BÀI TẬP CHƯƠNG I.

1. a) Nếu $P \to Q$ là sai $\to$ P = True và Q = False

Thấy $P \wedge Q$ = False.

         $\bar{P} \vee Q$ = False

         $Q \to P$ là true.

b)

① $((P \wedge Q) \wedge R) \to (S \vee M)$

Nếu biểu thức mệnh đề trên sai thì.

     $P \wedge Q \wedge R$ là đúng

     $S \vee M$ phải sai.

Thấy $\begin{cases} P = Q = R = T. \\ S = M = F. \end{cases}$

② $(P \wedge (Q \wedge R)) \to (S \oplus M)$

Nếu biểu thức trên sai thì.

     $P \wedge Q \wedge R$ phải đúng

     $S \oplus M$ phải sai.

Thấy $\begin{cases} P = Q = R = T. \\ \left[\begin{array}{l} S = M = T. \\ S = M = F. \end{array} \right. \end{cases}$

2. Q là T

Ta có $(Q \to ((\bar{P} \vee R) \wedge \bar{S})) \wedge (\bar{S} \to (\bar{R} \wedge Q))$

Muốn mệnh đề trên đúng thì:

$$\begin{cases} (\bar{P} \vee R) \wedge \bar{S} = T \\ \\ \bar{S} \to (\bar{R} \wedge Q) = T \end{cases} \iff \begin{cases} \bar{S} = T \iff S = F \\ \bar{P} \vee R = T \\ \\ \bar{R} \wedge Q = T \end{cases}$$

$$\iff \begin{cases} S = F \\ \bar{P} = T \iff P = F \\ \bar{R} = T \iff R = F \end{cases}$$

BÀI TẬP BỔ SUNG.

Câu 1

a) $(p \wedge q) \to (p \to q) \Leftrightarrow$ True

$\Leftrightarrow \bar{p} \vee \bar{q} \vee \bar{p} \vee q \Leftrightarrow$ True

$\Leftrightarrow \bar{p} \vee \bar{p} \vee (\bar{q} \vee q) \Leftrightarrow$ True

$\Leftrightarrow \bar{p} \vee \bar{p} \vee$ True $\Leftrightarrow$ True

$\Leftrightarrow$ True $\Leftrightarrow$ True.

b) $\overline{(p \to q)} \to \bar{q} \Leftrightarrow$ True

Ta có: VT $\Leftrightarrow \bar{p} \vee q \vee \bar{q} \Leftrightarrow \bar{p} \vee (q \vee \bar{q}) \Leftrightarrow \bar{p} \vee$ True

$\Leftrightarrow$ True $\Leftrightarrow$ VP.

c) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r \Leftrightarrow$ True

Ta có VT $\Leftrightarrow [\overline{(p \vee q)} \vee \overline{(\bar{p} \vee r)} \vee \overline{(\bar{q} \vee r)}] \vee r$.

$\Leftrightarrow [(\bar{p} \wedge \bar{q}) \vee (p \wedge \bar{r}) \vee (q \wedge \bar{r})] \vee r$

$\Leftrightarrow [\overline{(p \vee q)} \vee [\bar{r} \wedge (p \vee q)]] \vee r$

$\Leftrightarrow \overline{(p \vee q)} \vee \bar{r} \vee r \Leftrightarrow$ True $\Leftrightarrow$ VP.

d) $((\overline{(r \vee q) \vee \bar{q}})) \wedge ((\bar{p} \vee \bar{q}) \to (p \wedge q \wedge r)) \Leftrightarrow$ False

$\Leftrightarrow \bar{r} \wedge (\bar{q} \wedge q) \wedge ((\bar{p} \vee \bar{q}) \to (p \wedge q \wedge r)) \Leftrightarrow$ False

$\Leftrightarrow$ False $\Leftrightarrow$ False

e) $\bar{p} \wedge \overline{(p \wedge q)} \wedge \overline{(p \wedge \bar{\lambda})} \wedge \left(\left(\left(\bar{q} \to \lambda\right) \vee \overline{\left(q \vee (\lambda \wedge s\right)}\right.\right.$

$\left.\left.\vee \overline{(\lambda \wedge \bar{s})}\right)\right) \wedge p) \Leftrightarrow$ True

Đặt $A = (\bar{q} \to \lambda) \vee \overline{(q \vee (\lambda \wedge s) \vee (\lambda \wedge \bar{s}))}$

Ta có $VT \Leftrightarrow \bar{p} \wedge (\bar{p} \vee \bar{q}) \wedge (\bar{p} \vee \lambda) \wedge (A \wedge q)$

$\Leftrightarrow \bar{p} \wedge (\bar{p} \vee \lambda) \wedge (\lambda \wedge p) \Leftrightarrow \bar{p} \wedge A \wedge p \Leftrightarrow$ False $= VP$

f) $\left(\left((p \vee q) \wedge (p \vee \bar{q})\right) \vee q \vee (\bar{\lambda} \wedge q)\right) \wedge \left((p \to q) \wedge (\bar{q} \wedge (\lambda \vee \bar{q}))\right)$

$\Leftrightarrow$ False

Ta có Vế trái $\Leftrightarrow (p \vee q) \wedge \left((\bar{p} \vee q) \wedge \bar{q})\right)$

$\Leftrightarrow (p \vee q) \wedge (\bar{q} \wedge \bar{p}) \Leftrightarrow (p \vee q) \wedge \overline{(q \wedge p)}$

$\Leftrightarrow$ False $\Leftrightarrow VP$.

Câu 2

a) $(p \to \lambda) \wedge (q \to \lambda) \Leftrightarrow (p \vee q) \to \lambda$.

$\Leftrightarrow (\bar{p} \vee \lambda) \wedge (\bar{q} \vee \lambda) \Leftrightarrow (\bar{p} \wedge \bar{q}) \vee \lambda$

$\Leftrightarrow \lambda \vee (\bar{p} \wedge \bar{q}) \Leftrightarrow \overline{(p \wedge q)} \vee \lambda$

b) $(p \to q) \vee (p \to \lambda) \Leftrightarrow p \to (q \vee \lambda)$

$\Leftrightarrow (\bar{p} \vee q) \vee (\bar{p} \vee \lambda) \Leftrightarrow \bar{p} \vee q \vee \lambda$

$\Leftrightarrow \bar{p} \vee q \vee \lambda \Leftrightarrow \bar{p} \vee q \vee \lambda$

c) $((\overline{(p \vee q)} \vee (\overline{p} \vee q)) \wedge \overline{q} \Leftrightarrow (p \rightarrow q) \wedge (\overline{q} \wedge (r \vee \overline{q}))$

$\Leftrightarrow (((\overline{p} \wedge \overline{q}) \vee \overline{p} \vee q)) \wedge \overline{q} \Leftrightarrow (\overline{p} \vee q) \wedge \overline{q}$

$\Leftrightarrow (\overline{p} \vee q) \wedge \overline{q} \Leftrightarrow (\overline{p} \wedge \overline{q}) \vee (q \wedge \overline{q})$

$\Leftrightarrow \overline{p} \wedge \overline{q} \Leftrightarrow \overline{p} \wedge \overline{q}$

d) $(\overline{(\overline{(r \vee q)} \wedge q)} \vee \overline{p}) \Leftrightarrow ((\overline{p} \vee \overline{q}) \rightarrow (p \wedge q \wedge x))$

$\Leftrightarrow (\overline{q} \vee \overline{p}) \Leftrightarrow (\overline{p} \vee \overline{q}) \vee (p \wedge q \wedge x)$

$\Leftrightarrow q \wedge p \Leftrightarrow (p \vee q) \vee (p \vee q \wedge x)$

$\Leftrightarrow q \wedge p \Leftrightarrow p \vee q$

e) $p \vee ((p \wedge q) \vee (p \wedge \overline{x})) \Leftrightarrow p \wedge ((\overline{q} \rightarrow x) \vee \overline{(q \vee (x \wedge s) \vee (x \wedge \overline{s}))})$

$\Leftrightarrow p \vee (p \wedge (q \vee \overline{x})) \Leftrightarrow p \wedge ((q \vee x) \vee \overline{q} \wedge (\overline{x} \vee s) \wedge (\overline{r} \vee \overline{s}))$

$\Leftrightarrow p \Leftrightarrow p \wedge ((q \vee x) \vee (\overline{q} \wedge \overline{r} \vee (s \wedge \overline{s})))$

$\Leftrightarrow p \Leftrightarrow p \wedge ((q \vee x) \vee (\overline{q \vee x}))$

$\Leftrightarrow p \Leftrightarrow p$

Câu 3

Đặt $P(x) = \{x$ là hoàn hảo $\}$

$F(x) = \{$ x là bạn của bạn $\}$

a) $\exists x\ P(x)$ hoặc $\forall x\ \overline{P(x)}$

b) $\overline{\forall x\ P(x)}$ hoặc $\exists x\ \overline{P(x)}$

c) $\forall x\ (F(x) \to P(x))$

d) $\exists x\ (F(x) \wedge P(x))$

e) $\forall x\ (F(x) \wedge P(x))$

f) $\overline{\forall x\ F(x)} \vee \exists x\ \overline{P(x)}$

   hoặc $\exists x\ \overline{F(x)} \vee \exists x\ \overline{P(x)}$

Câu 4

a) $\neg p$

b) $p \wedge \neg q$

c) $p \to q$

d) $\neg p \to \neg q$

e) $p \to q$

f) $q \wedge \neg p$

g) $q \to p$

Câu 5.

a) $r \wedge \bar{p}$

b) $\bar{p} \wedge q \wedge r$

c) $r \to (q \Leftrightarrow \neg q)$

d) $\bar{q} \wedge \bar{p} \wedge r$

e) $q \to (\bar{r} \wedge \bar{p})$

f) $(p \wedge r) \to \bar{q}$