

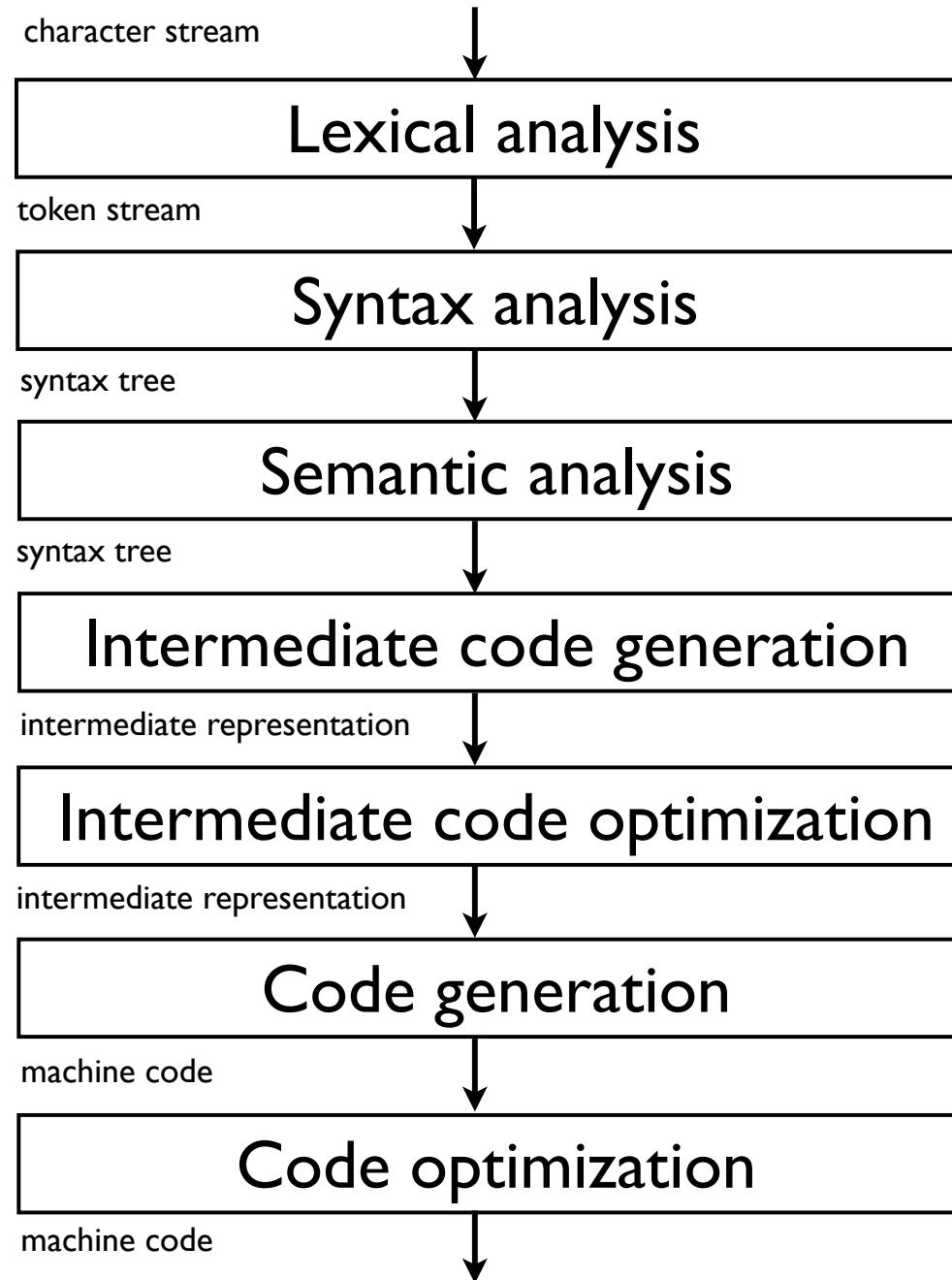
# Part 2

## Lexical analysis

# Outline

1. Principle
2. Regular expressions
3. Analysis with non-deterministic finite automata
4. Analysis with deterministic finite automata
5. Implementing a lexical analyzer

# Structure of a compiler

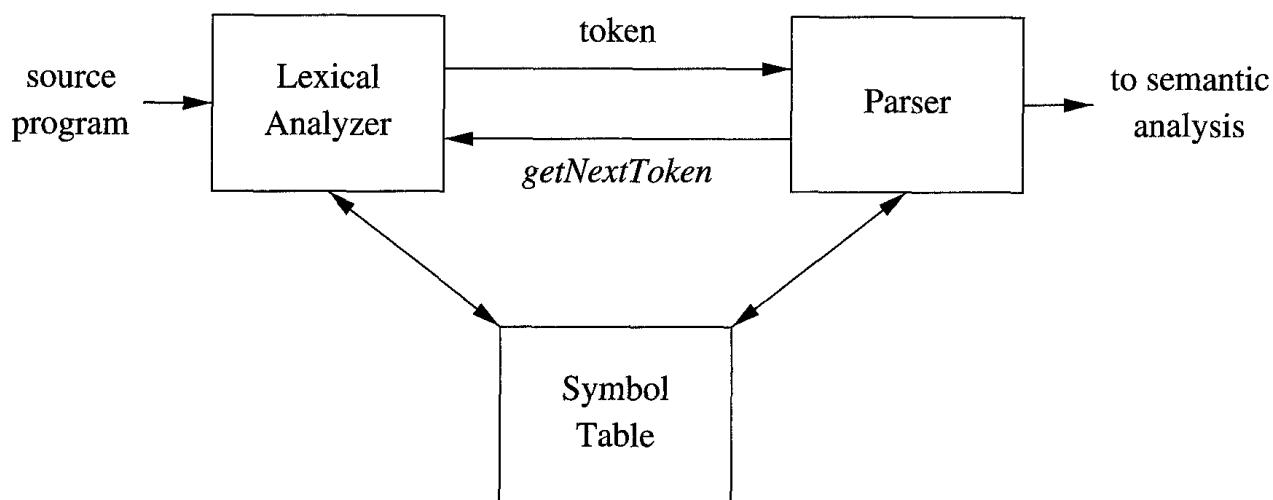


# Lexical analysis or scanning

## ■ Goals of the lexical analysis

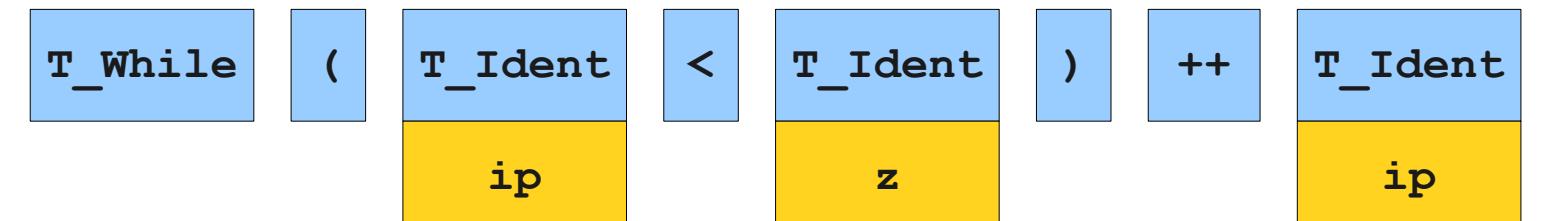
- ▶ Divide the character stream into meaningful sequences called **lexemes**.
- ▶ Label each lexeme with a **token** that is passed to the parser (syntax analysis)
- ▶ Remove non-significant blanks and comments
- ▶ Optional: update the symbol tables with all identifiers (and numbers)

## ■ Provide the interface between the source program and the parser



(Dragonbook)

# Example

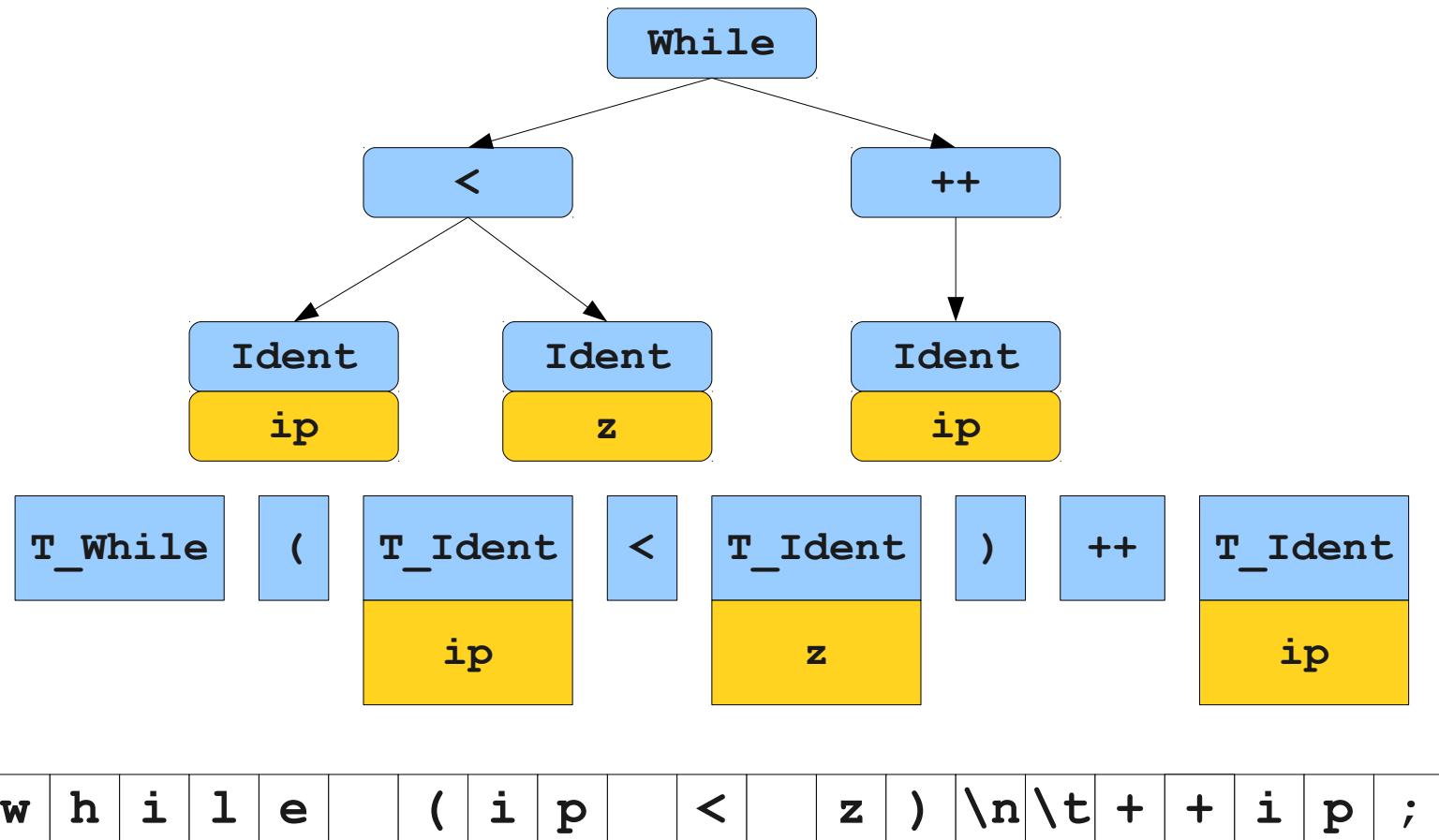


w	h	i	l	e	(	i	p	<	z	)	\n	t	+	+	i	p	;
---	---	---	---	---	---	---	---	---	---	---	----	---	---	---	---	---	---

```
while (ip < z)
    ++ip;
```

(Keith Schwarz)

# Example



```
while (ip < z)
    ++ip;
```

(Keith Schwarz)

# Lexical versus syntax analysis

Why separate lexical analysis from parsing?

- Simplicity of design: simplify both the lexical analysis and the syntax analysis.
- Efficiency: specialized techniques can be applied to improve lexical analysis.
- Portability: only the scanner needs to communicate with the outside

# Tokens, patterns, and lexemes

- A **token** is a  $\langle name, attribute \rangle$  pair. Attribute might be multi-valued.
  - ▶ Example:  $\langle Ident, ip \rangle$ ,  $\langle Operator, < \rangle$ ,  $\langle ")" \rangle$ ,  $\langle NIL \rangle$
- A **pattern** describes the character strings for the lexemes of the token.
  - ▶ Example: a string of letters and digits starting with a letter,  $\{<, >, \leq, \geq, ==\}$ ,  $"")$ .
- A **lexeme** for a token is a sequence of characters that matches the pattern for the token
  - ▶ Example: **ip**, “**<**”, “**)**” in the following program

```
while (ip < z)
      ++ip
```

# Defining a lexical analysis

1. Define the set of tokens
2. Define a pattern for each token (ie., the set of lexemes associated with each token)
3. Define an algorithm for cutting the source program into lexemes and outputting the tokens

# Choosing the tokens

- Very much dependent on the source language
- Typical token classes for programming languages:
  - ▶ One token for each keyword
  - ▶ One token for each “punctuation” symbol (left and right parentheses, comma, semicolon...)
  - ▶ One token for identifiers
  - ▶ Several tokens for the operators
  - ▶ One or more tokens for the constants (numbers or literal strings)
- Attributes
  - ▶ Allows to encode the lexeme corresponding to the token when necessary. Example: pointer to the symbol table for identifiers, constant value for constants.
  - ▶ Not always necessary. Example: keyword, punctuation...

# Describing the patterns

- A pattern defines the set of lexemes corresponding to a token.
- A lexeme being a string, a pattern is actually a [language](#).
- Patterns are typically defined through [regular expressions](#) (that define regular languages).
  - ▶ Sufficient for most tokens
  - ▶ Lead to efficient scanner

# Reminder: languages

- An **alphabet**  $\Sigma$  is a set of characters

*Example:*  $\Sigma = \{a, b\}$

- A **string** over  $\Sigma$  is a finite sequence of elements from  $\Sigma$

*Example:*  $aabba$

- A **language** is a set of strings

*Example:*  $L = \{a, b, abab, babbba\}$

- **Regular languages:** a subset of all languages that can be defined by regular expressions

## Reminder: regular expressions

- Any character  $a \in \Sigma$  is a regular expression  $L = \{a\}$
- $\epsilon$  is a regular expression  $L = \{\epsilon\}$
- If  $R_1$  and  $R_2$  are regular expressions, then
  - ▶  $R_1 R_2$  is a regular expression  
 $L(R_1 R_2)$  is the concatenation of  $L(R_1)$  and  $L(R_2)$
  - ▶  $R_1 | R_2$  ( $= R_1 \cup R_2$ ) is a regular expression  
 $L(R_1 | R_2) = L(R_1) \cup L(R_2)$
  - ▶  $R_1^*$  is a regular expression  
 $L(R_1^*)$  is the Kleene closure of  $L(R_1)$
  - ▶  $(R_1)$  is a regular expression  
 $L((R_1)) = L(R_1)$
- Example: a regular expression for even numbers:

$$(+| - | \epsilon)(0|1|2|3|4|5|6|7|8|9)^*(0|2|4|6|8)$$

# Notational conveniences

- Regular definitions:

*letter* → A|B|...|Z|a|b|...|z

*digit* → 0|1|...|9

*id* → *letter*(*letter*|*digit*)\*

- One or more instances:  $r^+ = rr^*$
- Zero or one instance:  $r? = r|\epsilon$
- Character classes:

[abc]=a|b|c

[a-z]=a|b|...|z

[0-9]=0|1|...|9

# Examples

- Keywords:  
if, while, for, ...
- Identifiers:  
 $[a-zA-Z_][a-zA-Z_0-9]^*$
- Integers:  
 $[+-]?[0-9]^+$
- Floats:  
 $[+-]?(([0-9]^+ ([0-9]*)>|[0-9]^+)([eE][+-]?[0-9]^+)?)$
- String constants:  
“([a-zA-Z0-9]|\\[a-zA-Z])^”

# Algorithms for lexical analysis

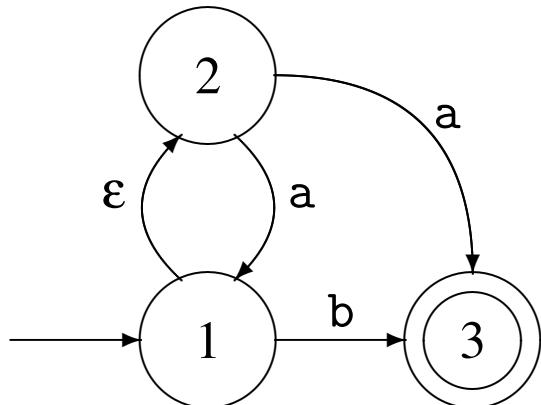
- How to perform lexical analysis from token definitions through regular expressions?
- Regular expressions are equivalent to finite automata, deterministic (DFA) or non-deterministic (NFA).
- Finite automata are easily turned into computer programs
- Two methods:
  1. Convert the regular expressions to an NFA and simulate the NFA
  2. Convert the regular expression to an NFA, convert the NFA to a DFA, and simulate the DFA.

## Reminder: non-deterministic automata (NFA)

A non-deterministic automaton is a five-tuple  $M = (Q, \Sigma, \Delta, s_0, F)$  where:

- $Q$  is a finite set of states,
- $\Sigma$  is an alphabet,
- $\Delta \subset (Q \times (\Sigma \cup \{\epsilon\}) \times Q)$  is the transition relation,
- $s \in Q$  is the initial state,
- $F \subseteq Q$  is the set of accepting states

Example:

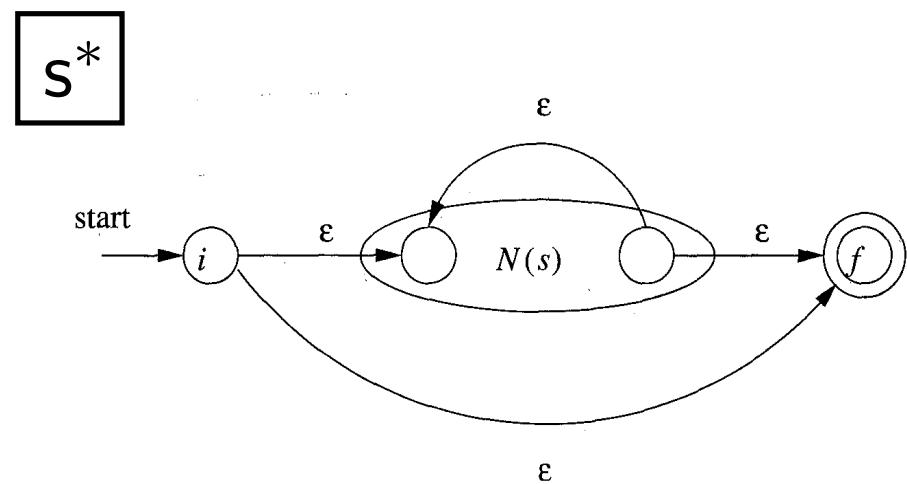
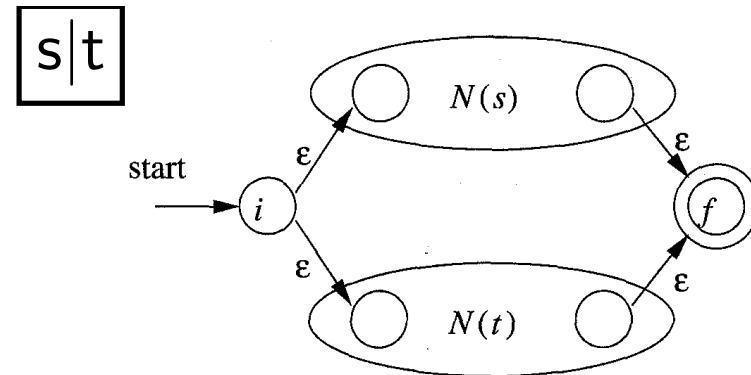
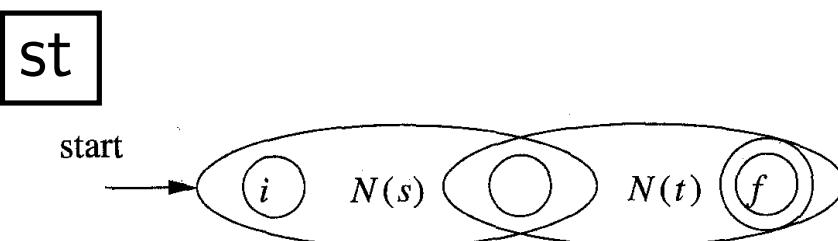
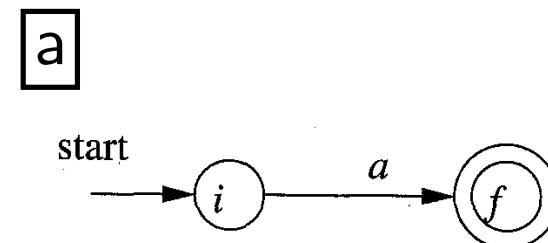
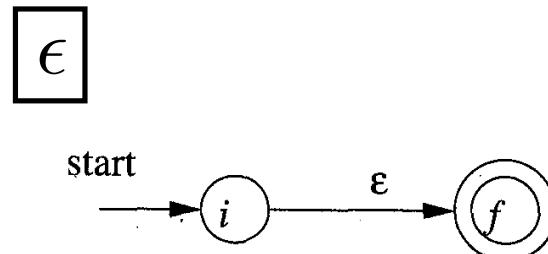


State	Transition table		
	a	b	\u03b5
1	\emptyset	{3}	{2}
2	{1,3}	\emptyset	\emptyset
3	\emptyset	\emptyset	\emptyset

(Mogensen)

# Reminder: from regular expression to NFA

A regular expression can be transformed into an equivalent NFA

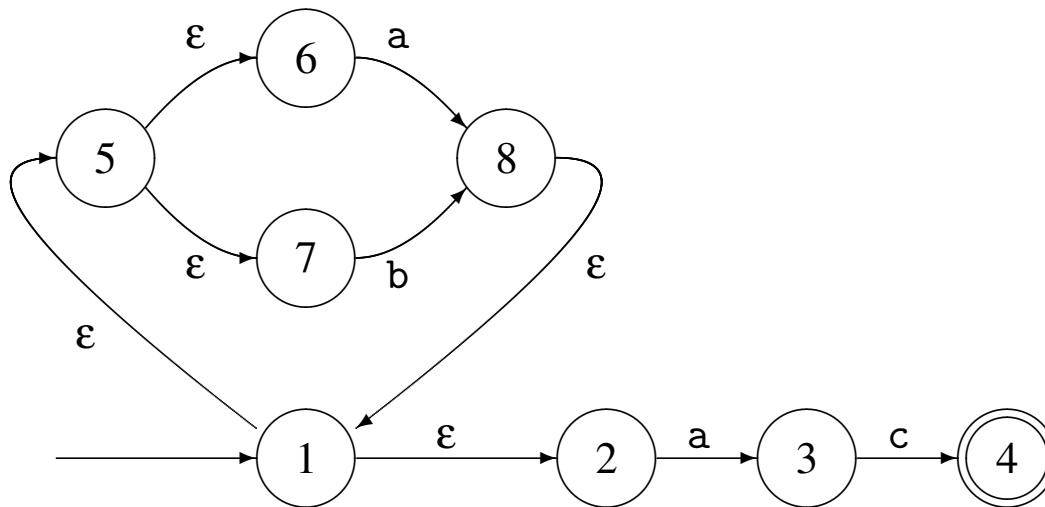


(Dragonbook)

# Reminder: from regular expression to NFA

Example:  $(a|b)^*ac$

(Mogensen)



The NFA  $N(r)$  for an expression  $r$  is such that:

- $N(r)$  has at most twice as many states as there are operators and operands in  $R$ .
- $N(r)$  has one initial state and one accepting state (with no outgoing transition from the accepting state and no incoming transition to the initial state).
- Each (non accepting) state in  $N(r)$  has either one outgoing transition or two outgoing transitions, both on  $\epsilon$ .

# Simulating an NFA

Algorithm to check whether an input string is accepted by the NFA:

```
1)  $S = \epsilon\text{-closure}(s_0);$ 
2)  $c = \text{nextChar}();$ 
3) while ( $c \neq \text{eof}$ ) {
4)      $S = \epsilon\text{-closure}(\text{move}(S, c));$ 
5)      $c = \text{nextChar}();$ 
6) }
7) if ( $S \cap F \neq \emptyset$ ) return "yes";
8) else return "no";
```

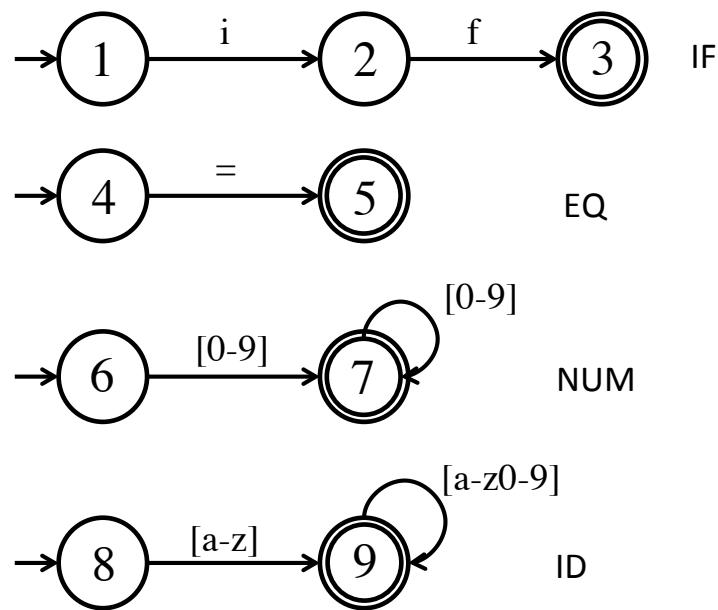
(Dragonbook)

- $\text{nextChar}()$ : returns the next character on the input stream
- $\text{move}(S, c)$ : returns the set of states that can be reached from states in  $S$  when observing  $c$ .
- $\epsilon\text{-closure}(S)$ : returns all states that can be reached with  $\epsilon$  transitions from states in  $S$ .

# Lexical analysis

- What we have so far:
  - ▶ Regular expressions for each token
  - ▶ NFAs for each token that can recognize the corresponding lexemes
  - ▶ A way to simulate an NFA
- How to combine these to cut apart the input text and recognize tokens?
- Two ways:
  - ▶ Simulate all NFAs in turn (or in parallel) from the current position and output the token of the first one to get to an accepting state
  - ▶ Merge all NFAs into a single one with labels of the tokens on the accepting states

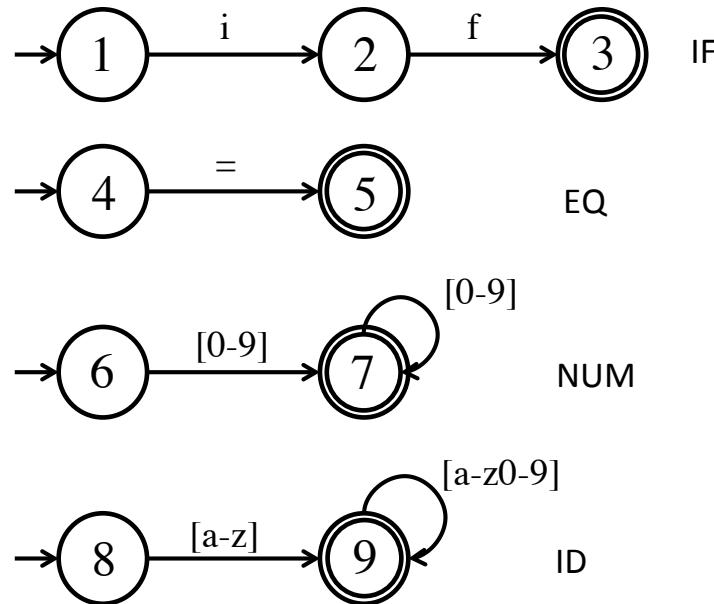
# Illustration



- Four tokens:  $\text{IF}=\text{if}$ ,  $\text{ID}=[\text{a-z}][\text{a-z0-9}]^*$ ,  $\text{EQ}='='$ ,  $\text{NUM}=[0-9]^+$
- Lexical analysis of  $x = 60$  yields:

$$\langle \text{ID}, x \rangle, \langle \text{EQ}, = \rangle, \langle \text{NUM}, 60 \rangle$$

## Illustration: ambiguities



- Lexical analysis of *ifu26 = 60*
- Many splits are possible:

$\langle IF \rangle, \langle ID, u26 \rangle, \langle EQ \rangle, \langle NUM, 60 \rangle$

$\langle ID, ifu26 \rangle, \langle EQ \rangle, \langle NUM, 60 \rangle$

$\langle ID, ifu \rangle, \langle NUM, 26 \rangle, \langle EQ \rangle, \langle NUM, 6 \rangle, \langle NUM, 0 \rangle$

....

# Conflict resolutions

- Principle of the **longest matching prefix**: we choose the longest prefix of the input that matches any token
- Following this principle, *ifu26 = 60* will be split into:

$\langle ID, ifu26 \rangle, \langle EQ \rangle, \langle NUM, 60 \rangle$

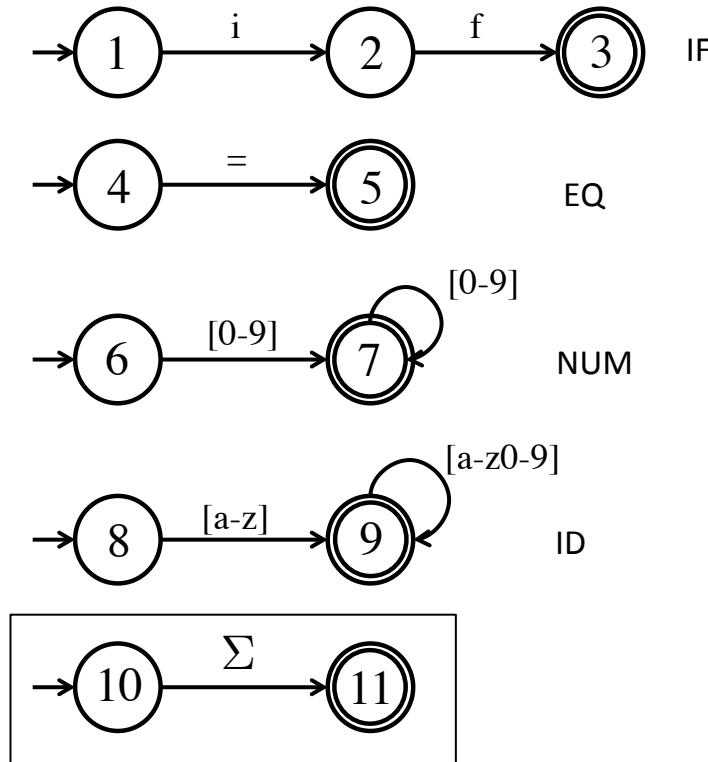
- How to implement?
  - ▶ Run all NFAs in parallel, keeping track of the last accepting state reached by any of the NFAs
  - ▶ When all automata get stuck, report the last match and restart the search at that point
- Requires to retain the characters read since the last match to re-insert them on the input
  - ▶ In our example, '=' would be read and then re-inserted in the buffer.

# Other source of ambiguity

- A lexeme can be accepted by two NFAs
  - ▶ Example: keywords are often also identifiers (*if* in the example)
- Two solutions:
  - ▶ Report an error (such conflict is not allowed in the language)
  - ▶ Let the user decide on a priority order on the tokens (eg., keywords have priority over identifiers)

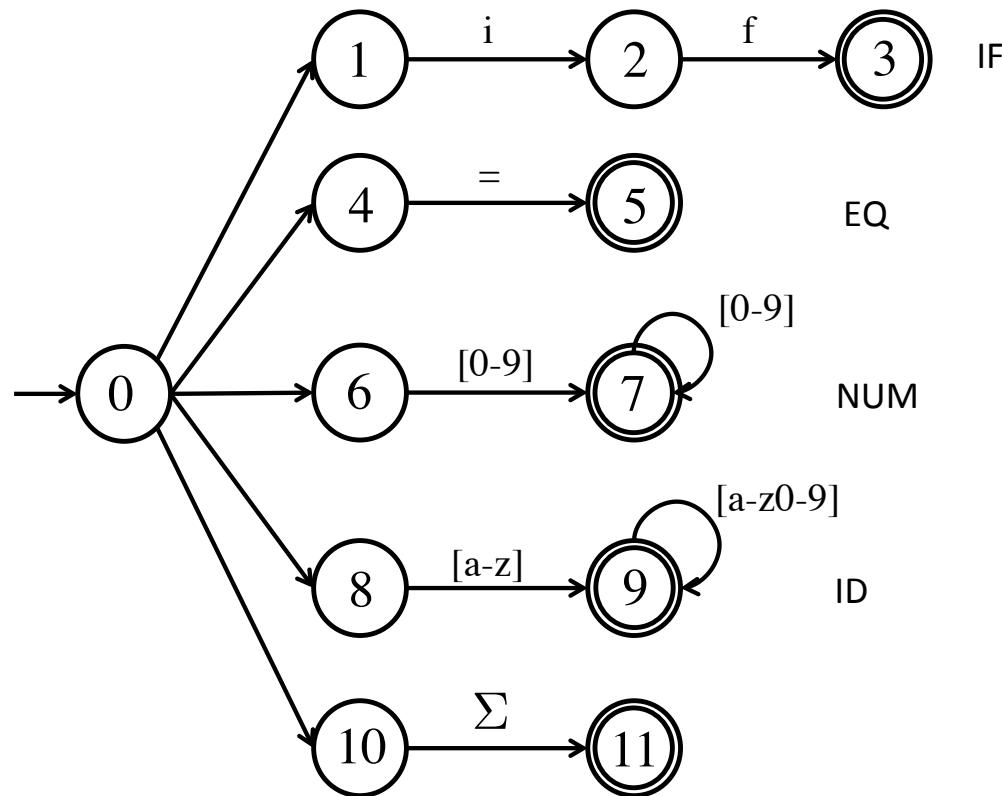
# What if nothing matches

- What if we can not reach any accepting states given the current input?
- Add a “catch-all” rule that matches any character and reports an error



# Merging all automata into a single NFA

- In practice, all NFAs are merged and simulated as a single NFA
- Accepting states are labeled with the token name



# Lexical analysis with an NFA: summary

- Construct NFAs for all regular expression
- Merge them into one automaton by adding a new start state
- Scan the input, keeping track of the last known match
- Break ties by choosing higher-precedence matches
- Have a catch-all rule to handle errors

# Computational efficiency

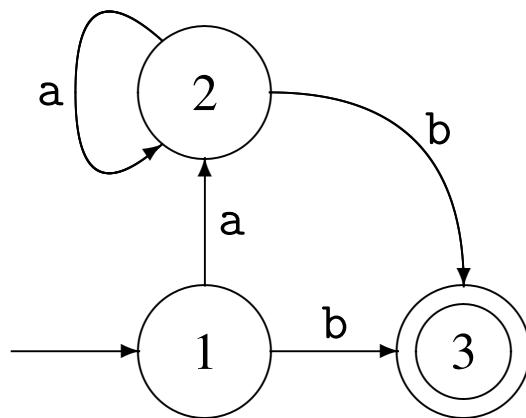
```
1)   $S = \epsilon\text{-closure}(s_0);$ 
2)   $c = nextChar();$ 
3)  while ( $c \neq \text{eof}$ ) {
4)       $S = \epsilon\text{-closure}(move(S, c));$ 
5)       $c = nextChar();$ 
6)  }
7)  if ( $S \cap F \neq \emptyset$ ) return "yes";
8)  else return "no";
```

(Dragonbook)

- In the worst case, an NFA with  $|Q|$  states takes  $O(|S||Q|^2)$  time to match a string of length  $|S|$
- Complexity thus depends on the number of states
- It is possible to reduce complexity of matching to  $O(|S|)$  by transforming the NFA into an equivalent deterministic finite automaton (DFA)

## Reminder: deterministic finite automaton

- Like an NFA but the transition relation  $\Delta \subset (Q \times (\Sigma \cup \{\epsilon\}) \times Q)$  is such that:
  - ▶ Transitions based on  $\epsilon$  are not allowed
  - ▶ Each state has at most one outgoing transition defined for every letter
- Transition relation is replaced by a transition function
$$\delta : Q \times \Sigma \rightarrow Q$$
- Example of a DFA



(Mogensen)

## Reminder: from NFA to DFA

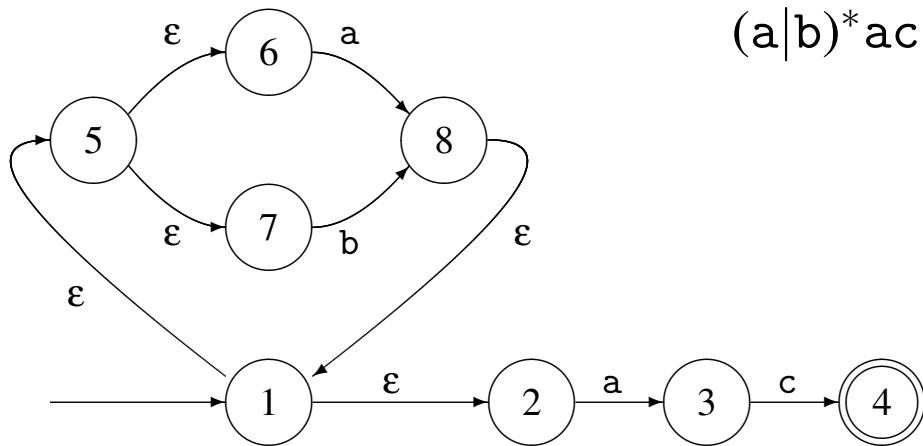
- DFA and NFA (and regular expressions) have the same expressive power
- An NFA can be converted into a DFA by the [subset construction method](#)
- Main idea: mimic the simulation of the NFA with a DFA
  - ▶ Every state of the resulting DFA corresponds to a set of states of the NFA. First state is  $\epsilon$ -closure( $s_0$ ).
  - ▶ Transitions between states of DFA correspond to transitions between set of states in the NFA:

$$\delta(S, c) = \epsilon\text{-closure}(\text{move}(S, c))$$

- ▶ A set of the DFA is accepting if any of the NFA states that it contains is accepting
- See [INFO0016](#) or the reference book for more details

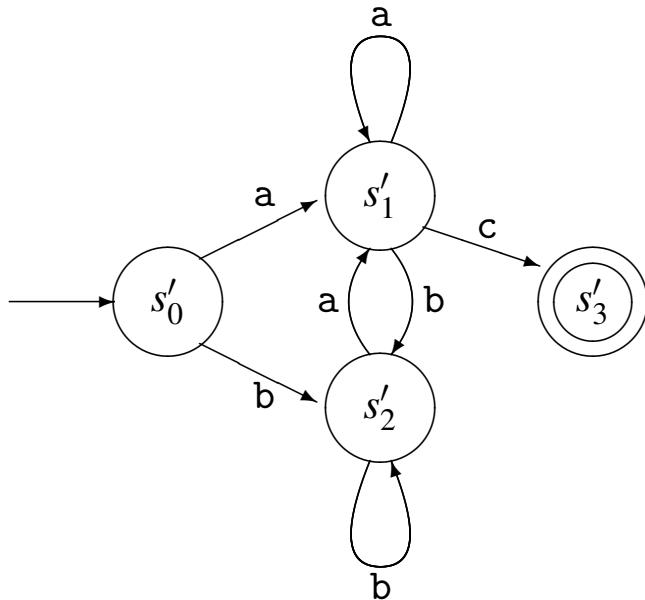
# Reminder: from NFA to DFA

NFA



$(a|b)^*ac$

DFA



$s'_0 \quad \{1, 2, 5, 6, 7\}$

$s'_1 \quad \{3, 8, 1, 2, 5, 6, 7\}$

$s'_2 \quad \{8, 1, 2, 5, 6, 7\}$

$s'_3 \quad \{4\}$

(Mogensen)

# Simulating a DFA

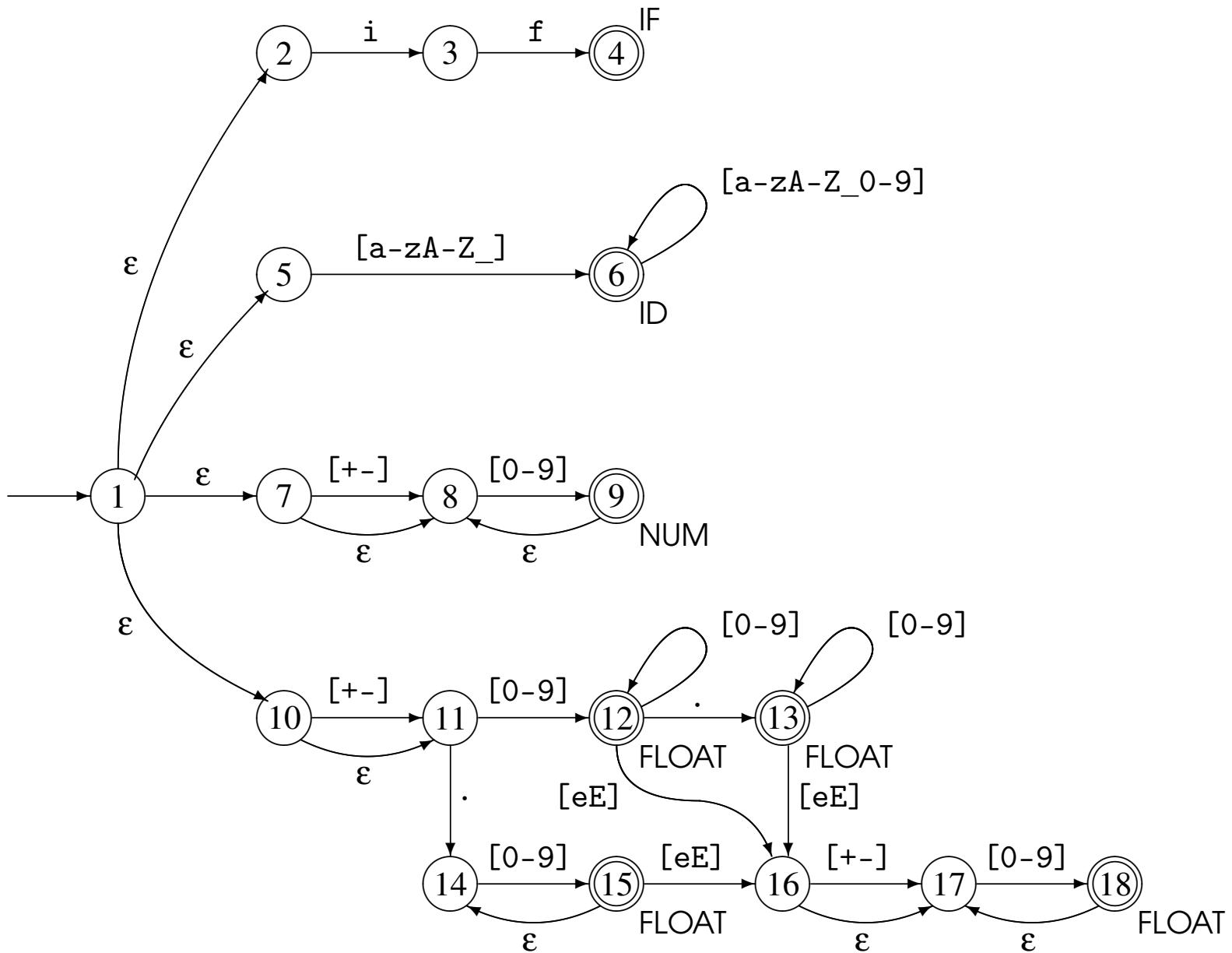
```
s = s0;
c = nextChar();
while ( c != eof ) {
    s = move(s, c);
    c = nextChar();
}
if ( s is in F ) return "yes";
else return "no";
```

- Time complexity is  $O(|S|)$  for a string of length  $|S|$
- Now independent of the number of states

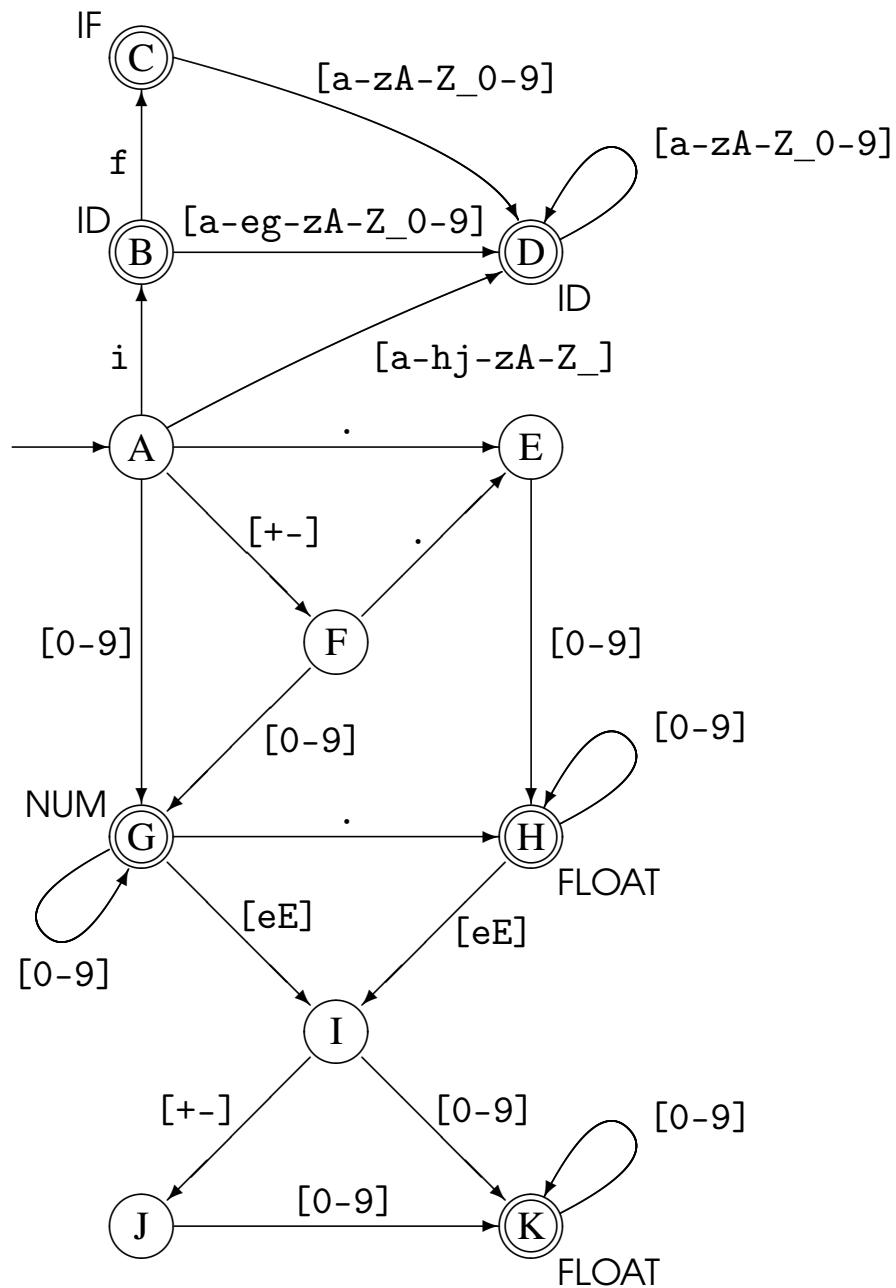
# Lexical analysis with a DFA: summary

- Construct NFAs for all regular expressions
- Mark the accepting states of the NFAs by the name of the tokens they accept
- Merge them into one automaton by adding a new start state
- Convert the combined NFA to a DFA
- Convey the accepting state labeling of the NFAs to the DFA (by taking into account precedence rules)
- Scanning is done like with an NFA

## Example: combined NFA for several tokens



# Example: combined DFA for several tokens



Try lexing on the strings:

- *if17*
- *3e-y*

# Speed versus memory

- The number of states of a DFA can grow exponentially with respect to the size of the corresponding regular expression (or NFA)
- We have to choose between low-memory and slow NFAs and high-memory and fast DFAs.

Note:

- It is possible to minimise the number of states of a DFA in  $O(n \log n)$  (Hopcroft's algorithm<sup>1</sup>)
  - ▶ Theory says that any regular language has a unique minimal DFA
  - ▶ However, the number of states may remain exponential in the size of the regular expression after minimization

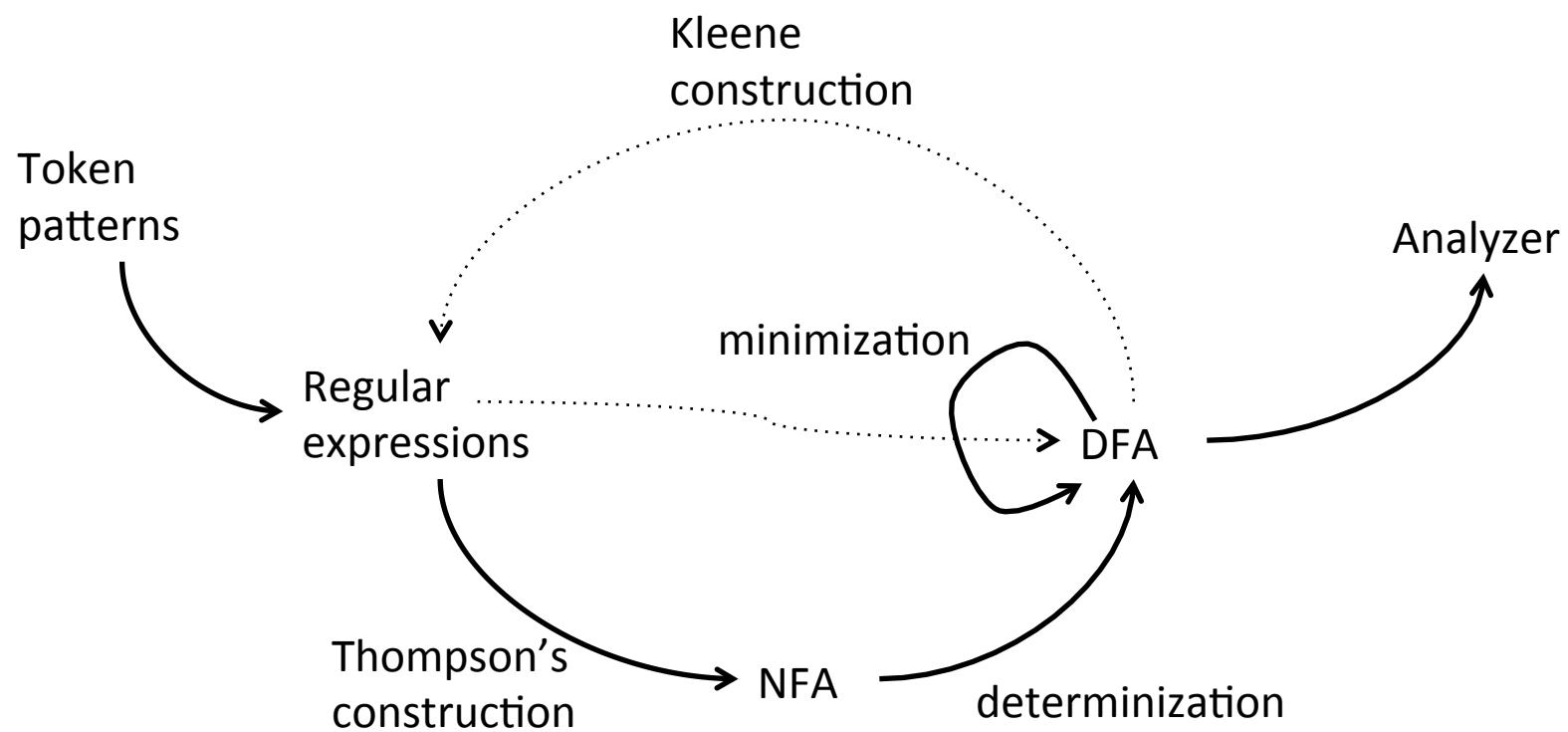
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<sup>1</sup>[http://en.wikipedia.org/wiki/DFA\\_minimization](http://en.wikipedia.org/wiki/DFA_minimization)

# Keywords and identifiers

- Having a separate regular expression for each keyword is not very efficient.
- In practice:
  - ▶ We define only one regular expression for both keywords and identifiers
  - ▶ All keywords are stored in a (hash) table
  - ▶ Once an identifier/keyword is read, a table lookup is performed to see whether this is an identifier or a keyword
- Reduces drastically the size of the DFA
- Adding a keyword requires only to add one entry in the hash table.

# Summary



# Some language specificities

Language specificities that make lexical analysis hard:

- Whitespaces are irrelevant in Fortran.

```
DO 5 I = 1,25
```

```
D05I = 1.25
```

- PL/1: keywords can be used as identifiers:

```
IF THEN THEN THEN = ELSE; ELSE ELSE = IF
```

- Python block defined by indentation:

```
if w == z:
```

```
    a = b
```

```
else:
```

```
    e = f
```

```
g = h
```

(the lexical analyser needs to record current indentation and output a token for each increase/decrease in indentation)

(Keith Schwarz)

# Some language specificities

- Sometimes, nested lexical analyzers are needed
- For example, to deal with nested comments:

```
/* /* where do my comments end? here? */ or here? */
```

- ▶ As soon as /\* is read, switch to another lexical analyzer that
  - ▶ only reads /\* and \*/,
  - ▶ counts the level of nested comments at current position (starting at 0),
  - ▶ get back to the original analyzer when it reads \*/ and the level is 0
- Other example: Javadoc (needs to interpret the comments)

NB: How could you test if your compiler accepts nested comments without generating a compilation error?

```
int nest = /*/*/0*/**/1;
```

# Implementing a lexical analyzer

- In practice (and for your project), two ways:
  - ▶ Write an ad-hoc analyser
  - ▶ Use automatic tools like (F)LEX.
- First approach is more tedious. It is only useful to address specific needs.
- Second approach is more portable

# Example of an ad-hoc lexical analyser

(source: <http://dragonbook.stanford.edu/lecture-notes.html>)

## Definition of the token classes (through constants)

```
#define T_SEMICOLON    ';'      // use ASCII values for single char tokens
#define T_LPAREN        '('
#define T_RPAREN        ')'
#define T_ASSIGN         '='
#define T_DIVIDE         '/'

...
#define T_WHILE          257      // reserved words
#define T_IF             258
#define T_RETURN         259
...

#define T_IDENTIFIER     268      // identifiers, constants, etc.
#define T_INTEGER         269
#define T_DOUBLE          270
#define T_STRING          271

#define T_END            349      // code used when at end of file
#define T_UNKNOWN         350      // token was unrecognized by scanner
```

# Example of an ad-hoc lexical analyser

## Structure for tokens

```
struct token_t {
    int type;                      // one of the token codes from above
    union {
        char stringValue[256];    // holds lexeme value if string/identifier
        int intValue;            // holds lexeme value if integer
        double doubleValue;      // holds lexeme value if double
    } val;
};
```

## Main function

```
int main(int argc, char *argv[])
{
    struct token_t token;

    InitScanner();
    while (ScanOneToken(stdin, &token) != T_END)
        ; // this is where you would process each token
    return 0;
}
```

# Example of an ad-hoc lexical analyser

## Initialization

```
static void InitScanner()
{
    create_reserved_table(); // table maps reserved words to token type
    insert_reserved("WHILE", T WHILE)
    insert_reserved("IF", T IF)
    insert_reserved("RETURN", T RETURN)
    ....
}
```

# Example of an ad-hoc lexical analyser

## Scanning (single-char tokens)

```
static int ScanOneToken(FILE *fp, struct token_t *token)
{
    int i, ch, nextch;

    ch = getc(fp);      // read next char from input stream
    while (isspace(ch)) // if necessary, keep reading til non-space char
        ch = getc(fp); // (discard any white space)

    switch(ch) {
        case '/': // could either begin comment or T_DIVIDE op
            nextch = getc(fp);
            if (nextch == '/' || nextch == '*')
                ; // here you would skip over the comment
            else
                ungetc(nextch, fp); // fall-through to single-char token case
        case ';': case ',': case '=': // ... and other single char tokens
            token->type = ch; // ASCII value is used as token type
            return ch; // ASCII value used as token type
    }
}
```

# Example of an ad-hoc lexical analyser

## Scanning: keywords

```
case 'A': case 'B': case 'C': // ... and other upper letters
    token->val.stringValue[0] = ch;
    for (i = 1; isupper(ch = getc(fp)); i++) // gather uppercase
        token->val.stringValue[i] = ch;
    ungetc(ch, fp);
    token->val.stringValue[i] = '\0'; // lookup reserved word
    token->type = lookup_reserved(token->val.stringValue);
    return token->type;
```

## Scanning: identifier

```
case 'a': case 'b': case 'c': // ... and other lower letters
    token->type = T_IDENTIFIER;
    token->val.stringValue[0] = ch;
    for (i = 1; islower(ch = getc(fp)); i++)
        token->val.stringValue[i] = ch; // gather lowercase
    ungetc(ch, fp);
    token->val.stringValue[i] = '\0';
    if (lookup_symtab(token->val.stringValue) == NULL)
        add_symtab(token->val.stringValue); // get symbol for ident
    return T_IDENTIFIER;
```

# Example of an ad-hoc lexical analyser

## Scanning: number

```
case '0': case '1': case '2': case '3': //.... and other digits
    token->type = T_INTEGER;
    token->val.intValue = ch - '0';
    while (isdigit(ch = getc(fp))) // convert digit char to number
        token->val.intValue = token->val.intValue * 10 + ch - '0';
    ungetc(ch, fp);
    return T_INTEGER;
```

## Scanning: EOF and default

```
case EOF:
    return T_END;

default: // anything else is not recognized
    token->val.intValue = ch;
    token->type = T_UNKNOWN;
    return T_UNKNOWN;
```

# Flex

- flex is a free implementation of the Unix lex program
- flex implements what we have seen:
  - ▶ It takes regular expressions as input
  - ▶ It generates a combined NFA
  - ▶ It converts it to an equivalent DFA
  - ▶ It minimizes the automaton as much as possible
  - ▶ It generates C code that implements it
  - ▶ It handles conflicts with the longest matching prefix principle and a preference order on the tokens.
- More information
  - ▶ <http://flex.sourceforge.net/manual/>

# Input file

- Input files are structured as follows:

```
%{  
    Declarations  
}%  
    Definitions  
%%  
    Rules  
%%  
    User subroutines
```

- Declarations and User subroutines are copied without modifications to the generated C file.
- Definitions specify options and name definitions (to simplify the rules)
- Rules: specify the patterns for the tokens to be recognized

# Rules

- In the form:

```
pattern1 action1  
pattern2 action2  
...
```

- Patterns are defined as regular expressions. Actions are blocks of C code.
- When a sequence is read that matches the pattern, the C code of the action is executed
- Examples:

```
[0-9]+ {printf("This is a number");}  
[a-z]+ {printf("This is symbol");}
```

# Regular expressions

- Many shortcut notations are permitted in regular expressions:
  - ▶ [], -, +, \*, ?: as defined previously
  - ▶ .: a dot matches any character (except newline)
  - ▶ [^x]: matches the complement of the set of characters in x (ex: all non-digit characters [^0-9]).
  - ▶ x{n,m}: x repeated between n and m times
  - ▶ "x": matches x even if x contains special characters (ex: "x\*" matches x followed by a star).
  - ▶ {name}: replace with the pattern defined earlier in the definition section of the input file

# Interacting with the scanner

- User subroutines and action may interact with the generated scanner through global variables:
  - ▶ `yylex`: scan tokens from the global input file `yyin` (defaults to `stdin`). Continues until it reaches the end of the file or one of its actions executes a return statement.
  - ▶ `yytext`: a null-terminated string (of length `yyleng`) containing the text of the lexeme just recognized.
  - ▶ `yylval`: store the attributes of the token
  - ▶ `yylloc`: location of the tokens in the input file (line and column)
  - ▶ ...

## Example 1: hiding numbers

- hide-digits.l:

```
%%  
[0-9]+ printf("?",);  
. ECHO;
```

- To build and run the program:

```
% flex hide-digits.l  
% gcc -o hide-digits lex.yy.c ll  
% ./hide-digits
```

## Example 2: wc

- count.l:

```
%{  
    int numChars = 0, numWords = 0, numLines = 0;  
}  
%%  
\n        {numLines++; numChars++;}  
[^ \t\n]+ {numWords++; numChars += yyleng;}  
.         {numChars++;}  
%%  
  
int main() {  
    yylex();  
    printf("%d\t%d\t%d\n", numChars, numWords, numLines);  
}
```

- To build and run the program:

```
% flex count.l  
% gcc -o count lex.yy.c ll  
% ./count < count.l
```

## Example 3: typical compiler

```
%{  
    /* definitions of manifest constants  
    LT, LE, EQ, NE, GT, GE,  
    IF, THEN, ELSE, ID, NUMBER, RELOP */  
}  
  
/* regular definitions */  
delim      [ \t\n]  
ws         {delim}+  
letter     [A-Za-z]  
digit      [0-9]  
id          {letter}({letter}|{digit})*  
number     {digit}+(\.{digit}+)?(E[+-]?)?{digit}+)?  
  
%%  
  
{ws}        {/* no action and no return */}  
if          {return(IF);}  
then        {return(THEN);}  
else        {return(ELSE);}  
{id}         {yyval = (int) installID(); return(ID);}  
{number}    {yyval = (int) installNum(); return(NUMBER);}  
"<"         {yyval = LT; return(RELOP);}  
"<="        {yyval = LE; return(RELOP);}  
"="          {yyval = EQ; return(RELOP);}  
"<>"       {yyval = NE; return(RELOP);}  
">"         {yyval = GT; return(RELOP);}  
">="        {yyval = GE; return(RELOP);}
```

## Example 3: typical compiler

### User defined subroutines

```
%%

int installID() /* function to install the lexeme, whose
                  first character is pointed to by yytext,
                  and whose length is yyleng, into the
                  symbol table and return a pointer
                  thereto */
}

int installNum() /* similar to installID, but puts numerical
                  constants into a separate table */
}
```