A Study of Belief Revision Postulates in Multi-Agent Systems

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Abstract

In this paper, we investigate the belief revision problem in multi-agent systems, i.e., what will be the beliefs of all agents after one agent gains the belief in some fluent formula. We propose a generalization of the classical AGM postulates, and of the Darwiche-Pearl postulates for iterated revision, to the multi-agent context. We provide an example of a simple, generalized "full-meet" multi-agent belief revision operator and prove that it satisfies all of the generalized postulates. We then present an event model based belief operator and prove that the proposed operator satisfes the generalized AGM and Darwiche-Pearl postulates in multi-agent systems.

1 Introduction

Belief revision has been an active research topic in KRR. Classical belief revision deals with the problem of identifying an agent's beliefs in the presence of a new piece of information. It assumes that the agent belief is represented by a logical theory. Major KR representating languages such as propositional logic, general logical language, situation calculs, or logic programming has been considered such as in (Alchourrón, Gärdenfors, and Makinson 1985; Gardenfors and Makinson 1988; Katsuno and Mendelzon 1992; Delgrande et al. 2008; Aravanis and Peppas 2017; Aravanis, Peppas, and Williams 2018; Shapiro et al. 2011; Witteveen, van der Hoek, and de Nivelle 1994). The vibrant history of belief revision, up to 2010, and its related problems have been highlighted in the special issue of the *Journal of Philosophical Logic* (Fermé and Hansson 2011).

Formally, a belief revision operator is defined as a mapping of pairs of knowledge bases and formulae into knowledge bases, i.e., $*$ – denote a belief operator – is defined as $* : \mathcal{L} \times \mathcal{L} \mapsto \mathcal{L}$ where $\mathcal{L}$ denotes the set of knowledge bases or formulae in a logical language. Studies in belief revision are often concerned with identifying general properties of a revision operator or defining a concerte one.

In their seminal work, Alchourrón, Gärdenfors, and Makinson (1985) proposed the foundational properties, referred to as *AGM Postulates*, that a rational belief revision operator should satisfy. Darwiche and Pearl (1997) introduced four additional properties, referred to as *DP Postulates*, for iterated belief revision that focuses on sequences of revisions. These postulates have been extensively studied by several authors. For example, concrete revision operators based on the distance between models of a knowledge base and a formula for revision are proposed in (Dalal 1988; Satoh 1988); approach to belief revision using possible worlds is discussed in (Katsuno and Mendelzon 1992); belief revision in the context of answer set programming is studied in (Aravanis and Peppas 2017; Delgrande et al. 2008); iterated belief revision or belief change in the context of a dynamic world is also studied extensively (e.g., (Baltag and Smets 2008; Hunter and Delgrande 2011; Peppas 2014; Shapiro et al. 2011; Tardivo et al. 2021)); etc.

It is worth noticing that belief revision in multi-agent systems has been considered very early on but most of the early work in this direction such as (Dragoni and Puliti 1994; Dragoni and Giorgini 1996; Liu and Williams 1999; Malheiro, Jennings, and Oliveira 1994) focus on maintaining the consistency of the local knowledge bases of agents in a distributed manner. Interest in reasoning about beliefs and knowledge and multi-agent epistemic planning has led to many proposals for updating the beliefs of multiple agents in dynamic environments (Baral et al. 2022; Baltag, Moss, and Solecki 1998; Herzig, Lang, and Marquis 2005; van Benthem 2007; van Benthem, van Eijck, and Kooi 2006; Lorini and Schwarzentruber 2021; van Ditmarsch, van der Hoek, and Kooi 2007). However, most of these approaches focus on belief updates. To the best of our knowledge, there exists no study on the AGM and the DP postulates in belief revision in multi-agent environements.

In this paper, [ST: need to change this paragraph] we investigate the belief revision problem in multi-agent systems, i.e., what will be the beliefs of all agents after one agent gains the belief in some fluent formula. We propose a generalization of the classical AGM postulates, and of the Darwiche-Pearl postulates for iterated revision, to the multi-agent context. We provide an example of a simple, generalized "full-meet" multi-agent belief revision operator and prove that it satisfies all of the generalized AGM postulates. We also show that it satisfies some, but not all, generalized postulates for iterated mutli-agent belief revision. We then present an event model based belief operator and prove that the proposed operator satisfes the generalized AGM and Darwiche-Pearl postulates in multi-agent systems.

[ST: The paper is organized as follows. ...]

2 Preliminaries

We start with a review of the basic notions and notations for multi-agent belief models and classical single-agent belief revision.

If A is a set of formulae, $Cn(A)$ is the set of logical consequences of A. Cn satisfies the following properties: (*i*) $A \subseteq Cn(A)$; (*ii*) if $A \subseteq B$ then $Cn(A) \subseteq Cn(B)$; (*ii*) $Cn(A) = Cn(Cn(A))$.

Cn is assumed to be supraclassical, i.e. if p can be derived from A by classical truth-functional logic, then $p \in Cn(A)$; A is a belief set if and only if $A = Cn(A)$.

In what follows, K will denote a belief set, $X \vdash p$ is an alternative notation for $p \in Cn(X)$, and $X \nvdash p$ for $p \notin Cn(X)$. $Cn(\emptyset)$ is the set of tautologies.

[MT: In the rest of the paper, should we replace $\models$ by $\vdash$ everywhere else other than when we say $(M,s) \models$ ..., to use different symbols for the two.]

The expansion of K by a formula p, i.e. the operation that just adds p and removes nothing, is denoted $K + p$, and defined as follows: $K + p = Cn(K \cup \{p\})$.

2.1 Belief Formula

A *multi-agent* domain $\langle \mathcal{A}, \mathcal{F} \rangle$ includes a finite and non-empty set of agents $\mathcal{A}$ and a set of fluents $\mathcal{F}$ encoding properties of the world. *Belief formulae* over $\langle \mathcal{A}, \mathcal{F} \rangle$ are defined by the BNF:

$$\varphi \ ::= \ p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \mathbf{B}_i\varphi$$

where $p \in \mathcal{F}$ is a fluent and $i \in \mathcal{A}$. We also use other standard connectives like $\rightarrow$ and $\leftrightarrow$ through their usual abbreviation. We refer to a belief formula which does not contain any occurrence of $\mathbf{B}_i$ as *a fluent formula*. In addition, for a formula φ and a non-empty set $\alpha \subseteq \mathcal{A}$, $\mathbf{B}_\alpha\varphi$ and $\mathbf{C}_\alpha\varphi$ denote $\bigwedge_{i\in\alpha} \mathbf{B}_i\varphi$ and $\bigwedge_{k=1}^{\infty} \mathbf{B}_\alpha^k\varphi$, where $\mathbf{B}_\alpha^1\varphi$=$\mathbf{B}_\alpha\varphi$ and $\mathbf{B}_\alpha^k\varphi$=$\mathbf{B}_\alpha^{k-1}\mathbf{B}_\alpha\varphi$ for $k > 1$, respectively. $\mathcal{L}_\mathcal{A}$ denotes the set of belief formulae over $\langle \mathcal{A}, \mathcal{F} \rangle$.

Satisfaction of belief formulae is defined over *pointed Kripke structures* (Fagin et al. 1995). A Kripke structure M is a tuple $\langle W, \{R_a\}_{a\in\mathcal{A}}, \pi \rangle$, where W is a set of worlds, $\pi : W \mapsto 2^{\mathcal{F}}$ is a function that associates an interpretation of $\mathcal{F}$ to each element of W, and for $a \in \mathcal{A}$, $R_i \subseteq W \times W$ is a binary relation over W. For $u \in W$ and a fluent formula φ, $M[\pi](u)$ and $M[\pi](u)(\varphi)$ denote the interpretation associated to u via π and the truth value of φ with respect to $M[\pi](u)$. For a world $s \in W$, referred to as *true state of the world*, (M, s) is a *pointed Kripke structure*.

The satisfaction relation $\models$ between belief formulae and a state (M, s) is defined as follows:

(i) $(M, s) \models p$ if p is a fluent and $M[\pi](s) \models p$;

(ii) $(M, s) \models \mathbf{B}_i\varphi$ if $\forall t.[(s, t) \in R_i \Rightarrow (M, t) \models \varphi]$;

(iii) $(M, s) \models \neg\varphi$ if $(M, s) \not\models \varphi$;

(iv) $(M, s) \models \varphi_1 \vee \varphi_2$ if $(M, s) \models \varphi_1$ or $(M, s) \models \varphi_2$;

(v) $(M, s) \models \varphi_1 \wedge \varphi_2$ if $(M, s) \models \varphi_1$ and $(M, s) \models \varphi_2$.

Two pointed Kripke structures $\langle W, \{R_a\}_{a\in\mathcal{A}}, \pi \rangle$ and $\langle W', \{R'_a\}_{a\in\mathcal{A}}, \pi' \rangle$ are *bisimilar* if there is a relation $\mathcal{Z} \subseteq W \times W'$ such that for all $(w, w') \in \mathcal{Z}$:

1. $M\pi[w] = M'\pi'[w']$;
2. for each $w_1 \in W$ such that $R_a(w, w_1)$, $R'_a(w', w'_1)$ for some $(w_1, w'_1) \in \mathcal{Z}$;
3. for each $w'_1 \in W'$ such that $R'_a(w', w'_1)$, $R_a(w, w_1)$ for some $(w_1, w'_1) \in \mathcal{Z}$.

2.2 Belief Revision Postulates

We follow (Darwiche and Pearl 1997) and state the standard postulates on the basis of *epistemic states* M, which implicitly determine a set of beliefs K_M. Two epistemic states are *equivalent*, written $M_1 \equiv M_2$, iff $K_{M_1} = K_{M_2}$, that is, they entail the same beliefs. The intuitive meaning of $M * p$ is to revise the beliefs so as to ensure that K_{M*p} contains p and is consistent (unless p is inconsistent). Alchourrón, Gärdenfors, and Makinson (1985), aka AGM, proposed the following eight basic postulates for one-shot belief revision:

- *Closure*: $K_{M*p} = Cn(K_{M*p})$
- *Success*: $p \in K_{M*p}$
- *Inclusion*: $K_{M*p} \subseteq K_M + p$
- *Vacuity*: if $\neg p \notin K$ then $K_{M*p} = K_M + p$
- *Consistency*: K_{M*p} is consistent if p is consistent
- *Extensionality*: if $\models p \leftrightarrow q$ then $M * p \equiv M * q$
- *Superexpansion*: $K_{M*(p\wedge q)} \subseteq K_{M*p} + q$
- *Subexpansion*: $K_{M*p} + q \subseteq K_{M*(p\wedge q)}$ if $\neg q \notin K_{M*p}$

Darwiche and Pearl (1997) augmented the AGM framework by four postulates for iterated revision:

- *DP1*: if $q \models p$ then $K_{(M*p)*q} = K_{M*q}$
- *DP2*: if $q \models \neg p$ then $K_{(M*p)*q} = K_{M*q}$
- *DP3*: if $p \in K_{M*q}$ then $p \in K_{(M*p)*q}$
- *DP4*: if $\neg p \notin K_{M*q}$ then $\neg p \notin K_{(M*p)*q}$

The following postlate strengthens DP3 and DP4 (Jin and Thielscher 2007):

- *Independence*: if $\neg p \notin K_{M*q}$ then $p \in K_{(M*p)*q}$

3 Multi-agent Belief Revision

We begin by discussing the basic concepts for multi-agent belief revision based on a single, pointed Kripke model representing the beliefs of all agents. Thereafter, we propose generalizations of all the AGM postulates and postulates for iterated revision, including the postulate of Independence.

3.1 Basic concepts

Belief Sets A *belief set* is represented by a single, pointed Kripke structure (M, s) and is the set of all formulae entailed by (M, s):

$$K_{(M,s)} \ = \ \{\varphi \in \mathcal{L}_\mathcal{A} \mid (M, s) \ \models \ \varphi\}$$

In the following, we will assume that for any pointed Kripke structure (M, s) in consideration, s is the true state of the world.

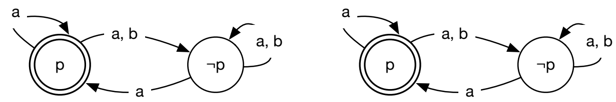

Figure 1: Two pointed Kripke structures (M_1, s_1) and (M_2, s_2), determining different multi-agent belief sets $K_1 = K_{(M_1,s)}$ and $K_2 = K_{(M_2,s)}$.

Fig. 1 shows two examples of pointed Kripke structures side by side. The correspondinhg belief sets both include that agent b (falsely) believes in $\neg p$ while a does not believe in either p or $\neg p$. Also, in both belief sets, b believes that a doesn't believe in either, that is, $\mathbf{B}_b[(\neg\mathbf{B}_a p) \wedge (\neg\mathbf{B}_a \neg p)]$ The two differ in that only in the first belief set, a believes that b believes $\neg p$. Formally, $K_1 \models \mathbf{B}_a\mathbf{B}_b\neg p$ whereas $K_2 \models \neg\mathbf{B}_a\mathbf{B}_b\neg p$.

Note that two multi-agent Kripke models that are bisimilar define the same belief set, provided the two designated worlds coincide in their interpretation. Even when the two structures are dissimilar, they still induce the same belief set if they entail the same belief formulas from the respective true state of the world.

Deductive closure of a belief set We consider the deductive closure of a belief set, *Cn*(K), w.r.t. entailment relation "$\models$". It follows immediately that any belief set satisfies *Cn*$(K) = K$.

Consistent beliefs/belief sets Agents can have inconsistent beliefs: By definition, $\mathbf{B}_a\bot \in K_{(M,s)}$ if, and only if, $R_a(s) = \emptyset$ in M. Agents can also believe that other agents have inconsistent beliefs etc., but a belief set itself is always consistent because $M[\pi](s) \not\models \bot$ for any pointed Kripke structure. Hence, the concept of consistency in the classical AGM postulates needs to be adapted to consistency of the beliefs of individual agents in the multi-agent setting when representing a belief set by a single Kripke structure.

Revision With the aim to model sensing actions in a multi-agent environment, we consider revising belief sets to reflect the result of an agent making an observation about the environment. Formally, a belief set will be revised by *first-degree belief formulae* according to the following definition:

$$\mathcal{B}_{\mathcal{A},\mathcal{F}} = \{\mathbf{B}_a\varphi \mid a \in \mathcal{A},\ \varphi \text{ fluent formula over } \mathcal{F}\}$$

The belief set after revision by a first-degree belief $\mathbf{B}_a\varphi$ should again be represented by a single, pointed Kripke structure, denoted by $(M, s) * \mathbf{B}_a\varphi$. We will simply write $K * \mathbf{B}_a\varphi$ to refer to the belief set $K_{(M,s)*\mathbf{B}_a\varphi}$ when it is clear from the context that (M, s) is the underlying Kripke model determining the belief set K.

Subset relation over belief sets The postulates also require us to define the concept of a subset relation among belief sets. Because every belief set is represented by a single Kripke structure, we cannot define this relation based on the set of all formulae entailed, since otherwise the subset relation would be satisfied only if the two Kripke structures entail identical sets of formulae. Therefore, and in line with the definiton of revision formulae, we consider all formulae of the form $\mathbf{B}_a\varphi$, where $a \in \mathcal{A}$ and φ is a fluent formula over $\mathcal{F}$, when comparing two belief sets:

$$K_{(M_1,s_1)} \subseteq K_{(M_2,s_2)}$$
iff
$$\forall\, \mathbf{B}_a\varphi \in \mathcal{B}_{\mathcal{A},\mathcal{F}}.[(M_1, s_1) \models \mathbf{B}_a\varphi \Rightarrow (M_2, s_2) \models \mathbf{B}_a\varphi]$$

Recall, for example, the belief sets represented in Fig. 1. Although $K_1 \neq K_2$, both of them entail the same first-degree belief formulas. This follows from the fact that for both Kripke structures (M, s), we have

$$\exists(s, w), (s, w') \in R_A.\ M[\pi]w \models p \wedge M[\pi]w' \models \neg p$$
$$\wedge\, \forall(s, w) \in R_B.\ M[\pi]w \models p$$

Hence, in both models we have that $(M, s) \models \mathbf{B}_A\varphi$ iff φ is a tautological fluent formula while $(M_i, s_i) \models \mathbf{B}_B\varphi$ iff $p \models \varphi$. Consequently, $K_1 \subseteq K_2$ and $K_2 \subseteq K_1$.

Minimal belief sets A consequence of this definition is that the "smallest" (w.r.t. the subset relation) representable belief sets are exactly those that include only first-degree belief formulas of the form $\mathbf{B}_a\varphi$ with φ a propositional tautology over $\mathcal{F}$. There are different Kripke structures that can be used to represent this set; a generic one can be constructed as $M_\emptyset = \langle W, \{R_a\}_{a\in\mathcal{A}}, \pi\rangle$ with $W = 2^{\mathcal{F}}$, $R_i = W \times W$ for all $i \in \mathcal{A}$, and $\pi(w) = w$.

Lemma 1. *For any $s \in 2^{\mathcal{F}}$ we have that $K_{(M_\emptyset,s)} \subseteq K$ for all belief sets K.*

It is worth stressing that not all smallest belief sets are equal as they can be based on structurally different Kripke models and hence contain different nested beliefs. For example, $(M_\emptyset, s)$ always entails $\mathbf{B}_a\neg\mathbf{B}_b\varphi$ for all agents $a, b \in \mathcal{A}$ and non-tautological fluent formulae φ, that is, every agent believes that no agent believes in anything other than tautological properties about the environment. This may not be the case in other minimal belief sets.

Expansion A key concept in the classical AGM postulates is the expansion of a belief set by a new belief. The intuition behind this concept is to add a new belief while retaining the existing beliefs, together with all the logical consequences of the old and new beliefs, but in a minimal fashion. In order to generalise this concept to multi-agent beliefs given by a pointed Kripke structure, the intuition behind the following formal definition is that $K+\mathbf{B}_a\varphi$ is obtained by constructing a new structure that's a combination of: (1) M (the old structure); (2) a new true state of the world s'; and (3) a structure that is obtained from M by: (i) removing all links labeled a into a world in which φ is false, and (ii) links labeled a going from s' to worlds in which there is a link labeled a.

Formally, let $M = (W, \{R_a\}_{a\in\mathcal{A}}, \pi)$, then *expanding* a pointed Kripke model (M, s) by a first-degree belief formula results in the pointed Kripke model $(M, s) + \mathbf{B}_a\varphi = (M', s')$ with $M' = (W', \{R'_a\}_{a\in\mathcal{A}}, \pi')$ such that

- $W' = W \cup W^r \cup \{s'\}$ with $W^r = \{s^r \mid s \in W\}$ – the replicate of W, where s' is a new world symbol that does not occur in $W \cup W^r$;
- for $w \in W$ and $w' \in \{w, w^r\}$, $\pi'(w') = \pi(w)$; and $\pi'(s') = \pi(s)$;

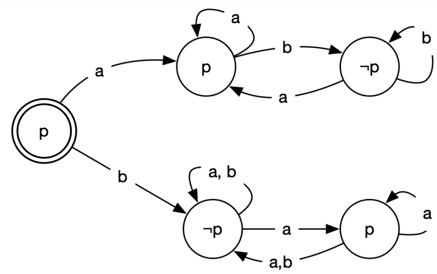

Figure 2: The pointed Kripke structure resulting from expanding (M_1, s_1) in Fig. 1 by $\mathbf{B}_a p$. The new designated world is linked via a to the replicate of the old Kripke structure (top) and via b to the old Kripke structure itself (bottom). All a-links to $\neg p$-worlds have been removed in the replicated part. As a result, a now believes in p (but also still in $\mathbf{B}_b \neg p$).

- for $x \in \mathcal{A} \setminus \{a\}$,
  - if $(u, v) \in R_x$ then (u, v) and (u^r, v^r) belong to R'_x,
  - if $(s, v) \in R_x$ then (s', v) belongs to R'_x;
- for $x = a$,
  - if $(u, v) \in R_x$ and $\pi(v) \models \varphi$ then (u, v) and (u^r, v^r) belong to R'_x;
  - if $(u, v) \in R_x$ and $\pi(v) \not\models \varphi$ then (u, v) belongs to R'_x;
  - if $(s, v) \in R_x$ and $\pi(v) \models \varphi$ then (s', v^r) belongs to R'_x.

Let K be a belief set that is represented by the pointed Kripke model (M, s), and let $\mathbf{B}_a\varphi \in \mathcal{B}_{\mathcal{A},\mathcal{F}}$, then the *expansion* $K + \mathbf{B}_a\varphi$ is defined as the belief set $K_{(M',s')}$ where $(M, s') = (M, s) + \mathbf{B}_a\varphi$.

As an example, Fig. 2 depicts the results of expanding the Kripke structure to the left in Fig. 1 by $\mathbf{B}_a p$. Obviously, a now believes in p, i.e. $\mathbf{B}_a p \in K_1 + \mathbf{B}_a p$, since the only a-accessible worlds from s' satisify p. We can also see that b retains exactly the old beliefs, e.g., $\mathbf{B}_b \neg p \in K_1 + \mathbf{B}_A p$ and also $\mathbf{B}_b((\neg \mathbf{B}_a p) \wedge (\neg \mathbf{B}_a \neg p)) \in K_1 + \mathbf{B}_a p$. This is so because the b-accessible worlds from s' are exactly those that were accessible from s previously, and with the same structure. Meanwhile, a also retained the belief that b believes in $\neg p$, that is, $K_1 + \mathbf{B}_a p \models \mathbf{B}_a \mathbf{B}_b \neg p$.

The next lemma shows that expanding a belief set with $\mathbf{B}_a\varphi$ does not change the first-degree beliefs of other agents.

Lemma 2. *For any formula ψ and agent $x \in \mathcal{A} \setminus \{a\}$, $(M, s) \models B_x\psi$ iff $(M', s') \models B_x\psi$.*

Proof. By construction of $(M, s) + \mathbf{B}_a\varphi$, we have that $(s', u) \in R'_x$ iff $(s, u) \in R_x$. Therefore, $(M, s) \models B_x\psi$ iff $(M', s') \models B_x\psi$. □

An expansion of a belief set with $\mathbf{B}_a\varphi$ can result in a having inconsistent beliefs or believing in φ as shown in the next lemma.

Lemma 3. *If $R'_a(s') = \emptyset$ then $(M', s') \models \mathbf{B}_a\bot$, i.e., (M', s') is inconsistent. If $R'_a(s') \neq \emptyset$ then $(M', s') \models \mathbf{B}_a\varphi$.*

Proof. By definion of the entailment of formulae from a pointed Kripke structure, if $R'_a(s') = \emptyset$ then $(M', s') \models \mathbf{B}_a\varphi$ and $(M', s') \models \mathbf{B}_a\neg\varphi$ for any formula φ. Thus, $(M', s') \models \mathbf{B}_a\bot$.

Assume that $R'_a(s') \neq \emptyset$. By construction, for every u^r such that $(s', u^r) \in R'_A$, we have that $(s, u) \in R_A$ and $\pi(u) \models \varphi$. This implies that $\pi(u^r) \models \varphi$, and thus, $(M', s') \models \mathbf{B}_A\varphi$ because $R'_a(s') \neq \emptyset$. □

The next lemma shows that the first-degree beliefs of agent a are exactly the logical consequences of its old beliefs plus the new belief.

Lemma 4. *If $R'_a(s') \neq \emptyset$ then $(M', s') \models \mathbf{B}_a\psi$ if, and only if, $\psi \in Cn\left(\{\varphi\} \cup \{\phi \in \mathcal{B}_{\mathcal{A},\mathcal{F}} \mid \mathbf{B}_a\phi \in K_{(M,s)}\}\right)$.*

Proof. The proof of this lemma is similar to the proof of Lemma 3 with the observation that for $\psi \in Cn\left(\varphi \cup \{\phi \in \mathcal{B}_{\mathcal{A},\mathcal{F}} \mid \mathbf{B}_a\phi \in K_{(M,s)}\}\right)$, $(s', u^r) \in R'_a$ and $\pi'(u^r) \models \psi$ iff $(s, u) \in R_a$ and $\pi(u) \models \psi$. □

3.2 Postulates for one-shot multi-agent BR

Using the basic concepts for multi-agent belief sets represented by a single, pointed Kripke strucure, we discuss generalizations of the standard postulates, beginning with the AGM postlates for for one-shot revision.

Closure A revised belief set is deductively closed, denoted by *Cn*, under the modal logic being interpreted:

$$K * \mathbf{B}_a\varphi = Cn(K * \mathbf{B}_a\varphi)$$

Since the set of belief formulas entailed by a single, pointed Kripke structure is deductively closed, this postulate holds under the assumption that a revised belief set is represented by a pointed Kripke model.

Success The result of revising a Kripke model by a first-degree belief formula should include the new belief:

$$\mathbf{B}_a\varphi \in K * \mathbf{B}_a\varphi$$

Inclusion A revised Kripke model should only contain first-degree belief formulas that are included in the expanded Kripke model:

$$K * \mathbf{B}_a\varphi \subseteq K + \mathbf{B}_a\varphi$$

From Lemma 2 and 4 it follows that under the Inclusion principle, a revised Kripke model contains only first-degree belief formulae that follow logically from the old and new first-degree beliefs.

Our generalized Inclusion postulate does not make any assumptions about second- or higher-degree belief formulas. In particular it allows for another agent, b, to change its beliefs about agent a believing in φ. Hence, $K * \mathbf{B}_a\varphi$ may contain beliefs that are not included in $K + \mathbf{B}_a\varphi$ if these are not first-order beliefs.

Vacuity If a Kripke model is revised by a first-degree belief that is consistent with the current beliefs, then the result should contain all first-degree belief formulas that are included in the expanded Kripke model:

$$\mathbf{B}_a\neg\varphi \notin K \;\Rightarrow\; K + \mathbf{B}_a\varphi \subseteq K * \mathbf{B}_a\varphi$$

By Lemma 2 and 4 it follows that under the Vacuity principle, a revised Kripke model contains all first-degree belief formulae that follow logically from the old and new first-degree beliefs, provided the old beliefs did not include the opposite of the new belief.

It should be noted that Vacuity and Inclusion together are a weaker requirement than stipulating that revision be identical to expansion in case a new belief is consistent with the old ones. They merely postulate that the first-degree beliefs are the same, while they do not demand anything about other belief formulae. In particular, they do not prescribe the specific structure from the definition of expansion of a pointed Kripke model with a new belief.

Similar to the generalized Inclusion princple, the postulate does allow for belief revision operators in which other agents change their belief about the belief of agent a in φ as a result of this revision.

Consistency 1 If a new belief $\mathbf{B}_a\varphi$ is consistent, then any revision by this new belief should result in a consistent belief for agent a:

$$\not\models \varphi \rightarrow \bot \;\Rightarrow\; K * \mathbf{B}_a\varphi \not\models \mathbf{B}_a\bot$$

It is worth noting that this does not postulate overall consistency of beliefs as it cannot be generally assumed that one agent changing their beliefs would always mean that any other agent that may have had inconsistent beliefs would automatically end up with consistent beliefs too. However, while it is possible that other agents change their beliefs about agent a's beliefs, it is reasonable to postulate that they do not end up with incosnsistent beliefs as a result if they had consistent beliefs beforehand. For this reason, we suggest the following additional postulate on consistency.

Consistency 2 If a new belief $\mathbf{B}_a\varphi$ is consistent, then any agent with consistent beliefs will have consistent beliefs after the revision:

$$\mathbf{B}_b\bot \notin K \wedge \not\models \varphi \rightarrow \bot \;\Rightarrow\; K * \mathbf{B}_a\varphi \not\models \mathbf{B}_b\bot$$

Extensionality If two new belief formulae are logically equivalent w.r.t. the underlying modal logic, then a belief set revised by either of the two should give the same result:

$$\models \mathbf{B}_a\varphi \leftrightarrow \mathbf{B}_b\psi \;\Rightarrow\; K * \mathbf{B}_a\varphi = K * \mathbf{B}_b\psi$$

It is easy to see that $\mathbf{B}_a\varphi$ and $\mathbf{B}_b\psi$ are logically equivalent under *any* pointed Kripke structure if, and only if, $a = b$ and φ and ψ are logically equivalent fluent fromulae.

Superexpansion Revising a belief set by a conjunction $\varphi \wedge \psi$ of two new beliefs of an agent a should not result in more first-degree beliefs than the expansion by $\mathbf{B}_a\psi$ of the result of revising by $\mathbf{B}_a\varphi$:

$$K * \mathbf{B}_a(\varphi \wedge \psi) \subseteq (K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$$

Similar to Inclusion and Vacuity, this generalized postulate does not make any assumptions about second- or higher-degree belief formulas of any agent. This is also true for the following counterpart, subexpansion.

Subexpansion If the second belief $\mathbf{B}_a\psi$ is consistent with the result of revising a belief set by the first belief $\mathbf{B}_a\varphi$, then revision by the conjunction of the two should not result in fewer first-degree beliefs than the expansion by $\mathbf{B}_a\psi$ of the result of revising by $\mathbf{B}_a\varphi$:

$$\mathbf{B}_a\neg\psi \notin K * \mathbf{B}_a\varphi \;\Rightarrow\; (K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi \subseteq K * \mathbf{B}_a(\varphi \wedge \psi)$$

3.3 Postulates for iterated multi-agent BR

DP1 – Successive revision respect Revision by one belief followed by revising by a stronger belief makes the first revision redundant:

$$\models \mathbf{B}_b\psi \rightarrow \mathbf{B}_a\varphi \;\Rightarrow\; (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi = K * \mathbf{B}_b\psi$$

DP2 – Irrelevance of superseded beliefs If for two successive revisions, the second belief contradicts the first one, then the resulting beliefs for all agents will be the same as if the original would have been revised according to the second belief only.

$$\models \mathbf{B}_b\psi \rightarrow \mathbf{B}_a\neg\varphi \;\Rightarrow\; (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi = K * \mathbf{B}_b\psi$$

DP3 – Consistency preservation across revisions If one agent's first-degree belief would be contained in the belief set after revision by any other first-degree belief, then in case the former is used to revise the belief set first, that first belief should be preserved through the second revision:

$$\mathbf{B}_a\varphi \in K * \mathbf{B}_b\psi \;\Rightarrow\; \mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi$$

DP4 – Minimal change when reaffirming a belief After two consecutive revisions, the first agent should not end up believing the opposite unless they would do so if the belief set was revised by the second belief only:

$$\mathbf{B}_a\neg\varphi \notin K * \mathbf{B}_b\psi \;\Rightarrow\; \mathbf{B}_a\neg\varphi \notin (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi$$

Independence After two consecutive revisions, the belief of the first agent should be preserved unless they would believe the opposite if the belief set was revised by the second belief only:

$$\mathbf{B}_a\neg\varphi \notin K * \mathbf{B}_b\psi \;\Rightarrow\; \mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi$$

Interestingly, while in the classical, single-agent case the Independence postulate strengthens both DP3 and DP4 (Jin and Thielscher 2007) for any operator that satisfies the AGM postulates, in the generalized case Independence only strengthens DP4.

Lemma 5. *Consider a belief revision operator that satisfies the generalized AGM postulates, then the operator satisfies multi-agent DP4 if it satisfies multi-agent Independence.*

Proof. Independence obviously implies DP4 unless $\{\mathbf{B}_a\phi, \mathbf{B}_a\neg\phi\} \subseteq (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi$. The latter would mean that a has inconsistent beliefs at the end of the two revisions, which according to Inconsistency 1 and 2 can only happen if $\varphi \models \bot$. This in turn implies $\mathbf{B}_a\neg\varphi \notin K * \mathbf{B}_b\psi$, thus DP4 holds vacuously in this case also. $\square$

It is noteworthy that Independence would not entail DP4 without the additional Consistency 2 postulate, which guarantees that a does not end up believing in both φ and $\neg\varphi$ as a result of further revision by $\mathbf{B}_b\psi$ after revising by $\mathbf{B}_a\varphi$.

Indepence does not imply DP3 even if an operator satisfies all multiagent AGM postulates, for the following reason: If $\mathbf{B}_a\bot \in K*\mathbf{B}_b\psi$ then Independence vacuously holds while DP3 is violated if $\mathbf{B}_a\bot \notin (K*\mathbf{B}_a\bot\varphi)*\mathbf{B}_b\psi$. This is possible for an operator that allows an agent to regain consistent beliefs through revising a multi-agent belief set by another agent's belief.

4 Multi-agent Belief Revision Operators

In this section, we discuss a generalization of the well-known "full-meet" revision operator for classical Belief Revision (Alchourrón and Makinson 1982) and show that this satisfies all generalized AGM postulates for multi-agent Belief Revision.

We define *multi-agent full meet revision*, denoted $*_{\text{fm}}$, as follows:

$$K_{(M,s)} *_{\text{fm}} \mathbf{B}_a\varphi = \begin{cases} K_{(M,s)} + \mathbf{B}_a\varphi & \text{if } \mathbf{B}_a\neg\varphi \notin K_{(M,s)} \\ K_{(M_\emptyset,s)} + \mathbf{B}_a\varphi & \text{otherwise} \end{cases}$$

The principle behind this definition is the same as for classical full-meet revision (Alchourrón, Gärdenfors, and Makinson 1985). If a new belief is consistent with the current beliefs, the underlying Kripke structure is simply expanded by that belief. Otherwise, the new belief is incorporated in the most conservative manner by starting with a minimal belief set, in which it is common knowledge that all agents believe in tautological fluent formulas only, and then expanding the underlying Kripke model by the new belief.

It should be noted, however, that unlike with the classical full-meet belief revision operator, the belief set resulting from revision by a belief that is inconsistent with the old beliefs does entail more beliefs than follow logically from the new one. This is so because the underlying Kripke structure makes strong assumptions about higher-degree beliefs. This notwithstanding, the generalized full-meet operator can be proved to satisfy all of the generalized AGM postulates for iterated multi-agent belief revision.

Closure By our definition, it is clear that

$$K *_{\text{fm}} \mathbf{B}_a\varphi = Cn(K *_{\text{fm}} \mathbf{B}_a\varphi).$$

Success By definition, if (M', s') is the result of expanding a belief set by $\mathbf{B}_a\varphi$, then $\pi(v) \models \varphi$ for all $v^r \in W'$ such that $(s', v^r) \in R'_a$. This implies $K *_{\text{fm}} \mathbf{B}_a\varphi \models \mathbf{B}_a\varphi$.

Inclusion

1. Suppose that $\mathbf{B}_a\neg\varphi \notin K_{(M,s)}$, then the definition of $*_{\text{fm}}$ implies that $K_{(M,s)} *_{\text{fm}} \mathbf{B}_a\varphi \subseteq K_{(M,s)} + \mathbf{B}_a\varphi$.
2. If $\mathbf{B}_a\neg\varphi \in K_{(M,s)}$, then
   (a) $R'_a(s') = \emptyset$ where $(M', s') = (M, s) + \mathbf{B}_a\varphi$, hence $K + \mathbf{B}_a\varphi \models \mathbf{B}_a\bot$ by Lemma 3, which implies that $K_{(M,s)} + \mathbf{B}_a\varphi \models \mathbf{B}_a\psi$ for any fluent formula ψ;
   (b) for $x \in \mathcal{A} \setminus \{a\}$, $(M_\emptyset, s) \models \mathbf{B}_x\psi$ if, and only if, $\models \psi$; by Lemma 2 this holds for $(M_\emptyset, s) + \mathbf{B}_a\varphi$ as well, hence $K_{(M_\emptyset,s)} + \mathbf{B}_a\varphi \models \mathbf{B}_b\psi$ only for tautologies ψ.

   Taken together, $K_{(M_\emptyset,s)} + \mathbf{B}_a\varphi \subseteq K_{(M,s)} + \mathbf{B}_a\varphi$, hence $K_{(M,s)} *_{\text{fm}} \mathbf{B}_a\varphi \subseteq K_{(M,s)} + \mathbf{B}_a\varphi$.

Vacuity This holds by definition.

Consistency 1 Suppose $\mathbf{B}_a\varphi$ is consistent, i.e., $\not\models \varphi \rightarrow \bot$.

1. If $\mathbf{B}_a\neg\varphi \notin K_{(M,s)}$, then $(s', v^r) \in R'_a$ for some $v^r \in W'$ such that $\pi[v^r] \models \varphi$, where $(M', s') = (M, s) + \mathbf{B}_a\varphi$. Hence, agent a has cosistent beliefs in $K_{(M,s)} *_{\text{fm}} \mathbf{B}_a\varphi$.
2. If $\mathbf{B}_a\neg\varphi \in K_{(M,s)}$, then $(s', v^r) \in R'_a$ for some $v^r \in W'$ such that $\pi[v^r] \models \varphi$, where $(M', s') = (M_\emptyset, s) + \mathbf{B}_a\varphi$. Hence, agent a has cosistent beliefs in $K_{(M,s)} *_{\text{fm}} \mathbf{B}_a\varphi$.

Consistency 2 If $\mathbf{B}_a\varphi$ is consistent with K then the beliefs of any agent other than a do not change as a result of expanding (M, s) by $\mathbf{B}_a\varphi$. If $\mathbf{B}_a\varphi$ is inconsistent, then any agent other than a has tautological beliefs only, hence does not have inconsistent beliefs.

Extensionality If $\models \varphi \leftrightarrow \psi$ then $(M, s) + \mathbf{B}_a\varphi$ and $(M, s) + \mathbf{B}_a\psi$ are identical Kripke structures for any (M, s) by definiton.

Superexpansion

1. Suppose $\mathbf{B}_a\neg(\varphi \wedge \psi) \notin K$. Since K is deductively closed, it follows that $\mathbf{B}_a\neg\varphi \notin K$. Hence, agent a has consistent beliefs in $K + \mathbf{B}_a(\varphi \wedge \psi)$, in $K + \mathbf{B}_a\varphi$, and in $(K + \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$. By Lemma 4 it follows that $(M, s) + \mathbf{B}_a(\varphi \wedge \psi) \models \mathbf{B}_a\chi$ iff $\chi \in Cn(\{\varphi \wedge \psi\} \cup \{\phi \in \mathcal{B}_{\mathcal{A},\mathcal{F}} \mid \mathbf{B}_a\phi \in K\})$. This is equivalent to $\chi \in Cn(\{\psi\} \cup Cn(\{\varphi\} \cup \{\phi \in \mathcal{B}_{\mathcal{A},\mathcal{F}} \mid \mathbf{B}_a\phi \in K\}))$, which in turn by Lemma 4 is equivalent to $(K + \mathbf{B}_a\varphi) + \mathbf{B}_a\psi \models \mathbf{B}_a\chi$. By Lemma 2 it follows that all other agents too have the same beliefs in $K + \mathbf{B}_a(\varphi \wedge \psi)$ and in $(K + \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$. We conclude that $K *_{\text{fm}} \mathbf{B}_a(\varphi \wedge \psi) \subseteq (K *_{\text{fm}} \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$.
2. Suppose $\mathbf{B}_a\neg(\varphi \wedge \psi) \in K$. By definition of $(M_\emptyset, s) + \mathbf{B}_a(\varphi \wedge \psi)$ and Lemma 4 it follows that, for any first-degree belief, $\mathbf{B}_a\phi \in K *_{\text{fm}} \mathbf{B}_a(\varphi \wedge \psi)$ iff $\phi \in Cn(\varphi \wedge \psi)$, and for $x \in \mathcal{A} \setminus \{a\}$, $\mathbf{B}_x\phi \in K *_{\text{fm}} \mathbf{B}_a(\varphi \wedge \psi)$ iff $\models \phi$.
   (a) Suppose $\mathbf{B}_a\neg\varphi \notin K$, then $(K *_{\text{fm}} \mathbf{B}_a\varphi) + \mathbf{B}_a\psi = (K + \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$. From $\mathbf{B}_a\neg(\varphi \wedge \psi) \in K$ it follows that $\mathbf{B}_a\neg\psi \in K + \mathbf{B}_a\varphi$. Hence, a has inconsistent beliefs in $(K + \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$ while the beliefs of all other agents $x \in \mathcal{A} \setminus \{a\}$ are the same in $(K + \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$ and $K + \mathbf{B}_a(\varphi \wedge \psi)$.
   (b) Suppose $\mathbf{B}_a\neg\varphi \in K$, then $(K *_{\text{fm}} \mathbf{B}_a\varphi) + \mathbf{B}_a\psi = (K_{(M_\emptyset,s)} + \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$. Hence, a's first-degree beliefs in $(K *_{\text{fm}} \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$ are exactly the logical consequences of $\varphi \wedge \psi$ while the beliefs of all other agents $x \in \mathcal{A} \setminus \{a\}$ are the same in $(K *_{\text{fm}} \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$ and $K + \mathbf{B}_a(\varphi \wedge \psi)$.

   Taken together, $K *_{\text{fm}} \mathbf{B}_a(\varphi \wedge \psi) \subseteq (K *_{\text{fm}} \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$.

Subexpansion The proof is identical to cases 1 and 2(b) for superexpansion.

Much like the standard full-meet revision, the generalized version does not satisfy all of the (extended) postulates for iterated revision.

DP1 – Successive revision respect Suppose $\mathbf{B}_a\psi$ is consistent with K, then so is $\mathbf{B}_a\varphi$ since $\psi \models \varphi$. It follows that $K *_{\text{fm}} \mathbf{B}_a\psi = K_{(M,s)+\mathbf{B}_a\psi}$ and $(K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_a\psi = K_{((M,s)+\mathbf{B}_a\varphi)+\mathbf{B}_a\psi}$. The two Kripke structures are bisimilar except for unreachable worlds, hence they entail the same set of belief formulas:

- $(M_\varphi, s_\varphi) = (M, s) + \mathbf{B}_a\varphi$ contains the worlds W in M plus a replicate W_φ of these worlds such that a world is a-accessible from s_φ if, and only if, it is in W_φ and satisfies φ. $(M', s') = (M_\varphi, s_\varphi) + \mathbf{B}_a\psi$ contains the worlds $W \cup W_\varphi$ plus a replicate of these, $W' \cup W'_\varphi$, such that a world is a-accessible from s' iff it is in $W' \cup W'_\varphi$ and satisfies $\varphi \wedge \psi$, which is equivalent to saying it satisfies ψ since $\psi \models \varphi$; while none of the worlds in W_φ are reachable from s' because the only links from s_φ into worlds in W_φ are labeled with agent a.
- $(M_\psi, s_\psi) = (M, s) + \mathbf{B}_a\psi$ contains the worlds W in M plus a replicate W_ψ of these worlds such that a world is a-accessible from s_ψ iff it is in W_ψ and satisfies ψ.

It is easy to define a bisimulation for all the worlds reachable from s' and s_ψ, respectively, by which each world in W is identified with itself and its replica in W', and each world in W_ψ is identified with the corresponding world in W'_φ.

Suppose $\mathbf{B}_a\varphi$ is inconsistent with K, then so is $\mathbf{B}_a\psi$ since $\psi \models \varphi$. It follows that $K *_{\text{fm}} \mathbf{B}_a\psi = K_{(M_\emptyset,s)+\mathbf{B}_a\psi}$ and $(K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_a\psi = K_{((M_\emptyset,s)+\mathbf{B}_a\varphi)+\mathbf{B}_a\psi}$. Similar to the above it can be shown that the resulting pointed Kripke models entail the same set of beliefs.

Suppose $\mathbf{B}_a\varphi$ is consistent with K but $\mathbf{B}_a\psi$ is not, then $B_a\psi$ is also inconsistent with $K + \mathbf{B}_a\varphi$. It follows that $(K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_a\psi = K_{(M_\emptyset,s)+\mathbf{B}_a\psi} = K *_{\text{fm}} \mathbf{B}_a\psi$.

DP2 – Irrelevance of superseded beliefs Generalized full-meet does *not* satisfy weak DP, for the following reason: If $\mathbf{B}_b\psi$ is consistent with the current belief set K then $K *_{\text{fm}} \mathbf{B}_b\psi = K + \mathbf{B}_b\psi$, hence by Lemma 2, all agents retain all their beliefs when revising K set by b's new belief. But if K is revised by $\mathbf{B}_a\varphi$ first and $\models \mathbf{B}_b\psi \rightarrow \mathbf{B}_a\varphi$ holds then $(K *_{\text{fm}} \mathbf{B}_b\varphi) *_{\text{fm}} \mathbf{B}_b\psi = K_{(M_\emptyset,s)} + \mathbf{B}_b\psi$, which implies that all other agents' beliefs have been erased. The fact that full-meet does not satisfy DP2 mirrors a result in classical, single-agent belief revision (Jin and Thielscher 2007).

For this reason, full-meet multiagent belief revision satisfies only one direction of DP2, namely, revising by a belief that is then superseded by a second, contradictory belief never introduces more beliefs than revision with the second belief directly. To show this, we make a case distinction:

1. Suppose $\models \varphi \leftrightarrow \bot$ then $\mathbf{B}_a\neg\varphi \in K$, hence $K *_{\text{fm}} \mathbf{B}_a\varphi = K_{(M_\emptyset,s)} + \mathbf{B}_a\bot$, that is, there is no a-accessible world in s while all other agents' first-degree beliefs are tautological fluent formulas. Consequently, $(K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_b\psi \subseteq K *_{\text{fm}} \mathbf{B}_b\psi$.
2. Otherwise, $\models \mathbf{B}_b\psi \rightarrow \mathbf{B}_a\varphi$ implies $a = b$ and $\psi \models \neg\varphi$, hence $\varphi \models \neg\psi$. It follows that $\mathbf{B}_b \not\psi \in K *_{\text{fm}} \mathbf{B}_a\varphi$, hence $(K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_b\psi = K_{(M_\emptyset,s)} + \mathbf{B}_b\psi$, which implies $(K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_b\psi \subseteq K *_{\text{fm}} \mathbf{B}_b\psi$.

DP3 – Consistency preservation across revisions If $\models \varphi \leftrightarrow \top$ then DP3 holds trivially. Otherwise, $\mathbf{B}_a\varphi \in K *_{\text{fm}} \mathbf{B}_b\psi$ implies that $\mathbf{B}_b\psi$ is consistent with $K * B_a\varphi$ unless $\models \psi \rightarrow \bot$. But if the latter is true, then $K *_{\text{fm}} \mathbf{B}_b\psi = (K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_\psi = K_{(M_\emptyset,s)+\mathbf{B}_b\bot}$. In either case, DP3 follows.

DP4 – Minimal change when reaffirming a belief It is easy to see that DP4 holds if $\mathbf{B}_b\psi$ is inconsistent with $K *_{\text{fm}} \mathbf{B}_a\varphi$. In case it is consistent, DP4 follows from Lemma 2 and 4.

Independence While generalized full-meet satisfies both DP3 and DP4, it does *not* satisfy the stricter postulate of Independence: Consider two agents $a \neq b$ and a satisfiable but non-tautological fluent formula φ. Suppose further that $\mathbf{B}_a\varphi$ is consistent with K but $\mathbf{B}_b\psi$ is not, then $\mathbf{B}_b\psi$ is also inconsistent with $K *_{\text{fm}} \mathbf{B}_a\varphi$. It follows that $\mathbf{B}_a\varphi \notin (K *_{\text{fm}} \mathbf{B}_a\varphi) *_{\text{fm}} \mathbf{B}_b\psi$. But in this case, a only believes in tautological fluent formulae in the revised set $K *_{\text{fm}} \mathbf{B}_b\psi = K_{(M_\emptyset,s)} + \mathbf{B}_b\psi$, hence $\mathbf{B}_a\neg\varphi \notin K *_{\text{fm}} \mathbf{B}_b\psi$, thus violating the postulate.

5 Formal Assessment of Multi-agent Belief Revision Frameworks

Update Models A set $\{p \rightarrow \varphi \mid p \in \mathcal{F}, \varphi \in \mathcal{L}_\mathcal{A}\}$ is called an $\mathcal{L}_\mathcal{A}$*-substitution* (or substitution, for short). For each substitution sub and each $p \in \mathcal{F}$, we assume that sub contains exactly one formula $p \rightarrow \varphi$. For simplicity of the presentation, we often omit $p \rightarrow \varphi$ in a substitution. $SUB_{\mathcal{L}_\mathcal{A}}$ denotes the set of all substitutions. A substitution is used to encode changes caused by an action occurrence. A formula $p \rightarrow \varphi$ in a substitution states the condition (φ) under which p will become true.

Definition 1 (Update Model)**.** *An* update model $\mathbf{\Sigma}$ *is a tuple* $\langle \Sigma, \{E_a\}_{a\in\mathcal{A}}, pre, sub\rangle$ *where*

- Σ *is a set, whose elements are called* events*;*
- *each* E_a *is a binary relation on* Σ*;*
- $pre : \Sigma \rightarrow \mathcal{L}_\mathcal{A}$ *is a function mapping each event* $e \in \Sigma$ *to a formula in* $\mathcal{L}_\mathcal{A}$*; and*
- $sub : \Sigma \rightarrow SUB_{\mathcal{L}_\mathcal{A}}$.

An update instance ω *is a pair* $(\mathbf{\Sigma}, e)$ *where* $\mathbf{\Sigma}$ *is an update model and* $e \in \Sigma$ *(or a* designated event*). An* update template *is a pair* $(\mathbf{\Sigma}, \Gamma)$ *where* $\mathbf{\Sigma}$ *is an update model with the set of events* Σ *and* $\Gamma \subseteq \Sigma$*.*

Definition 2 (Updates by an Update Model)**.** *Given an update model* $\mathbf{\Sigma} = \langle \Sigma, R_1, \ldots, R_n, pre, sub\rangle$ *and a Kripke structure* $M = (W, \{R_a\}_{a\in\mathcal{A}}, \pi)$*. The* update operator *induced by* $\mathbf{\Sigma}$ *defines a Kripke structure* $M' = M \otimes \mathbf{\Sigma} = (W', \{R'_a\}_{a\in\mathcal{A}}, \pi'))$*, where:*

- $W' = \{(s, \tau) \mid s \in W, \tau \in \Sigma, (M, s) \models pre(\tau)\}$*;*
- $((s, \tau), (s', \tau')) \in R'_a$ *iff* $(s, \tau), (s', \tau') \in W'$*,* $(s, s') \in R_a$ *and* $(\tau, \tau') \in E_a$*;*

- $\forall f \in \mathcal{F}.[\pi'((s,\tau)) \models f$ *iff* $f \rightarrow \varphi \in sub(\tau), (M,s) \models \varphi]$.

For an agent $a \in \mathcal{A}$ to revise a conjunction of fluent literals φ, we define $\mathbf{\Sigma}^a(\varphi)$, called an *event-model for revision of* $\mathbf{B}_a\varphi$, as the event model $\langle \Sigma, \{E_a\}_{a\in\mathcal{A}}, pre, sub\rangle$ where $\Sigma = \{\sigma, \tau, \delta, \gamma, \sigma_a, \epsilon\}$ and

- $E_a = \{(\sigma, \sigma_a), (\tau, \sigma_a), (\delta, \delta), (\lambda, \lambda), (\epsilon, \epsilon)\}$;
- $E_x = \{(\sigma, \epsilon), (\tau, \epsilon), (\delta, \epsilon), (\lambda, \epsilon), (\epsilon, \epsilon)\}$ for $x \in \mathcal{A} \setminus \{a\}$;
- $pre(\sigma) = \varphi \wedge \neg \mathbf{B}_a \neg\varphi$, $pre(\tau) = \neg\varphi \wedge \neg\mathbf{B}_a\neg\varphi$, $pre(\delta) = \varphi \wedge \mathbf{B}_a\neg\varphi$, $pre(\lambda) = \neg\varphi \wedge \mathbf{B}_a\neg\varphi$, and $pre(\epsilon) = \top$; and
- $sub(\eta) = \emptyset$ for $\eta \in \Sigma \setminus \{\eta\}$ and $sub(\eta) = \{f \rightarrow \top \mid f \in \mathcal{F}, \varphi \models f\} \cup \{f \rightarrow \bot \mid f \in \mathcal{F}, \varphi \models \neg f\}$.

The model is depicted in Figure 3 where squares depict events and squares with double stroke line are designated events. Intuitively, $\mathbf{\Sigma}^a(\varphi)$ is an event model that aims to capture the changes in a's beliefs after it revises its beliefs with $\mathbf{B}_a\varphi$.

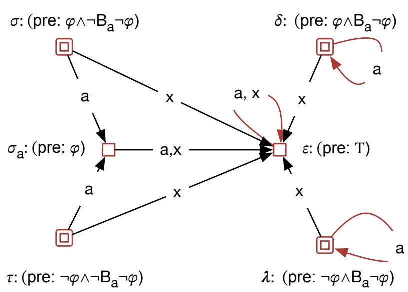

Figure 3: Revision Event Model for $\mathbf{B}_a\varphi$

Definition 3. *Let (M,s) be a pointed Kripke structor φ be a fluent literal. The event based revision operator $*$ of a revision of $\mathbf{B}_a\varphi$ in (M,s) is defined by*

$$(M,s) * \mathbf{B}_a\varphi = (M,s) \otimes (\mathbf{\Sigma}^a(\varphi), \{\sigma, \tau, \delta, \gamma\})$$

In the following, we will show that our resvision operator indeed satisfies the AGM+DP postulates. Before we discuss them in details, let us observe that for every (M,s), there exists a unique event $\eta \in \{\sigma, \tau, \delta, \gamma\}$ such that $(M,s) \models pre(\eta)$. Therefore, (s,η) will be a state in $(M,s) * \mathbf{B}_a\varphi$. From now on, we will refer to $(M,s) * \mathbf{B}_a\varphi$ as (M',s') where $s' = (s,\eta)$ and $M' = (W', \{R'_x\}_{x\in\mathcal{A}}, \pi')$ when no confusion is possible and assume that $M = (W, \{R_x\}_{x\in\mathcal{A}}, \pi)$. We first prove that beliefs of other agents do not change.

Lemma 6. *For every agent $x \in \mathcal{A}\setminus\{a\}$ and a fluent formula ψ, $(M',s') \models \mathbf{B}_x\psi$ iff $(M,s) \models \mathbf{B}_x\psi$.*

Proof. By the construction of (M',s'), we have that $s' = (s,\eta)$ such that $(M,s) \models pre(\eta)$. Furthermore, $(u,\tau) \in W'$ for every $u \in W$ and $((s,\eta),(u,\rho)) \in R'_x$ iff $(s,u) \in R_x$. Therefore, $(M',s') \models \mathbf{B}_x\psi$ iff $(M,s) \models \mathbf{B}_x\psi$. □

Closure By our definition, it is clear that

$$K * \mathbf{B}_a\varphi = Cn(K * \mathbf{B}_a\varphi).$$

Success Assume that K is consistent. We show that $K * \mathbf{B}_a\varphi \models \mathbf{B}_a\varphi$. We consider the four cases:

- $(M,s) \models \varphi \wedge \neg\mathbf{B}_a\neg\varphi$, i.e., a does not believe in $\neg\varphi$ before the revision. This implies that $s' = (s,\sigma)$. Furthermore, because K is consistent, we have that $R_a(s) \neq \emptyset$ and there exists some $u \in W$ such that $(s,u) \in R_a$ and $\pi(u) \models \varphi$. This means that $((s,\sigma),(u,\sigma_a)) \in R'_a$. In addition, if $((s,\sigma),(u,\rho)) \in R'_a$ then $\rho = \sigma_a$, and hence, $\pi(u) \models \varphi$, because E_a contains only one element related to σ, (σ,σ_a), and $(s,u) \in R_a$. Thus, we have that $(M',s') \models \mathbf{B}_a\varphi$.
- $(M,s) \models \neg\varphi \wedge \neg\mathbf{B}_a\neg\varphi$. This implies that $s' = (s,\tau)$. Similar arguments to the previous case allow us to show that $R'_a((s') \neq \emptyset$ and if $((s,\tau),(u,\rho)) \in R'_a$ then $\rho = \sigma_a$ and $(s,u) \in R_a$ and $\pi(u) \models \varphi$, which implies that $(M',s') \models \mathbf{B}_a\varphi$.
- $(M,s) \models \varphi \wedge \mathbf{B}_a\neg\varphi$. This implies that $s' = (s,\delta)$. Because of the loop (δ,δ) in E_a, we have that $((s,\delta),(s,\delta)) \in R'_a$. Furthermore, because $(M,s) \models \mathbf{B}_a\neg\varphi$, for every $u \in R_a$, $(u,\delta) \notin W'$, and therefore, R'_a is the singleton $\{((s,\delta),(s,\delta))\}$ which implies that $(M',s') \models \mathbf{B}_a\varphi$.
- $(M,s) \models \neg\varphi \wedge \mathbf{B}_a\neg\varphi$. This implies that $s' = (s,\lambda)$. Because of the loop (λ,λ) in E_a, we have that $((s,\lambda),(s,\lambda)) \in R'_a$, i.e., $R'_a(s') \neq \emptyset$. Furthermore, if $((s,\lambda),(u,\rho)) \in R'_a$ then $\rho = \lambda$, because E_a contains only (λ,λ) and $(s,u) \in R_a$ and thus $(M,s) \models \neg\varphi \wedge \mathbf{B}_a\neg\varphi$. This implies that for every $u' \in R'_a(s')$, $\pi'(u') \models \varphi$ due to the substitution for λ. Thus, we have that $(M',s') \models \mathbf{B}_a\varphi$.

Inclusion Due to Lemma 2 and Lemma 6, it suffices to show that if $\mathbf{B}_a\eta \in K * \mathbf{B}_a\varphi$ then $\mathbf{B}_a\eta \in K + \mathbf{B}_a\varphi$ holds. Assume that K is represented by (M,s). To distinguis the structure resulting from the expansion $+$ and the revision $*$, we denote $K + \mathbf{B}_a\varphi$ and $K * \mathbf{B}_a\varphi$ with (M^+, s^+) and (M^*, s^*), respectively. By definition of $K + \mathbf{B}_a\varphi$, we have that $R^+_a(s^+) = \emptyset$, i.e., $K + \mathbf{B}_a\varphi$ is inconsistent if $(M,s) \models \mathbf{B}_a\neg\varphi$. Thus, the postulate holds trivially in this case. Therefore, we only need to consider the cases that $(M,s) \models \neg\mathbf{B}_a\neg\varphi$. In both cases, as shown above, we have that $s^* = (s,\eta)$ for $\eta \in \{\sigma,\tau\}$ and $((s,\eta),(u,\sigma_a)) \in R^*$ implies that $(s,u) \in R_a$ and $\pi(u) \models \varphi$. This implies that $(s', u^r) \in R^+_a$. On the other hand, if $(s', u^r) \in R^+_a$ then we can also conclude that $((s,\eta),(u,\sigma_a)) \in R^*_a$. This implies that $(M^+, s^+) \models \mathbf{B}_a\eta$ iff $(M^*, s^*) \models \mathbf{B}_a\eta$.

Vacuity The proof of this postulate is the second part of the proof for **Inclusion**.

Consistency $K * \mathbf{B}_a\varphi$ is consistent if $\mathbf{B}_a\varphi$ and K are consistent. Assume that K is represented by (M,s). Since K and $\mathbf{B}_a\varphi$ are consistent, $R_a(s) \neq \emptyset$ and exactly one out of four cases in the proof for **Success** occurs. In any case, (M',s') is defined and $R'_a(s') \neq \emptyset$. Together with Lemma 6, we have that $K * \mathbf{B}_a\varphi$ is consistency.

Extensionality Assume that $\models \mathbf{B}_a\varphi \leftrightarrow \mathbf{B}_b\psi$. This implies that $\varphi \leftrightarrow \psi$ and $a = b$. Assume that K is represented by (M,s). Let us denote $\mathbf{\Sigma}^a(\varphi)$ and $\mathbf{\Sigma}^a(\psi)$ by $\mathbf{\Sigma}^1$

and $\mathbf{\Sigma}^2$. Let us denote with $(M^1, s^1) = (M, s) * \mathbf{B}_a\varphi$ and $(M^2, s^2) = (M, s) * \mathbf{B}_a\psi$. We will attach the superscript ^{i} ($i \in \{1, 2\}$) to each element of the models $\mathbf{\Sigma}^i$ or (M^i, s^i), e.g., σ^1 and σ^2 denotes the event σ in $\mathbf{\Sigma}^1$ and $\mathbf{\Sigma}^2$, respectively. We have that for any $u \in W$ and $\eta \in \{\sigma, \tau, \delta, \gamma, \epsilon\}$, $(u, \eta^1) \in W^1$ iff $(u, \eta^2) \in W^2$ because $\pi(u) \models \varphi$ and $(M, u) \models B_a\varphi$ iff $\pi(u) \models \psi$ and $(M, u) \models B_a\psi$ as $\varphi \leftrightarrow \psi$ and $\mathbf{B}_a\varphi \leftrightarrow \mathbf{B}_a\psi$. Similarly, $((u, \eta^1), (v, \xi^1)) \in R^1_x$ iff $((u, \eta^2), (v, \xi^2)) \in R^2_x$. This shows that there is a bijection between (M^1, s^1) and (M^2, s^2), i.e., $*$ satisfies this postulate.

Superexpansion Consider a fluent formula η, Lemma 6 and Lemma 2 shows that $K * \mathbf{B}_a(\varphi \wedge \psi) \models \mathbf{B}_x\eta$ iff $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi \models \mathbf{B}_x\eta$ for $x \neq a$. So, we just need to prove $K * \mathbf{B}_a(\varphi \wedge \psi) \models \mathbf{B}_a\eta$ implies that $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi \models \mathbf{B}_a\eta$, i.e., $*$ satisfies this postulate.

Let us denote $\mathbf{\Sigma}^a(\varphi \wedge \psi)$ and $\mathbf{\Sigma}^a(\varphi)$ by $\mathbf{\Sigma}^1$ and $\mathbf{\Sigma}^2$, $(M^1, s^1) = (M, s) * \mathbf{B}_a(\varphi \wedge \psi)$, and $(M^2, s^2) = (M, s) * \mathbf{B}_a\varphi$. In the following, we will attach the superscript ^{i} ($i \in \{1, 2\}$) to each element of the models $\mathbf{\Sigma}^i$ or (M^i, s^i), e.g., σ^1 and σ^2 denotes the event σ in $\mathbf{\Sigma}^1$ and $\mathbf{\Sigma}^2$, respectively. Furthermore, (M', s') denotes $(M^2, s^2) + \mathbf{B}_a\psi$. The proof considers four cases, similar to the proof of **Success**.

1. $(M, s) \models (\varphi \wedge \psi) \wedge \neg\mathbf{B}_a\neg(\varphi \wedge \psi)$. In this case, $s^1 = (s, \sigma^1)$ and $(s^1, u^1) \in R^1_a$ where $u^1 = (u, \sigma^1_a)$ iff $(s, u) \in R_a$ and $\pi(u) \models (\varphi \wedge \psi)$. Since $(M, s) \models \neg\mathbf{B}_a\neg(\varphi \wedge \psi)$, we have that $(M, s) \models \neg\mathbf{B}_a\neg\varphi$. This implies that $s^2 = (s, \sigma^2)$, and $(s^2, u^2) \in R^2_a$ where $u^2 = (u, \sigma^1_a)$ iff $(s, u) \in R_a$ and $\pi(u) \models \varphi$. So, $(s^1, u^1) \in R^1_a$ implies that $(s^2, u^2) \in R^2_a$. This shows that $K * \mathbf{B}_a(\varphi \wedge \psi) \models \mathbf{B}_a\eta$ implies that $(K * \mathbf{B}_a\varphi) \models \mathbf{B}_a\eta$, which, together with Lemma 3, shows that $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi \models \mathbf{B}_a\eta$.
2. $(M, s) \models \neg(\varphi \wedge \psi) \wedge \neg\mathbf{B}_a\neg(\varphi \wedge \psi)$. In this case, $s^1 = (s, \tau^1)$ and $(s^1, u^1) \in R^1_a$ iff $(s, u) \in R_a$ and $\pi(u) \models (\varphi \wedge \psi)$. Furthermore, $(M, s) \models \neg\mathbf{B}_a\neg\varphi$. It means that $s^2 = (s, \sigma^2)$ or $s^2 = (s, \tau^2)$. In either case, $(s^2, u^2) \in R^2_a$ where $u^2 = (u, \sigma^2_a)$ iff $(s, u) \in R_a$ and $\pi(u) \models \varphi$. Similar to the first case, we have the conclusion of the postulate.
3. $(M, s) \models (\varphi \wedge \psi) \wedge \mathbf{B}_a\neg(\varphi \wedge \psi)$. In this case, $s^1 = (s, \delta^1)$ and $R^1_a = \{(s^1, s^1)\}$. There are two cases:
   (a) $(M, s) \models \varphi \wedge \mathbf{B}_a\neg\varphi$. We then have, $s^2 = (s, \delta^2)$ and $R^2_a = \{(s^2, s^2)\}$. Because $\pi^1(s^1) = \pi^2(s^2)$, the conclusion of the postulate holds.
   (b) $(M, s) \models \varphi \wedge \neg\mathbf{B}_a\neg\varphi$. This implies that for every u such that $(s, u) \in R_a$, if $\pi(u) \models \varphi$ then $\pi(u) \models \neg\psi$. Therefore, $s^2 = (s, \sigma^2)$, and $(s^2, u^2) \in R^2_a$ where $u^2 = (u, \sigma^2_a)$ iff $(s, u) \in R_a$ and $\pi(u) \models \neg\psi$, i.e., $\pi^2(u^2) \models \neg\psi$ for every $(s^2, u^2) \in R^2_a$. This means that a is inconsistent in $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$, and thus, the postulate also holds in this case.
4. $(M, s) \models \neg(\varphi \wedge \psi) \wedge \mathbf{B}_a\neg(\varphi \wedge \psi)$. In this case, $s^1 = (s, \lambda^1)$ and $R^1_a = \{(s^1, s^1)\}$ where $\pi^1(s^1)$ is the result of the substitution that ensures $\varphi \wedge \psi$ is true. Consider the following situations:
   (a) $(M, s) \models \mathbf{B}_a\neg\varphi$. If $(M, s) \models \varphi$ then we have that $s^2 = (s, \delta^2)$ and $R^2_a = \{(s^2, s^2)\}$ which concludes the postulate because $\pi^1(s^1) = \pi^2(s^2)$. If $(M, s) \models \neg\varphi$, $s^2 = (s, \lambda^2)$ and $R^2_a = \{(s^2, s^2)\}$ where $\pi^2(s^2)$ is the result of the substitution that ensures φ is true. If $(M^2, s^2) \models \neg\psi$ then a is inconsistent in $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$. Otherwise, it is easy to see that $\pi^2(s^2) = \pi^1(s^1)$ which concludes the postulate.
   (b) $(M, s) \models \neg\mathbf{B}_a\neg\varphi$. The proof is then similar to the proof of the second subcase of the previous case.

[ST: Might be used for space saving: We omit the detailed proof for brevity. We note the key points of this proof are:

- either $((s, \eta^2), (u, \rho^2)) \in R^2_a$ for every $((s, \eta^1), (u, \rho^1)) \in R^1_a$: this shows that $(M^1, s^1) \models \mathbf{B}_a\eta$ implies $(M^2, s^2) \models \mathbf{B}_a\eta$, which, together with Lemma 3, shows that the postulate holds.
- or for every $((s, \eta^2), (u, \rho^2)) \in R^2_a$, $(M^2, (u, \rho^2)) \models \neg\psi$, which means that $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi$ is consistent, i.e., the postulate holds trivially.

]

Subexpansion The proof of this postulate is similar to the proof of **Superexpansion** where we show that, under the assumption that $\mathbf{B}_a\neg\psi \notin K * \mathbf{B}_a\varphi$, which eliminates the cases where a is inconsitent in $K * \mathbf{B}_a\varphi + \mathbf{B}_a\psi$ (Case 3(a) and 4(a)) with the following remark: in the remaining cases, we have that $(s^2, u^2) \in R^a_a$ iff $(s^1, u^1) \in R^1_a$ and $\pi^2(u^2) = \pi^1(u^1)$.

[ST: in addition to the conclusion that $((s, \eta^2), (u, \rho^2)) \in R^2_a$ if $((s, \eta^1), (u, \rho^1)) \in R^1_a$, it also holds that $((s, \eta^1), (u, \rho^1)) \in R^1_a$ if $((s, \eta^2), (u, \rho^2)) \in R^2_a$. This helps prove the postulate.]

DP1 – Successive revision respect Since $\models \mathbf{B}_b\psi \rightarrow \mathbf{B}_a\varphi$, we have that $\psi \rightarrow \varphi$ and a and b are identical. Therefore, we will show that $*$ satisfies this postulate by showing that if $\models \mathbf{B}_a\psi \rightarrow \mathbf{B}_a\varphi$ holds then $(K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi = K * \mathbf{B}_a\psi$. Assume that (M, s) represents K. Let us denote with $\mathbf{\Sigma}^1$ and $\mathbf{\Sigma}^2$ the event model $\mathbf{\Sigma}^a(\varphi)$ and $\mathbf{\Sigma}^a(\psi)$, respectively. Let $(M^0, s^0) = (M, s) * \mathbf{B}_a\varphi$, $(M^1, s^1) = (M^0, s^0) * \mathbf{B}_a\psi$, and $(M^2, s^2) = (M, s) * \mathbf{B}_A\psi$. Furthemore, $M^i = (W^i, \{R^i_x\}_{x\in\mathcal{A}}, \pi^i)$.

Again, we will need to consider four cases, similar to the proof of **Success**, following the structure of $\mathbf{\Sigma}^a(\psi)$.

1. $(M, s) \models \psi \wedge \neg\mathbf{B}_a\neg\psi$. In this case, $s^2 = (s, \sigma^2)$ and $(s^2, u^2) \in R^2_a$ where $u^2 = (u, \sigma^2_a)$ iff $(s, u) \in R_a$ and $\pi(u) \models \psi$. Since $(M, s) \models \neg\mathbf{B}_a\neg\psi$, we have that $(M, s) \models \varphi \wedge \neg\mathbf{B}_a\neg\varphi$. This implies that $s^0 = (s, \sigma^0)$, and $(s^0, u^0) \in R^0_a$ where $u^0 = (u, \sigma^0_a)$ iff $(s, u) \in R_a$ and $\pi(u) \models \varphi$. Furthermore, $s^1 = ((s, \sigma^0_a), \sigma^1)$ and $(s^1, u^1) \in R^1_a$ where $u^1 = ((u, \sigma^0_a), \sigma^1_a)$ then $\pi(u) \models \psi$. This means that for every $u^2 = (u, \sigma^2_a)$ such that $(s^2, u^2) \in R^2_a$, $(s^1, u^1) \in R^1_a$ and vice versa. This proves the consequence of the postulate.
2. $(M, s) \models \neg\psi \wedge \neg\mathbf{B}_a\neg\psi$. In this case, $s^2 = (s, \tau^2)$ and $(s^2, u^2) \in R^2_a$ iff $(s, u) \in R_a$ and $\pi(u) \models \psi$. Furthermore, $(M, s) \models \neg\mathbf{B}_a\neg\varphi$. It means that $s^0 = (s, \sigma^0)$ or $s^0 = (s, \tau^0)$. In either case, $s^1 = (s^0, \sigma^1)$ and $(s^1, u^1) \in R^1_a$ where $u^1 = (u^0, \sigma^1_a)$ and $u^0 = (u, \sigma^0_a)$ iff $(s, u) \in R_a$ and $\pi(u) \models \varphi$. Similar to the first case, we can conclude that the postulate is correct.
3. $(M, s) \models \psi \wedge \mathbf{B}_a\neg\psi$. In this case, $s^2 = (s, \delta^2)$ and $R^2_a = \{(s^2, s^2)\}$. There are two cases:

(a) $(M, s) \models \varphi \wedge \mathbf{B}_a\neg\varphi$. We then have, $s^0 = (s, \delta^0)$ and $R_a^0 = \{(s^0, s^0)\}$. This leads to, $s^1 = (s^0, \delta^1)$ and $R_a^1 = \{(s^1, s^1)\}$. Because $\pi^1(s^1) = \pi^2(s^2) = \pi(s)$, the conclusion of the postulate holds.

(b) $(M, s) \models \varphi \wedge \neg\mathbf{B}_a\neg\varphi$. This implies that for every u such that $(s, u) \in R_a$, $\pi(u) \models \neg\psi$. Therefore, $s^0 = (s, \sigma^0)$, and $(s^0, u^0) \in R_a^0$ where $u^0 = (u, \sigma_a^0)$ iff $(s, u) \in R_a$ and $\pi(u) \models \neg\psi$, i.e., $\pi^0(u^0) \models \neg\psi$ for every $(s^0, u^0) \in R_a^0$. This means that $(M_0, s_0) \models \psi \wedge \mathbf{B}_a\neg\psi$, and hence, $s_1 = (s^0, \delta^0)$ and $R_a^1 = \{(s^1, s^1)\}$. Again, because $\pi^1(s^1) = \pi^2(s_2) = \pi(s)$, the postulate is correct in this case.

4. $(M, s) \models \neg\psi \wedge \mathbf{B}_a\neg\psi$. In this case, $s^2 = (s, \lambda^2)$ and $R_a^2 = \{(s^2, s^2)\}$ where $\pi^2(s^2)$ is the result of the substitution that ensures ψ is true. We consider the following situations:

(a) $(M, s) \models \mathbf{B}_a\neg\varphi$. If $(M, s) \models \varphi$ then we have that $s^0 = (s, \delta^0)$ and $R_a^0 = \{(s^0, s^0)\}$. Since $(M, s) \models \neg\psi$, $s^1 = (s^0, \lambda_1)$ and $\pi^1(s^1)$ is the result of the substitution that ensures ψ is true, i.e., $\pi^1(s^1) = \pi^2(s^2)$. The postulate is correct is this case.

(b) $(M, s) \models \neg\mathbf{B}_a\neg\varphi$. The proof of this case is similar to the proof for Case 3(b).

DP2 – Irrelevance of superseded beliefs As in **DP1**, $\models \mathbf{B}_b\psi \to \mathbf{B}_a\neg\varphi$ implies that $a = b$ and $\psi \to \neg\varphi$. As such, we need to show that $\mathbf{B}_a\psi \models \neg\mathbf{B}_a\varphi \;\Rightarrow\; (K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi = K * \mathbf{B}_a\psi$. We will use the same notations as in the proof of **DP1**.

DP3 – Consistency preservation across revisions The operator $*$ satisfies this postulate due to Lemma 6 and the **Success** postulate: $\mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi)$.

DP4 – Minimal change when reaffirming a belief This postulate is similar to **DB3**, $\mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi)$ and $\mathbf{B}_a\neg\varphi \notin (K * \mathbf{B}_a\varphi)$. Therefore, $\mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi$ and $\mathbf{B}_a\neg\varphi \notin (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi$.

Independence Similar to **DP4**.

6 On DP2 Postulate

Suppose that we have a pointed Kripke structure (M, s) and an agent a. A belief revision operator $*$ will necessarily compose of a component for dealing with R_a accessibility relation of a and another one for dealing with π the interpretation of the worlds in M. Obviously, the second component can be borrowed from classical belief revision operators. For now, let us assume that this component is one of the classical belief revision operators.

For an arbitrary formula η, we denote with $R_a \mid \eta$ the set of $\{(s, u) \in R_a \mid \pi(u) \models \eta\}$. Assume that $*$ is a belief revision operator. Let $(M', s') = (M, s) * \mathbf{B}_a\varphi$. As a revision operator, it is necessarily to have $(M', s') \models \mathbf{B}_a\varphi$. Therefore, we have the following possibilities:

1. *Deleting and revising*: this operator behaves as follows
   - If $R_a \mid \varphi \neq \emptyset$ then $*$ should remove all $R_a \mid \neg\varphi$ and the result is $R'_a \mid \varphi = R_a \mid \varphi$ (this is an abuse of notation; precisely, it should have been (*a*) for all $u \in R_a(s)$ there exists $u' \in R'_a(s')$ such that $\pi(u) = \pi'(u')$; and (*b*) for all $u' \in R'_a(s')$ there exists $u \in R_a(s)$ such that $\pi'(u') = \pi(u)$);
   - If $R_a \mid \varphi = \emptyset$ then $*$ should result in $R'_a \mid \varphi = (R_a \mid \neg\varphi)*\varphi$ where for each $u \in R_a \mid \neg\varphi$ needs to be revised in such a way that $\pi'(u) \models \varphi$.
2. *Revising*: this operator behaves as follows: $R_a = R'_a$ and $*$ should revising every $u \in R_a \mid \neg\varphi$ so that $\pi'(u) \models \varphi$.

For a set of worlds S, let us denote with $S*\varphi = \{u*\varphi \mid u \in S\}$. We next refer to these operators by DR- and R-operator and write $*_{DR}$ and $*_R$ to denote them. They are depicted in Figure 4.

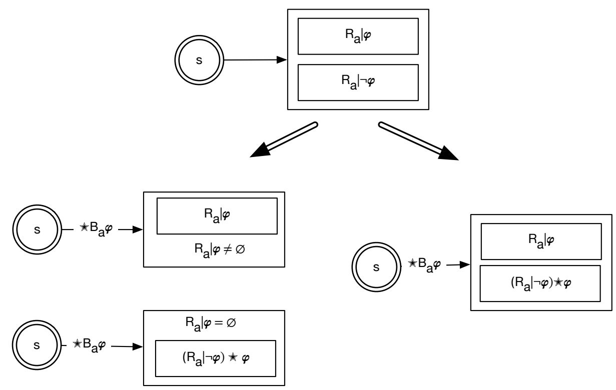

Figure 4: Two Possibilities: DR (left) vs R (right)

In the spirit of belief revision principle, we will need to discuss what should be considered as "*minimal change*" in multi-agent setting. It is obvious that

- if $R_a|\varphi = \emptyset$ or $R_a|\neg\varphi = \emptyset$ then $*_{DR} = *_R$, i.e., $(M, s) *_{DR} \mathbf{B}_a\varphi = (M, s) *_R \mathbf{B}_a\varphi$;
- if $R_a|\varphi \neq \emptyset$ and $R_a|\neg\varphi \neq \emptyset$ then $R'_a \mid \varphi \subset R''_a \mid \varphi$ where $(M', s') = (M, s) *_{DR} \mathbf{B}_a\varphi$ and $(M'', s'') = (M, s) *_R \mathbf{B}_a\varphi$, i.e., $(M', s') \models \mathbf{B}_a\eta$ implies $(M'', s'') \models \mathbf{B}_a\eta$.

We will show next that any *DR*-operator will not satisfy the DP2 postulate. Assume that $\psi \to \neg\varphi$. Consider a pointed Kripke structure (M, s) that satisfies the following properties:

- $R_1 = R_a \mid \psi \wedge \neg\varphi \neq \emptyset$
- $R_2 = R_a \mid \neg\psi \wedge \varphi \neq \emptyset$
- $R_3 = R_a \mid \neg\psi \wedge \neg\varphi \neq \emptyset$

This implies that $R_a \mid \psi \neq \emptyset$ and $R_a \mid \neg\psi \neq \emptyset$, i.e., $R'_a \mid \psi = R_a \mid \psi = R_1$ where $(M', s') = (M, s)*_{DR}\mathbf{B}_a\psi$. Note that we write $R'_a \mid \psi = R_a \mid \psi = R_1$ to mean that the set of interpretations on one side quals the set of interprtations on the other side.

Let $(M^*, s^*) = (M, s) *_{DR} \mathbf{B}_a\varphi$. Then, we have that $R_a^* \mid \varphi = R_a \mid \neg\psi \wedge \varphi = R_2$. In other words, $(M^*, s^*) \models \mathbf{B}_a\neg\psi$. Let $(M'', s'') = (M^*, s^*) *_{DR} \mathbf{B}_a\psi$. We have that $R''_a \mid \psi = \psi *_{DR} (R_a \mid \neg\psi \wedge \varphi) = \psi *_{DR} R_2$.

It is easy to see that there exists such (M, s) such that $R_1 \neq \psi *_{DR} R_2$ for any available classical revision operator $*_{DR}$. An example is shown in Figure 5, assuming that $p \to \neg q$. On the left is the successive application of $*_{DR}$ on φ

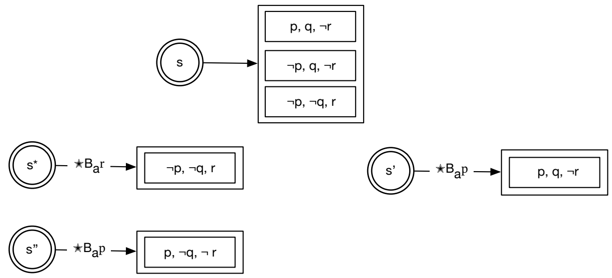

Figure 5: DR-revision does not satisfy DP2

then ψ, i.e., $((M, s) *_{DR} \mathbf{B}_a\varphi) *_{DR} \psi$ and on the right is the application of $*_{DR}$ on $\mathbf{B}_a\psi$.

[ST: It is interesting that R-operator will not satisfy **Vacuity** for the expansion operator defined as in the previous section. The main reason is that the expansion operator does remove all elements in $R_a \mid \neg\varphi$ when $(M, s) \models \neg\mathbf{B}_a\neg\varphi$. Thus, only one direction could be satisfied.
]

6.1 A R-Operator that Satisfies DP2

Let $*_C$ be a mapping from interpretations to interpretations that satisfies the following conditions

- $u *_C \psi \models \varphi$ if $\varphi \wedge \psi$ is consistent and $u \models \varphi$;
- $u *_C \varphi = u *_C (\varphi \wedge \psi)$ if $\varphi \wedge \psi$ is consistent and $u \models \varphi$;
- $(u *_C \varphi) *_C \psi = u *_C \psi$ if $\psi \rightarrow \varphi$ and $u \models \psi \rightarrow \varphi$.

For an agent $a \in \mathcal{A}$ to revise a conjunction of fluent literals φ, we define $\mathbf{\Sigma}^a(\varphi)$, called an *event-model for revision of* $\mathbf{B}_a\varphi$, as the event model $\langle \Sigma, \{E_a\}_{a\in\mathcal{A}}, pre, sub\rangle$ where $\Sigma = \{\sigma, \tau, \epsilon\}$ and

- $E_a = \{(\sigma, \tau), (\tau, \epsilon), (\epsilon, \epsilon)\}$;
- $E_x = \{(\sigma, \epsilon), (\tau, \epsilon), (\epsilon, \epsilon)\}$ for $x \in \mathcal{A} \setminus \{a\}$;
- $pre(\sigma) = pre(\epsilon) = pre(\tau) = \top$; and
- $sub(\sigma) = sub(\epsilon) = \emptyset$ and $sub(\tau)$ is defined via $*_C$.

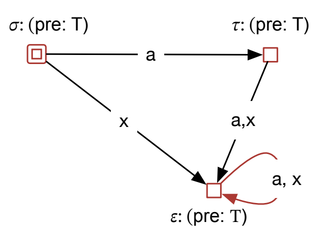

Figure 6: An R-Operator for DP2

Definition 4. *Let (M, s) be a pointed Kripke structor φ be a fluent literal. The event based revision operator $*$ of a revision of $\mathbf{B}_a\varphi$ in (M, s) is defined by*

$$(M, s) * \mathbf{B}_a\varphi = (M, s) \otimes (\mathbf{\Sigma}^a(\varphi), \{\sigma\})$$

In the following, we will show that our resvision operator indeed satisfies several AGM+DP postulates. Before we discuss them in details, let us observe that for every (M, s), (s, σ) will be a world in $(M, s) * \mathbf{B}_a\varphi$. From now on, we assume that K is represented by (M, s). We will also refer to $(M, s) * \mathbf{B}_a\varphi$ as (M^φ, s^φ) where $s^\varphi = (s, \sigma)$ and $M^\varphi = (W^\varphi, \{R^\varphi_x\}_{x\in\mathcal{A}}, \pi^\varphi)$ when no confusion is possible and assume that $M = (W, \{R_x\}_{x\in\mathcal{A}}, \pi)$. Successive revisions such as $(K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi$ will be denoted by $((M^\varphi)^\psi, (s^\varphi)^\psi)$ etc. We first prove that beliefs of other agents do not change.

Lemma 7. *For every agent $x \in \mathcal{A} \setminus \{a\}$ and a fluent formula ψ, $(M^\varphi, s^\varphi) \models \mathbf{B}_x\psi$ iff $(M, s) \models \mathbf{B}_x\psi$.*

Proof. By the construction of (M^φ, s^φ), we have that $s^\varphi = (s, \sigma)$. Furthermore, $(u, \epsilon) \in W^\varphi$ for every $u \in W$ and $((s, \sigma), (u, \epsilon)) \in R^\varphi_x$ iff $(s, u) \in R_x$. Therefore, $(M^\varphi, s^\varphi) \models \mathbf{B}_x\psi$ iff $(M, s) \models \mathbf{B}_x\psi$. ☐

Closure By our definition, it is clear that

$$K * \mathbf{B}_a\varphi = Cn(K * \mathbf{B}_a\varphi).$$

Success Assume that K is consistent. We show that $K * \mathbf{B}_a\varphi \models \mathbf{B}_a\varphi$. This is obvious because for every $((s, \sigma), (u, \tau)) \in R^\varphi_a$, $\pi^\varphi((u, \tau)) = u *_C \varphi$ and, by definition, $u *_C \varphi \models \varphi$, i.e., $\pi^\varphi((u, \tau)) \models \varphi$. The consistency of K ensures that $R^\varphi_a \neq \emptyset$. This concludes the proof of this postulate.

Inclusion Due to Lemma 2 and Lemma 7, it suffices to show that if $\mathbf{B}_a\eta \in K * \mathbf{B}_a\varphi$ then $\mathbf{B}_a\eta \in K + \mathbf{B}_a\varphi$ holds. Assume that K is represented by (M, s). We refer to $K + \mathbf{B}_a\varphi$ by (M', s') as it is defined. By definition of $K + \mathbf{B}_a\varphi$, we have that $R'_a(s') = \emptyset$, i.e., $K + \mathbf{B}_a\varphi$ is inconsistent if $(M, s) \models \mathbf{B}_a\neg\varphi$. Thus, the postulate holds trivially in this case. Therefore, we only need to consider the cases that $(M, s) \models \neg\mathbf{B}_a\neg\varphi$. This implies that $R_a \mid \varphi \neq \emptyset$. By the construction of $K + \mathbf{B}_a\varphi$ and $K * \mathbf{B}_a\varphi$, we can conclude that if $(s', u^r) \in R'_a$ then $((s, \sigma), (u, \tau)) \in R^\varphi_a$ and $\pi'(u^r) = \pi^\varphi((u, \tau))$. This implies that if $\mathbf{B}_a\eta \in K * \mathbf{B}_a\varphi$ then $\mathbf{B}_a\eta \in K + \mathbf{B}_a\varphi$.

Vacuity It follows from the above proof that **Vacuity** does not hold for this operator because there exists some $\mathbf{B}_a\psi \in K + \mathbf{B}_a\varphi$ such that $B_a\psi \notin K * \mathbf{B}_a\varphi$. This can happen when $R_a \mid \neg\varphi \neq \emptyset$, and thus, there exists some $((s, \sigma), (u, \tau)) \in R^\varphi_a$ such that $(s', u^r) \in R'_a$. For example, consider the Kripke structure in Figure 5, assuming there is no constraints among the propositions, then $(M, s) * \mathbf{B}_a r \models \neg\mathbf{B}_a\neg p$ while $(M, s) + \mathbf{B}_a r \models \mathbf{B}_a\neg p$.

Consistency Since K and $\mathbf{B}_a\varphi$ are consistent, $R_a(s) \neq \emptyset$ and therefore $R^\varphi_a((s, \sigma)) \neq \emptyset$. Together with Lemma 7, we have that $K * \mathbf{B}_a\varphi$ is consistency.

Extensionality Assume that $\models \mathbf{B}_a\varphi \leftrightarrow \mathbf{B}_b\psi$. This implies that $\varphi \leftrightarrow \psi$ and $a = b$. Assume that K is represented by (M, s). We have that for any $u \in W$ and $\eta \in \{\sigma, \tau, \epsilon\}$, $(u, \eta^\varphi) \in W^\varphi$ iff $(u, \eta^\psi) \in W^\psi$. Furthermore, $((u, \eta^\varphi), (v, \xi^\varphi)) \in R^\varphi_x$ iff $((u, \eta^\psi), (v, \xi^\psi)) \in R^\psi_x$. This shows that there is a bijection between (M^φ, s^φ) and (M^ψ, s^ψ), i.e., $*$ satisfies this postulate.

Superexpansion Consider a fluent formula η, Lemma 7 and Lemma 2 shows that $K * \mathbf{B}_a(\varphi \wedge \psi) \models \mathbf{B}_x\eta$ iff $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi \models \mathbf{B}_x\eta$ for $x \neq a$. So, we just need to prove $K * \mathbf{B}_a(\varphi \wedge \psi) \models \mathbf{B}_a\eta$ implies that $(K * \mathbf{B}_a\varphi) + \mathbf{B}_a\psi \models$

$\mathbf{B}_a\eta$, i.e., $*$ satisfies this postulate when $\varphi \wedge \psi$ and (M, s) are consistent.

Let us denote $(M^\psi, s^\psi) + \mathbf{B}_a\psi$ by (M', s'). By construction, for $x \in \{\varphi \wedge \psi, \psi\}$, $s^x = (s, \sigma^x)$ and $(s^x, u^x) \in R_a^x$ where $u^x = (u, \tau^x)$ iff $(s, u) \in R_a$. Furthermore, $\pi^{\varphi\wedge\psi}(u^{\varphi\wedge\psi}) = u *_C (\varphi \wedge \psi)$ and $\pi^\psi(u^\psi) = u *_C \varphi$. By the assumption of $*_C$, $(M^{\varphi\wedge\psi}, s^{\varphi\wedge\psi}) \models \mathbf{B}_a\eta$ implies that $(M^\psi, s^\psi) \models \mathbf{B}_a\eta$. Lemmas 3-4 imply that $(M', s') \models \mathbf{B}_a\eta$, and thus, the postulate holds.

Subexpansion Similar to the **Vacuity** postulate, this operator does not satisfy this postulate. The main reason is that when $K * \mathbf{B}_a\varphi + \psi$ is consistent then the set of $R_a^{\varphi\wedge\psi}$ can be a proper superset of R'_a, using the same notations as in the previous paragraph.

DP1 – Successive revision respect Since $\models \mathbf{B}_b\psi \rightarrow \mathbf{B}_a\varphi$, we have that $\psi \rightarrow \varphi$ and a and b are identical. Therefore, we will show that $*$ satisfies this postulate by showing that if $\models \mathbf{B}_a\psi \rightarrow \mathbf{B}_a\varphi$ holds then $(K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi = K * \mathbf{B}_a\psi$.

By construction, $x \in (W^\varphi)^\psi$ has the form $((x, \eta^\varphi), \gamma^\psi)$ for $\eta, \gamma \in \{\sigma, \tau, \epsilon\}$. Furthermore, $(s^\varphi)^\psi = ((s, \sigma^\varphi), \sigma^\psi)$ and $((s^\varphi)^\psi, (u^\varphi)^\psi \in (R^\varphi)_a^\psi$ where $(u^\varphi)^\psi = ((u, \tau^\varphi), \tau^\psi)$ and $u \in W$ and $(\pi^\varphi)^\psi((u^\varphi)^\psi) = \pi^\varphi((u, \tau^\varphi)) *_C \psi = u *_C \varphi *_C \psi = u *_C \psi$ (assumption about $*_C$) iff $(s, u) \in R_a$ and $(s^\psi, u^\psi) \in R_a^\psi$ where $s^\psi = (s, \sigma^\psi)$ and $u^\psi = (u, \tau^\psi)$. This proves the consequence of the postulate.

DP2 – Irrelevance of superseded beliefs $\mathbf{B}_b\psi \models \neg\mathbf{B}_a\varphi$ implies that $a = b$ and $\psi \rightarrow \neg\varphi$. As such, we need to show that $\mathbf{B}_a\psi \models \neg\mathbf{B}_a\varphi \Rightarrow (K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi = K * \mathbf{B}_a\psi$.

By construction of $*$, it is easy to see that $s^x = (s, \sigma^x)$ and any world in M^x ($x \in \{\varphi, \psi\}$) has the form (u, η^x) and any world in $(M^\varphi)^\psi$ has the form $((u, \eta^\varphi), \gamma^\psi)$ for some $u \in W$ and $\eta, \gamma \in \{\sigma, \tau, \epsilon\}$.

We have that $(s^\varphi, u^\varphi) \in R_a^\varphi$ iff there exists some $u \in W$ such that $(s, u) \in R_a$, $u^\varphi = (u, \tau^\varphi)$. Because $sub(\tau)(u) = u *_C \varphi$, $\pi^\varphi(u) \models \varphi \wedge \neg\psi$.

By definition, $(s^\varphi)^\psi = (s^\varphi, \sigma^\psi)$. We have that
$((s^\varphi)^\psi, (u^\varphi)^\psi) \in (R^\varphi)_a^\psi$
iff there exists some $u^\varphi \in W^\varphi$ such that $(s^\varphi, u^\varphi) \in R_a^\varphi$, $(u^\varphi)^\psi = (u^\varphi, \tau^\psi)$ and $(\pi^\varphi)^\psi(\tau^\psi)((u^\varphi)^\psi) = u^\varphi *_C \psi$
iff there exists some $u \in W$ such that $(s, u) \in R_a$, $u^0\varphi = (u, \tau^\varphi)$ and $\pi^\varphi(\tau^\varphi)(u) = u *_C \varphi$
iff there exists some $u \in W$ such that $(s, u) \in R_a$, and $(\pi^\varphi)^\psi(\tau^\psi)(u) = (u *_C \varphi) *_C \psi = u *_C \psi = \pi^\psi(\tau^\psi)(u)$
iff $(s^\psi, u^\psi) \in R_a^\psi$ where $s^\psi = (s, \sigma^\psi)$ and $u^\psi = (u, \tau^\psi)$.

The above concludes that $(K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi = K * \mathbf{B}_a\psi$.

DP3 – Consistency preservation across revisions The proof for this postulate follows from Lemma 7 and the **Success** postulate for $a \neq b$. If $a = b$, it means that we need to show that $\mathbf{B}_a\varphi \in K * \mathbf{B}_a\psi \Rightarrow \mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi$. This is obvious because $\mathbf{B}_a\varphi \in K * \mathbf{B}_a\psi$ implies that $\varphi \wedge \psi$ is consistent. A similar argument to that in the proof of **DP1** shows that $\mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi) * \mathbf{B}_a\psi$.

DP4 – Minimal change when reaffirming a belief Clearly, this should be considered only if $a \neq b$. $\mathbf{B}_a\neg\varphi \notin K * \mathbf{B}_b\psi$ implies that there exists some $(s, u) \in R_a$ such that $u *_C \psi \models \varphi$. By construction of $K * \mathbf{B}_a\psi$, this means that $u \models \varphi$. This implies that $u *_C \varphi = u$. Thus, there exists some $(u^\varphi)^\psi \in (W^\varphi)^\psi$ such that $((s^\varphi)^\psi, (u^\varphi)^\psi) \in (R^\varphi)_a^\psi$ and $(u^\varphi)^\psi \models \varphi$. This proves the postulate.

Independence Lemma 7 guarantees that $\mathbf{B}_a\varphi \in (K * \mathbf{B}_a\varphi) * \mathbf{B}_b\psi$ if $a \neq b$. When $s = b$, the condition $\mathbf{B}_a\neg\varphi \notin K * \mathbf{B}_a\psi$ implies that $\varphi \wedge \psi$ is consistent. Therefore, for every $(u^\varphi)^\psi \in (W^\varphi)^\psi$ such that $((s^\varphi)^\psi, (u^\varphi)^\psi) \in (R^\varphi)_a^\psi$ and $(u^\varphi)^\psi \models \varphi$. This proves the postulate.

7 Discussion

References

Alchourrón, C. E., and Makinson, D. 1982. On the logic of theory change: Contraction functions and their associated revision functions. *Theoria* 48(1):14–37.

Alchourrón, C. E.; Gärdenfors, P.; and Makinson, D. 1985. On the logic of theory change: Partial meet contraction and revision functions. *J. Symb. Log.* 50(2):510–530.

Aravanis, T. I., and Peppas, P. 2017. Belief revision in answer set programming. In *Proceedings of the 21st Pan-Hellenic Conference on Informatics, Larissa, Greece, September 28-30, 2017*, 2:1–2:5.

Aravanis, T. I.; Peppas, P.; and Williams, M. 2018. Iterated belief revision and dalal's operator. In *Proceedings of the 10th Hellenic Conference on Artificial Intelligence, SETN 2018, Patras, Greece, July 09-12, 2018*, 26:1–26:4.

Baltag, A., and Smets, S. 2008. A qualitative theory of dynamic interactive belief revision. In *Proc. of 7th LOFT*, Texts in Logic and Games 3, 13–60. Amsterdam University Press.

Baltag, A.; Moss, L.; and Solecki, S. 1998. The logic of public announcements, common knowledge, and private suspicions. In *7th TARK*, 43–56.

Baral, C.; Gelfond, G.; Pontelli, E.; and Son, T. C. 2022. An action language for multi-agent domains. *Artif. Intell.* 302:103601.

Dalal, M. 1988. Investigations into theory of knowledge base revision. In *Proc. AAAI*, 449–479.

Darwiche, A., and Pearl, J. 1997. On the logic of iterated belief revision. *Artif. Intell.* 89(1-2):1–29.

Delgrande, J. P.; Schaub, T.; Tompits, H.; and Woltran, S. 2008. Belief revision of logic programs under answer set semantics. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Eleventh International Conference, KR 2008, Sydney, Australia, September 16-19, 2008*, 411–421.

Dragoni, A. F., and Giorgini, P. 1996. Belief revision through the belief-function formalism in a multi-agent environment. In Müller, J. P.; Wooldridge, M. J.; and Jennings, N. R., eds., *Intelligent Agents III, Agent Theories, Architectures, and Languages, ECAI '96 Workshop (ATAL), Budapest, Hungary, August 12-13, 1996, Proceedings*, volume 1193 of *Lecture Notes in Computer Science*, 103–115. Springer.

Dragoni, A. F., and Puliti, P. 1994. Distributed belief revision versus distributed truth maintenance. In *Sixth International Conference on Tools with Artificial Intelligence, ICTAI '94, New Orleans, Louisiana, USA, November 6-9, 1994*, 499–505. IEEE Computer Society.

Fagin, R.; Halpern, J.; Moses, Y.; and Vardi, M. 1995. *Reasoning about Knowledge*. MIT press.

Fermé, E. L., and Hansson, S. O. 2011. Editorial introduction - 25 years of AGM theory. *J. Philos. Log.* 40(2):113–114.

Gardenfors, P., and Makinson, D. 1988. Revisions in knowledge systems using epistemic entrenchment. In *Proceedings of the 2nd international conference on theoretical aspects of reasoning about knowledge*, 1413–1419.

Herzig, A.; Lang, J.; and Marquis, P. 2005. Action Progression and Revision in Multiagent Belief Structures. In *Sixth Workshop on Nonmonotonic Reasoning, Action, and Change (NRAC)*.

Hunter, A., and Delgrande, J. P. 2011. Iterated belief change due to actions and observations. *J. Artif. Intell. Res. (JAIR)* 40:269–304.

Jin, Y., and Thielscher, M. 2007. Iterated belief revision, revised. *Artificial Intelligence* 171(1):1–18.

Katsuno, H., and Mendelzon, A. O. 1992. Propositional knowledge base revision and minimal change. *Artif. Intell.* 52(3):263–294.

Liu, W., and Williams, M. 1999. A framework for multi-agent belief revision, part I: the role of ontology. In Foo, N. Y., ed., *Advanced Topics in Artificial Intelligence, 12th Australian Joint Conference on Artificial Intelligence, AI '99, Sydney, Australia, December 6-10, 1999, Proceedings*, volume 1747 of *Lecture Notes in Computer Science*, 168–179. Springer.

Lorini, E., and Schwarzentruber, F. 2021. Multi-agent belief base revision. In Zhou, Z., ed., *Proceedings of the Thirtieth International Joint Conference on Artificial Intelligence, IJCAI 2021, Virtual Event / Montreal, Canada, 19-27 August 2021*, 1959–1965. ijcai.org.

Malheiro, B.; Jennings, N. R.; and Oliveira, E. 1994. Belief revision in multi-agent systems. In Cohn, A. G., ed., *Proceedings of the Eleventh European Conference on Artificial Intelligence, Amsterdam, The Netherlands, August 8-12, 1994*, 294–298. John Wiley and Sons, Chichester.

Peppas, P. 2014. A panorama of iterated revision. Springer Netherlands. 71–94.

Satoh, K. 1988. Nonmonotonic reasoning by minimal belief revision. In *Proc. FGCS*, 455–462. Springer.

Shapiro, S.; Pagnucco, M.; Lespérance, Y.; and Levesque, H. J. 2011. Iterated belief change in the situation calculus. *Artif. Intell.* 175(1):165–192.

Tardivo, F.; Pham, L.; Son, T. C.; and Pontelli, E. 2021. A logic programming approach to regression based repair of incorrect initial belief states. In Morales, J. F., and Orchard, D. A., eds., *Practical Aspects of Declarative Languages - 23rd International Symposium, PADL 2021, Copenhagen, Denmark, January 18-19, 2021, Proceedings*, volume 12548 of *Lecture Notes in Computer Science*, 73–89. Springer.

van Benthem, J.; van Eijck, J.; and Kooi, B. P. 2006. Logics of communication and change. *Inf. Comput.* 204(11):1620–1662.

van Benthem, J. 2007. Dynamic logic of belief revision. *Journal of Applied Non-Classical Logics* 17(2):129–155.

van Ditmarsch, H.; van der Hoek, W.; and Kooi, B. 2007. *Dynamic Epistemic Logic*. Springer.

Witteveen, C.; van der Hoek, W.; and de Nivelle, H. 1994. Revision of non-monotonic theories. In MacNish, C.; Pearce, D.; and Pereira, L. M., eds., *Logics in Artificial Intelligence, European Workshop, JELIA '94, York, UK, September 5-8, 1994, Proceedings*, volume 838 of *Lecture Notes in Computer Science*, 137–151. Springer.