



baitapmaunhapth

Toán rời rạc (Trường Đại học Cần Thơ)



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BÀI TẬP MỆNH ĐỀ

9/

- a. $(P \wedge Q) \rightarrow P$
 $= \Gamma(P \wedge Q) \vee P$
 $= \Gamma P \vee P \vee \Gamma Q$
 $= T \vee \Gamma Q$
 $= T \mid$
- b. $P \rightarrow (\Gamma P \rightarrow P)$
 $= P \rightarrow (\Gamma \Gamma P \vee P)$
 $= P \rightarrow P$
 $= \Gamma P \vee P$
 $= T$
- c. $P \rightarrow ((Q \rightarrow (P \wedge Q)))$
 $= P \rightarrow (\Gamma Q \vee (P \wedge Q))$
 $= P \rightarrow ((\Gamma Q \vee P) \wedge (\Gamma Q \vee Q))$
 $= P \rightarrow ((\Gamma Q \vee P) \wedge T)$
 $= P \rightarrow (\Gamma Q \vee P)$
 $= \Gamma P \vee (\Gamma Q \vee P)$
 $= \Gamma P \vee P \vee \Gamma Q$
 $= T \vee \Gamma Q$
 $= T$
- d. $\Gamma(P \vee \Gamma Q) \rightarrow \Gamma P$
 $= (\Gamma P \vee \Gamma Q) \vee \Gamma P$
 $= T \vee \Gamma Q$
 $= T$
 $= ((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \rightarrow (\Gamma P \vee R)$
 $= (((\Gamma P \wedge \Gamma Q) \vee (\Gamma P \wedge R)) \vee ((Q \wedge \Gamma Q) \vee (Q \wedge R))) \rightarrow (\Gamma P \vee R)$
 $= ((\Gamma P \wedge (\Gamma Q \vee R)) \vee (F \vee (Q \wedge R))) \rightarrow (\Gamma P \vee R)$
 $= ((\Gamma P \wedge (\Gamma Q \vee R)) \vee (Q \wedge R)) \rightarrow (\Gamma P \vee R)$
 $= (((Q \wedge R) \vee \Gamma P) \wedge ((Q \wedge R) \vee (\Gamma Q \vee R))) \rightarrow (\Gamma P \vee R)$
 $= (((Q \wedge R) \vee \Gamma P) \wedge$
- e. $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
 $= \Gamma((P \rightarrow Q) \wedge (Q \rightarrow R)) \vee (P \rightarrow R)$
 $= (\Gamma(P \rightarrow Q) \vee \Gamma(Q \rightarrow R)) \vee (P \rightarrow R)$
 $= (\Gamma(\Gamma P \vee Q) \vee \Gamma(\Gamma Q \vee R)) \vee (P \rightarrow R)$
 $= ((\Gamma P \wedge \Gamma Q) \vee (Q \wedge \Gamma R)) \vee (P \rightarrow R)$
 $= ((\Gamma P \wedge \Gamma Q) \vee (Q \wedge \Gamma R)) \vee (\Gamma P \vee R)$
 $= (((\Gamma P \vee Q) \wedge (\Gamma P \vee R)) \wedge ((\Gamma Q \vee R) \wedge (\Gamma Q \vee \Gamma R))) \vee (\Gamma P \vee R)$
 $= (((\Gamma P \vee Q) \wedge (\Gamma P \vee R)) \wedge (T \wedge (\Gamma Q \vee \Gamma R))) \vee (\Gamma P \vee R)$
 $= (((\Gamma P \vee Q) \wedge (\Gamma P \vee R)) \wedge (\Gamma Q \vee \Gamma R)) \vee (\Gamma P \vee R)$
 $= ((\Gamma P \vee Q) \wedge ((\Gamma P \vee R) \wedge (\Gamma Q \vee \Gamma R))) \vee (\Gamma P \vee R)$
 $= ((\Gamma P \vee Q) \wedge (\Gamma R \vee (P \wedge \Gamma Q))) \vee (\Gamma P \vee R)$
 $= (((\Gamma P \vee Q) \wedge \Gamma R) \vee ((\Gamma P \vee Q) \wedge (P \wedge \Gamma Q))) \vee (\Gamma P \vee R)$

$$a/ F = P \wedge (Q \vee R) \quad G = (P \wedge Q) \vee R$$

$$F \rightarrow G$$

$$\begin{aligned} \Leftrightarrow & (P \wedge (Q \vee R)) \rightarrow ((P \wedge Q) \vee R) \\ & = \neg (P \wedge (Q \vee R)) \vee ((P \wedge Q) \vee R) \\ & = \neg P \vee \neg (Q \vee R) \vee ((P \wedge Q) \vee R) \\ & = \neg P \vee \neg Q \wedge \neg R \vee ((P \wedge Q) \vee R) \\ & = \neg (P \wedge Q) \wedge \neg R \vee ((P \wedge Q) \vee R) \\ & = \neg ((P \wedge Q) \vee R) \vee ((P \wedge Q) \vee R) \\ & = T \end{aligned}$$

Vậy biểu thức mệnh đề G là hệ quả của F

$$\begin{aligned}
& b/ F = (P \rightarrow Q) \wedge (Q \rightarrow R) \quad G = P \rightarrow (Q \rightarrow R) \\
& \quad F \rightarrow G \\
& \Leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow (Q \rightarrow R)) \\
& \quad = ((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \rightarrow (P \rightarrow (\Gamma Q \vee R)) \\
& \quad = ((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \rightarrow (\Gamma P \vee (\Gamma Q \vee R)) \\
& \quad = \Gamma((\Gamma P \vee Q) \wedge (\Gamma Q \vee R)) \vee (\Gamma P \vee (\Gamma Q \vee R)) \\
& \quad = \Gamma(\Gamma P \vee Q) \vee \Gamma(\Gamma Q \vee R) \vee (\Gamma P \vee (\Gamma Q \vee R)) \\
& \quad = \Gamma(\Gamma P \vee Q) \vee (\Gamma(\Gamma Q \vee R) \vee (\Gamma Q \vee R)) \vee \Gamma P \\
& \quad = \Gamma(\Gamma P \vee Q) \vee \Gamma P \vee T \\
& \quad = \Gamma(\Gamma P \vee Q) \vee T \\
& \quad = T
\end{aligned}$$

Vậy biểu thức mệnh đề G là hệ quả của F

$$\begin{aligned}
& c/ F = P \wedge Q \quad G = (\Gamma P \rightarrow Q) \vee (P \rightarrow \Gamma Q) \\
& \quad F \rightarrow G \\
& \Leftrightarrow (P \wedge Q) \rightarrow ((\Gamma P \rightarrow Q) \vee (P \rightarrow \Gamma Q)) \\
& \quad = (P \wedge Q) \rightarrow ((P \vee Q) \vee (\Gamma P \vee \Gamma Q)) \\
& \quad = \Gamma(P \wedge Q) \vee ((P \vee Q) \vee (\Gamma P \vee \Gamma Q)) \\
& \quad = T \vee (\Gamma P \vee \Gamma Q) \\
& \quad = T
\end{aligned}$$

Vậy biểu thức mệnh đề G là hệ quả của F

11/

$$\begin{aligned}
& a/ ((P \vee Q) \wedge \Gamma(P \wedge Q)) \Leftrightarrow P \\
& VT = (P \vee Q) \wedge \Gamma(P \wedge Q) \\
& \quad = (P \vee Q) \wedge (P \vee \Gamma Q) \\
& \quad = P \vee (Q \wedge \Gamma Q) \\
& \quad = P \vee F \\
& \quad = P = VP
\end{aligned}$$

$$\begin{aligned}
& b/ (\Gamma(\Gamma((P \vee Q) \wedge R)) \vee \Gamma Q) \Leftrightarrow Q \wedge R \\
& VT = \Gamma(\Gamma(\Gamma((P \vee Q) \wedge R)) \vee \Gamma Q) \\
& \quad = ((P \vee Q) \wedge R) \wedge Q \\
& \quad = (P \vee Q) \wedge Q \wedge R \\
& \quad = Q \wedge R = VP
\end{aligned}$$

$$\begin{aligned}
& c/ ((P \vee Q) \wedge (P \vee \Gamma Q)) \vee Q \Leftrightarrow P \vee Q \\
& VT = ((P \vee Q) \wedge (P \vee \Gamma Q)) \vee Q
\end{aligned}$$

$$\begin{aligned}
& = (P \vee (Q \wedge \Gamma Q)) \vee Q \\
& = (P \vee F) \vee Q \\
& = P \vee Q = VP
\end{aligned}$$

$d/\Gamma(P \vee Q) \vee ((\Gamma P \wedge Q) \vee \Gamma Q) \Leftrightarrow \Gamma(Q \wedge P)$
 $VT = \Gamma(P \vee Q) \vee ((\Gamma P \wedge Q) \vee \Gamma Q)$
 $= \Gamma(P \vee Q) \vee ((\Gamma Q \vee \Gamma P) \wedge (\Gamma Q \vee \Gamma Q))$
 $= \Gamma(P \vee Q) \vee ((\Gamma Q \vee \Gamma P) \wedge T)$
 $= \Gamma(P \vee Q) \vee (\Gamma Q \vee \Gamma P)$
 $= \Gamma(Q \vee P) \vee \Gamma(Q \wedge P)$
 $= \Gamma((Q \vee P) \wedge (Q \wedge P))$
 $= \Gamma(((Q \vee P) \wedge Q) \wedge P)$
 $= \Gamma(Q \wedge P) = VP$

Câu 1/TR.150

$a/(p \wedge q) \rightarrow (p \rightarrow q) \Leftrightarrow \text{True}$
 $VT = (p \wedge q) \rightarrow (p \rightarrow q)$
 $= \Gamma(p \wedge q) \vee (\Gamma p \vee \Gamma q)$
 $= \Gamma p \vee \Gamma q \vee \Gamma p \vee \Gamma q$
 $= \Gamma p \vee \Gamma p \vee T$
 $= T = VP$

$b/\Gamma(p \rightarrow q) \rightarrow \Gamma q \Leftrightarrow T$
 $VT = \Gamma(p \rightarrow q) \rightarrow \Gamma q$
 $= (p \rightarrow q) \vee \Gamma q$
 $= (\Gamma p \vee \Gamma q) \vee \Gamma q$
 $= \Gamma p \vee T$
 $= T = VP$

$c/[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r \Leftrightarrow T$
 $VT = [(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
 $= [(p \vee q) \wedge (\Gamma p \vee \Gamma r) \wedge (\Gamma q \vee \Gamma r)] \rightarrow r$
 $= [(p \vee q) \wedge (r \vee \Gamma(p \vee q))] \rightarrow r$
 $= [((p \vee q) \wedge r) \vee F] \rightarrow r$
 $= ((p \vee q) \wedge r) \rightarrow r$
 $= \Gamma((p \vee q) \wedge r) \vee r$
 $= \Gamma(p \vee q) \vee \Gamma r \vee r$
 $= \Gamma(p \vee q) \vee T$
 $= T = VP$

$d/(\Gamma((r \vee q) \vee \Gamma q)) \wedge ((\Gamma p \vee \Gamma q) \rightarrow (p \wedge q \wedge r)) \Leftrightarrow F$
 $VT = (\Gamma((r \vee q) \vee \Gamma q)) \wedge ((\Gamma p \vee \Gamma q) \rightarrow (p \wedge q \wedge r))$
 $= (\Gamma(r \vee q) \wedge q) \wedge (\Gamma(\Gamma p \vee \Gamma q) \vee (p \wedge q \wedge r))$
 $= (\Gamma r \wedge \Gamma q \wedge q) \wedge ((p \wedge q) \vee (p \wedge q \wedge r))$
 $= (\Gamma r \wedge \Gamma q \wedge q) \wedge (p \wedge q)$
 $= \Gamma r \wedge \Gamma q \wedge q \wedge p \wedge q$
 $= \Gamma r \wedge F \wedge p \wedge q$
 $= F = VP$

$$\begin{aligned}
& e/\Gamma p \wedge \Gamma (p \wedge q) \wedge \Gamma (p \wedge \Gamma r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s)))) \wedge p \Leftrightarrow F \\
& VT = \Gamma p \wedge \Gamma (p \wedge q) \wedge \Gamma (p \wedge \Gamma r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s)))) \wedge p \\
& = \Gamma p \wedge (\Gamma p \vee \Gamma q) \wedge (\Gamma p \vee r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s)))) \wedge p \\
& = \Gamma p \wedge (\Gamma p \vee r) \wedge (((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s)))) \wedge p \\
& = \Gamma p \wedge (((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s)))) \wedge p \\
& = (\Gamma p \wedge p) \wedge (((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s)))) \\
& = F \wedge (((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s)))) \\
& = F = VP
\end{aligned}$$

$$\begin{aligned}
& f/(((p \vee q) \wedge (p \vee \Gamma q)) \vee q \vee (\Gamma r \wedge q)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (r \vee \Gamma q))) \Leftrightarrow F \\
& VT = (((p \vee q) \wedge (p \vee \Gamma q)) \vee q \vee (\Gamma r \wedge q)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (r \vee \Gamma q))) \\
& = ((p \vee (q \wedge \Gamma q)) \vee q \vee (\Gamma r \wedge q)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (r \vee \Gamma q))) \\
& = (p \vee q \vee (\Gamma r \wedge q)) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (r \vee \Gamma q))) \\
& = (p \vee ((q \vee \Gamma r) \wedge (q \vee q))) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (r \vee \Gamma q))) \\
& = (p \vee (q \wedge (q \vee \Gamma r))) \wedge ((p \rightarrow q) \wedge (\Gamma q \wedge (r \vee \Gamma q))) \\
& = ((p \vee q) \wedge (\Gamma p \vee q)) \wedge (\Gamma q \wedge (r \vee \Gamma q)) \\
& = (q \vee (p \wedge \Gamma p)) \wedge (\Gamma q \wedge (r \vee \Gamma q)) \\
& = (q \wedge \Gamma q) \wedge (r \vee \Gamma q) \\
& = F \wedge (r \vee \Gamma q) \\
& = F = VP
\end{aligned}$$

Câu 2/TR.150

Điều phải chứng minh = dpcm

$$\begin{aligned}
& a/(p \rightarrow r) \wedge (q \rightarrow r) \Leftrightarrow (p \vee q) \rightarrow r \\
& (\Gamma p \vee r) \wedge (\Gamma q \vee r) \Leftrightarrow \Gamma (p \vee q) \vee r \\
& r \vee (\Gamma p \wedge \Gamma q) \Leftrightarrow \Gamma (p \vee q) \vee r \\
& r \vee \Gamma (p \vee q) \Leftrightarrow \Gamma (p \vee q) \vee r \text{ (dpcm)}
\end{aligned}$$

$$\begin{aligned} b / (p \rightarrow q) \vee (p \rightarrow r) &\Leftrightarrow p \rightarrow (q \vee r) \\ (\Gamma p \vee q) \vee (\Gamma p \vee r) &\Leftrightarrow \Gamma p \vee (q \vee r) \\ \Gamma p \vee (q \vee r) &\Leftrightarrow \Gamma p \vee (q \vee r) \text{ (dpcm)} \end{aligned}$$

$$\begin{aligned} c / ((\Gamma (p \vee q) \vee (\Gamma p \vee q)) \wedge \Gamma q) &\Leftrightarrow (p \rightarrow q) \wedge (\Gamma q \wedge (r \vee \Gamma q)) \\ (\Gamma p \wedge \Gamma q) \vee \Gamma p \vee q &\wedge \Gamma q \Leftrightarrow (\Gamma p \vee q) \wedge \Gamma q \\ (\Gamma p \vee q) \wedge \Gamma q &\Leftrightarrow (\Gamma p \vee q) \wedge \Gamma q \text{ (dpcm)} \end{aligned}$$

$$\begin{aligned} d / \Gamma (\Gamma (r \vee q) \wedge q) \vee \Gamma p &\Leftrightarrow ((\Gamma p \vee \Gamma q) \rightarrow (p \wedge q \wedge r)) \\ ((r \vee q) \wedge q) \wedge p &\Leftrightarrow ((p \wedge q) \vee (p \wedge q \wedge r)) \\ (p \wedge q) &\Leftrightarrow (p \wedge q) \text{ (dpcm)} \end{aligned}$$

$$\begin{aligned} e / p \vee ((p \wedge q) \vee (p \wedge \Gamma r)) &\Leftrightarrow p \wedge ((\Gamma q \rightarrow r) \vee \Gamma (q \vee (r \wedge s) \vee (r \wedge \Gamma s))) \\ VP = p \wedge ((q \vee r) \vee \Gamma (q \vee (r \wedge (s \vee \Gamma s)))) & \\ = p \wedge ((q \vee r) \vee \Gamma (q \vee r)) & \\ = p \wedge T & \\ = p & \end{aligned}$$

$$\begin{aligned} VT &= p \vee (p \wedge (q \vee \Gamma r)) \\ &= p \\ \rightarrow VT &= VP \text{ (dpcm)} \end{aligned}$$