Lecture 4: Model Free Control

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- Structure closely follows much of David Silver's Lecture 5. For additional reading please see SB Sections 5.2-5.4, 6.4, 6.5, 6.7

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Class Structure

- Last time: Policy evaluation with no knowledge of how the world works (MDP model not given)
- This time: Control (making decisions) without a model of how the world works
- Next time: Value function approximation

Evaluation to Control

- Last time: how good is a specific policy?
  - Given no access to the decision process model parameters
  - Instead have to estimate from data / experience
- Today: how can we learn a good policy?

Recall: Reinforcement Learning Involves

- Optimization
- Delayed consequences (planning)
- Exploration
- Generalization not yet

Today: Learning to Control Involves

- Optimization: Goal is to identify a policy with high expected rewards (similar to Lecture 2 on computing an optimal policy given decision process models)
- Delayed consequences: May take many time steps to evaluate whether an earlier decision was good or not
- Exploration: Necessary to try different actions to learn what actions can lead to high rewards

Today: Model-free Control

- Generalized policy improvement
- Importance of exploration
- Monte Carlo control
- Model-free control with temporal difference (SARSA, Q-learning)
- Maximization bias

Model-free Control Examples

- Many applications can be modeled as a MDP: Backgammon, Go, Robot locomation, Helicopter flight, Robocup soccer, Autonomous driving, Customer ad selection, Invasive species management, Patient treatment
- For many of these and other problems either:
    - MDP model is unknown but can be sampled
    - MDP model is known but it is computationally infeasible to use directly, except through sampling

On and Off-Policy Learning

- On-policy learning
  - Direct experience
  - Learn to estimate and evaluate a policy from experience obtained from following that policy
- Off-policy learning
  - Learn to estimate and evaluate a policy using experience gathered from following a different policy

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Recall Policy Iteration

$|A|^{|S|}$

$\pi(s) = a \quad \forall s$ random

monotonic policy improvement

- Initialize policy π
- Repeat:
  - Policy evaluation: compute $\underline{V^\pi}$
  - Policy improvement: update π

$$\pi'(s) = \arg\max_a \underbrace{R(s,a)} + \gamma \sum_{s' \in S} \underbrace{P(s'|s,a)} V^\pi(s') = \arg\max_a Q^\pi(s,a)$$

- Now want to do the above two steps without access to the true dynamics and reward models
- Last lecture introduced methods for model-free policy evaluation

Model Free Policy Iteration

- Initialize policy π
- Repeat:
  - Policy evaluation: compute Q^{π}
  - Policy improvement: update π

$Q(s, a)$

MC for On Policy Q Evaluation

V : $N(s)$ $G(s)$

Initialize $N(s,a) = 0,\ G(s,a) = 0,\ Q^\pi(s,a) = 0,\ \forall s \in S,\ \forall a \in A$

Loop

- Using policy π sample episode $i = s_{i,1}, a_{i,1}, r_{i,1}, s_{i,2}, a_{i,2}, r_{i,2}, \dots, s_{i,T_i}$
  (s, a)
- $G_{i,t} = r_{i,t} + \gamma r_{i,t+1} + \gamma^2 r_{i,t+2} + \cdots \gamma^{T_i - 1} r_{i,T_i}$
- For each **state,action** (s,a) visited in episode i
  - For **first or every** time t that (s,a) is visited in episode i
    - $N(s,a) = N(s,a) + 1,\ G(s,a) = G(s,a) + G_{i,t}$
    - Update estimate $Q^\pi(s,a) = G(s,a)/N(s,a)$

Model-free Generalized Policy Improvement

$\pi(s) \rightarrow a$

- Given an estimate $Q^{\pi_i}(s, a)\ \forall s, a$
- Update new policy

$$\pi_{\underline{i+1}}(s) = \arg\max_a Q^{\pi_i}(s, a) \tag{1}$$

Model-free Policy Iteration

- Initialize policy π
- Repeat:
  - Policy evaluation: compute Q^π
  - Policy improvement: update π given Q^π

- May need to modify policy evaluation:
  - If π is deterministic, can't compute $Q(s, a)$ for any $a \neq \pi(s)$
- How to interleave policy evaluation and improvement?
  - Policy improvement is now using an estimated Q

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Policy Evaluation with Exploration

$Q^\pi(s,a)\ \forall s\ \forall a$

- Want to compute a model-free estimate of $\underline{Q}^\pi$
- In general seems subtle
  - Need to try all (s, a) pairs but then follow π
  - Want to ensure resulting estimate Q^π is good enough so that policy improvement is a monotonic operator
- For certain classes of policies can ensure all (s,a) pairs are tried such that asymptotically Q^π converges to the true value

ϵ-greedy Policies

- Simple idea to balance exploration and exploitation
- Let $|A|$ be the number of actions
- Then an ϵ-greedy policy w.r.t. a state-action value $Q^\pi(s,a)$ is $\pi(a|s) =$

$$\begin{cases} \text{w/prob } 1-\epsilon & \arg\max_a Q^\pi(s,a) \\ \text{else } a & \text{with prob } \frac{\epsilon}{|A|} \end{cases}$$

Check Your Understanding: MC for On Policy Q Evaluation

Initialize $N(s, a) = 0$, $G(s, a) = 0$, $Q^{\pi}(s, a) = 0$, $\forall s \in S$, $\forall a \in A$
Loop

- Using policy π sample episode $i = s_{i,1}, a_{i,1}, r_{i,1}, s_{i,2}, a_{i,2}, r_{i,2}, \ldots, s_{i,T_i}$
- $G_{i,t} = r_{i,t} + \gamma r_{i,t+1} + \gamma^2 r_{i,t+2} + \cdots \gamma^{T_i - 1} r_{i,T_i}$
- For each **state,action** (s, a) visited in episode i
  - For **first or every** time t that (s, a) is visited in episode i
    - $N(s, a) = N(s, a) + 1$, $G(s, a) = G(s, a) + G_{i,t}$
    - Update estimate $Q^{\pi}(s, a) = G(s, a)/N(s, a)$

- Mars rover with new actions:
  - $r(-, a_1) = [\ 1\ 0\ 0\ 0\ 0\ 0\ +10]$, $r(-, a_2) = [\ 0\ 0\ 0\ 0\ 0\ 0\ +5]$, $\gamma = 1.$
- Assume current greedy $\pi(s) = a_1\ \forall s$, $\epsilon = .5$
- Sample trajectory from ϵ-greedy policy
- Trajectory = (s_3, a_1, 0, s_2, a_2, 0, s_3, a_1, 0, s_2, a_2, 0, s_1, a_1, 1, terminal)
- First visit MC estimate of Q of each (s, a) pair?

$Q(-, a_1) = [1\ 0\ 1\ 0\ 0\ 0\ 0]$ $Q(-, a_2) = [0\ 1\ 0\ 0\ 0\ 0\ 0]$

Monotonic[1] ϵ-greedy Policy Improvement

Theorem

For any ϵ-greedy policy π_i, the ϵ-greedy policy w.r.t. Q^{π_i}, π_{i+1} is a monotonic improvement $V^{\pi_{i+1}} \geq V^{\pi_i}$

greedy

$$
\begin{aligned}
Q^{\pi_i}(s, \pi_{i+1}(s)) &= \sum_{a \in A} \pi_{i+1}(a|s) Q^{\pi_i}(s,a) \\
&= \underbrace{(\epsilon/|A|) \sum_{a \in A} Q^{\pi_i}(s,a)}_{\text{random w/ prob } \epsilon} + (1-\epsilon) \max_a Q^{\pi_i}(s,a)
\end{aligned}
$$

$$= \frac{|\epsilon|}{|A|} \sum_a Q^{\pi_i}(s,a) + 1-\epsilon \max_a Q^{\pi_i}(s,a) \frac{1-\epsilon}{1-\epsilon}$$

$$= \quad '' \quad + (1-\epsilon) \max_a Q^{\pi_i}(s,a) \left[\frac{\sum_a (\pi_i(a|s) - \epsilon/|A|)}{1-\epsilon} \right] = 1$$

$$\geq \quad '' \quad + \frac{(1-\epsilon)}{1-\epsilon} \sum_a \left(\pi_i(a|s) - \frac{\epsilon}{|A|}\right) Q^{\pi_i}(s,a)$$

$$= \frac{|\epsilon|}{|A|} \sum_a Q^{\pi_i}(s,a) + \sum_a \pi_i(a|s) Q^{\pi_i}(s,a) - \frac{\epsilon}{|A|} \sum Q^{\pi_i}(s,a)$$

$$= \sum_a \pi_i(a|s) Q^{\pi_i}(s,a) = V^{\pi_i}$$

- Therefore $V^{\pi_{i+1}} \geq V^{\pi}$ (from the policy improvement theorem)

[1]The theorem assumes that Q^{π_i} has been computed exactly.

Monotonic[1] ϵ-greedy Policy Improvement

Theorem

For any ϵ-greedy policy π_i, the ϵ-greedy policy w.r.t. Q^{π_i}, π_{i+1} is a monotonic improvement $V^{\pi_{i+1}} \geq V^{\pi}$

$$
\begin{aligned}
Q^{\pi_i}(s, \pi_{i+1}(s)) &= \sum_{a \in A} \pi_{i+1}(a|s) Q^{\pi_i}(s,a) \\
&= (\epsilon/|A|) \sum_{a \in A} Q^{\pi_i}(s,a) + (1-\epsilon) \max_a Q^{\pi_i}(s,a) \\
&= (\epsilon/|A|) \sum_{a \in A} Q^{\pi_i}(s,a) + (1-\epsilon) \max_a Q^{\pi_i}(s,a) \frac{1-\epsilon}{1-\epsilon} \\
&= (\epsilon/|A|) \sum_{a \in A} Q^{\pi_i}(s,a) + (1-\epsilon) \max_a Q^{\pi_i}(s,a) \sum_{a \in A} \frac{\pi_i(a|s) - \frac{\epsilon}{|A|}}{1-\epsilon} \\
&\geq \frac{\epsilon}{|A|} \sum_{a \in A} Q^{\pi_i}(s,a) + (1-\epsilon) \sum_{a \in A} \frac{\pi_i(a|s) - \frac{\epsilon}{|A|}}{1-\epsilon} Q^{\pi_i}(s,a) \\
&= \sum_{a \in A} \pi_i(a|s) Q^{\pi_i}(s,a) = V^{\pi_i}(s)
\end{aligned}
$$

- Therefore $V^{\pi_{i+1}} \geq V^{\pi}$ (from the policy improvement theorem)

[1] The theorem assumes that Q^{π_i} has been computed exactly.

Greedy in the Limit of Infinite Exploration (GLIE)

Definition of GLIE

- All state-action pairs are visited an infinite number of times

$$\lim_{i \to \infty} N_i(s, a) \to \infty$$

- Behavior policy converges to greedy policy

$$\lim_{i \to \infty} \pi(a|s) \to \arg\max_{w.prob\ 1} Q(s,a)$$

- A simple GLIE strategy is ϵ-greedy where ϵ is reduced to 0 with the following rate: $\epsilon_i = 1/i$

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Monte Carlo Online Control / On Policy Improvement

```
Initialize Q(s,a) = 0, N(s,a) = 0 ∀(s,a), Set ϵ = 1, k = 1
π_k = ϵ-greedy(Q) // Create initial ϵ-greedy policy        1−ϵ argmax_a Q
                                                            ϵ/|A| a
loop
    Sample k-th episode (s_{k,1}, a_{k,1}, r_{k,1}, s_{k,2}, ..., s_{k,T}) given π_k
    G_{k,t} = r_{k,t} + γ r_{k,t+1} + γ^2 r_{k,t+2} + ··· γ^{T_i−1} r_{k,T_i}
    for t = 1, ..., T do
        if First visit to (s,a) in episode k then          ← could do every visit
            N(s,a) = N(s,a) + 1
            Q(s_t, a_t) = Q(s_t, a_t) + 1/N(s,a) (G_{k,t} − Q(s_t, a_t))
        end if
    end for
    k = k + 1, ϵ = 1/k
    π_k = ϵ-greedy(Q) // Policy improvement
end loop
```

Check Your Understanding: MC for On Policy Control

- Mars rover with new actions:
  - $r(-, a_1) = [\ 1\ 0\ 0\ 0\ 0\ 0\ +10]$, $r(-, a_2) = [\ 0\ 0\ 0\ 0\ 0\ 0\ +5]$, $\gamma = 1$.
- Assume current greedy $\pi(s) = a_1\ \forall s$, ϵ=.5
- Sample trajectory from ϵ-greedy policy
- Trajectory $= (s_3, a_1, 0, s_2, a_2, 0, s_3, a_1, 0, s_2, a_2, 0, s_1, a_1, 1, \text{terminal})$
- First visit MC estimate of Q of each (s, a) pair?
- $Q^{\epsilon-\pi}(-, a_1) = [1\ 0\ 1\ 0\ 0\ 0\ 0]$, $Q^{\epsilon-\pi}(-, a_2) = [0\ 1\ 0\ 0\ 0\ 0\ 0]$
- What is $\pi(s) = \arg\max_a Q^{\epsilon-\pi}(s, a)\ \forall s$?

- What is new ϵ-greedy policy, if $k = 3$, $\epsilon = 1/k$

GLIE Monte-Carlo Control

Theorem

GLIE Monte-Carlo control converges to the optimal state-action value function $Q(s, a) \rightarrow Q^*(s, a)$

Model-free Policy Iteration

- Initialize policy π
- Repeat:
  - Policy evaluation: compute Q^π
  - Policy improvement: update π given Q^π

bootstrapping

- What about TD methods?

$$V^\pi(s) = V^\pi(s) + \alpha(\underbrace{r + \gamma V^\pi(s')}_{\text{sampling expectation}} - V^\pi(s))$$

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Model-free Policy Iteration with TD Methods

- Use temporal difference methods for policy evaluation step
- Initialize policy π
- Repeat:
  - Policy evaluation: compute Q^{π} using temporal difference updating with ϵ-greedy policy
  - Policy improvement: Same as Monte carlo policy improvement, set π to ϵ-greedy (Q^{π})

General Form of SARSA Algorithm

Set initial ϵ-greedy policy π randomly, $t = 0$, initial state $s_t = s_0$
Take $a_t \sim \pi(s_t)$ // Sample action from policy
Observe (r_t, s_{t+1})
loop
    Take action $a_{t+1} \sim \pi(s_{t+1})$
    Observe (r_{t+1}, s_{t+2})
    Update Q given $(s_t, a_t, r_t, s_{t+1}, a_{t+1})$:
    $$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + (1-\alpha)\big(r_t + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)\big)$$
    Perform policy improvement:
    $$\pi(s_t) = \arg\max Q(s_t, a_t) \quad + \epsilon\text{-greedy}$$
    $t = t + 1$
end loop

General Form of SARSA Algorithm

$(1-\alpha)V + \alpha(r + \gamma Q$

$V + \alpha(r + \gamma Q - V)$

Set initial ϵ-greedy policy π, $t = 0$, initial state $s_t = s_0$
Take $a_t \sim \pi(s_t)$ // Sample action from policy
Observe (r_t, s_{t+1})
loop
  Take action $a_{t+1} \sim \pi(s_{t+1})$
  Observe (r_{t+1}, s_{t+2})
  $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha(r_t + \gamma Q(\overset{s_{t+1}}{\cancel{s_{t+2}}}, a_{t+1}) - Q(s_t, a_t))$
  $\pi(s_t) = \arg\max_a Q(s_t, a)$ w.prob $1 - \epsilon$, else random
  $t = t + 1$
end loop

What are the benefits to improving the policy after each step?
What are the benefits to updating the policy less frequently?

Convergence Properties of SARSA

Theorem

SARSA for finite-state and finite-action MDPs converges to the optimal action-value, $Q(s,a) \rightarrow Q^*(s,a)$, under the following conditions:

1. The policy sequence $\pi_t(a|s)$ satisfies the condition of GLIE
2. The step-sizes $\underbrace{\alpha_t}_{\text{learning rate param}}$ satisfy the Robbins-Munro sequence such that

$\alpha_t = 1/t$

$$\left.\begin{aligned} \sum_{t=1}^{\infty} \alpha_t &= \infty \\ \sum_{t=1}^{\infty} \alpha_t^2 &< \infty \end{aligned}\right]$$

empirically gen don't use this

Convergence Properties of SARSA

Theorem

SARSA for finite-state and finite-action MDPs converges to the optimal action-value, $Q(s, a) \to Q^*(s, a)$, under the following conditions:

1. The policy sequence $\pi_t(a|s)$ satisfies the condition of GLIE
2. The step-sizes α_t satisfy the Robbins-Munro sequence such that

$$\begin{aligned} \sum_{t=1}^{\infty} \alpha_t &= \infty \\ \sum_{t=1}^{\infty} \alpha_t^2 &< \infty \end{aligned}$$

Would one want to use a step size choice that satisfies the above in practice? Likely not.

Q-Learning: Learning the Optimal State-Action Value

- Can we estimate the value of the optimal policy π^* without knowledge of what π^* is?
- Yes! Q-learning
- Key idea: Maintain state-action Q estimates and use to bootstrap– use the value of the best future action
- Recall SARSA

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha((r_t + \gamma Q(s_{t+1}, \underline{a_{t+1}})) - Q(s_t, a_t)) \quad (2)$$

- Q-learning:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha((r_t + \gamma \max_{a'} \underline{Q(s_{t+1}, a')}) - Q(s_t, a_t)) \quad (3)$$

Off-Policy Control Using Q-learning

- In the prior slide assumed there was some π_b used to act
- π_b determines the actual rewards received
- Now consider how to improve the behavior policy (policy improvement)
- Let behavior policy π_b be ϵ-greedy with respect to (w.r.t.) current estimate of the optimal $Q(s, a)$

Q-Learning with ϵ-greedy Exploration

Initialize $Q(s,a), \forall s \in S, a \in A$ $t = 0$, initial state $s_t = s_0$
Set π_b to be ϵ-greedy w.r.t. Q
loop
    Take $a_t \sim \pi_b(s_t)$ // Sample action from policy
    Observe (r_t, s_{t+1})
    Update Q given (s_t, a_t, r_t, s_{t+1}):
    $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha(r + \max_a Q(s_t, a) - Q(s_t, a_t))$
    Perform policy improvement: set π_b to be ϵ-greedy w.r.t. Q
    π_b for s_t
    $t = t + 1$
end loop

Q-Learning with ϵ-greedy Exploration

Initialize $Q(s, a), \forall s \in S, a \in A$ $t = 0$, initial state $s_t = s_0$
Set π_b to be ϵ-greedy w.r.t. Q
loop
  Take $a_t \sim \pi_b(s_t)$ // Sample action from policy
  Observe (r_t, s_{t+1})
  $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha(r_t + \gamma \cancel{\arg} \max_a Q(s_{\overset{t+1}{\cancel{t}}}, a) - Q(s_t, a_t))$
  $\pi(s_t) = \arg\max_a Q(s_t, a)$ w.prob $1 - \epsilon$, else random
  $t = t + 1$
end loop

Does how Q is initialized matter?

Check Your Understanding: Q-learning

- Mars rover with new actions:
  - $r(-, a_1) = [\ 1\ 0\ 0\ 0\ 0\ 0\ +10]$, $r(-, a_2) = [\ 0\ 0\ 0\ 0\ 0\ 0\ +5]$, $\gamma = 1$.
- Assume current greedy $\pi(s) = a_1\ \forall s$, ϵ=.5
- Sample trajectory from ϵ-greedy policy
- Trajectory = (s_3, a_1, 0, s_2, a_2, 0, s_3, a_1, 0, s_2, a_2, 0, s_1, a_1, 1, terminal)
- New ϵ-greedy policy under MC, if $k = 3$, $\epsilon = 1/k$: with probability 2/3 choose π = [1 2 1 tie tie tie tie], else choose randomly
- Q-learning updates? Initialize $\epsilon = 1/k$, $k = 1$, and $\alpha = 0.5$
- π is random with probability ϵ, else $\pi = [\ 1\ 1\ 1\ 2\ 1\ 2\ 1]$
- First tuple: $(s_3, a_1, 0, s_2)$.
- Q-learning:
  $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha(r_t + \gamma \arg\max_a Q(s_{t_1}, a) - Q(s_t, a_t))$

Q-Learning with ϵ-greedy Exploration

- What conditions are sufficient to ensure that Q-learning with ϵ-greedy exploration converges to optimal Q^*?

  s, a ∞ often
  conditions on α $\leftarrow$ see SARSA

- What conditions are sufficient to ensure that Q-learning with ϵ-greedy exploration converges to optimal π^*?

  GLIE

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Maximization Bias[1]

- Consider single-state MDP ($|S| = 1$) with 2 actions, and both actions have 0-mean random rewards, ($\mathbb{E}(r|a = a_1) = \mathbb{E}(r|a = a_2) = 0$). $\gamma = 0$
- Then $Q(s, a_1) = Q(s, a_2) = 0 = V(s)$ optimal
- Assume there are prior samples of taking action a_1 and a_2
- Let $\hat{Q}(s, a_1), \hat{Q}(s, a_2)$ be the finite sample estimate of Q
- Use an unbiased estimator for Q: e.g. $\hat{Q}(s, a_1) = \frac{1}{n(s,a_1)} \sum_{i=1}^{n(s,a_1)} r_i(s, a_1)$
- Let $\hat{\pi} = \arg\max_a \hat{Q}(s, a)$ be the greedy policy w.r.t. the estimated $\hat{Q}$
- *Even though each estimate of the state-action values is unbiased*, the estimate of $\hat{\pi}$'s value $\hat{V}^{\hat{\pi}}$ can be biased:

$$
\begin{aligned}
\hat{V}^{\hat{\pi}} &= E[\max(\hat{Q}(a_1), \hat{Q}(a_2))] \\
&\boxed{\geq} \max[E(Q(a_1)), E(Q(a_2))] \\
&= \max[0, 0] \\
&= 0 \\
&= V^{\pi}
\end{aligned}
$$

[1]Example from Mannor, Simester, Sun and Tsitsiklis. Bias and Variance Approximation in Value Function Estimates. Management Science 2007

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Maximization Bias[2]

- Consider single-state MDP ($|S| = 1$) with 2 actions, and both actions have 0-mean random rewards, ($\mathbb{E}(r|a = a_1) = \mathbb{E}(r|a = a_2) = 0$).
- Then $Q(s, a_1) = Q(s, a_2) = 0 = V(s)$
- Assume there are prior samples of taking action a_1 and a_2
- Let $\hat{Q}(s, a_1), \hat{Q}(s, a_2)$ be the finite sample estimate of Q
- Use an unbiased estimator for Q: e.g. $\hat{Q}(s, a_1) = \frac{1}{n(s,a_1)} \sum_{i=1}^{n(s,a_1)} r_i(s, a_1)$
- Let $\hat{\pi} = \arg\max_a \hat{Q}(s, a)$ be the greedy policy w.r.t. the estimated $\hat{Q}$
- *Even though each estimate of the state-action values is unbiased*, the estimate of $\hat{\pi}$'s value $\hat{V}^{\hat{\pi}}$ can be biased:

[2]Example from Mannor, Simester, Sun and Tsitsiklis. Bias and Variance Approximation in Value Function Estimates. Management Science 2007

Double Learning

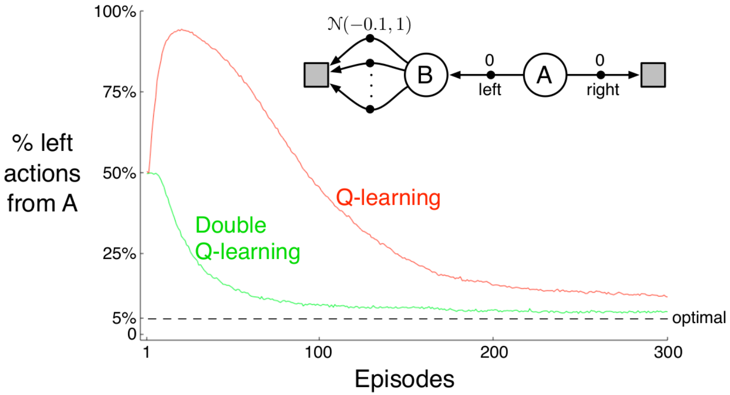
- The greedy policy w.r.t. estimated Q values can yield a maximization bias during finite-sample learning
- Avoid using max of estimates as estimate of max of true values
- Instead split samples and use to create two independent unbiased estimates of $Q_1(s_1, a_i)$ and $Q_2(s_1, a_i)$ $\forall a$.
  - Use one estimate to select max action: $a^* = \arg\max_a Q_1(s_1, a)$
  - Use other estimate to estimate value of a^*: $Q_2(s, a^*)$
  - Yields unbiased estimate: $\mathbb{E}(Q_2(s, a^*)) = Q(s, a^*)$
- Why does this yield an unbiased estimate of the max state-action value?
- If acting online, can alternate samples used to update Q_1 and Q_2, using the other to select the action chosen
- Next slides extend to full MDP case (with more than 1 state)

Double Q-Learning

Initialize $Q_1(s,a)$ and $Q_2(s,a), \forall s \in S, a \in A$ $t = 0$, initial state $s_t = s_0$
loop
    Select a_t using ϵ-greedy $\pi(s) = \arg\max_a Q_1(s_t, a) + Q_2(s_t, a)$
    Observe (r_t, s_{t+1})
    if (with 0.5 probability) **then**
        $Q_1(s_t, a_t) \leftarrow Q_1(s_t, a_t) + \alpha$
    else
        $Q_2(s_t, a_t) \leftarrow Q_2(s_t, a_t) + \alpha$
    end if
    $t = t + 1$
end loop

- Compared to Q-learning, how does this change the: memory requirements, computation requirements per step, amount of data required?

Double Q-Learning (Figure 6.7 in Sutton and Barto 2018)



Due to the maximization bias, Q-learning spends much more time selecting suboptimal actions than double Q-learning.

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What You Should Know

- Be able to implement MC on policy control and SARSA and Q-learning
- Compare them according to properties of how quickly they update, (informally) bias and variance, computational cost
- Define conditions for these algorithms to converge to the optimal Q and optimal π and give at least one way to guarantee such conditions are met.

Class Structure

- Last time: Policy evaluation with no knowledge of how the world works (MDP model not given)
- This time: Control (making decisions) without a model of how the world works
- **Next time: Value function approximation**

Backup Material, Not Expected to Cover in This Lecture

Recall: Off Policy, Policy Evaluation

- Given data from following a behavior policy π_b can we estimate the value V^{π_e} of an alternate policy π_e?
- Neat idea: can we learn about other ways to do things different than what we actually did?
- Discussed how to do this for Monte Carlo evaluation
- Used Importance Sampling
- First see how to do off policy evaluation with TD

Importance Sampling for Off Policy TD (Policy Evaluation)

- Recall the Temporal Difference (TD) algorithm which is used to incremental model-free evaluation of a policy π_b. Precisely, given a state s_t, an action a_t sampled from $\pi_b(s_t)$ and the observed reward r_t and next state s_{t+1}, TD performs the following update:

$$V^{\pi_b}(s_t) = V^{\pi_b}(s_t) + \alpha(r_t + \gamma V^{\pi_b}(s_{t+1}) - V^{\pi_b}(s_t)) \tag{4}$$

- Now want to use data generated from following π_b to estimate the value of different policy π_e, V^{π_e}
- Change TD target $r_t + \gamma V(s_{t+1})$ to weight target by single importance sample ratio
- New update:

$$V^{\pi_e}(s_t) = V^{\pi_e}(s_t) + \alpha \left[\frac{\pi_e(a_t|s_t)}{\pi_b(a_t|s_t)} (r_t + \gamma V^{\pi_e}(s_{t+1}) - V^{\pi_e}(s_t)) \right] \tag{5}$$

Importance Sampling for Off Policy TD Cont.

- Off Policy TD Update:

$$V^{\pi_e}(s_t) = V^{\pi_e}(s_t) + \alpha \left[\frac{\pi_e(a_t|s_t)}{\pi_b(a_t|s_t)} (r_t + \gamma V^{\pi_e}(s_{t+1}) - V^{\pi_e}(s_t)) \right] \quad (6)$$

- Significantly lower variance than MC IS. (Why?)
- Does π_b need to be the same at each time step?
- What conditions on π_b and π_e are needed for off policy TD to converge to V^{π_e}?