Tran Hai Long 5130203/20102

Report: Binary Classification and Statistical Learning Theory

1. Problem of Binary Classification Binary classification is one of the primary tasks in machine learning. In this context, we have an input space X and an output space Y, where Y is binary, represented as {-1, 1}. The aim is to find a function $f: X \to Y$ that classifies each point in X into one of the two classes. Mathematically, this classification is formulated by assuming the existence of an unknown probability distribution P on X x Y, where the data points are drawn i.i.d (independent and identically distributed) according to P. The objective is to minimize the classification error, defined by the risk function: $R(f) = E_P [l(f(X), Y)]$ where l is a loss function that measures the cost of misclassification, often 0-1 loss. Given a set of training data $\{(X1, Y1), ..., (Xn, Yn)\}$, binary classification seeks to find a function that minimizes the empirical risk: $Remp(f) = (1/n) \text{ sum } l(f(Xi), Yi)$

2. Mathematical Framework in Statistical Learning Theory (SLT) SLT provides the theoretical foundations for solving the problem of binary classification by introducing concepts such as VC-dimension, generalization error, and empirical risk minimization (ERM). The Bayes classifier f_Bayes is the optimal classifier that minimizes the expected risk. Since the underlying distribution P is unknown, SLT uses the ERM principle to approximate the optimal classifier by minimizing the empirical risk on the training data. SLT also ensures that the classifier's performance generalizes well to unseen data by providing bounds on the generalization error. Through VC theory, SLT formalizes the trade-off between model complexity and generalization, helping to avoid overfitting and underfitting.