\pi)p_1^d}{1+r}$$

where: $p_1^u = \max(0, X - S_1^u)$
$p_1^d = \max\left(0, X - S_1^d\right)$

Video: https://youtu.be/bXEC-78y_AU

ALTERNATIVE INVESTMENTS

Learning Module 1: Categories, Characteristics, and Compensation Structure of Alternative Investments

No formula.

Learning Module 2: Performance Calculation and Appraisal of Alternative Investments

$$Sharpe\ ratio = \frac{Portfolio\ return - Risk\ free\ rate}{Standard\ deviation\ of\ portfolio}$$

$$Sortino\ ratio = \frac{Portfolio\ return - Risk\ free\ rate}{Downside\ deviation\ of\ portfolio}$$

$$MAR\ ratio = \frac{Portfolio\ average\ compounded\ annual\ return\ since\ inception}{Maximum\ drawdown\ since\ inception}$$

$$Calmar\ ratio = \frac{Portfolio\ average\ compounded\ annual\ return\ for\ a\ period}{Maximum\ drawdown\ during\ period}$$

Multiple on Invested Capital

$$MOIC = \frac{Realized\ value\ of\ investment + Unrealized\ value\ of\ investment}{Total\ amount\ of\ invested\ capital}$$

Calculating Hedge Fund Fees and Returns

Management Fee Based on Beginning Market Value

$$\begin{matrix} Management \\ Fee \end{matrix} = \begin{matrix} \%Management \\ Fee \end{matrix} \times \begin{matrix} Beginning\ Market \\ Value \end{matrix}$$

Management Fee Based on Ending Market Value

$$\begin{matrix} Management \\ Fee \end{matrix} = \begin{matrix} \%Management \\ Fee \end{matrix} \times \begin{matrix} Ending\ Market \\ Value \end{matrix}$$

*Incentive Fee Calculated **Independent** of Management Fee*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times Gain$$

*Incentive Fee Calculated **Net** of Management Fee*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times (Gain - Management\ Fee)$$

*Incentive Fee with **Hard Hurdle** (**Independent** of Management Fee)*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times (Gain - Hurdle)$$

*Incentive Fee with **Hard Hurdle** (**Net** of Management Fee)*

$$\begin{matrix} Incentive \\ Fee \end{matrix} = \begin{matrix} \%Incentive \\ Fee \end{matrix} \times (Gain - Management\ Fee - Hurdle)$$

Note: 1) No incentive is paid if hedge fund incurs loss for the year.
2) Gain may be subject to high watermark.

Video: https://youtu.be/0DKmCgsAAdc

Learning Module 3: Private Capital, Real Estate, Infrastructure, Natural Resources, and Hedge Funds

No formula.

PORTFOLIO MANAGEMENT

Learning Module 1: Portfolio Management: An Overview

No formula.

Learning Module 2: Portfolio Risk and Return: Part I

Time-Weighted Rate of Return

$$r_{TW} = [(1 + r_1) \times (1 + r_2) \times \ldots (1 + r_N)]^{1/N-1}$$

where: r_i = Time-weighted return for year i

Annualized Return

$$r_{annual} = \left(1 + r_{weekly}\right)^{52} - 1$$

$$r_{annual} = \left(1 + r_{monthly}\right)^{12} - 1$$

Holding Period Return

$$R = \frac{Price_1 - Price_0 + Income_1}{Price_0}$$

Real Returns

$$r_{real} = \frac{1 + r_{nominal}}{1 + \pi}$$

where π = inflation rate

Indifference Curves

Utility function, $U = E(r) - \frac{1}{2}A\sigma^2$

where: U = utility of an investment
$E(r)$ = expected return
σ^2 = variance of the investment (use decimal)
A = measure of risk aversion

Capital Allocation Line (CAL)

$$E(R_p) = R_f + \left[\frac{E(R_i) - R_f}{\sigma_i}\right]\sigma_p$$

where: $\frac{E(R_i)-R_f}{\sigma_i}$ = market price of risk

Two-asset portfolio

Portfolio expected return, $E(R_p) = w_1R_1 + w_2R_2$

Portfolio variance, $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1, R_2)$

Portfolio standard deviation, $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(R_1, R_2)}$

Note: 1) $Cov(R_1, R_2) = \rho_{12}\sigma_1\sigma_2$

2) *n* securities requires *n* variances and $\frac{n(n-1)}{2}$ covariances

Video: https://youtu.be/lUwulZ9ONC0

A portfolio that consists of the two-asset portfolio and the risk-free asset will have:

$$E(R) = w_PR_P + w_FR_F$$

$$\sigma(R) = w_P\sigma_P$$

Portfolio of Many Risky Assets

$$\sigma_p^2 = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N}\overline{Cov} = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N}\rho\bar{\sigma}^2$$

Condition for adding new asset to current portfolio

$$\frac{E(R_{new}) - R_f}{\sigma_{new}} > \frac{E(R_p) - R_f}{\sigma_p} \times \rho_{new,p}$$

Learning Module 3: Portfolio Risk and Return: Part II

Capital Market Line (CML)

$$E(R_p) = w_f R_f + (1 - w_f)E(R_m) = R_f + \left[\frac{E(R_m) - R_f}{\sigma_m}\right]\sigma_p$$

$$\sigma_p = (1 - w_f)\sigma_m$$

Beta of security *i*

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

Portfolio beta, $\beta_p = \sum_{i=1}^{n} w_i \beta_i$

Total variance = Systematic variance + Nonsystematic variance

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_e^2$$

Total risk, $\sigma_i = \sqrt{\beta_i^2 \sigma_m^2 + \sigma_e^2}$

Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

The Market Model

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Portfolio Performance Appraisal Measures

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

$$\text{Treynor ratio} = \frac{R_p - R_f}{\beta_p}$$

$$M^2 = (R_p - R_f)\frac{\sigma_m}{\sigma_p} + R_f$$

$$M^2\ alpha = \left[(R_p - R_f)\frac{\sigma_m}{\sigma_p} + R_f\right] - R_m$$

Jensen's Alpha, $\alpha_p = R_p - [R_f + \beta_p(R_m - R_f)]$

Security Characteristic Line (SCL)

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f)$$

$$\text{Information ratio} = \frac{\alpha_i}{\sigma_{ei}}$$

Note: The ratio above should be called the Appraisal ratio but for consistency with the textbook, I kept the name as Information ratio.

Learning Module 4: Basics of Portfolio Planning and Construction

No formula.

Learning Module 5: The Behavioral Biases of Individuals

No formula.

Learning Module 6: Introduction to Risk Management

No formula.

Learning Module 7: Technical Analysis

Head and Shoulders Pattern

$$Price\ target = Neckline - (Head - Neckline)$$

Inverse Head and Shoulders Pattern

$$Price\ target = Neckline + (Neckline - Head)$$

Double Top Pattern

$$Price\ target = Valley\ low - (High\ of\ double\ top - Valley\ low)$$

Rate of Change (ROC) Oscillator

$$M = (V - Vx) \times 100$$

where: M = Momentum oscillator value (oscillates above and below 0)
V = Most recent closing price
Vx = Closing price x days ago, typically 10 days

$$M = \frac{V}{Vx} \times 100$$

where: M = Momentum oscillator value (oscillates above and below 100)

Relative Strength Index

$$RSI = 100 - \frac{100}{1 + RS}$$

where:
$$RS = \frac{\Sigma(Up\ changes\ for\ the\ period\ under\ consideration)}{\Sigma(Down\ changes\ for\ the\ period\ under\ consideration)}$$

Stochastic Oscillator

$$\%K = 100\left(\frac{C - L14}{H14 - L14}\right)$$

where:
C = Latest closing price
$L14$ = Lowest price in the past 14 days
$H14$ = Highest price in the past 14 days
$\%D$ = Average of the last three %K values calculated daily

Learning Module 8: Fintech in Investment Management

No formula.