Cargo-Cult Complexity Testing

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INTRODUCTION TO ALGORITHMS

THIRD EDITION

The master theorem

The master method depends on the following theorem.

Theorem 4.1 (Master theorem)
Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n) ,$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ has the following asymptotic bounds:

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \lg n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta(f(n))$. ■

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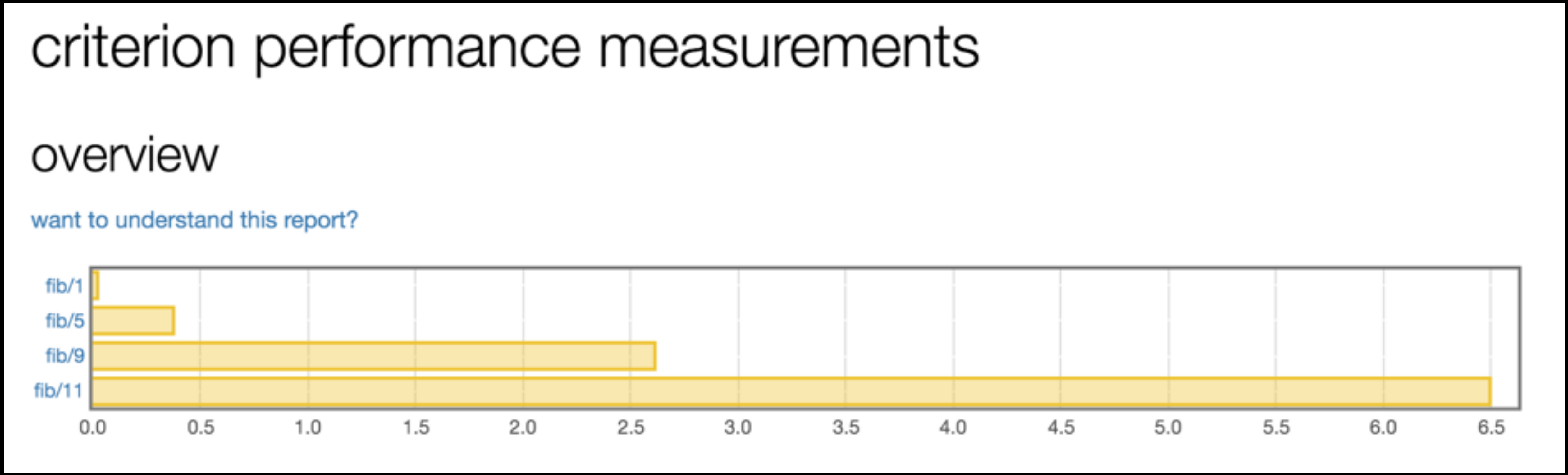
Just Run It And See



Generate Inputs

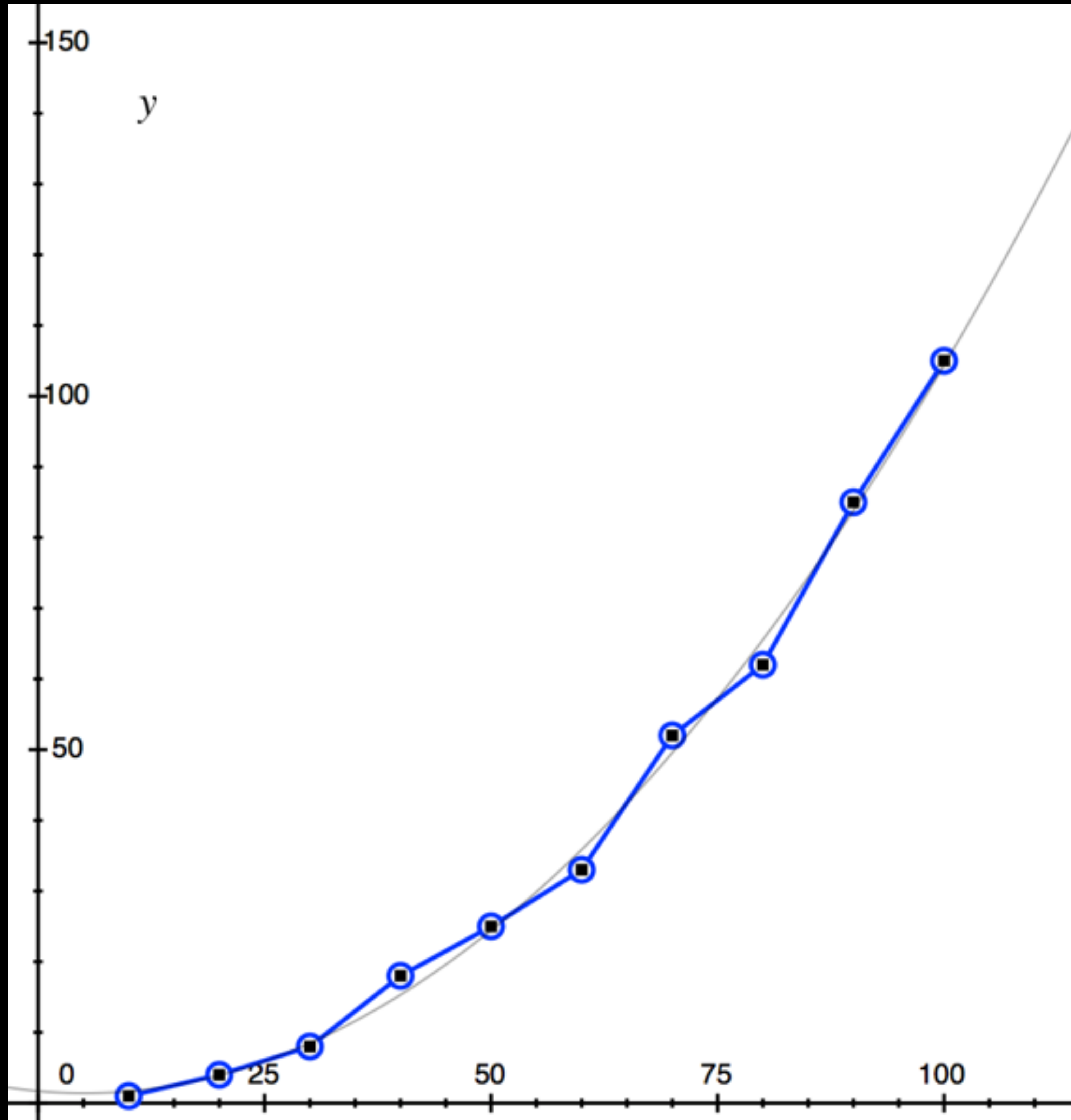
Integer input size n:

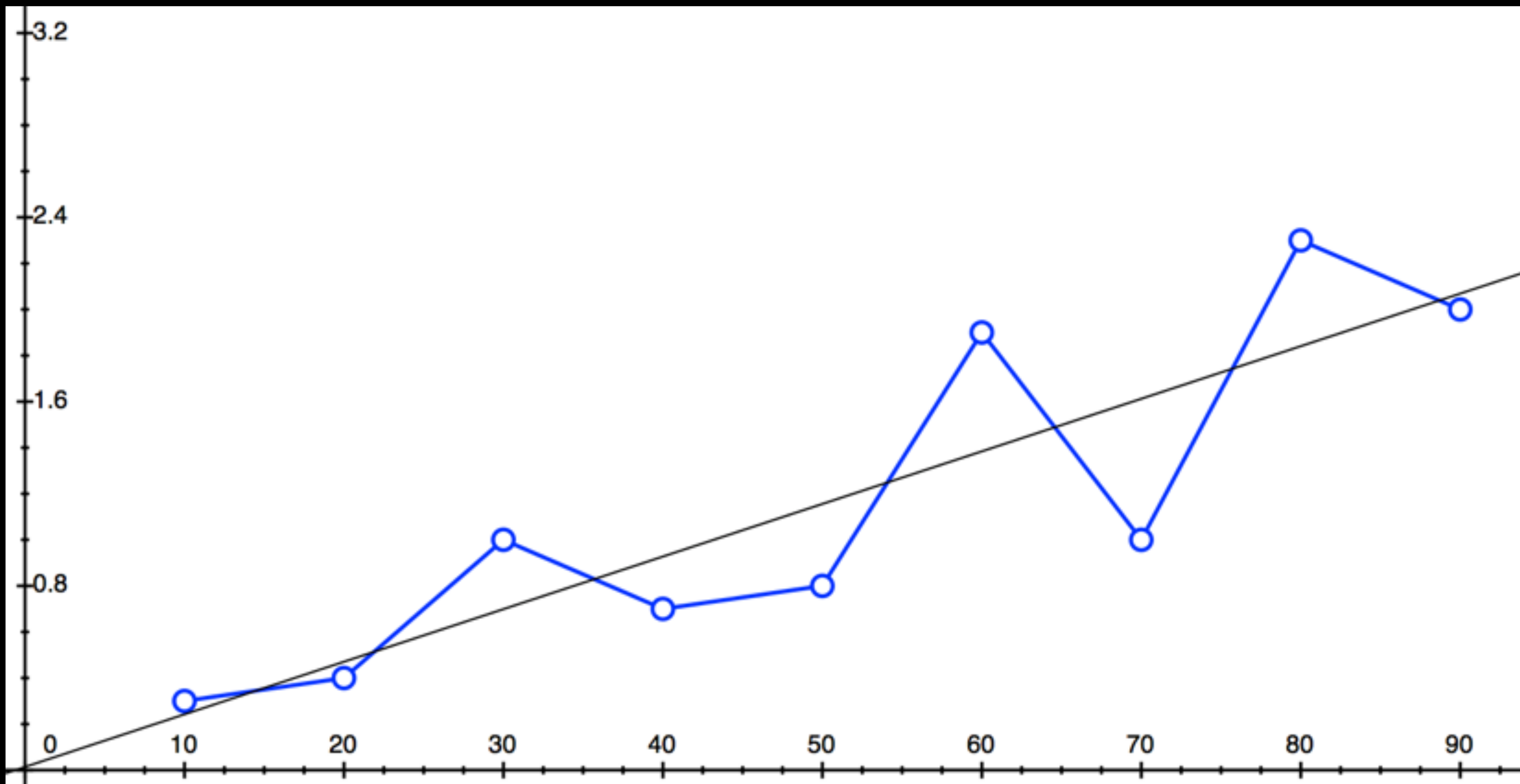
```
:: Gen Int
return n
```

List input size n:

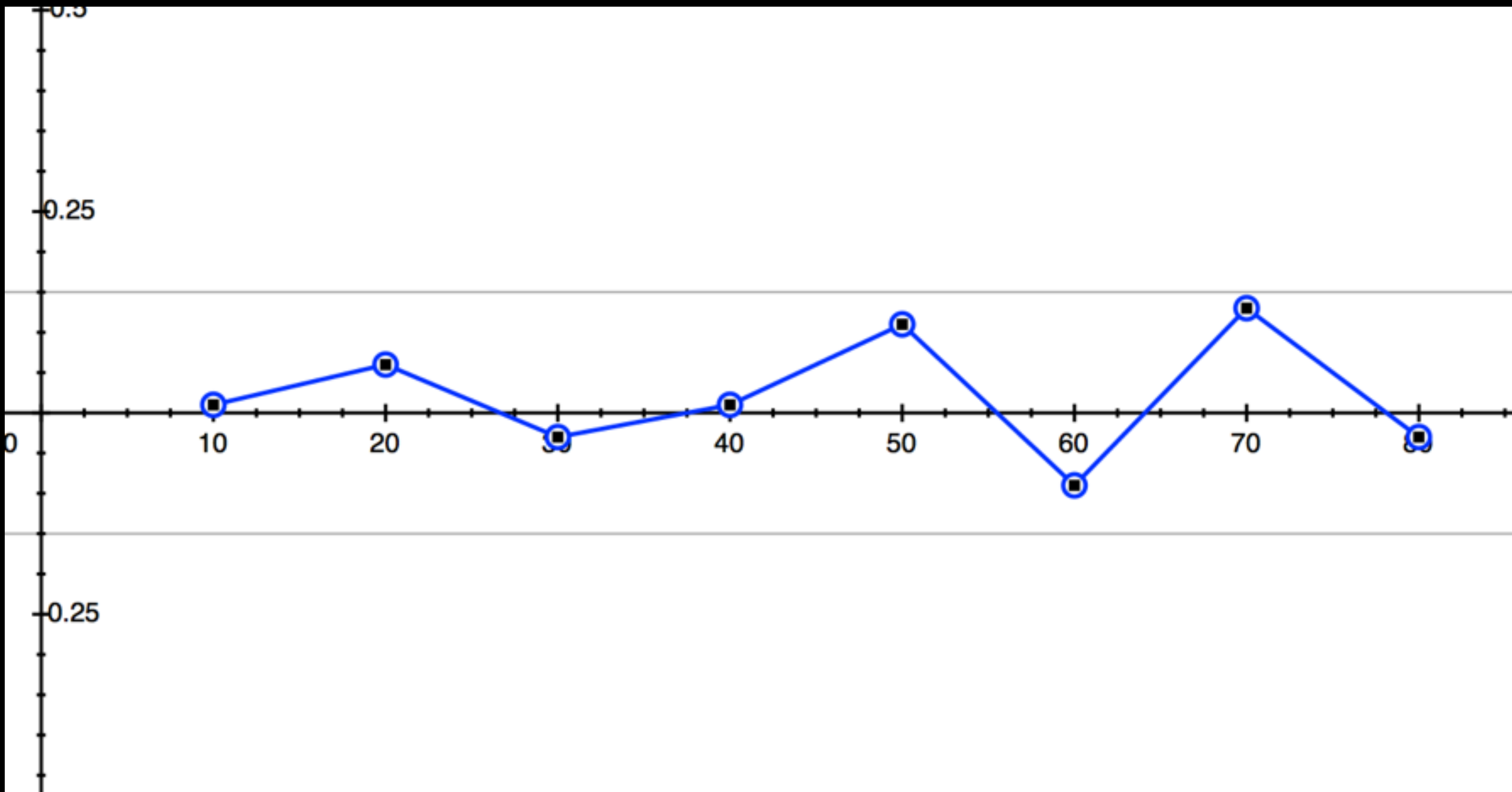
```
:: Arbitrary a => Gen [a]
take n <$> arbitrary
```

Demo 1
(generate points)


150
y
100
50
0
25
50
75
100



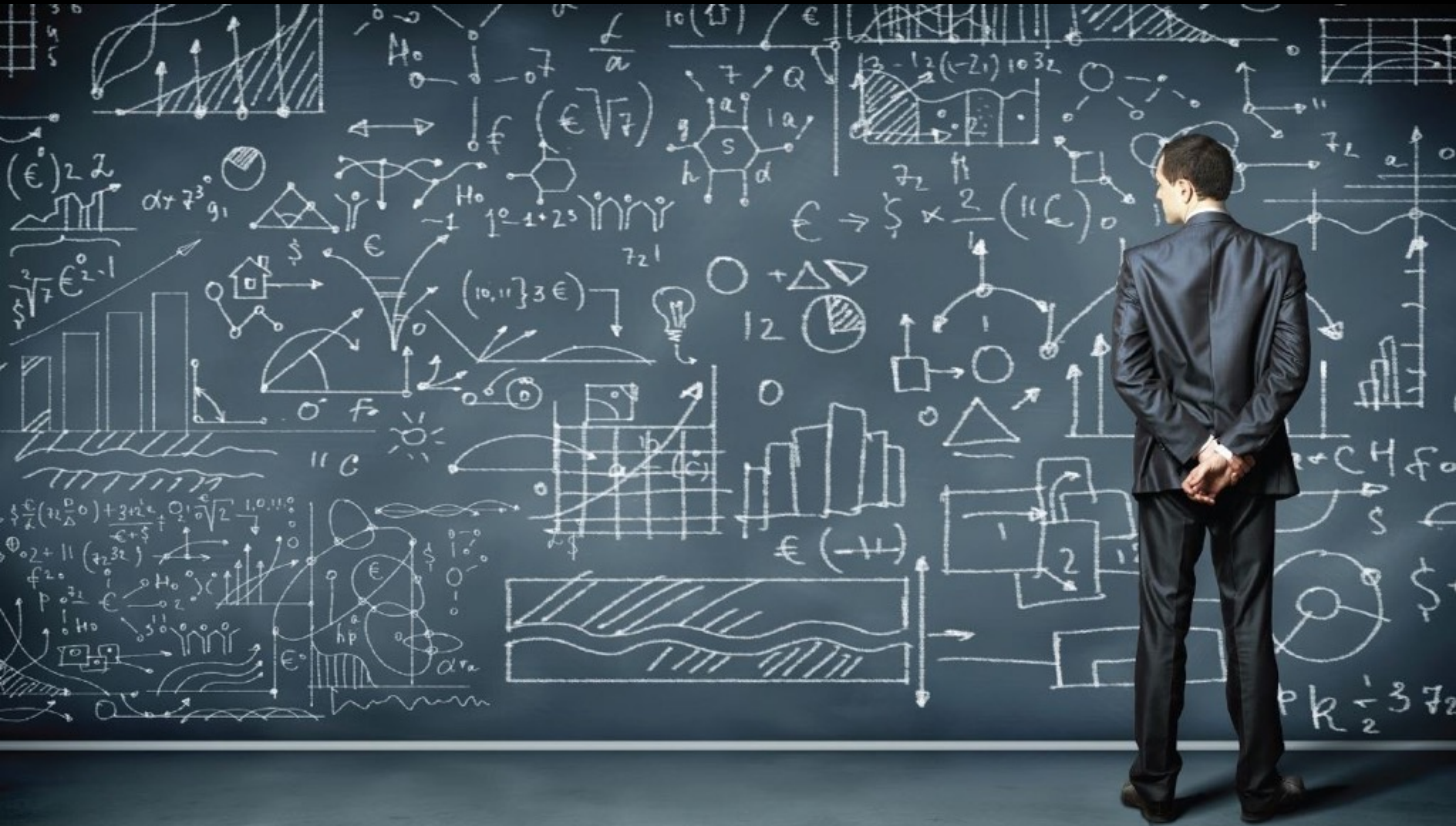
3.2
2.4
1.6
0.8
0
10
20
30
40
50
60
70
80
90


0.25
-0.25
0
10
20
40
50
60
70

Find a polynomial order that fits best

```haskell
-- | Estimate the polynomial order for some points, given
--   a margin of error.
--
polyOrder :: Double -> [Point] -> Maybe Order
polyOrder epsilon points@(_:_:_:_)
  | isConstant points
  = Just 0
  | otherwise
  = fmap succ $ polyOrder epsilon $ deriv points
  where
   isConstant derivs
    = sd (fmap snd derivs) < epsilon
polyOrder _ _
  = Nothing
```

Demo 2
(naive)



Use curve-fitting to try and fit every type of curve

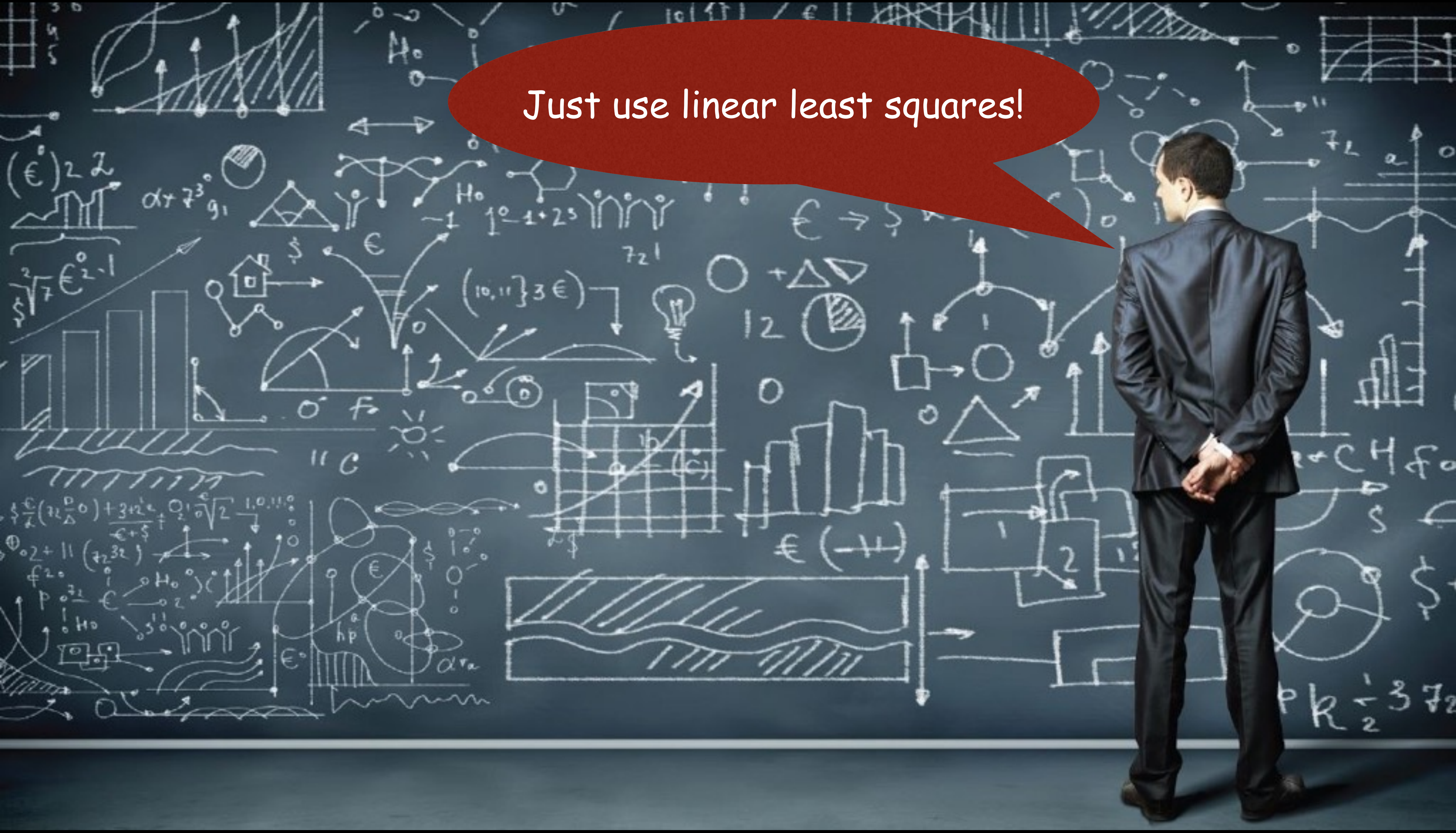
Use the R-squared statistic to determine their goodness of fit!

Curve Fitting
with nls()?

R's non-linear least squares method for curve-fitting

Curve Fitting with nls()?

"If there was a strategy that was both good and general -- one that always worked - it would already be implemented in every nonlinear least squares program and starting values would be a non-issue."


Just use linear least squares!

lm()

```
> summary(lm(y ~ I(x^2)+I(x)))

Call:
lm(formula = y ~ I(x^2) + I(x))

Residuals:
       Min         1Q     Median         3Q        Max 
-2.627e-04 -1.723e-04 -1.097e-05  1.229e-04  5.871e-04 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.641e-04  1.669e-04   2.182  0.04439 *  
I(x^2)       1.073e-05  1.867e-06   5.747    3e-05 ***
I(x)        -1.497e-04  3.843e-05  -3.895  0.00129 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0002174 on 16 degrees of freedom

Multiple R-squared:  0.8397,  Adjusted R-squared:  0.8197

F-statistic: 41.91 on 2 and 16 DF,  p-value: 4.361e-07
```

HaskellR

```
[r|
  summary(lm(y ~ I(x^2) + I(x)))$adj.r.squared
|]
```

HaskellR

```haskell
lm' :: Order -> [Double] -> [Double] -> R s (R.SomeSEXP s)
lm' order x y
  = do [r| x = x_hs |]
       [r| y = y_hs |]
       case order of
         Constant    -> [r|summary(lm(y ~ 1))$adj.r.squared|]
         LogN        -> [r|summary(lm(y ~ I(log(x))))$adj.r.squared|]
         NLogN       -> [r|summary(lm(y ~ I(x * log(x))))$adj.r.squared|]
         Linear      -> [r|summary(lm(y ~ I(x)))$adj.r.squared|]
         Quadratic   -> [r|summary(lm(y ~ I(x^2) + I(x)))$adj.r.squared|]
         Cubic       -> [r|summary(lm(y ~ I(x^3) + I(x^2) + I(x)))$adj.r.squared|]
         Quartic     -> [r|summary(lm(y ~ I(x^4) + I(x^3) + I(x^2) + I(x)))$adj.r.squared|]
         Exp         -> [r|summary(lm(log(y) ~ x))$adj.r.squared|]
```

Demo 3
(the sorts)

Demo 4
(fibonacci)

Demo 5
(codensity!?)

“Honourable” mention

計

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Hakaru

```haskell
-- | Given a complexity order and some data, generate a curve of
--   that order that fit the data.
--
curveFit :: Order -> Fit -> IO (Double -> Double)
curveFit thing (Fit xs ys dropn taken)
 = do l <- mcmc (measureForOrder thing xs ys)
                (map (Just . toDyn . Lebesgue) ys)
      let means = expectations $ take taken $ drop dropn l
      return $ mkCurve thing means
```

/end