CFA® Program
Level II

FORMULA SHEET (2024) Version 2.0

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FOR REFERENCE ONLY

(Note: Formula Sheet is not provided in the CFA exam)

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CFA Level 2 – Formula Sheet (2024)

Setting Up the Texas BA II Plus Financial Calculator

Video: https://youtu.be/0MS8d8QOFmc

QUANTITATIVE METHODS

Learning Module 1 | Basics of Multiple Regression and Underlying Assumptions

$$Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \cdots + +b_kX_{ki} + \varepsilon_i \qquad i = 1, 2, 3, \dots, n$$

where:
Y = dependent variable
X = independent variable
b_0 = intercept
$b_1, b_2, \dots, b_k$ = slope coefficients
ε = error term
n = number of observations
k = number of independent variables
$b_0, b_1, b_2, \dots, b_k$ = regression coefficients

$$Variation\ of\ Y = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

Learning Module 2 | Evaluating Regression Model Fit and Interpreting Model Results

Coefficient of determination, R^2

$$R^2 = \frac{Sum\ of\ Squares\ Regression}{Sum\ of\ Squares\ Total} = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

$$Sum\ of\ Squares\ Total, \qquad SST = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

$$Sum\ of\ Squares\ Regression, \qquad SSR = \sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2$$

$$Sum\ of\ Squares\ Error, \qquad SSE = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$$

$$Adjusted\ R^2, \quad \bar{R}^2 = 1 - \left[\frac{SSE/(n-k-1)}{SST/(n-1)}\right] = 1 - \left(\frac{n-1}{n-k-1}\right)(1 - R^2)$$

Akaike's information criterion (AIC)

$$AIC = n\ln\left(\frac{Sum\ of\ squares\ error}{n}\right) + 2(k+1)$$

where:
$n =$ Sample size
$k =$ Number of independent variables in the model

Schwarz's Bayesian information criterion (BIC of SBC)

$$BIC = n\ln\left(\frac{Sum\ of\ squares\ error}{n}\right) + \ln(n)\,(k+1)$$

F-distributed test statistic for jointly omitted variables

$$F = \frac{(Sum\ of\ squares\ error\ restricted\ model - Sum\ of\ squares\ unrestricted)/q}{Sum\ of\ squares\ unrestricted\ model/(n-k-1)}$$

where:
$q =$ Number of restrictions (i.e., number of variables omitted in the restricted model compared to the unrestricted model)

$H_0: b_m = b_{m+1} = \cdots = b_{m+q-1} = 0$
H_a: At least one of the q slopes $\neq 0$

F-test for joint test of slope coefficients

ANOVA	df	SS	MS	*F*
Regression	k	SSR	SSR/k	$\frac{SSR/k}{SSE/(n-k-1)}$
Residual	$n-k-1$	SSE	$SSE/(n-k-1)$	
Total	$n-1$	SST		

$$F\ statistic = \frac{Mean\ Square\ Regression}{Mean\ Square\ Error} = \frac{SSR/k}{SSE/(n-k-1)}$$

$H_0: b_1 = b_2 = \cdots = b_k = 0$
H_a: At least one $b_j \neq 0$

t-test statistic for slope coefficient

$$t = \frac{\hat{b}_j - B_j}{s_{\hat{b}_j}}$$

where:
$\hat{b}_j$ = Regression estimate of b_j
B_j = Hypothesized value of coefficient j
$s_{\hat{b}_j}$ = Estimated standard error of $\hat{b}_j$

Video (Simple Linear Regression): https://youtu.be/uR_9im2JP18

Learning Module 3 | Model Misspecification

Breusch-Pagan Test

$$Test\ Statistic, \qquad \chi^2_{BP,k} = nR^2$$

where:
R^2 = R-squared between squared residuals and independent variables

Variance Inflation Factor (VIF)

$$VIF_j = \frac{1}{1 - R_j^2}$$

where:
R_j^2 = Variation in X_j explained by the other $k - 1$ independent variables

Learning Module 4 | Extensions of Multiple Regression

Detecting Influential Points
Sum of individual leverages for all observations $= k + 1$

If observation's leverage $> 3\left(\frac{k+1}{n}\right) \Rightarrow$ Potentially influential observation

Studentized Deleted Residual, t_{i^*}

$$t_{i^*} = \frac{e_i^*}{s_{e^*}} = \frac{e_i}{\sqrt{MSE_{(i)}(1 - h_{ii})}} \sqrt{\frac{n - k - 1}{SSE(1 - h_{ii}) - e_i^2}}$$

where:
e_i^* = The residual with the ith observation deleted
s_{e^*} = The standard deviation of the residuals
k = The number of independent variables
$MSE_{(i)}$ = Mean squared error of the regression model that deletes the ith observation
h_{ii} = The leverage value for the ith observation

Cook's Distance

$$D_i = \frac{e_i^2}{(k+1)MSE}\left[\frac{h_{ii}}{(1-h_{ii})^2}\right]$$

where:
e_i = Residual for observation i
k = The number of independent variables
MSE = Mean square error of the estimated regression model
h_{ii} = The leverage value for the ith observation

If $D_i > \sqrt{k/n}$, then ith observation is highly likely to be an influential data point

Logistic Regression (Logit)

$$\ln\left(\frac{P}{1-P}\right) = b_0 + b_1X_1 + b_2X_2 + \cdots + b_kX_k + \varepsilon$$

$$P = \frac{1}{1+\exp[-(b_0 + b_1X_1 + b_2X_2 + \cdots + b_kX_k + \varepsilon)]}$$

$$\ln\left(\frac{P}{1-P}\right) = Log\ odds$$

$$Odds\ ratio = e^{b_i}$$

Likelihood ratio (LR) test

$$LR = -2(Log\ likelihood\ restricted\ model - Log\ likelihood\ unrestricted\ model)$$

Learning Module 5 | Time-Series Analysis

Linear Trend Models

$$Y_t = b_0 + b_1t + \varepsilon_t \qquad t = 1,2,\ldots,T$$

t = time (independent variable)

Log-Linear Trend Models

$$Y_t = e^{b_0+b_1t} \qquad t = 1,2,\ldots,T$$

$$\ln Y_t = b_0 + b_1t$$

Growth rate of $Y = e^{b_1} - 1$

p-th order autoregressive model, $AR(p)$

$$x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-2} + \cdots + b_p x_{t-p} + \varepsilon_t$$

Test statistic for autocorrelation of residuals

$$t = \frac{Residual\ autocorrelation - 0}{Standard\ error} = \frac{Residual\ autocorrelation}{1/\sqrt{T}}$$

where:

$$\rho_{\varepsilon,k} = \frac{Cov(\varepsilon_t, \varepsilon_{t-k})}{\sigma_\varepsilon^2} = k^{th}\ order\ autocorrelation\ of\ the\ residual$$

Mean reverting level for AR(1) model

$$x_t = \frac{b_0}{1 - b_1}$$

Root Mean Squared Error

$$RMSE = \sqrt{\frac{Squared\ error}{n}}$$

Dickey and Fuller Unit-Root Test

$$x_t - x_{t-1} = b_0 + g_1 x_{t-1} + \varepsilon_t$$

where:

$g_1 = b_1 - 1$

ARCH(1):

$$\hat{\sigma}_{t+1}^2 = a_0 + a_1 \hat{\epsilon}_t^2$$

Learning Module 6 | Machine Learning

Neural Networks

$$\begin{matrix} New\ network \\ weight \end{matrix} = Old\ weight - Learning\ rate \times \begin{pmatrix} Partial\ derivative\ of\ the \\ total\ error\ with\ respect \\ to\ the\ old\ weight \end{pmatrix}$$

Learning Module 7 | Big Data Projects

Normalization of variable X

$$X_{i\,(normalized)} = \frac{X_i - X_{min}}{X_{max} - X_{min}}$$

Standardization of variable X

$$X_{i\,(standardized)} = \frac{X_i - \mu}{\sigma}$$

$$Precision, P = \frac{True\ Positive}{True\ Positive + False\ Positive}$$

$$Recall, R = \frac{True\ Positive}{True\ Positive + False\ Negative}$$

$$Accuracy = \frac{True\ Positive + True\ Negative}{True\ Positive + False\ Positive + True\ Negative + False\ Negative}$$

$$F1\ Score = \frac{2 \times Precision \times Recall}{Precision + Recall}$$

$$False\ Positive\ Rate, FPR = \frac{False\ Positive}{True\ Negative + False\ Positive}$$

$$True\ Positive\ Rate, TPR = \frac{True\ Positive}{True\ Positive + False\ Negative}$$

$$Document\ Frequency, DF = \frac{Sentence\ Count\ with\ Word}{Total\ Number\ of\ Sentences}$$

$$Inverse\ Document\ Frequency, IDF = \log\left(\frac{1}{DF}\right)$$

$$TF\text{-}IDF = TF \times IDF$$

ECONOMICS

Learning Module 1 | Currency Exchange Rates: Understanding Equilibrium Value

Cross Rates

When given $\frac{A}{C}$ and $\frac{C}{B}$, then $\frac{A}{B} = \frac{A}{C} \times \frac{C}{B}$

When given $\frac{A}{C}$ and $\frac{B}{C}$, then $\frac{A}{B} = \frac{A}{C} \times \frac{1}{\left(\frac{B}{C}\right)}$

Currency pair	Bid	Bid/Ask
A/B	x	y
B/A	$1/y$	$1/x$

Video: https://youtu.be/wyDKKPkPhzw

Arbitrage Opportunities Between Dealers and Interbank

Video: https://youtu.be/Lqo9UZ3yyEA

Covered Interest Rate Parity

$$F_{f/d} = S_{f/d}\left[\frac{1 + i_f\left(\frac{Actual}{360}\right)}{1 + i_d\left(\frac{Actual}{360}\right)}\right]$$

$$\frac{F_{f/d} - S_{f/d}}{S_{f/d}} = \frac{(i_f - i_d)\left(\frac{Actual}{360}\right)}{1 + i_d\left(\frac{Actual}{360}\right)}$$

Video: https://youtu.be/9jOzFA9GuHU

Mark-to-Market Value of a Forward Contract

Original position: **Long** base currency *d* forward at forward rate $F_{0,f/d}$ (Offer side)

$$Value\ of\ Long\ Forward = \frac{\left(F_{t,f/d} - F_{0,f/d}\right) \times Contract\ Size}{1 + i_f\left(\frac{Remaining\ days\ to\ maturity}{360}\right)}$$

$F_{t,f/d}$ = Forward rate at valuation date, t (Bid side)

Video: https://youtu.be/wLqyZRrutAc

Uncovered Interest Rate Parity

$$E(S_{f/d}) = S_{f/d}\left[\frac{1 + i_f\left(\frac{Actual}{360}\right)}{1 + i_d\left(\frac{Actual}{360}\right)}\right]$$

$$\%\Delta S^e_{f/d} = \frac{(i_f - i_d)\left(\frac{Actual}{360}\right)}{1 + i_d\left(\frac{Actual}{360}\right)} \approx (i_f - i_d)\left(\frac{Actual}{360}\right)$$

Video (Carry Trade): https://youtu.be/_26fG3Zvzyg

Absolute PPP

$$S_{f/d} = \frac{P_f}{P_d}$$

Relative PPP

$$\%\Delta S_{f/d} = \frac{(\pi_f - \pi_d)\left(\frac{Actual}{360}\right)}{1 + \pi_d\left(\frac{Actual}{360}\right)} \approx (\pi_f - \pi_d)\left(\frac{Actual}{360}\right)$$

Ex ante PPP

$$\%\Delta S^e_{f/d} \approx (\pi^e_f - \pi^e_d)\left(\frac{Actual}{360}\right)$$

International Fisher Effect

$$i_f - i_d = \pi^e_f - \pi^e_d$$

where:
π^e = Expected inflation rate
π = Actual inflation rate

Mundell-Fleming Model

Bonus Video: https://youtu.be/xNo3GpWYgKA

Learning Module 2 | Economic Growth and the Investment Decision

Grinold-Kroner Model

$$E(R_e) = DY + \Delta(P/E) + i + g - \Delta S$$

where:
$E(R_e)$ = Expected equity return
DY = Dividend yield
$\Delta(P/E)$ = Expected repricing
i = Expected inflation rate
g = Real economic growth rate
ΔS = Change in shares outstanding

Dilution effect

$$\Delta S = Net\ buyback + Relative\ dynamism$$

Cobb-Douglas Production Function

$$Y = TK^{\alpha}L^{1-\alpha} \qquad \text{where a < 1}$$

where:
Y = Output
α = Share of output allocated to capital (K)
$1-\alpha$ = share of output allocated to labor (L)
T = total factor productivity (TFP), represents technological progress of the economy

$$\textbf{Output per worker} = \frac{Y}{L} = T\left(\frac{K}{L}\right)^{\alpha}$$

$$\begin{matrix} Percentage\ change\ in \\ labor\ productivity \end{matrix} = \begin{matrix} Percentage\ change\ in \\ total\ factor\ productivity \end{matrix} + \begin{matrix} Percentage\ change\ in \\ capital\ deepening \end{matrix}$$

$$\frac{\Delta(Y/L)}{Y/L} = \frac{\Delta T}{T} + \alpha\frac{\Delta(K/L)}{K/L}$$

Marginal product of capital, MPK

$$MPK = \alpha\left(\frac{Y}{K}\right)$$

Amount of output that is allocated to providers of capital, *a*

$$\alpha = \frac{rK}{Y}$$

Growth Accounting equation:

$$\frac{\Delta Y}{Y} = \frac{\Delta T}{T} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

Growth rate in potential GDP = Long-term growth rate of **labor force** + Long-term growth rate in **labor productivity**

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{\Delta(Y/L)}{Y/L}$$

$$Labor\ force\ participation = \frac{Labor\ force}{Working\ age\ population}$$

Sustainable growth rate of **output per capita**

$$g^* = \frac{\theta}{1-\alpha}$$

Sustainable growth rate of output (Steady state growth rate)

$$G^* = \frac{\theta}{1-\alpha} + n$$

Equilibrium output-to-capital ratio (in **steady state**):

$$\frac{Y}{K} = \frac{1}{s}\left[\frac{\theta}{1-\alpha} + n + \delta\right]$$

where:
θ = growth rate of TFP
α = elasticity of output with respect to capital
s = fraction of income (Y) that is saved
δ = rate of depreciation of physical capital stock
n = labor supply growth = $\%\Delta L$

Endogeneous Growth Model

Production function:

$$y_e = ck_e$$

Growth rate of output per capita:

$$\frac{\Delta y_e}{y_e} = \frac{\Delta k}{k_e} = sc - \delta - n$$

where:
y_e = output per worker
k_e = capital per worker
c = marginal product of capital in the aggregate economy (constant)

FINANCIAL STATEMENT ANALYSIS

Learning Module 1 | Intercorporate Investments

Investments in Associates (Equity Method)

$$\begin{matrix} Ending \\ investment\ in \\ associates \end{matrix} = \begin{matrix} Beginning \\ investment\ in \\ associates \end{matrix} + \begin{matrix} Share\ of \\ net\ income \end{matrix} - \begin{matrix} Share\ of \\ dividend \\ received \end{matrix} - \begin{matrix} Amortization\ of\ excess \\ purchase\ price \end{matrix}$$

Impact on **Investor's** Income Statement

$$= \begin{matrix} Share\ of \\ net\ income \end{matrix} - \begin{matrix} Amortization\ of\ excess \\ purchase\ price \end{matrix} - \begin{matrix} Share\ of\ Unrealized\ profit\ from \\ downstream\ or\ upstream\ sale \end{matrix}$$

Business Combinations (Acquisition Method)

Excess purchase price
= Acquisition price – %Ownership × **Book** value of net identifiable assets

Partial Goodwill
= Acquisition price – (%Ownership × **Fair** value of identifiable net assets)
= Acquisition price – (%Ownership × **Book value** of identifiable net assets)
– (%Ownership × Excess purchase price attributable to identifiable net assets)

$$Non\ controlling\ interest = \%NCI \times Fair\ value\ of\ identifiable\ net\ assets$$

Full Goodwill
= Fair value of entity – **Fair** value of net identifiable assets

$$Non\ controlling\ interest = \%NCI \times Fair\ value\ of\ entity$$

Video: https://youtu.be/RgxmPbx4-0o

IFRS

$$\begin{matrix} Impairment \\ loss \end{matrix} = \begin{matrix} Carrying\ value\ of \\ cash\ generating\ unit \end{matrix} - \begin{matrix} Recoverable\ amount\ of \\ cash\ generating\ unit \end{matrix}$$

US GAAP

$$\begin{matrix} Implied \\ goodwill \end{matrix} = \begin{matrix} Fair\ value\ of \\ reporting\ unit \end{matrix} - \begin{matrix} Fair\ value\ of\ reporting\ unit's \\ identifiable\ net\ assets \end{matrix}$$

$$\begin{matrix} Impairment \\ loss \end{matrix} = \begin{matrix} Carrying\ value \\ of\ goodwill \end{matrix} - \begin{matrix} Implied \\ goodwill \end{matrix}$$

Learning Module 2 | Employee Compensation - Post-Employment and Share-Based

Share-Based Compensation Accounting

$$Compensation\ expense = \frac{Fair\ value\ of\ award\ on\ grant\ date}{Vesting\ period}$$

Treasury Stock Method

$$\begin{matrix} Diluted \\ shares \\ outstanding \end{matrix} = \begin{matrix} Basic \\ shares \\ outstanding \end{matrix} + \begin{matrix} Shares\ issued\ from \\ conversion\ or\ exercise \\ of\ share\ based\ awards \end{matrix} - \frac{\begin{pmatrix} Proceeds\ from \\ conversion\ or\ exercise \\ of\ share\ based\ awards \end{pmatrix}}{\begin{matrix} Average\ share\ price\ for \\ the\ reporting\ period \end{matrix}}$$

$$\begin{matrix} Assumed \\ proceeds \end{matrix} = \begin{matrix} Cash\ proceeds \\ from\ exercise \end{matrix} + \begin{matrix} Average\ unrecognized \\ share\text{-}based \\ compensation\ expense \end{matrix}$$

Forecasting Shares Outstanding With Share-Based Awards

$$\begin{matrix} Basic\ shares \\ outstanding, \\ end\ of\ period \end{matrix} = \begin{matrix} Basic\ shares \\ outstanding, \\ beginning\ of\ period \end{matrix} + RSUs\ vested + \begin{matrix} Share\ options \\ exercised \end{matrix} + \begin{matrix} Shares\ issued\ from \\ secondaries, \\ acquisitions,\ etc. \end{matrix} - \begin{matrix} Share \\ repurchases \end{matrix}$$

Financial Reporting for Defined Benefit Pension Plans

$$Funded\ status = Fair\ value\ of\ plan\ assets - Pension\ obligation$$

$$\begin{matrix} Ending\ fair\ value \\ of\ plan\ assets \end{matrix} = \begin{matrix} Beginning\ fair\ value \\ of\ plan\ assets \end{matrix} + Contributions + \begin{matrix} Actual\ return \\ on\ plan\ assets \end{matrix} - \begin{matrix} Benefits \\ paid \end{matrix}$$

$$\begin{matrix} Ending\ pension \\ obligation \end{matrix} = \begin{matrix} Beginning\ pension \\ obligation \end{matrix} + \begin{matrix} Service \\ costs \end{matrix} + \begin{matrix} Interest \\ cost \end{matrix} + \begin{matrix} Actuarial \\ loss/(gain) \end{matrix} - \begin{matrix} Benefits \\ paid \end{matrix}$$

$$Interest\ cost = \begin{matrix} Beginning\ pension \\ obligation \end{matrix} \times \begin{matrix} Yield\ on\ investment\ grade \\ corporate\ bond \end{matrix}$$

IFRS Only

In income statement,

$$Reported\ pension\ expense = \begin{matrix}Current\ and\ past\\ service\ costs\end{matrix} + \begin{matrix}Net\ interest\\ cost/(income)\end{matrix}$$

$$\begin{matrix}Net\ interest\\ cost/(income)\end{matrix} = \left(\begin{matrix}Beginning\ pension\\ obligation\end{matrix} - \begin{matrix}Beginning\ fair\ value\\ of\ plan\ assets\end{matrix}\right) \times \begin{matrix}Yield\ on\ investment\\ grade\ corporate\ bond\end{matrix}$$

Learning Module 3 | Multinational Operations

Net assets = Total assets – Total liabilities

Net monetary assets = Monetary assets – Monetary liabilities

Current Rate Method

Currency translation adjustment
= Total assets of foreign subsidiary in parent currency terms
— Total liabilities of foreign subsidiary in parent currency terms
— Shareholder capital of foreign subsidiary in parent currency terms
— Other equity items of foreign subsidiary in parent currency terms

Hyperinflationary Environment

IFRS

$$\begin{matrix}Restatement\ factor\ for\\ monetary\ assets\ \&\ liabilities\end{matrix} = \frac{End\ of\ period\ price\ index}{End\ of\ period\ price\ index}$$

$$\begin{matrix}Restatement\ factor\ for\\ non\text{-}monetary\ assets\ \&\ liabilities\end{matrix} = \frac{End\ of\ period\ price\ index}{Beginning\ of\ period\ price\ index}$$

$$\begin{matrix}Restatement\ factor\ for\\ income\ statement\ items\end{matrix} = \frac{End\ of\ period\ price\ index}{Average\ price\ index\ for\ the\ period}$$

Learning Module 4 | Analysis of Financial Institutions

$$\begin{matrix} Total\ Tier\ 1 \\ Capital \end{matrix} = \begin{matrix} Common\ Equity \\ Tier\ 1\ Capital \end{matrix} + \begin{matrix} Additional \\ Tier\ 1\ Capital \end{matrix}$$

$$\begin{matrix} Total\ Regulatory \\ Capital \end{matrix} = \begin{matrix} Total\ Tier\ 1 \\ Capital \end{matrix} + \begin{matrix} Total\ Tier\ 2 \\ Capital \end{matrix}$$

$$\begin{matrix} Common\ Equity \\ Tier\ 1\ Ratio \end{matrix} = \frac{Common\ Equity\ Tier\ 1\ Capital}{Risk\ Weighted\ Assets} \geq 4.5\%$$

$$\begin{matrix} Tier\ 1 \\ Ratio \end{matrix} = \frac{Total\ Tier\ 1\ Capital}{Risk\ Weighted\ Assets} \geq 6.0\%$$

$$\begin{matrix} Total\ Capital \\ Ratio \end{matrix} = \frac{Total\ Regulatory\ Capital}{Risk\ Weighted\ Assets} \geq 8.0\%$$

$$\begin{matrix} Liquidity\ Coverage \\ Ratio, LCR \end{matrix} = \frac{High\ Quality\ Liquid\ Assets}{Expected\ cash\ outflows}$$

$$\begin{matrix} Number\ of\ days\ that\ bank\ can\ withstand \\ a\ stress\ level\ volume\ of\ cash\ outflows \end{matrix} = LCR \times 30$$

Number of days that bank can withstand a stress level volume of cash outflows for ($LCR \times 30$) days.

$$\begin{matrix} Net\ Stable\ Funding \\ Ratio, NSFR \end{matrix} = \frac{Available\ Stable\ Funding}{Required\ Stable\ Funding}$$

Property and Casualty Companies

$$\begin{matrix} Loss\ and\ loss\ adjustment \\ expense\ ratio \end{matrix} = \frac{Loss\ expense + Loss\ adjustment\ expense}{Net\ premiums\ earned}$$

$$\begin{matrix} Underwriting \\ expense\ ratio \end{matrix} = \frac{Underwriting\ expense}{Net\ premiums\ written}$$

$$\begin{matrix} Combined \\ ratio \end{matrix} = \begin{matrix} Loss\ and\ loss\ adjustment \\ expense\ ratio \end{matrix} + \begin{matrix} Underwriting \\ expense\ ratio \end{matrix}$$

$$\begin{matrix} Dividends\ to\ policyholders \\ (shareholders)\ ratio \end{matrix} = \frac{Dividends\ to\ policyholders\ (shareholders)}{Net\ premiums\ earned}$$

$$\begin{matrix} Combined\ ratio \\ \boldsymbol{after\ dividends} \end{matrix} = \begin{matrix} Combined \\ ratio \end{matrix} + \begin{matrix} Dividends\ to\ policyholders \\ (shareholders)\ ratio \end{matrix}$$

Learning Module 5 | Evaluating Quality of Financial Reports

Beneish Model

$$M\text{-}score = -4.84 + 0.920\,(DSR) + 0.528\,(GMI) + 0.404\,(AQI) + 0.892\,(SGI) + 0.115\,(DEPI) - 0.172\,(SGAI) + 4.670\,(Accruals) - 0.327\,(LEVI)$$

where:

$$\text{DSR (day sales receivable index)} = \frac{Receivables_t/Sales_t}{Receivables_{t-1}/Sales_{t-1}}$$

$$\text{GMI (gross margin index)} = \frac{GM_{t-1}}{GM_t}$$

$$\text{AQI (asset quality index)} = \frac{[1-(PPE_t + CA_t)/TA_t]}{[1-(PPE_{t-1} + CA_{t-1})/TA_{t-1}]}$$

$$\text{SGI (sales growth index)} = \frac{Sales_t}{Sales_{t-1}}$$

$$\text{DEPI (depreciation index)} = \frac{Depreciation_{t-1}}{Depreciation_t}$$

$$\text{SGAI (sales, general, and administrative expenses index)} = \frac{SGA_t/Sales_t}{SGA_{t-1}/Sales_{t-1}}$$

$$\text{Accruals} = \frac{Income\ before\ extraordinary\ items\ - Cash\ from\ operations}{Total\ assets}$$

$$\text{LEVI (leverage index)} = \frac{Leverage_t}{Leverage_{t-1}}$$

Earnings Persistence

$$Earnings_{t+1} = \alpha + \beta(Earnings_t) + \varepsilon$$

$$Earnings_{t+1} = \alpha + \beta_1(Cash\ flow_t) + \beta_2(Accruals_t) + \varepsilon$$

Cash-flow-based accruals = NI – (CFO + CFI)

Learning Module 6 | Integration of Financial Statement Analysis Techniques

$$\begin{aligned}\begin{matrix}Net\ Operating\\ Assets\ (NOA)\end{matrix} &= \begin{matrix}Operating\\ Assets\end{matrix} - \begin{matrix}Operating\\ Liabilities\end{matrix}\\ &= \left(\begin{matrix}Total\\ Assets\end{matrix} - \begin{matrix}Cash\ and\\ Short\text{-}term\ Investments\end{matrix}\right) - \left(\begin{matrix}Total\\ Liabilities\end{matrix} - \begin{matrix}Total\\ Debt\end{matrix}\right)\end{aligned}$$

$$\begin{matrix}Balance\text{-}sheet\text{-}based\\ accruals\ ratio\end{matrix} = \frac{NOA_t - NOA_{t-1}}{(NOA_t + NOA_{t-1})/2}$$

$$\begin{matrix}Cash\text{-}flow\text{-}based\\ accruals\ ratio\end{matrix} = \frac{NI_t - (CFO_t + CFI_t)}{(NOA_t + NOA_{t-1})/2}$$

Learning Module 7 | Financial Statement Modeling

Growth Relative to GDP Growth approach

If company's revenue is forecast to grow at K bps above the nominal GDP growth rate ($g\%$), then company's revenue growth rate $= g\% + \frac{K}{100}\%$

If company's revenue is forecast to grow $H\%$ faster than the nominal GDP growth rate ($g\%$), then company's revenue growth rate $= g\% \times \left(1 + \frac{H}{100}\right)$

Market Growth and Market Share approach

$$Forecast\ revenue = Market\ share\ (in\ \%) \times Industry\ revenue$$

Return on Invested Capital

$$\mathrm{ROIC} = \frac{NOPLAT}{Invested\ Capital}$$

where:

$NOPLAT$ = Net operating profit less adjusted taxes

$Invested\ Capital = Operating\ assets - Operating\ liabilities$

CORPORATE ISSUERS

Learning Module 1 | Analysis of Dividends and Share Repurchases

Dividend Payout Policies

Target payout adjustment model (Lintner model)

$$\begin{matrix}Expected\\dividend\end{matrix} = \begin{matrix}Last\\dividend\end{matrix} + \left(\begin{matrix}Expected\\Earnings\end{matrix} \times \begin{matrix}Target\ payout\\ratio\end{matrix} - \begin{matrix}Last\\dividend\end{matrix}\right) \times \begin{matrix}Adjustment\\factor\end{matrix}$$

where:

$$Adjustment\ factor = \frac{1}{Number\ of\ years\ for\ adjustment\ to\ take\ place}$$

Constant dividend payout ratio policy

$$Dividend = \begin{matrix}Dividend\\payout\ ratio\end{matrix} \times \begin{matrix}Current\\earnings\end{matrix}$$

Video: https://youtu.be/hhcvNiTpZX4

EPS and BVPS After Share Repurchase

$$EPS\ after\ buyback = \frac{Earnings\ before\ buyback - After\ tax\ cost\ of\ funds}{Shares\ outstanding\ after\ buyback}$$

Video: https://youtu.be/Pd0-QQF-VhQ

$$BVPS\ after\ buyback = \frac{Book\ Value\ before\ buyback - Value\ of\ share\ buyback}{Shares\ outstanding\ after\ buyback}$$

Analysis of Dividend Safety

$$Dividend\ payout\ ratio = \frac{Dividends}{Net\ Income}$$

$$Dividend\ coverage\ ratio = \frac{Net\ Income}{Dividends}$$

$$FCFE\ coverage\ ratio = \frac{FCFE}{Dividends + Share\ repurchases}$$

Learning Module 3 | Cost of Capital: Advanced Topics

Weighted average cost of capital

$$WACC = w_d r_d(1-t) + w_p r_p + w_e r_e$$

where:
w_d = Weight of debt in capital structure
w_p = Weight of preferred equity in capital structure
w_e = Weight of common equity in capital structure
r_d = Pre-tax cost of debt
r_p = Cost of preferred equity
r_e = Cost of common equity

Cost of debt, $r_d = r_f + Credit\ spread$

Cost of equity, $r_e = r_f + ERP + IRP$

where:
ERP = Equity risk premium = $\begin{matrix} Benchmark\ index \\ return \end{matrix} - \begin{matrix} Risk\ free \\ rate \end{matrix}$
IRP = Idiosyncratic risk premium

Leases

$$\begin{matrix} Present\ Value\ of \\ Lease\ Payments \end{matrix} + \begin{matrix} Present\ Value\ of \\ Residual\ Value\ to \\ Lessor \end{matrix} = \begin{matrix} Fair\ Value\ of \\ Leased\ Asset \end{matrix} + \begin{matrix} Lessor's\ Direct \\ Initial\ Costs \end{matrix}$$

Equity Risk Premium

Historical Approach (Ex-Post)

$$ERP = \begin{matrix} Average\ benchmark \\ index\ return \end{matrix} - \begin{matrix} Average\ risk \\ free\ rate \end{matrix}$$

Gordon Growth model

$$ERP = \frac{D_1}{V_0} + g - r_f$$

Grinold-Kroner Model

$$ERP = [DY + \text{Expected repricing} + Earnings\ growth\ per\ share] - r_f$$

$$ERP = [DY + \Delta(P/E) + i + g - \Delta S] - r_f$$

$$Earnings\ growth\ per\ share = i + g - \Delta S$$

where:
DY = Dividend yield of market index
$\Delta(P/E)$ = Expected growth rate in P/E
i = Expected inflation = $\frac{1+YTM_{Treasury\ bond}}{1+YTM_{TIPS}} - 1$
g = Expected growth rate in real earnings per share
ΔS = Expected change in shares outstanding ($\Delta S > 0$ for share issuance; $\Delta S < 0$ for share buyback)

Cost of Equity

Gordon Growth Model

$$r_e = \frac{D_1}{P_0} + g$$

Two-Stage DDM

$$P_0 = \sum_{i=1}^{n} \frac{D_t}{(1+r_e)^t} + \frac{P_n}{(1+r_e)^n}$$

Bond Yield Plus Risk Premium Approach (BYPRP)

$$r_e = r_d + Risk\ premium$$

where r_d = Cost of company's long-term debt

Capital Asset Pricing Model (CAPM)

$$r_e = r_f + \beta \times ERP$$

Fama-French model

Three-factor model

$$r_e = r_f + \beta_1 ERP + \beta_2 SMB + \beta_3 HML$$

Five-factor model

$$r_e = r_f + \beta_1 ERP + \beta_2 SMB + \beta_3 HML + \beta_4 RMW + \beta_5 CMA$$

where:
SMB = Size premium
HML = Value premium
RMW = Profitability premium
CMA = Investment premium

Expanded CAPM

$$r_e = r_f + \beta_{peer}(ERP) + SP + IP + SCRP$$

where:
SP = Size premium (for smaller, privately held companies)
IP = Industry risk premium
$SCRP$ = Company-specific risk premium

Build-Up Approach

$$r_e = r_f + ERP + SP + SCRP$$

Country Spread Model

$$ERP = \begin{matrix} ERP\ for\ a \\ developed\ market \end{matrix} + \lambda \times Country\ risk\ premium$$

where:
λ = Level of exposure of the company in the local country
$Country\ risk\ premium = Sovereign\ yield\ spread$

$$\begin{matrix} Sovereign\ yield \\ spread \end{matrix} = \begin{matrix} Yield\ on\ emerging\ market\ bonds \\ (denominated\ in\ the\ currency \\ of\ the\ developed\ market) \end{matrix} - \begin{matrix} Yield\ on\ developed\ market \\ government\ bonds \end{matrix}$$

Aswath Damodaran's CRP

$$Country\ risk\ premium = Sovereign\ yield\ spread \times \frac{\sigma_{Equity}}{\sigma_{Bond}}$$

where:
σ_{Equity} = Volatility of the local country's equity market
σ_{Bond} = Volatility of the local country's bond market

International CAPM

$$E(r_e) = r_f + \beta_G\left[E\left(r_{gm}\right) - r_f\right] + \beta_C\left[E(r_C) - r_f\right]$$

where:
$E\left(r_{gm}\right) - r_f$ = Risk premium of a global index
r_C = Wealth-weighted foreign currency index return

Learning Module 4 | Corporate Restructuring

Evaluating Materiality Based on Size

For **Acquisition/Divestiture**:

$$\frac{Value\ of\ transaction}{Enterprise\ value\ of\ acquiring\ company}$$

For **Cost Restructuring**:

$$\frac{Cost\ savings}{Sales}$$

Premium Paid Analysis

$$Takeover\ premium,\ PRM = \frac{DP - SP}{SP}$$

where:
$DP =$ Deal price per share of the target company
$SP =$ Unaffected stock price of the target company (i.e., pre-announcement)

EQUITY VALUATION

Learning Module 1 | Equity Valuation Applications and Processes

$V_E - P = (V - P) + (V_E - V)$

where:
V_E = Estimated intrinsic value
P = Market price
V = Intrinsic value

Conglomerate discount = Sum-of-the-parts value – Market value

Learning Module 2 | Discounted Dividend Valuation

Discounted Dividend Valuation

$$V_0 = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}$$

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \cdots + \frac{D_n}{(1+r)^n} + \frac{P_n}{(1+r)^n}$$

Gordon Growth Model

$$V_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$

Fixed-rate perpetual preferred stock

$$V_0 = \frac{D}{r}$$

Value of stock = Value of a company with **zero-growth** + Present value of growth opportunities (PVGO)

$$V_0 = \frac{E_1}{r} + PVGO$$

$$\frac{V_0}{E_1} = \frac{P_0}{E_1} = \frac{1}{r} + \frac{PVGO}{E_1}$$

If dividend and earnings growth rate is constant,

$$r = \frac{D_1}{P_0} + g$$

Two-Stage Dividend Discount Model

$$V_0 = \sum_{t=1}^{n} \frac{D_0(1+g_s)^t}{(1+r)^t} + \frac{D_0(1+g_s)^n(1+g_L)}{(1+r)^n(r-g_L)}$$

Video: https://youtu.be/7vXWsTKiSPE

The H-Model

$$V_0 = \frac{D_0(1+g_L) + D_0H(g_S-g_L)}{r-g_L}$$

where:
H = half-life in years of the high-growth period
g_S = Short-term growth-rate
g_L = Long-term growth rate

Video: https://youtu.be/IAMFZXSPKOY

PRAT model

Sustainable growth rate, $g = b \times ROE$

Video: https://youtu.be/MnfRRRhuGpA

$$g = \frac{NI - Dividends}{NI} \times \frac{NI}{Sales} \times \frac{Sales}{TA} \times \frac{TA}{TE}$$

Learning Module 3 | Free Cash Flow Valuation

Free Cash Flow to the Firm (FCFF) Valuation Approach

$$Firm\ Value = \sum_{t=1}^{\infty} \frac{FCFF_t}{(1+WACC)^t}$$

If non-operating assets = 0

Equity Value = Firm Value – Market Value of Debt

FCFE Valuation Approach

$$Equity\ Value = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r)^t}$$

Single-Stage (Constant Growth) FCFF and FCFE Model

FCFF Valuation Approach

$$Firm\ Value = \frac{FCFF_1}{WACC - g} = \frac{FCFF_0(1+g)}{WACC - g}$$

FCFE Valuation Approach

$$Equity\ Value = \frac{FCFE_1}{r - g} = \frac{FCFE_0(1+g)}{r - g}$$

Free cash flow to the Firm, FCFF

FCFF = NI + NCC + Int(1 – Tax Rate) – FCInv – WCInv
= CFO + Int(1 – Tax Rate) – FCInv
= EBIT(1 – Tax Rate) + Dep – FCInv – WCInv
= EBITDA(1 – Tax Rate) + Dep(Tax Rate) – FCInv – WCInv

where:
NI = Net income available to common shareholders
NCC = Net noncash charges (e.g. depreciation)
Int = Interest expense
FCInv = Fixed capital investments = Maintenance Capex + Growth Capex
$= \Delta Gross\ PPE = \Delta Net\ PPE + Depreciation$
WCInv = Investment in working capital

Free cash flow to the Equity, FCFE

FCFE = FCFF – Int(1 – Tax Rate) + Net borrowing
= CFO – FCInv + Net borrowing

where:
Net borrowing = Debt issued – Debt repaid

Video: https://youtu.be/rtlvly6Fl0A

If (FCInv – Dep) and WCInv funded using Debt (based on debt ratio):

FCFE = NI + Dep – FCInv – WCInv + Net borrowing

where:
Net borrowing = DR(FCInv – Dep) + DR(WCInv)
$DR = Debt\ ratio = \frac{Debt}{Assets}$

If company issues preferred shares:

FCFF = CFO + Int(1 – Tax Rate) + **Preferred dividends** – FCInv

Two-Stage Free Cash Flow Models

$$Firm\ value = \sum_{t=1}^{n} \frac{FCFF_t}{(1+WACC)^t} + \frac{FCFF_{n+1}}{(WACC-g)}\left[\frac{1}{(1+WACC)^n}\right]$$

$$Equity\ value = \sum_{t=1}^{n} \frac{FCFE_t}{(1+r)^t} + \frac{FCFE_{n+1}}{(r-g)}\left[\frac{1}{(1+r)^n}\right]$$

Value of Firm = Value of operating assets + Value of nonoperating assets
(PV of FCFF)

Learning Module 4 | Market-Based Valuation Price and Enterprise Value Multiples

Enterprise value, EV = Market value of **common stock**
+ Market value of **preferred equity**
+ Market value of **debt** + Minority interest
– Cash and Short-term investments

	Actual	Justified
Trailing P/E	$\frac{Market\ price\ per\ share}{EPS\ over\ previous\ 12\ months}$	$\frac{(1-b)(1+g)}{r-g}$
Leading P/E	$\frac{Market\ price\ per\ share}{Forecasted\ EPS\ over\ next\ 12\ months}$	$\frac{1-b}{r-g}$
P/B	$\frac{Market\ price\ per\ share}{Book\ value\ per\ share}$	$\frac{ROE-g}{r-g}$ Video: https://youtu.be/c0vmCUtDpZs
P/S	$\frac{Market\ price\ per\ share}{Sales\ per\ share}$	$\frac{V_0}{S_0} = \frac{E_0}{S_0} \times \frac{(1-b)(1+g)}{r-g}$ or $\frac{V_1}{S_1} = \frac{E_1}{S_1} \times \frac{1-b}{r-g}$

	Actual	Justified
Trailing D/P	$\frac{4 \times Most\ recent\ quarterly\ dividend}{Market\ price\ per\ share}$	$\frac{r-g}{1+g}$
Leading D/P	$\frac{Forecast\ dividends\ over\ the\ next\ year}{Market\ price\ per\ share}$	$r-g$
Earnings yield	$\frac{EPS}{Price\ per\ share}$	$\frac{r-g}{(1-b)(1+g)}$

Underlying Earnings = EPS – non recurring gains + non recurring loss

Normalized Earnings

Method 1: Average EPS Approach

$$\text{Normalized EPS} = \frac{1}{n}\sum_{i=1}^{n} EPS_i$$

Method 2: Average ROE Approach

$$\text{Normalized EPS} = \frac{1}{n}\sum_{i=1}^{n} ROE_i \times Current\ Book\ value\ per\ share$$

Price-to-Earnings Growth (PEG) Ratio

$$PEG\ ratio = \frac{P/E\ ratio}{g\ (in\ \%)}$$

Momentum Indicators

Earnings surprise = Reported EPS – Expected EPS

$$\boldsymbol{Scaled\ earnings\ surprise} = \frac{Reported\ EPS - Expected\ EPS}{\sigma(Analyst\ forecast\ EPS)}$$

Standardized unexpected earnings (SUE) $= \frac{Earnings\ Surprise}{\sigma(Earnings\ Surprise)}$

Portfolio P/E

$$Weighted\ harmonic\ mean = \frac{1}{\sum_{i=1}^{n} \frac{w_i}{X_i}}$$

where:

w_i = Weight of stock *i* in portfolio

X_i = P/E of stock *i*

Learning Module 5 | Residual Income Valuation

Economic Value Added (EVA)

$$EVA_t = EBIT_t(1 - T) - (WACC \times Invested\ Capital_{t-1})$$

Market Value Added (MVA)

$$MVA_t = Market\ value\ of\ Firm_t - Invested\ Capital_t$$

Residual Income, RI

$$RI_t = E_t - (r \times B_{t-1}) = (ROE - r) \times B_{t-1}$$

Residual Income Model

$$V_0 = B_0 + \left[\frac{RI_1}{(1+r)^1} + \frac{RI_2}{(1+r)^2} + \frac{RI_3}{(1+r)^3} + \cdots\right]$$

Video: https://youtu.be/O0KTBkEtP9M

Single-stage residual income valuation model

$$V_0 = B_0 + \frac{(ROE - r) \times B_0}{r - g} = B_0 + \frac{RI_1}{r - g}$$

Video: https://youtu.be/82GJu5umrB0

Tobin's Q

$$Tobin's\ Q = \frac{Market\ value\ of\ debt + Market\ value\ of\ equity}{Replacement\ cost\ of\ total\ assets}$$

Continuing Residual Income

$$V_0 = B_0 + \sum_{t=1}^{T-1} \frac{RI_t}{(1+r)^t} + \frac{RI_T}{(1 + r - \omega)(1+r)^{T-1}} \qquad 0 \le \omega \le 1$$

ω = Persistence factor

If RI **declines to Long-run level** in mature industry, with premium over book value

$$V_0 = B_0 + \sum_{t=1}^{T} \frac{RI_t}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T}$$

Video: https://youtu.be/vhRW3q70E0w

Clean surplus relationship:

$$B_t = B_{t-1} + E_t - Div_t$$

Learning Module 6 | Private Company Valuation

Capitalized Cash Flow Method (CCM)

$$Firm\ value = \frac{FCFF_0(1+g)}{WACC-g} \rightarrow \begin{matrix}Equity\\ value\end{matrix} = \begin{matrix}Firm\\ value\end{matrix} - \begin{matrix}Market\ value\\ of\ Debt\end{matrix}$$

$$Equity\ value = \frac{FCFE_0(1+g)}{r-g}$$

Excess Earnings Method (EEM)

$$\begin{matrix}\boldsymbol{Excess}\\ \boldsymbol{earnings}\end{matrix} = \begin{matrix}Normalized\\ earnings\end{matrix} - \begin{matrix}Earnings\ required\ to\ provide\\ the\ required\ rate\ of\ return\ on\\ \boldsymbol{working\ capital}\ \text{and}\ \boldsymbol{fixed\ assets}\end{matrix}$$

$$Value\ of\ the\ intangible\ assets = \frac{(Excess\ Earnings)_1}{k-g}$$

Value of the firm = Working capital + Fixed assets + Intangible Assets

Video: https://youtu.be/137ga1xgAbA

Control Premium

$$\begin{matrix}Equity\ value\\ (with\ control\ premium)\end{matrix} = \begin{matrix}Equity\ value\\ (without\ control\ premium)\end{matrix} \times (1 + Control\ premium)$$

$$Adjusted\ control\ premium = Control\ premium \times \left(1 + \frac{Debt}{Assets}\right)$$

Discount for Lack of Control and Marketability

Discount for Lack of Control (DLOC)

$$DLOC = 1 - \frac{1}{1 + Control\ premium}$$

Total discount = 1 – (1 – DLOC)(1 – DLOM)

$$DLOM = \frac{Value\ of\ ATM\ option}{Share\ price}$$

FIXED INCOME

Learning Module 1 | The Term Structure and Interest Rate Dynamics

Forward Pricing Model

$$DF_B = DF_A \times F_{A,B-A}$$

where:

$$DF_B = \frac{1}{(1+z_B)^B}$$

$$F_{A,B-A} = \frac{1}{\left(1+f_{A,B-A}\right)^{B-A}}$$

Forward Rate Model

$$(1+z_B)^B = (1+z_A)^A\left(1+f_{A,B-A}\right)^{B-A}$$

where:
z_B = Spot rate for period B
$f_{A,B-A} = (B-A)$ forward rate that starts in period A

Calculating spot rate from one-period forward rates

$$z_T = \left[(1+z_1)\left(1+f_{1,1}\right)\left(1+f_{2,1}\right)\ldots\left(1+f_{T-1,1}\right)\right]^{1/T} - 1$$

<u>Boostrapping Spot Rates From Par Rates</u>

Video: https://youtu.be/-FnweFO172Q

Fixed swap rate

$$s_T = \frac{1-DF_T}{\sum_{t=1}^{T} DF_t} = \frac{1 - Discount\ Factor\ of\ Last\ Payment}{Sum\ of\ Discount\ Factors}$$

Swap spread = YTM of swap rate – YTM of government bond (same maturity)

TED spread = LIBOR – YTM of T-bill (same maturity)

LIBOR-OIS spread = LIBOR – OIS Fixed rate

For **Parallel shifts** in yield curve:

$$\%\Delta PV = -ModDur \times \Delta YTM \qquad \Delta PV = -ModDur \times \Delta YTM \times PV_0$$

$$\%\Delta PV = -EffDur \times \Delta Curve \qquad \Delta PV = -EffDur \times \Delta Curve \times PV_0$$

Non-parallel shifts (i.e. change in slope or curvature):

$$\%\Delta PV = -KeyRateDuration \times \Delta Key\ Rate$$

Bond Risk Premium

$$Bond\ risk\ premium = \begin{matrix} Yield\ of\ default\ free \\ long\ term\ bond \end{matrix} - \begin{matrix} Yield\ of\ default\ free \\ short\ term\ bond \end{matrix}$$

Learning Module 2 | The Arbitrage-Free Valuation Framework

Arbitrage-free Value of Bond

$$V_0 = \frac{C}{(1+z_1)^1} + \frac{C}{(1+z_2)^2} + \cdots + \frac{FV + C}{(1+z_n)^n}$$

where:
z_n = Spot rate for period *n*

Backward Induction Valuation Methodology

$$Bond\ value\ at\ any\ node = \frac{(0.5 \times V_H + 0.5 \times V_L) + C}{1+i}$$

where:
V_H = bond's value if the higher forward rate is realized one year hence
V_L = bond's value if the lower forward rate is realized one year hence
C = coupon payment that is not dependent on interest rates

Video (Backward Induction Valuation): https://youtu.be/DhAVQ3hIXlQ

Video (Backward Induction with Financial Calculator): https://youtu.be/FycX2UwJxCM

Video (Pathwise Valuation): https://youtu.be/3oM-220oi7o

Binomial Interest Rate Tree

$i_{1,H} = i_{1,L}e^{2\sigma}$

$i_{2,HH} = i_{2,LL}e^{4\sigma} \qquad i_{2,HL} = i_{2,LL}e^{2\sigma}$

$i_{3,HHH} = i_{3,LLL}e^{6\sigma} \qquad i_{3,HHL} = i_{3,LLL}e^{4\sigma} \qquad i_{3,LLH} = i_{3,LLL}e^{2\sigma}$

Equilibrium Term Structure Models

Cox-Ingersoll-Ross (CIR) Model

$$dr = k(\theta - r_t)dt + \sigma\sqrt{r_t}dz$$

Vasicek Model

$$dr = k(\theta - r_t)dt + \sigma dz$$

where:
k = Speed of reversion (> 0)
θ = Long-run interest rate
σ = Interest rate volatility

Arbitrage Free Models

Ho-Lee Model

$$dr_t = \theta_t dt + \sigma dz_t$$

Kalotay-Williams-Fabozzi (KWF) Model

$$d(\ln r_t) = \theta_t dt + \sigma dz_t$$

where:
θ_t = Time-dependent drift term

Learning Module 3 | Valuation and Analysis - Bonds with Embedded Options

Callable and Putable Bonds

Value of **callable** bond = Value of straight bond – Value of **issuer call option**

Value of **putable** bond = Value of straight bond + Value of **investor put option**

Video (Valuing a callable bond): https://youtu.be/lWLSodiqZaM

Video (Valuing a putable bond): https://youtu.be/qmUnAtpXIAg

$$Effective\ duration = \frac{(PV_-) - (PV_+)}{2 \times (\Delta Curve) \times PV_0}$$

$$Effective\ convexity = \frac{(PV_-) + (PV_+) - 2 \times PV_0}{PV_0 \times (\Delta Curve)^2}$$

Capped and Floored Floaters

Value of capped floater = Value of straight floater – Value of cap

Value of floored floater = Value of straight floater + Value of floor

Video (Valuing a capped floater): https://youtu.be/d4LNMdXV9vU

Video (Valuing a floored floater): https://youtu.be/YJZU0THHBNE

Convertible Bonds

$$\begin{matrix}Conversion\\value\end{matrix} = \begin{matrix}Underlying\\share\ price\end{matrix} \times \begin{matrix}Conversion\\ratio\end{matrix}$$

$$\begin{matrix}\boldsymbol{Minimum\ value}\\of\ convertible\ bond\end{matrix} = \boldsymbol{Max}\begin{bmatrix}\begin{matrix}Conversion\\value\end{matrix}, \begin{matrix}Value\ of\ underlying\\Straight\ bond\end{matrix}\end{bmatrix}$$

$$\begin{matrix}Market\ conversion\\price\end{matrix} = \frac{Convertible\ bond\ price}{Conversion\ ratio}$$

$$\begin{matrix}Market\ conversion\\premium\ per\ share\end{matrix} = \begin{matrix}Market\ conversion\\price\end{matrix} - \begin{matrix}Underlying\ share\\price\end{matrix}$$

$$\begin{matrix}Market\ conversion\\premium\ ratio\end{matrix} = \frac{Market\ conversion\ premium\ per\ share}{Underlying\ share\ price}$$

$$\begin{matrix}Premium\ over\\Straight\ value\end{matrix} = \frac{Convertible\ bond\ price}{Straight\ value} - 1$$

Convertible Bond (With No Additional Options)

$$\begin{matrix}Value\ of\\convertible\ bond\end{matrix} = \begin{matrix}Value\ of\\straight\ bond\end{matrix} + \begin{matrix}Value\ of\ call\\option\ on\\issuer's\ stock\end{matrix}$$

Callable Convertible Bond

$$\begin{matrix}Value\ of\\convertible\ bond\end{matrix} = \begin{matrix}Value\ of\\straight\ bond\end{matrix} + \begin{matrix}Value\ of\ call\\option\ on\\issuer's\ stock\end{matrix} - \begin{matrix}Value\ of\\issuer\ call\\option\end{matrix}$$

Putable Convertible Bond

$$\begin{matrix}Value\ of\\convertible\ bond\end{matrix} = \begin{matrix}Value\ of\\straight\ bond\end{matrix} + \begin{matrix}Value\ of\ call\\option\ on\\issuer's\ stock\end{matrix} + \begin{matrix}Value\ of\\investor\ put\\option\end{matrix}$$

Learning Module 4 | Credit Analysis Models

$$G\text{-}spread = YTM\ of\ Corporate\ bond - YTM\ of\ Government\ bond$$

$$\begin{matrix} Loss\ given \\ default \end{matrix} = \begin{matrix} Expected \\ exposure \end{matrix} \times \left(1 - \begin{matrix} Recovery \\ rate \end{matrix}\right)$$

$$\begin{matrix} Loss \\ severity \end{matrix} = 1 - \begin{matrix} Recovery \\ rate \end{matrix}$$

Expected Loss = Probability of Default × Loss Given Default

$$\begin{matrix} Fair\ value \\ of\ credit\ risky\ bond \end{matrix} = \begin{matrix} Fair\ value\ of\ bond \\ assuming\ no\ default \end{matrix} - \begin{matrix} Credit\ Valuation \\ Adjustment \end{matrix}$$

$$Credit\ valuation\ adjustment, \qquad CVA = \sum_{t=1}^{n} \frac{EL_t}{(1+rf_t)^t} = \sum_{t=1}^{n} \frac{POD_t \times LGD_t}{(1+rf_t)^t}$$

where:
EL_t = Expected loss of bond at time t
POD_t = Probability of default of bond at time t
LGD_t = Loss given default at time $t = Expected\ Exposure_t - Recovery_t$
rf_t = Risk-free rate at time t
n = Bond's remaining tenor

$$PV\ of\ expected\ loss\ for\ period\ t = \frac{EL_t}{(1+rf_t)^t}$$

$$POD_t = (1 - Hazard\ rate)^{t-1} \times Hazard\ rate$$

Approximation of credit spread ≈ Annual hazard rate × (1 – Recovery rate)

Video (Probability of Default): https://youtu.be/e7K4x48Eg4U

Video (Valuing a Credit Risky Bond – Zero Interest Rate Volatility): https://youtu.be/2I9bgu-o7aI

Video (YTM of Corporate Bonds – Default and Non-Default): https://youtu.be/K253Y7c2Yto

Expected percentage price change of a corporate bond

$$\sum Probability\ of\ credit\ migration \times \%\Delta P$$

where:
$\%\Delta P = -ModDur \times \Delta credit\ spread$

Structural Model

$$A_t = D(t,T) + S_t$$

In terms of...	Call options	Put options
Equity	$E(T) = Max[A(T) - K, 0]$	$E(T) = A(T) - K + Max[K - A(T), 0]$
Debt	$D(T) = A(T) - Max[A(T) - K, 0]$	$D(T) = K - Max[K - A(T), 0]$

where:
$S_t = Equity\ value\ at\ time\ t$
$A_T = Asset\ value\ at\ time\ T$
$K = Face\ value\ of\ debt$

Learning Module 5 | Credit Default Swaps

CDS payout amount = Payout ratio × Notional
= (1 – Recovery rate of CTD bond) × Notional

Upfront payment = PV of protection leg – PV of premium leg

$$\begin{matrix} Upfront \\ premium \end{matrix} = \begin{matrix} PV\ of \\ Credit\ Spread \end{matrix} - \begin{matrix} PV\ of \\ Fixed\ Coupon \end{matrix}$$
$$\approx \left(\begin{matrix} Credit \\ Spread \end{matrix} - \begin{matrix} Fixed \\ Coupon \end{matrix} \right) \times CDS\ Duration$$

$$Price\ of\ CDS\ per\ 100\ notional = 100 - Upfront\ premium$$

$$\begin{matrix} \%\ Change\ in \\ CDS\ price \end{matrix} = \begin{matrix} Change\ in \\ spread\ in\ bps \end{matrix} \times Duration$$

DERIVATIVES

Learning Module 1 | Pricing and Valuation of Forward Commitments

Forward Contracts

Forward Pricing:

$$F_0 = S_0(1+r)^T$$

$$F_0 = (S_0 + CC_0 - CB_0)(1+r)^T$$

$$F_0 = S_0 e^{r_c T}$$

$$F_0 = S_0 e^{(r_c + CC - CB)T}$$

where:
S_0 = Current spot price
F_0 = Forward price (set today)
r = Annually compounded risk-free rate
r_c = Continuously compounded risk-free rate
CC_0 = PV of Carry cost
CB_0 = PV of carry benefits
CC = Continuously compounded cost of carry
CB = Continuously compounded carry benefit

Forward Valuation (Long Position):

$$V_0 = 0$$

$$V_t = \frac{F_t - F_0}{(1+r)^{T-t}} = S_t - \frac{F_0}{(1+r)^{T-t}}$$

$$V_T = S_T - F_0$$

Forward Rate Agreement (FRA)

$$\boldsymbol{Long}\ FRA\ payoff\ at\ \boldsymbol{expiration}\ of\ FRA = \frac{Notional[L_m - FRA_0]t_m}{1 + D_m t_m}$$

$$FRA_0 = \left(\frac{1 + L_T t_T}{1 + L_h t_h} - 1\right)\left(\frac{1}{t_m}\right)$$

Valuation at time $t = g$ (prior to FRA expiration):

$$Value\ of\ Long\ FRA\ at\ g = \frac{Notional\left(FRA_g - FRA_0\right)t_m}{1 + D_{T-g}t_{T-g}}$$

where:
D_m = Discount rate for *m* periods at *t* = *h*
h = FRA tenor
m = Tenor of the underlying rate
$T = h + m$ = Maturity of underlying instrument

Video (Pricing an FRA): https://youtu.be/uBmAt_z9f3Y

Video (Valuing an FRA): https://youtu.be/AYKRVdaYvxY

Fixed Income Forwards and Futures

Pricing:

$$F_0 = \begin{matrix}Quoted\ futures\\ price\end{matrix} \times \begin{matrix}Conversion\\ factor\end{matrix}$$
$$= FV(B_0 + AI_0) - AI_T - FVCI$$

Valuation for fixed income **forward contracts**:

$$V_t = Present\ value\ of\ difference\ in\ forward\ prices$$
$$= PV[F_t - F_0]$$

Valuation for fixed income **futures contracts**:
V_t = Price change since previous day's settlement

where:
B_0 = Quoted bond price

$$AI = \frac{Number\ of\ accrued\ days\ since\ last\ coupon\ payment}{Total\ days\ during\ the\ coupon\ payment\ period} \times \frac{Annual\ coupon}{Coupon\ frequency}$$

Interest Rate Swaps (IRS)

$$FS = \frac{1 - PV_n}{\sum_{i=1}^{n} PV_i}$$

$$PV_i = \frac{1}{1 + Spot\ rate_i \left(\frac{Days\ to\ Maturity_i}{360}\right)}$$

Pay-fixed, receive-floating IRS

$$Value\ of\ Swap = Notional \times (FS_t - \boldsymbol{FS_0}) \sum_{i=1}^{n} PV_i$$

Receive-fixed, pay-floating IRS

$$Value\ of\ Swap = Notional \times (\boldsymbol{FS_0} - FS_t) \sum_{i=1}^{n} PV_i$$

Video (Pricing an Interest Rate Swap) : https://youtu.be/0QvtKZutr5E

Video (Valuing an Interest Rate Swap): https://youtu.be/_A2a909etvg

Currency Swap

Pricing for fixed leg of currency swap in currency a

$$FS_a = \frac{1 - PV_{n,a}}{\sum_{i=1}^{n} PV_{i,a}}$$

Value of a fixed-for-fixed currency swap

$$V_{CS} = Notional_a \times V_a - S_t \times Notional_b \times V_b$$

$$V_a = FS_a \sum_{i=1}^{n} PV_{i,a} + PV_{n,a} \times Par_a = Value\ of\ currency\ a\ leg\ (receive)$$

$$V_b = FS_b \sum_{i=1}^{n} PV_{i,b} + PV_{n,b} \times Par_b = Value\ of\ currency\ b\ leg\ (pay)$$

S_t = Spot exchange rate at time t (quoted as a/b)

Video (Pricing a currency swap): https://youtu.be/XZlxcVByc00

Video (Valuing a currency swap): https://youtu.be/3h4mElS48aA

Equity Swap

Value of equity swap (receive fixed-rate, pay equity return)

$$V_{EQ,t} = V_{FIX}(C_0) - \frac{S_t}{S_{t-1}} \times Notional - PV_t(Par - Notional)$$

$$Value\ of\ Equity\ Leg = \frac{S_t}{S_{t-1}} \times Notional$$

Cash flow for equity leg $= Notional \times Periodic\ equity\ return$

where:
$V_{FIX}(C_0) =$ Value at time t of a fixed-rate bond initiated with coupon C_0 at Time 0
$S_t =$ Current equity index level
S_{t-1} = Equity index level at last reset date

Learning Module 2 | Valuation of Contingent Claims

Hedge Ratio

$$h_{call} = \frac{c^+ - c^-}{S^+ - S^-} \geq 0$$

$$h_{put} = \frac{p^+ - p^-}{S^+ - S^-} \leq 0$$

No-arbitrage Approach:

$$c = h_{call}S + PV(-h_{call}S^+ + c^+) = h_{call}S + PV(-h_{call}S^- + c^-)$$

$$p = h_{put}S + PV\left(-h_{put}S^+ + p^+\right) = h_{put}S + PV\left(-h_{put}S^- + p^-\right)$$

Expectations Approach:

$$\pi = \frac{(1 + r) - d}{u - d}$$

where:
$u =$ Up factor
$d =$ Down factor
$r =$ Risk-free rate

One-period Binomial Model

$$c = \frac{\pi c^{+} + (1-\pi)c^{-}}{1+r}$$

$$p = \frac{\pi p^{+} + (1-\pi)p^{-}}{1+r}$$

where:
$\pi =$ Risk-neutral probability of an up-move

Note: For **interest rate options**, $\pi = 0.5$ and discount expected option payoff using the 1-period forward rates.

Video (Valuing interest rate options): https://youtu.be/X4R8j_cf8SA

Two-period Binomial Model:

$$c = \frac{\pi^2 c^{++} + 2\pi(1-\pi)c^{+-} + (1-\pi)^2 c^{--}}{(1+r)^2}$$

$$p = \frac{\pi^2 p^{++} + 2\pi(1-\pi)p^{+-} + (1-\pi)^2 p^{--}}{(1+r)^2}$$

For **2-period American-styled call option** with dividend in *t* = 1:

$$S^{+} = u \times (S - PV\ of\ dividends\ at\ risk\ free\ rate)$$

$$S^{-} = d \times (S - PV\ of\ dividends\ at\ risk\ free\ rate)$$

Video: https://youtu.be/U_XkIZjJIAU

Black-Scholes Option Pricing Model

$$c = SN(d_1) - Xe^{-rT}N(d_2)$$

$$p = Xe^{-rT}N(-d_2) - SN(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Put-call parity: $$p + S = c + Xe^{-rT}$$

- Hedge ratio for calls = $N(d_1)$
- Probability that the call option expires in the money = $N(d_2) = Prob(S_T > X)$
- Hedge ratio for puts = $N(d_1) - 1 = -N(-d_1)$
- Probability that the put option expires in the money = $1 - N(d_2)$

$$Prob(S_T < X) = N(-d_2)$$

BSM model with carry benefits

$$c = Se^{-\gamma T}N(d_1) - Xe^{-rT}N(d_2)$$

$$p = Xe^{-rT}N(-d_2) - Se^{-\gamma T}N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \gamma + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Put-call parity: $$p + Se^{-\gamma T} = c + Xe^{-rT}$$

Black Option Valuation Model

European Options on Futures

$$c = e^{-rT}[F_0(T)\,N(d_1) - XN(d_2)]$$

$$p = Xe^{-rT}N(-d_2) - Se^{-\gamma T}N(-d_1)$$

$$d_1 = \frac{\ln\left[\frac{F_0(T)}{X}\right] + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Put-call parity: $c = e^{-rT}[F_0(T) - X] + p$

Interest Rate Options

$$c = (AP)e^{-r(t_{j-1}+t_m)}\left[FRA\left(0, t_{j-1}, t_m\right)N(d_1) - R_X e^{-rT}N(d_2)\right]$$

$$p = (AP)e^{-r(t_{j-1}+t_m)}\left[R_X e^{-rT}N(-d_2) - FRA\left(0, t_{j-1}, t_m\right)N(-d_1)\right]$$

$$d_1 = \frac{\ln\left[\frac{FRA\left(0, t_{j-1}, t_m\right)}{X}\right] + \left(\frac{1}{2}\sigma^2\right)t_{j-1}}{\sigma\sqrt{t_{j-1}}}$$

$$d_2 = d_1 - \sigma\sqrt{t_{j-1}}$$

Payer Swaption

$$PAY_{SWN} = AP \times [R_{FIX}N(d_1) - R_X N(d_2)] \times \sum_{j=1}^{n} PV_j(1)$$

Receiver Swaption

$$REC_{SWN} = AP \times [R_X N(-d_2) - R_{FIX}N(-d_1)] \times \sum_{j=1}^{n} PV_j(1)$$

$$d_1 = \frac{\ln\left(\frac{R_{FIX}}{R_X}\right) + \left(\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Video (Interest Rate Options & Swaptions Equivalences:
https://youtu.be/uZQO50sEzso

Optimal Number of Hedging Units (for Delta Hedging)

$$N_H = -\frac{Portfolio\ delta}{Delta_H}$$

Video: https://youtu.be/v8RcvkQKFpw

Option Greeks

$Delta_{call} = e^{-\delta T} N(d_1)$

$Delta_{put} = -e^{-\delta T} N(-d_1)$

where:
$\delta =$ Continuously compounded dividend yield

$Gamma_{call} = Gamma_{put} = \frac{e^{-\delta T}}{S\sigma\sqrt{T}} N(d_1)$

$c = c_0 + Delta_{call} \times \Delta S + \frac{1}{2} Gamma_{call} \times (\Delta S)^2$

$p = p_0 + Delta_{put} \times \Delta S + \frac{1}{2} Gamma_{put} \times (\Delta S)^2$

ALTERNATIVE INVESTMENTS

Learning Module 1 | Introduction to Commodities and Commodity Derivatives

$$\begin{matrix}Futures\\price\end{matrix} = \begin{matrix}Spot\ price\ of\\physical\ commodity\end{matrix} + \begin{matrix}Storage\\costs\end{matrix} - \begin{matrix}Convenience\\yield\end{matrix}$$

$$\begin{matrix}Calendar\\spread\end{matrix} = \begin{matrix}Near\ term\\futures\ contract\\closing\ price\end{matrix} - \begin{matrix}Longer\ term\\futures\ contract\\closing\ price\end{matrix}$$

$$Price\ return = \frac{Current\ price - Previous\ price}{Previous\ price}$$

$$Roll\ return = \frac{\left(\begin{matrix}Near\ term\\futures\ contract\\closing\ price\end{matrix} - \begin{matrix}Longer\ term\\futures\ contract\\closing\ price\end{matrix}\right)}{\begin{matrix}Near\ term\\futures\ contract\\closing\ price\end{matrix}} \times \begin{matrix}Percentage\ of\ the\ position\\in\ the\ futures\ contract\\being\ rolled\end{matrix}$$

$$\begin{matrix}Total\\return\end{matrix} = \begin{matrix}Price\\return\end{matrix} + \begin{matrix}Roll\\return\end{matrix} + \begin{matrix}Collateral\\return\end{matrix} + \begin{matrix}Rebalancing\\return\ (for\ index\ only)\end{matrix}$$

Learning Module 2 | Overview of Types of Real Estate Investment

Net and Gross Leases

$$Net\ rent = Gross\ rent - Operating\ expenses$$

Retail Rent

$$\begin{matrix}Rent\ per\\square\ foot\end{matrix} = \begin{matrix}Minimum\ rent\\per\ square\ foot\end{matrix} + Share\% \times \left(\begin{matrix}Revenue\ per\\square\ foot\end{matrix} - \begin{matrix}Natural\ breakpoint\\per\ square\ foot\end{matrix}\right)$$

Appraisal-based index

$$Return = \frac{NOI - \begin{matrix}Capital\\Expenditures\end{matrix} + \left(\begin{matrix}Ending\\market\ value\end{matrix} - \begin{matrix}Beginning\\market\ value\end{matrix}\right)}{Beginning\ market\ value}$$

Learning Module 3 | Investments in Real Estate Through Publicly Traded Securities

Net Asset Value Approach

$$NAV\ per\ share = \frac{\begin{matrix}Value\ of\\ operating\ estate\end{matrix} + \begin{matrix}Value\ of\\ other\ assets\end{matrix} - \begin{matrix}Total\ debt\\ and\ liabilities\end{matrix}}{Number\ of\ shares\ outstanding}$$

If valuation of operating real estate is not provided:

$$Value\ of\ operating\ estate = \frac{NOI_1}{Cap\ rate}$$

Video: https://youtu.be/WncC3BZmfs8

$$NOI = \begin{matrix}Gross\ rental\\ revenue\end{matrix} - \begin{matrix}Estimated\ vacancy\ and\\ collections\ loss\end{matrix} - \begin{matrix}Operating\\ expenses\end{matrix}$$

Relative Value Approach

Funds from Operations:

$$FFO = Net\ income + \begin{matrix}Depreciation\\ and\ amortization\end{matrix} - \begin{matrix}Gains\ on\ sale\\ of\ property\end{matrix} + \begin{matrix}Loss\ on\ sale\\ of\ property\end{matrix}$$

Adjusted Funds from Operations:

$$AFFO = FFO - \begin{matrix}Non\ cash\\ rent\end{matrix} - \begin{matrix}Recurring\ capital\ expenditure\\ and\ leasing\ costs\end{matrix}$$

Two-Stage Dividend Discount Model

$$Value\ of\ a\ REIT\ share = PV\ of\ dividends + PV\ of\ terminal\ value$$

Learning Module 4 | Hedge Fund Strategies

Equity Market Neutral Pairs Trading

$$\begin{matrix}Amount\ of\ Short\ Position \\ in\ Overvalued\ Stock\end{matrix} = -\frac{Beta\ of\ undervalued\ stock \times Amount\ Invested}{Beta\ of\ overvalued\ stock}$$

Merger Arbitrage Strategy

For a stock-for-stock deal:

$$Payoff\ if\ merger\ is\ successful = (N_A \times P_A) - (N_T \times P_T)$$

where:
N_A = Number of acquirer's shares to short sell
P_A = Share price of acquirer post announcement
N_T = Number of target's shares to buy
P_T = Share price of target post announcement

Conditional Factor Risk Model

$$\begin{aligned} R_{i,t} = {} & \alpha_i + \beta_{i,1}(Factor\ 1)_t + \beta_{i,2}(Factor\ 2)_t + \cdots + \beta_{i,K}(Factor\ K)_t + \\ & + D_t\beta_{i,1}(Factor\ 1)_t + D_t\beta_{i,2}(Factor\ 2)_t + \cdots + D_t\beta_{i,K}(Factor\ K)_t + (error)_{i,t} \end{aligned}$$

where:
$R_{i,t}$ = Return of hedge fund i in period t
$\beta_{i,K}(Factor\ K)_t$ = Exposure to risk factor K for hedge fund i in period t during normal times
$D_t\beta_{i,K}(Factor\ K)_t$ = Incremental exposure to risk factor K for hedge fund i in period t during financial crisis periods
D_t = Dummy variable that equals 1 during financial crisis periods (0 otherwise)
α_i = Intercept for hedge fund i
$(error)_{i,t}$ = Random error with zero mean and standard deviation σ_i

PORTFOLIO MANAGEMENT

VOL5 Learning Module 1 | Exchange-Traded Funds: Mechanics and Applications

End-of-day ETF premium or discount (%)

$$\frac{ETF\ price - NAV\ per\ share}{NAV\ per\ share}$$

Intraday ETF premium or discount (%)

$$\frac{ETF\ price - Indicated\ NAV\ per\ share}{Indicated\ NAV\ per\ share}$$

$$Holding\ period\ cost\ (\%) = Round\ trip\ trade\ cost\ (\%) + Management\ fee\ (\%)$$

$$Round\ trip\ trade\ cost\ \% = One\ way\ commission\ \% \times 2 + Bid\ ask\ spread\ \%$$

VOL5 Learning Module 2 | Using Multifactor Models

Arbitrage Pricing Theory (APT)

$$E(R_p) = R_F + \lambda_1\beta_{p,1} + \cdots + \lambda_K\beta_{p,K}$$

where:
$E(R_p)$ = the expected return to portfolio *p*
R_F = the risk-free rate
$\beta_{p,j}$ = the **sensitivity** of the **portfolio** to **factor *j***
λ_j = the **expected reward** for bearing the risk of factor j
K = the **number of factors**

Carhart Four-Factor Model

$$E\left(R_p\right) = R_F + \beta_{p,1}RMRF + \beta_{p,2}SMB + \beta_{p,3}HML + \beta_{p,4}WML$$

where:
$RMRF$ = Return on a value-weighted equity index *minus* **one-month T-bill rate**
SMB = small minus big; **average return** on three **small-cap portfolios *minus*** the **average return** on three **large-cap portfolios**
HML = high minus low; **average return** on two **high book-to-market portfolios *minus*** **average return** on two **low book-to-market portfolios**
WML = winners minus losers, a **momentum factor**; return on a portfolio of past year's winners minus return on a portfolio of past year's losers.

Macroeconomic Factor Model

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \cdots + b_{iK}F_K + \varepsilon_i$$

where:
F_k = the **surprise in the factor *k***
b_{ik} = the **sensitivity** of the return on asset *i* to a surprise in factor *k*, *k* = 1, 2, ...,
a_i = Expected return on the portfolio

Fundamental Factor Model

$$b_{ik} = \frac{\text{Value of attribute } k \text{ for asset } i - \text{Average value of attribute } k}{\sigma(\text{Values of attribute } k)}$$

Return Attribution

$$\begin{aligned} Active\ return &= R_P - R_B \\ &= \sum_{k=1}^{K} \left[\begin{pmatrix} Portfolio \\ sensitivity \end{pmatrix}_k - \begin{pmatrix} Benchmark \\ sensitivity \end{pmatrix}_k \right] \times \begin{pmatrix} Factor \\ return \end{pmatrix}_k + \frac{Security}{selection} \end{aligned}$$

$$Tracking\ error, \qquad TE = s(R_P - R_B)$$

$$Information\ ratio, \qquad IR = \frac{\bar{R}_P - \bar{R}_B}{s(R_P - R_B)}$$

Active risk squared = Active factor risk + Active specific risk

VOL5 Learning Module 3 | Measuring and Managing Market Risk

Parametric VaR (Using Normal Distribution)

$$Value\ at\ Risk, \qquad VaR = -\left[E\left(R_p\right) - z \times \sigma_p\right] \times \begin{matrix} Portfolio \\ Value \end{matrix}$$

where:
$E\left(R_p\right)$ = Portfolio expected return
σ_p = Portfolio standard deviation

<u>Two-asset portfolio</u>:

$$E\left(R_p\right) = w_1 E(R_1) + w_2 E(R_2)$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

Scaling from **daily** returns to **annual** returns (Assuming 1 year = 250 trading days):

$$R_{daily} \times 250\ trading\ days$$

Scaling from **daily** standard deviation to **annual** standard deviaton:

$$\sigma_{daily} \times \sqrt{250}$$

$$Incremental\ VaR\ (IVaR) = VaR\ after\ change - VaR\ before\ change$$

Percentage change in bond price:

$$\frac{\Delta \mathrm{B}}{B} \approx -Duration\frac{\Delta y}{1+y} + \frac{1}{2}Convexity\left(\frac{\Delta y}{1+y}\right)^2$$

New call price: $c + \Delta c \approx c + \mathrm{Delta}_c(\Delta S) + \frac{1}{2}\mathrm{Gamma}_c(\Delta S)^2 + Vega_c(\Delta\sigma)$

New put price: $p + \Delta p \approx p + \mathrm{Delta}_p(\Delta S) + \frac{1}{2}\mathrm{Gamma}_p(\Delta S)^2 + Vega_p(\Delta\sigma)$

VOL5 Learning Module 4 | Backtesting and Simulation

No formula.

VOL6 Learning Module 1 | Economics and Investment Markets

One-period real-risk free rate:

$$l_{t,1} = \frac{1}{E_t\left[\widetilde{m}_{t,1}\right]} - 1$$

where:

$E_t\left[\widetilde{m}_{t,1}\right]$ = Inter-temporal rate of substitution

$$Price\ of\ risky\ asset = \frac{E\left[\tilde{P}_{t+1,s-1}\right]}{1 + l_{t,1}} + cov_t\left[\tilde{P}_{t+1,s-1}, \widetilde{m}_{t,1}\right]$$

where:

$\frac{E\left[\tilde{P}_{t+1,s-1}\right]}{1+l_{t,1}}$ = risk neutral present value

$cov_t\left[\tilde{P}_{t+1,s-1}, \widetilde{m}_{t,1}\right]$ = covariance between investor's inter-temporal rate of substitution and the random future price the investment at t + 1, based on the information available to investor today.

s = time to maturity of investment

Default-free nominal coupon-paying bond

$$P_t^i = \sum_{s=1}^{N} \frac{CF_{t+s}^i}{(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s})^s}$$

where:

$l_{t,s}$ = Real-risk free rate

$\theta_{t,s}$ = Expected inflation rate

$\pi_{t,s}$ = Uncertainty in future inflation rate

$\theta_{t,s} + \pi_{t,s}$ = Breakeven rate of inflaton

Short-dated nominal zero-coupon government bonds (e.g., T-bills)

$$P_t^i = \frac{CF_{t+s}^i}{(1 + l_{t,s} + \theta_{t,s})^s}$$

Taylor Rule

$$pr_t = I_t + \pi_t + 0.5(\pi_t - \pi_t^*) + 0.5(Y_t - Y_t^*)$$

where:

pr_t = policy rate at time *t*

I_t = level of **real** short-term interest rates that balance long-term savings and borrowing in the economy

π_t = rate of inflation

π_t^* = **target** rate of inflation

Y_t = logarithmic level of **actual GDP**

Y_t^* = logarithmic level of **potential real GDP**

$Y_t - Y_t^*$ = output gap

Corporate bond

$$P_t^i = \sum_{s=1}^{N} \frac{E_t\left[\widetilde{CF}_{t+s}^i\right]}{(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma_{t,s})^s}$$

where:

$\gamma_{t,s}$ = Credit premium

Equity

$$P_t^i = \sum_{s=1}^{\infty} \frac{E_t\left[\widetilde{CF}_{t+s}^i\right]}{(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma_{t,s} + \kappa_{t,s})^s}$$

$$P_t^i = \sum_{s=1}^{\infty} \frac{E_t\left[\widetilde{CF}_{t+s}^i\right]}{(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \lambda_{t,s})^s}$$

where:

$\kappa_{t,s}$ = Equity premium relative to risky bonds

$\lambda_{t,s} = \gamma_{t,s} + \kappa_{t,s}$ = Equity risk premium

Commercial Real Estate

$$P_t^i = \sum_{s=1}^{N} \frac{E_t\left[\widetilde{CF}_{t+s}^i\right]}{(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma_{t,s} + \kappa_{t,s} + \phi_{t,s})^s}$$

where:

$\phi_{t,s}$ = liquidity risk premium

VOL6 Learning Module 2 | Analysis of Active Portfolio Management

Active return, $R_A = R_P - R_B$

$Alpha, \alpha_p = R_p - \beta_p R_B$

$$Value\ added, \qquad R_A = \sum_{j=1}^{M} \Delta w_j R_{B,j} + \sum_{j=1}^{M} w_{P,j} R_{A,j}$$

$SR_P^2 = SR_B^2 + IR^2$

$\sigma^2(R_P) = \sigma^2(R_B) + \sigma^2(R_A)$

For **optimal** Sharpe ratio,

$$\sigma(R_A) = \frac{IR}{SR_B}\sigma\,(R_B)$$

$$Transfer\ Coefficient, \qquad TC = Corr\left(\frac{\mu_i}{\sigma_i}, \Delta w_i \sigma_i\right)$$

$$Information\ Coefficient, \qquad IC = Corr\left(\frac{R_{Ai}}{\sigma_i}, \frac{\mu_i}{\sigma_i}\right)$$

$IC \approx 2(Probability\ of\ right\ call) - 1$

Forecasted active return, $\mu_i = IC \times \sigma_i \times S_i$

where: S_i is set of standardized forecasts of expected returns across securities

Mean-variance optimal weights

$$\Delta w_i^* = \frac{\mu_i \sigma_A}{\sigma_i^2 IC\sqrt{BR}}$$

Full Fundamental Law

$$E(R_A) = TC \times IC\sqrt{BR}\sigma_A$$

$$IR = TC \times IC\sqrt{BR}$$

$$\sigma\,(R_A) = TC \times \frac{IR^*}{SR_B}\sigma\,(R_B)$$

$$SR_P^2 = SR_B^2 + (TC)^2(IR^*)^2$$

Performance Measurement

$$R_A = E(R_A|IC_R) + Noise$$

TC^2 = Proportion of variation in realized performance attributed to realized information coefficient

where:
IC_R = realized information coefficient

Ex-ante measurement of skill

$$E(R_A) = \frac{IC}{\sigma_{IC}}\sigma_A$$

Independence of Investment Decision

$$BR = \frac{N}{1 + (N - 1)\rho}$$