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## BÀI TẬP MÈNH ĐỀ

9/

- a. 
$$\begin{aligned} & (P \wedge Q) \rightarrow P \\ & = \Gamma(P \wedge Q) \vee P \\ & = \Gamma P \vee \Gamma Q \\ & = \Gamma \vee \Gamma Q \\ & = \Gamma \mid \end{aligned}$$
- b. 
$$\begin{aligned} & P \rightarrow (\Gamma P \rightarrow P) \\ & = P \rightarrow (\Gamma \Gamma P \vee P) \\ & = P \rightarrow P \\ & = \Gamma P \vee P \\ & = \Gamma \end{aligned}$$
- c. 
$$\begin{aligned} & P \rightarrow ((Q \rightarrow (P \wedge Q))) \\ & = P \rightarrow (\Gamma Q \vee (P \wedge Q)) \\ & = P \rightarrow ((\Gamma Q \vee P) \wedge (\Gamma Q \vee Q)) \\ & = P \rightarrow ((\Gamma Q \vee P) \wedge T) \\ & = P \rightarrow (\Gamma Q \vee P) \\ & = \Gamma P \vee (\Gamma Q \vee P) \\ & = \Gamma P \vee P \vee \Gamma Q \\ & = \Gamma \vee \Gamma Q \\ & = \Gamma \end{aligned}$$
- d. 
$$\begin{aligned} & \Gamma(P \vee \Gamma Q) \rightarrow \Gamma P \\ & = (\Gamma P \vee \Gamma Q) \vee \Gamma P \\ & = \Gamma \vee \Gamma Q \\ & = \Gamma \\ \\ & = ((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \rightarrow (\Gamma P \vee R) \\ & = (((\Gamma P \wedge \Gamma Q) \vee (\Gamma P \wedge R)) \vee ((Q \wedge \Gamma Q) \vee (Q \wedge R))) \rightarrow (\Gamma P \vee R) \\ & = ((\Gamma P \wedge (\Gamma Q \vee R)) \vee (F_v(Q \wedge R))) \rightarrow (\Gamma P \vee R) \\ & = ((\Gamma P \wedge (\Gamma Q \vee R)) \vee (Q \wedge R)) \rightarrow (\Gamma P \vee R) \\ & = (((Q \wedge R) \vee \Gamma P) \wedge ((Q \wedge R) \vee (\Gamma Q \vee R))) \rightarrow (\Gamma P \vee R) \\ & = (((Q \wedge R) \vee \Gamma P) \wedge (Q \wedge R)) \end{aligned}$$
- e. 
$$\begin{aligned} & ((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R) \\ & = \Gamma((P \rightarrow Q) \wedge (Q \rightarrow R)) \vee (P \rightarrow R) \\ & = (\Gamma(P \rightarrow Q) \vee \Gamma(Q \rightarrow R)) \vee (P \rightarrow R) \\ & = (\Gamma(\Gamma P \vee Q) \vee \Gamma(\Gamma Q \vee R)) \vee (P \rightarrow R) \\ & = ((P \wedge \Gamma Q) \vee (Q \wedge \Gamma R)) \vee (P \rightarrow R) \\ & = ((P \wedge \Gamma Q) \vee (Q \wedge \Gamma R)) \vee (\Gamma P \vee R) \\ & = (((P \vee Q) \wedge (P \vee \Gamma R)) \wedge ((Q \vee Q) \wedge (\Gamma Q \vee \Gamma R))) \vee (\Gamma P \vee R) \\ & = (((P \vee Q) \wedge (P \vee \Gamma R)) \wedge (T \wedge (\Gamma Q \vee \Gamma R))) \vee (\Gamma P \vee R) \\ & = (((P \vee Q) \wedge (P \vee \Gamma R)) \wedge (\Gamma Q \vee \Gamma R)) \vee (\Gamma P \vee R) \\ & = ((P \vee Q) \wedge ((P \vee \Gamma R) \wedge (\Gamma Q \vee \Gamma R))) \vee (\Gamma P \vee R) \\ & = ((P \vee Q) \wedge (\Gamma R \vee (P \wedge \Gamma Q))) \vee (\Gamma P \vee R) \\ & = (((P \vee Q) \wedge \Gamma R) \vee ((P \vee Q) \wedge (P \wedge \Gamma Q))) \vee (\Gamma P \vee R) \end{aligned}$$

$$\begin{aligned}
 a/ & F = P \wedge (Q \vee R) & G = (P \wedge Q) \vee R \\
 & F \rightarrow G \\
 \Leftrightarrow & (P \wedge (Q \vee R)) \rightarrow ((P \wedge Q) \vee R) \\
 & = \Gamma (P \wedge (Q \vee R)) \vee ((P \wedge Q) \vee R) \\
 & = \Gamma P \vee \Gamma (Q \vee R) \vee ((P \wedge Q) \vee R) \\
 & = \Gamma P \vee \Gamma Q \wedge \Gamma R \vee ((P \wedge Q) \vee R) \\
 & = \Gamma (P \wedge Q) \wedge \Gamma R \vee ((P \wedge Q) \vee R) \\
 & = \Gamma ((P \wedge Q) \vee R) \vee ((P \wedge Q) \vee R) \\
 & = T
 \end{aligned}$$

Vậy biểu thức mệnh đề G là hệ quả của F

$$\begin{aligned}
 b/ F &= (P \rightarrow Q) \wedge (Q \rightarrow R) & G &= P \rightarrow (Q \rightarrow R) \\
 &\quad F \rightarrow G \\
 \Leftrightarrow & ((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow (Q \rightarrow R)) \\
 & = ((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \rightarrow (P \rightarrow (\Gamma Q \vee R)) \\
 & = ((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \rightarrow (\Gamma P \vee (\Gamma Q \vee R)) \\
 & = \Gamma ((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \vee (\Gamma P \vee (\Gamma Q \vee R)) \\
 & = \Gamma (\Gamma P \vee Q) \vee \Gamma (\Gamma Q \vee R) \vee (\Gamma P \vee (\Gamma Q \vee R)) \vee \Gamma P \\
 & = \Gamma (\Gamma P \vee Q) \vee \Gamma P \vee \Gamma \\
 & = \Gamma (\Gamma P \vee Q) \vee \Gamma \\
 & = \Gamma
 \end{aligned}$$

Vậy biểu thức mệnh đề G là hệ quả của F

$$\begin{aligned}
 c/ F &= P \wedge Q & G &= (\Gamma P \rightarrow Q) \vee (P \rightarrow \Gamma Q) \\
 &\quad F \rightarrow G \\
 \Leftrightarrow & (P \wedge Q) \rightarrow ((\Gamma P \rightarrow Q) \vee (P \rightarrow \Gamma Q)) \\
 & = (P \wedge Q) \rightarrow ((P \vee Q) \vee (\Gamma P \vee \Gamma Q)) \\
 & = \Gamma (P \wedge Q) \vee ((P \vee Q) \vee (\Gamma P \vee \Gamma Q)) \\
 & = \Gamma \vee (\Gamma P \vee \Gamma Q) \\
 & = \Gamma
 \end{aligned}$$

Vậy biểu thức mệnh đề G là hệ quả của F

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$$\begin{aligned}
 a/ ((P \vee Q) \wedge \Gamma (\Gamma P \wedge Q)) &\Leftrightarrow P \\
 VT &= (P \vee Q) \wedge \Gamma (\Gamma P \wedge Q) \\
 &= (P \vee Q) \wedge (P \vee \Gamma Q) \\
 &= P \vee (Q \wedge \Gamma Q) \\
 &= P \vee F \\
 &= P = VP
 \end{aligned}$$

$$\begin{aligned}
 b/ (\Gamma ((P \vee Q) \wedge R) \vee \Gamma Q) &\Leftrightarrow Q \wedge R \\
 VT &= \Gamma ((P \vee Q) \wedge R) \vee \Gamma Q \\
 &= ((P \vee Q) \wedge R) \wedge Q \\
 &= (P \vee Q) \wedge Q \wedge R \\
 &= Q \wedge R = VP
 \end{aligned}$$

$$\begin{aligned}
 c/ ((P \vee Q) \wedge (P \vee \Gamma Q)) \vee Q &\Leftrightarrow P \vee Q \\
 VT &= ((P \vee Q) \wedge (P \vee \Gamma Q)) \vee Q
 \end{aligned}$$

$$\begin{aligned}
 &= (P \vee (Q \wedge \Gamma Q)) \vee Q \\
 &= (P \vee F) \vee Q \\
 &= P \vee Q = VP
 \end{aligned}$$

$d / \Gamma(P \vee Q) \vee ((\Gamma P \wedge Q) \vee \Gamma Q) \Leftrightarrow \Gamma(Q \wedge P)$   
 $\forall T = \Gamma(P \vee Q) \vee ((\Gamma P \wedge Q) \vee \Gamma Q)$   
 $= \Gamma(P \vee Q) \vee ((\Gamma Q \vee \Gamma P) \wedge (\Gamma Q \vee Q))$   
 $= \Gamma(P \vee Q) \vee ((\Gamma Q \vee \Gamma P) \wedge T)$   
 $= \Gamma(P \vee Q) \vee (\Gamma Q \vee \Gamma P)$   
 $= \Gamma(Q \vee P) \vee (\Gamma Q \wedge P)$   
 $= \Gamma((Q \vee P) \wedge (Q \wedge P))$   
 $= \Gamma(((Q \vee P) \wedge Q) \wedge P))$   
 $= \Gamma(Q \wedge P) = VP$

### Câu 1/TR.150

$a / (p \wedge q) \rightarrow (p \rightarrow q) \Leftrightarrow True$   
 $\forall T = (p \wedge q) \rightarrow (p \rightarrow q)$   
 $= \Gamma(p \wedge q) \vee (\Gamma p \vee q)$   
 $= \Gamma p \vee \Gamma q \vee \Gamma p \vee q$   
 $= \Gamma p \vee \Gamma p \vee T$   
 $= T = VP$

$b / \Gamma(p \rightarrow q) \rightarrow \Gamma q \Leftrightarrow T$   
 $\forall T = \Gamma(p \rightarrow q) \rightarrow \Gamma q$   
 $= (p \rightarrow q) \vee \Gamma q$   
 $= (\Gamma p \vee q) \vee \Gamma q$   
 $= \Gamma p \vee T$   
 $= T = VP$

$c / [(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r \Leftrightarrow T$   
 $\forall T = [(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$   
 $= [(p \vee q) \wedge (\Gamma p \vee r) \wedge (\Gamma q \vee r)] \rightarrow r$   
 $= [(p \vee q) \wedge (r \vee \Gamma (p \vee q))] \rightarrow r$   
 $= [((p \vee q) \wedge r) \vee F] \rightarrow r$   
 $= ((p \vee q) \wedge r) \rightarrow r$   
 $= \Gamma((p \vee q) \wedge r) \vee r$   
 $= \Gamma(p \vee q) \vee \Gamma r \vee r$   
 $= \Gamma(p \vee q) \vee T$   
 $= T = VP$

$d / (\Gamma((rvq) \vee \Gamma q)) \wedge ((\Gamma p \vee \Gamma q) \rightarrow (p \wedge q \wedge r)) \Leftrightarrow F$   
 $\forall T = (\Gamma((rvq) \vee \Gamma q)) \wedge ((\Gamma p \vee \Gamma q) \rightarrow (p \wedge q \wedge r))$   
 $= (\Gamma(rvq) \wedge q) \wedge (\Gamma(\Gamma p \vee \Gamma q) \vee (p \wedge q \wedge r))$   
 $= (\Gamma r \wedge \Gamma q \wedge q) \wedge ((p \wedge q) \vee (p \wedge q \wedge r))$   
 $= (\Gamma r \wedge \Gamma q \wedge q) \wedge (p \wedge q)$   
 $= \Gamma r \wedge \Gamma q \wedge q \wedge p \wedge q$   
 $= \Gamma r \wedge F \wedge p \wedge q$   
 $= F = VP$

$\vdash \Gamma p \wedge \Gamma(p \wedge q) \wedge \Gamma(p \wedge r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma(q \vee (r \wedge s)) \vee (r \wedge \Gamma s))) \wedge p \Leftrightarrow F$   
 $\vdash \Gamma p \wedge \Gamma(p \wedge q) \wedge \Gamma(p \wedge r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma(q \vee (r \wedge s)) \vee (r \wedge \Gamma s))) \wedge p$   
 $\vdash \Gamma p \wedge (\Gamma p \vee q) \wedge (\Gamma p \vee r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma(q \vee (r \wedge s)) \vee (r \wedge \Gamma s))) \wedge p$   
 $\vdash \Gamma p \wedge (\Gamma p \vee r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma(q \vee (r \wedge s)) \vee (r \wedge \Gamma s))) \wedge p$   
 $\vdash \Gamma p \wedge (((\Gamma q \rightarrow r) \vee \Gamma(q \vee (r \wedge s)) \vee (r \wedge \Gamma s))) \wedge p$   
 $\vdash (\Gamma p \wedge p) \wedge (((\Gamma q \rightarrow r) \vee \Gamma(q \vee (r \wedge s)) \vee (r \wedge \Gamma s)))$   
 $\vdash F \wedge (((\Gamma q \rightarrow r) \vee \Gamma(q \vee (r \wedge s)) \vee (r \wedge \Gamma s)))$   
 $\vdash F = VP$

$\vdash (((pvq) \wedge (pv \wedge q)) \vee q \vee (r \wedge q)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (rv \wedge q))) \Leftrightarrow F$   
 $\vdash (((pvq) \wedge (pv \wedge q)) \vee q \vee (r \wedge q)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (rv \wedge q)))$   
 $\vdash (((pv \wedge q) \wedge (q \wedge r)) \vee q \vee (r \wedge q)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (rv \wedge q)))$   
 $\vdash ((pv \wedge q) \wedge (q \wedge r)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (rv \wedge q)))$   
 $\vdash (pv \wedge (q \wedge r)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (rv \wedge q)))$   
 $\vdash (pv \wedge (q \wedge r)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (rv \wedge q)))$   
 $\vdash ((pvq) \wedge (pv \wedge p)) \wedge (\Gamma q \wedge (rv \wedge q))$   
 $\vdash (qv \wedge (p \wedge r)) \wedge (\Gamma q \wedge (rv \wedge q))$   
 $\vdash (q \wedge r) \wedge (rv \wedge q)$   
 $\vdash F \wedge (rv \wedge q)$   
 $\vdash F = VP$

## Câu 2/TR.150

Điều phải chứng minh=dpcm

$a/(p \rightarrow r) \wedge (q \rightarrow r) \Leftrightarrow (pvq) \rightarrow r$   
 $(\Gamma p \vee r) \wedge (\Gamma q \vee r) \Leftrightarrow \Gamma(pvq) \vee r$   
 $rv(\Gamma p \wedge \Gamma q) \Leftrightarrow \Gamma(pvq) \vee r$   
 $rv\Gamma(pvq) \Leftrightarrow \Gamma(pvq) \vee r$  (dpcm)

b/(p $\rightarrow$ q)v(p $\rightarrow$ r) $\Leftrightarrow$ p $\rightarrow$ (qvr)  
( $\Gamma$ p $\nu$ q)v( $\Gamma$ pvr) $\Leftrightarrow$  $\Gamma$ p $\nu$ (qvr)  
 $\Gamma$ p $\nu$ (qvr) $\Leftrightarrow$  $\Gamma$ p $\nu$ (qvr) (dpcm)

c/(( $\Gamma$ (p $\nu$ q)v( $\Gamma$ p $\nu$ q)) $\Lambda$  $\Gamma$ q) $\Leftrightarrow$ (p $\rightarrow$ q) $\Lambda$ (( $\Gamma$ q $\Lambda$ (rv $\Gamma$ q))  
( $\Gamma$ p $\Lambda$  $\Gamma$ q)v( $\Gamma$ p $\nu$ q) $\Lambda$  $\Gamma$ q $\Leftrightarrow$ ( $\Gamma$ p $\nu$ q) $\Lambda$  $\Gamma$ q  
( $\Gamma$ p $\nu$ q) $\Lambda$  $\Gamma$ q $\Leftrightarrow$ ( $\Gamma$ p $\nu$ q) $\Lambda$  $\Gamma$ q (dpcm)

d/ $\Gamma$ ((rvq) $\Lambda$ q)v $\Gamma$ p $\Leftrightarrow$ (( $\Gamma$ p $\nu$ v $\Gamma$ q) $\rightarrow$ (p $\Lambda$ q $\Lambda$ r))  
((rvq) $\Lambda$ q) $\Lambda$ p $\Leftrightarrow$ ((p $\Lambda$ q)v(p $\Lambda$ q $\Lambda$ r)  
(p $\Lambda$ q) $\Leftrightarrow$ (p $\Lambda$ q) (dpcm)

e/pv((p $\Lambda$ q)v(p $\Lambda$ r)) $\Leftrightarrow$ p $\Lambda$ (( $\Gamma$ q $\rightarrow$ r)v $\Gamma$ (qv(r $\Lambda$ s)v(r $\Lambda$  $\Gamma$ s)))  
 $\nabla$ P=p $\Lambda$ ((qvr)v $\Gamma$ (qv(r $\Lambda$ (sv $\Gamma$ s)))  
=p $\Lambda$ ((qvr)v $\Gamma$ (qvr))  
=p $\Lambda$ T  
=p

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$\nabla$ T=pv(p $\Lambda$ (qv $\Gamma$ r))  
=p  
 $\rightarrow$   $\nabla$ T= $\nabla$ P (dpcm)