Notes 2018, Part I

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Here is a collected notes on physics readings.

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§1 Problems

§1.1 Peskin

§1.1.1 Peskin Problem 11.1

Part a

Lemma 1.1
For a free field $\phi(x)$, the following holds:

$$\langle e^{i\phi(x)}e^{-i\phi(0)}\rangle = e^{D(x)-D(0)}$$

where $D(x) \equiv \langle\phi(x-y)\phi(y)\rangle_c$

Proof. Define Z(J) as:

$$Z(J) = \int d\phi \exp\left(i\int d^dx\phi D^{-1}\phi + J\phi)\right) \tag{1}$$

We prove by evaluating the time ordered correlator directly in the path integral:

$$\langle e^{i(\phi(x)-\phi(0))}\rangle = \frac{\int d\phi \exp\left(i\int d^dx\phi D^{-1}\phi + i(\phi(x)-\phi(0))\right)}{Z(0)} \tag{2}$$

$$= \frac{Z(J_0)}{Z(0)} \tag{3}$$

$$= e^{i\frac{1}{2}J_0 D J_0} \text{ (Formula for gaussian integrals)} \tag{4}$$

We note the equation works if we define $J_0(y) = \delta(x-y) - \delta(y)$. This is just a gaussian integral, with result being:

$$\langle e^{i(\phi(x)-\phi(0))}\rangle = \exp\left(i\frac{1}{2}\int dx'\int dy'(\delta(x-x')-\delta(x'))D(\delta(x-y')-\delta(y'))\right) \tag{5}$$

There are 2 terms for each propagator which don't vanish, at index x and 0, and they cancel the $\frac{1}{2}$:

$$\langle e^{i(\phi(x)-\phi(0))}\rangle = \exp\left(i\left(D(x)-D(0)\right)\right) \tag{6}$$

□

Part b The most general term is of the form:

$$\phi^n(\partial\phi)^(2m) \tag{7}$$

$n = 0$ because of U(1) symmetry. The remaining term has the following coupling dimensions:

$$\underbrace{[g]}_{\text{coupling dimension}} + 2m\left([\partial]+[\phi]\right) = d \tag{8}$$

$$g = d - 2m\left([\partial]+[\phi]\right) \geq 0 \tag{9}$$

$$[\partial] = 1 \tag{10}$$

$$[\phi] = \frac{d-2}{2} \text{ (Kinetic term is canonically normalized)} \tag{11}$$

$$m \leq \frac{d}{2(1+\frac{d-2}{2})} \tag{12}$$

$$m \leq 1 \tag{13}$$

This implies the most general renormalizable coupling is of the form:

$$\rho(\partial\phi)^2 \tag{14}$$

Part c This is just coulomb potential in d dimensions:

$$D(k^2) = \frac{1}{\rho k^2} \tag{15}$$
$$\rho \nabla^2 D(x) = \delta(x) \tag{16}$$
$$\tag{17}$$

What the previous equation states is that D(x) satisfies gauss's law (it solves poisson's equation), and we can therefore immediately obtain the potential.

We get that the following cases:

$$\begin{cases} D(x) = \frac{1}{\rho S_d r^{d-2}} \ (d > 2,\ S_d \text{ is the surface of a unit d dimensional sphere}) \\ D(x) = \frac{\ln(r)}{\rho S_d} \ (\text{d} = 2\ ) \end{cases} \tag{18}$$

The spin spin correlation is obtained from part a:

$$\begin{cases} D(x) \propto e^{\frac{1}{\rho x^{d-2}}} \ (\ d > 2) \\ D(x) \propto \frac{1}{\rho x} \ (\text{d} = 2) \\ D(x) \propto e^{-x} (\text{d} = 1) \end{cases} \tag{19}$$

We see that there is long range order in dimensions 3, 4 and greater. At dimension 2, we have algebraic order (conformal behavior). In dimensions 1, we have loss of long range order.