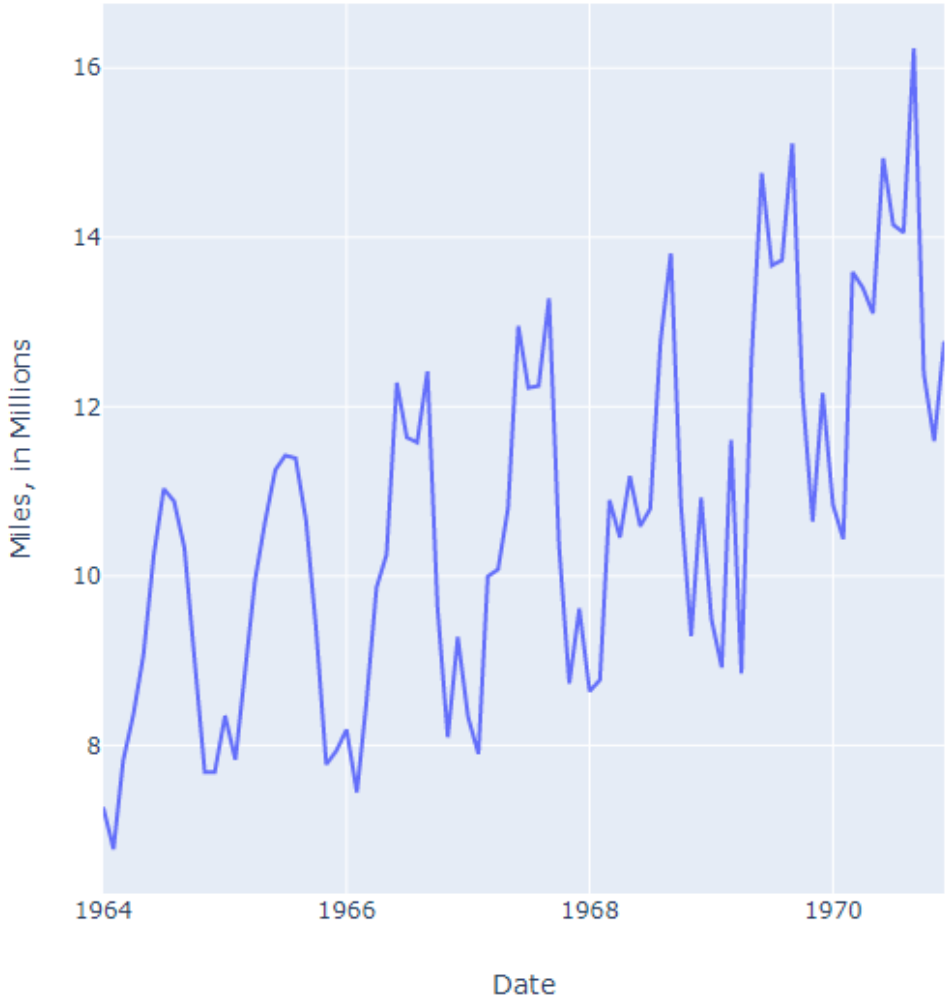
Trang Nguyen

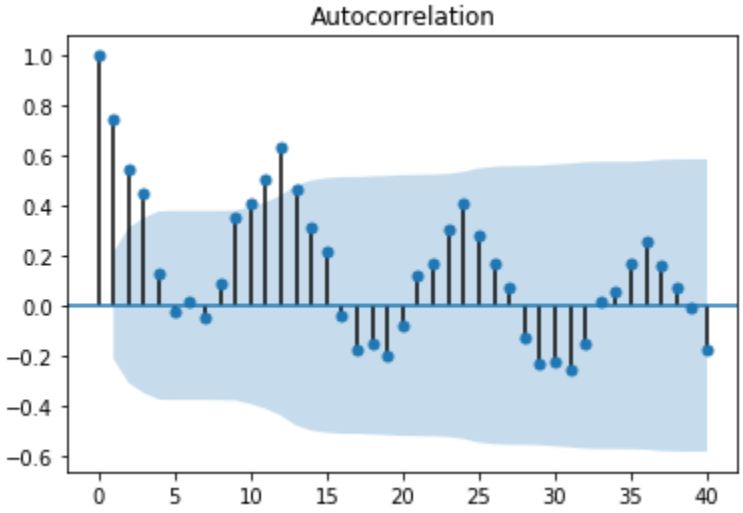
Time Series

Project 1

1.

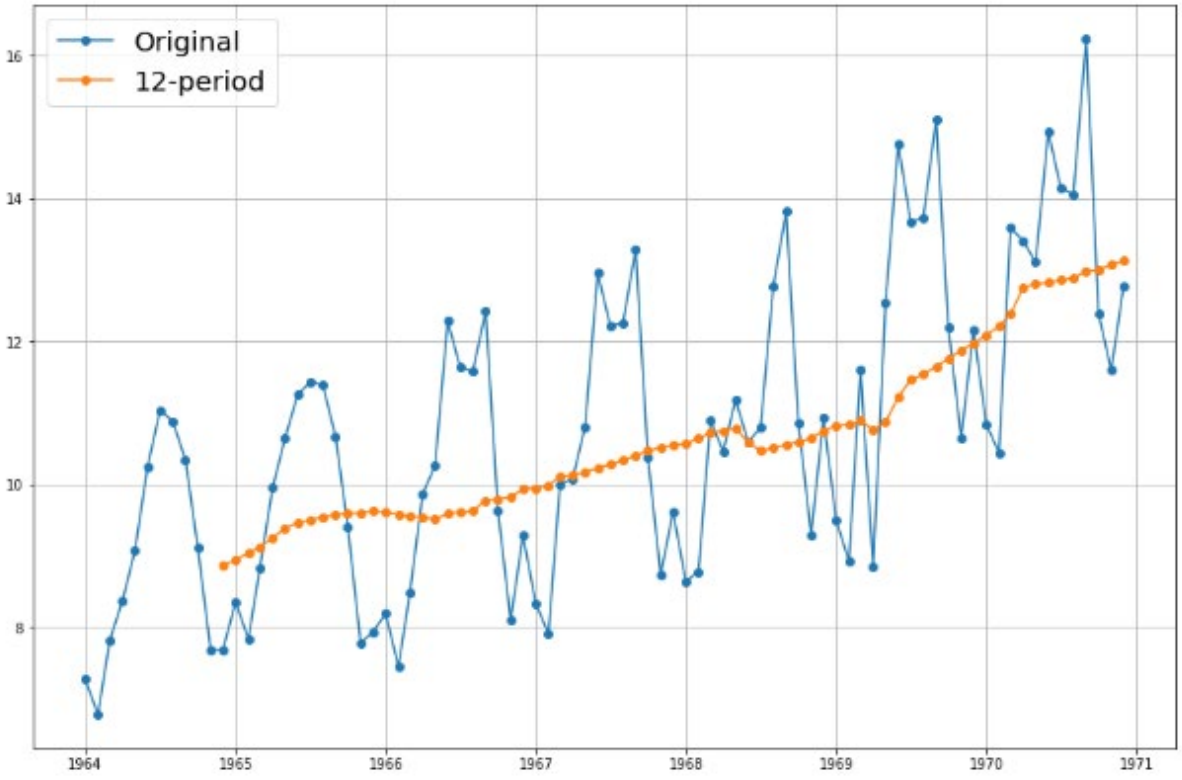


2. Autocorrelation function with lags = 40:



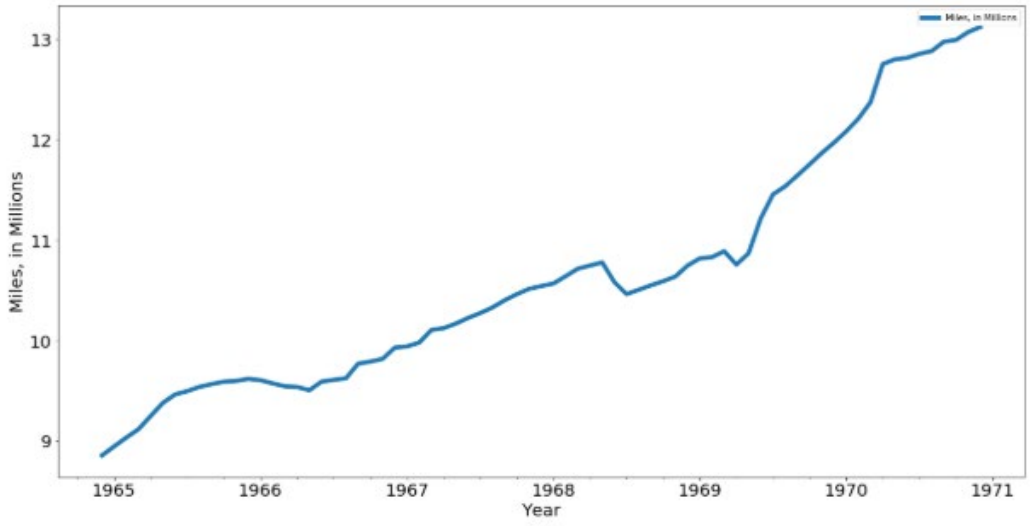
Since from one peak to another peak, we have 12 points. Therefore, the seasonal period is 12 here.

3.

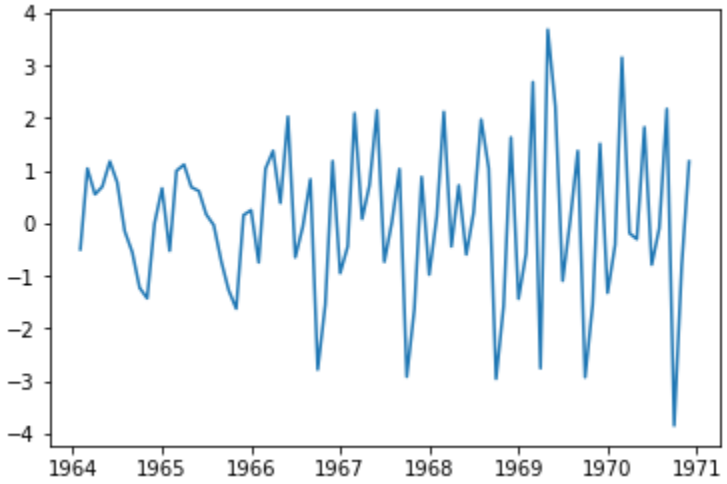


Since the seasonal period is 12, we can guess the suitable choice for the moving average window length is 12 since it will flat out the time series (get rid all of the variability of the time series) and give us the trend line.

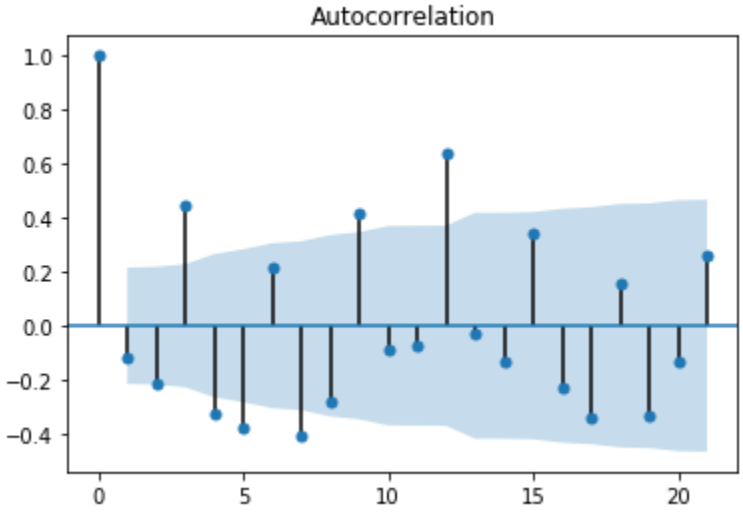
4. Overall, the trend line is nonlinear and increasing. From 1965 to first few months of 1966 (around March or April), the trend line has concave down shape. And from 1966 to 1968, the trend line is roughly increasing in linear form. Then the trend line is decreasing a little bit in the middle of 1968 and also decreasing in the middle of 1969 a little bit. After that, the trend line is roughly increasing.



5. The first difference plot:

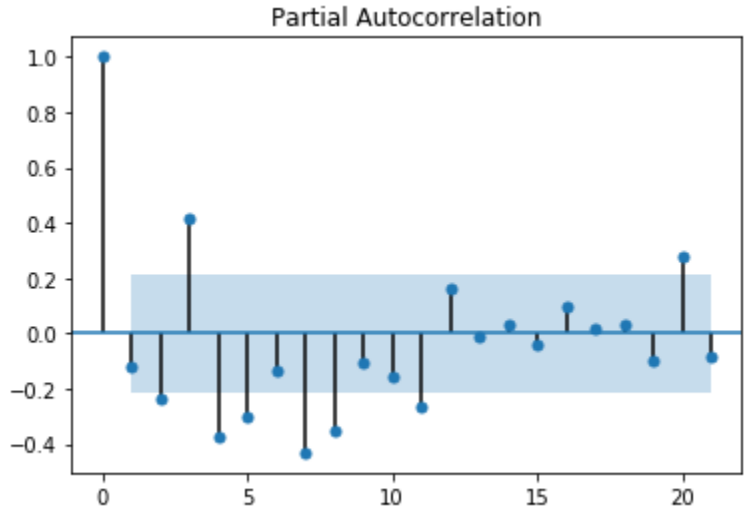


ACF plot:



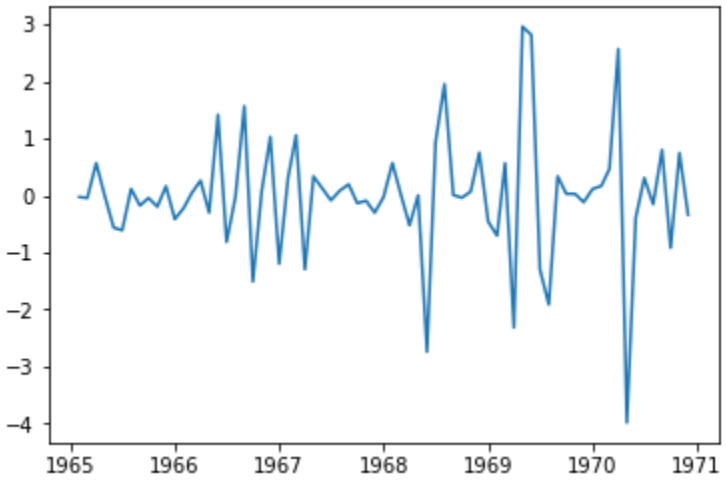
The significant lags based on ACF are 2, 3, 4, 5, 7, 9, 12

PACF plot:

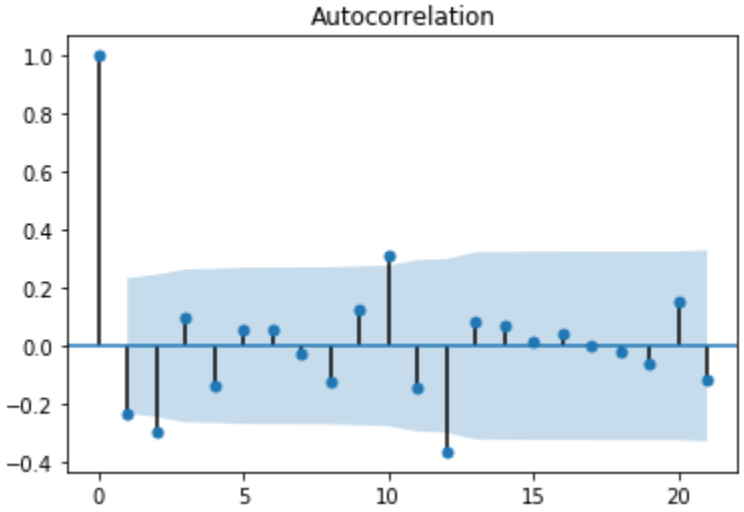


The significant lags based on PACF are 2, 3, 4, 5, 7, 8, 11, 20

6. The first seasonal difference plot:

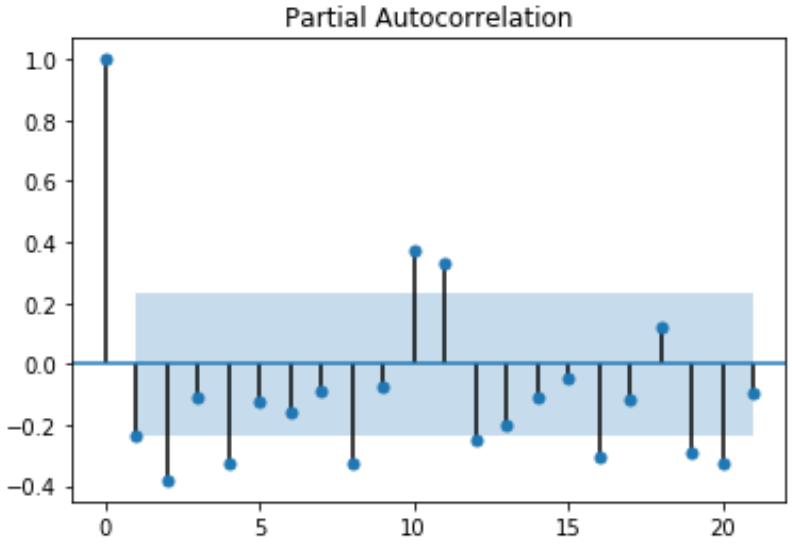


ACF plot:



The significant lags based on ACF are 1, 2, 10, 12

PACF plot:



The significant lags based on PACF are 2, 4, 8, 10, 11, 12, 16, 19, 20

7.

a.

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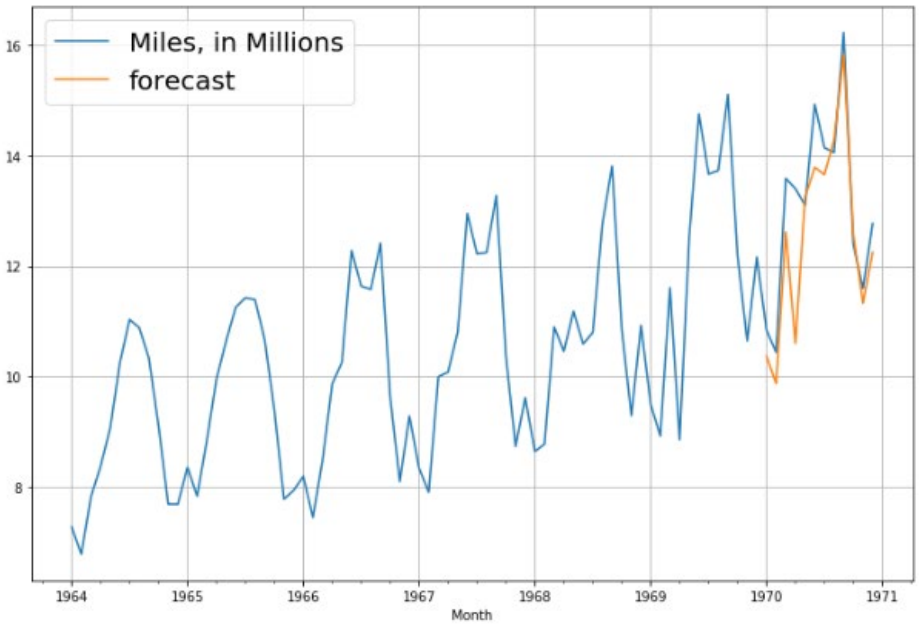
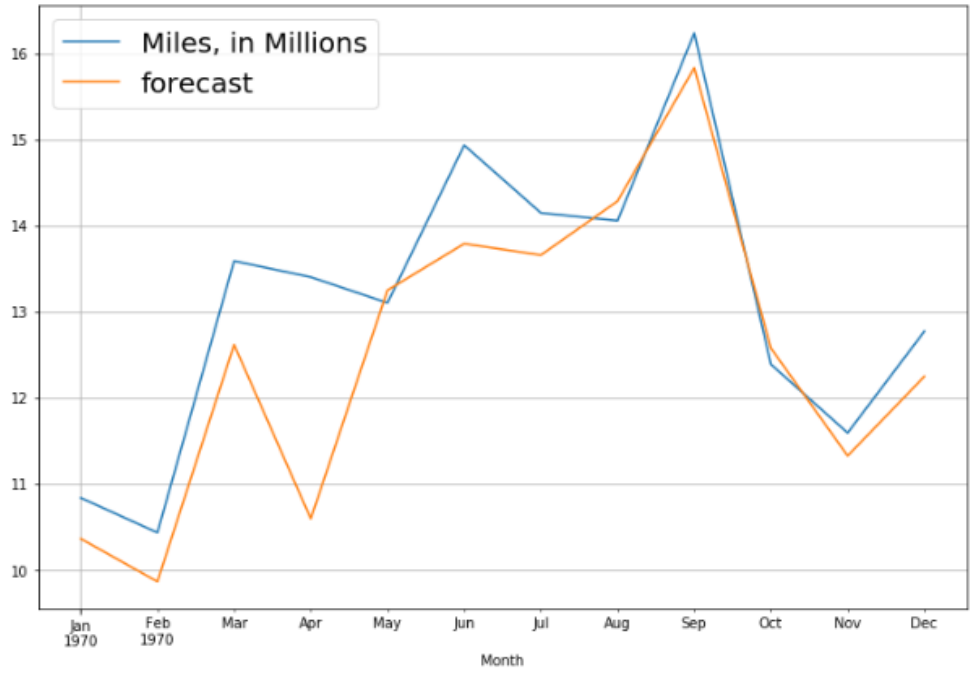
b. We know that when comparing two models, the lower AIC value the model has, the better the model is. Based on a., we sort the AIC value in ascending order to find the lowest AIC which is 145.848 here. In other words, the model with that AIC, which is ARIMA(2, 1, 3, 1, 1, 0), is the best model. So (2, 1, 3, 1, 1, 0) is my best choice of parameters.

8.



Close-up:



Even though the forecast does not really match with the actual data, the forecast still has roughly similar pattern, especially from August to December of 1970.

To improve the forecast, we can try time series cross-validation which we will separate the available data into training and test sets. We will have a series of test sets that each of the set will consist of a single observation and the corresponding training set will have the observations happened before the observation that form the test set. Then, we can compare some evaluation criteria, for example RMSE here, to find the best model using this time series cross-validation.

One other way is we can try different type of forecasting then compare them to find the best forecast. Since trend and seasonality pattern appear in the time series, we can also try

Holt-Winters Exponential Smoothing to see if it can be better than the ARIMA model with seasonality.