Conjecture

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Abstract

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1 INTRODUCTION.

I'd like to formulate a research question that so far I did not answer theoretically. Can this question be answered using a computer program?

Let M be a positive integer, $f_m = \overline{f_{-m}}, f_m$ are numbers, $-M \leq m \leq M, m$ are integers, $\bar{\cdot}$ stands for complex conjugate,

$$f(\theta) := \sum_{m=-M}^{M} f_m e^{im\theta}, \quad 0 < c_1 \leq f(\theta) \leq c_2, \tag{1.1}$$

c_1, c_2 are constants.

Let j_m be integers, such that

$$0 \leq j_m \leq n+2; \sum_{m=-M}^{M} j_m = n+2; \sum_{m=-M}^{M} m j_m = -n, \tag{1.2}$$

where

$$C^{n+2}_{\dots j_m \dots} := \frac{(n+2)!}{(j_{-M})!\dots\dots(j_m)!\dots(j_M)!} \tag{1.3}$$

is a multinomial coefficient:

$$(\sum_{m=-M}^{M} a_m)^n = \sum_{j_m} C^n_{\dots j_m \dots} \prod_{m=-M}^{M} a_m^{j_m} \tag{1.4}$$

Assume:

1. (1.2) and
2. $\sum_{j_m} C^{n+2}_{\dots j_m \dots} \prod_{m=-M}^{M} f_m^{j_m} = 0$, where 1. and 2. hold for all $n > n_0$, and n_0 is an arbitrary large fixed positive integer. Here M does not depend on n.

My Conjecture: Assumptions 1. and 2. imply that $f_m = 0 \quad \forall m \neq 0$.

I have proved this conjecture for M=1.

Can one design a computer program to prove this conjecture for any (finite) integer M? One may start with M=2. If this conjecture is proved, then an important open (since 1929) problem will be solved.

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