A Range-free Multiple Target Localization Algorithm Using Compressive Sensing Theory in Wireless Sensor Networks

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***Abstract*—Considering the incompleteness of localization information in wireless sensor networks, the sensor network monitoring region was divided into a plurality of small grids. Sensors and targets are randomly dropped in the grids. Defining the targets position information as a sparse vector, a range-free multiple target localization algorithm using compressive sensing theory (MTLCS) was proposed. Only targets number sensed by sensor nodes is needed in the algorithm. It doesn't depend on extra hardware measurements. MTLCS can provide the targets position with sparse detected information. The number of targets detected by sensor nodes was expressed as the product of measurement matrix, sparse matrix and sparse vector in compressive sensing theory. Targets are localized with the sparse signal reconstruction. In order to explore MTLCS performance, BP and OMP are applied to recover targets localization. In case of grid number *N*=20×20, simulation is done with different measurement noise, sensing radius, targets number and sensor numbers. In case of $0.6 \le r/n \le 0.8$, simulation results show that MTLCS has the localization error smaller than 30% without physical distance measurement. MTLCS can satisfy the requirements of target localization in wireless sensor network in the case of incomplete information.**

Keywords- multiple target localization; rang-free; sparse information; compressive sensing theory; wireless sensor networks

I. INTRODUCTION

Localization plays a vital role in wireless sensor networks design and application. It limits the development of research and application of wireless sensor network to a large extent, especially in unattended cases. Additionally, due to the constraints of energy and hardware, range-free localization becomes popular research area for wireless sensor networks [1]. Furthermore, limited by environmental factors and information extraction technology, the physical information for localization presents strong incompleteness [2], which brings great challenge for wireless sensor networks design.

Recently, compressive sensing theory has shown great potential applied value in the field of sparse signal image processing. Applying appropriate reconstruction algorithm, compressive sensing theory can recover complicated image information from less measurements [3-5]. In [6], wireless sensor network monitoring region was divided into N discrete grids, and target positions are modeled as a *N*-dimensional vector of *K*- sparse. The rationality of CS theory applying in the localization is demonstrated theoretically. A sparse recovery algorithm called greedy matching pursuit (GMP) is also proposed for target localization with good performance. In [7], target localization was modeled as an approximation problem for distributed sparse vector so that the cost of communication between nodes was reduced. In [8], a pre-processing procedure on the original measurements is introduced to match CS theory, and the target positions can be recovered by ℓ_1 minimization method. Due to the sparse vector being K ideal N-dimensional vector of 1- sparse, the communication overhead in the algorithm is too large. In [9], in order to acquire the optimal solution, network has to transmit a large amount of iterative information among sensor nodes, which also causes more energy consumption. Energy is a vital issue in wireless sensor networks design, simultaneously reducing the communication cost of network in sparse information is also an important job in localization mechanism design.

In this paper, considering the limit of the energy of wireless sensor network and information incompleteness, a new range-free multiple target localization algorithm using compressive sensing theory (MTLCS) is proposed. MTLCS uses targets number sensed by sensor node localizing targets. The targets position is defined as a sparse vector in the discrete space and the number of detected targets by sensor nodes is expressed as the product of measurement matrix, sparse matrix and sparse vector in compressive sensing theory. We recovered the target location with Basis Pursuit (BP) [10] and Orthogonal Matching Pursuit (OMP) [11] respectively. Localization performance is analyzed with different sensing radius, nodes quantity, targets quantity and measurement noise. Simulation results show the validity and superiority of MTLCS in targets localization.

II. NETWORK MODEL AND PARAMETER DEFINITION

A. Network Model

Sensor nodes are randomly deployed in the network. Each node is static and location-aware; targets are also in static state. For the convenience of study, the monitoring region is defined as the square area of $n\times n$, it is devided into N ($N= n\times n$) grids. M ($M<<N$) sensor nodes with position

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information are randomly deployed within some grids. Considering the effectiveness of network coverage (the monitored area is covered by a minimum number of sensors), we assume that there is at most one node for each grid. K ($K<M<<N$) targets are scattered in different grids, and there is no more than one target in each grid. Moreover, the real targets' position is assumed as the corresponding grid center. The diagram of the network is shown as Fig. 1.

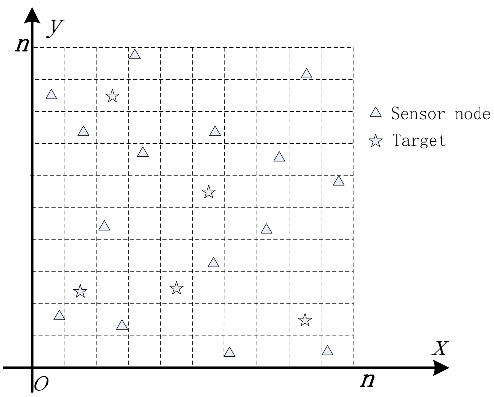


Figure 1. The diagram of network

B. Parameter Definition

The multi-targets localization will be constructed as a compressive sensing sparse signal reconstruction problem, and the relative parameters matrix involved in the problem is defined as following.

1) Sparse Vector of Target Position Information $\boldsymbol{s}$

K monitored targets coordinate the center of relative grids respectively. The information vector of target position is defined as $\boldsymbol{s}$, $\boldsymbol{s}=\{s_k, k=1, 2, \cdots, N\}^{\mathrm{T}}$, and $\boldsymbol{s}$ is a $N\times 1$ vector. If there is a target in k grid, $s_k=1$, otherwise $s_k=0$. Because there are K targets in the monitored area, only K elements of the vector $\boldsymbol{s}$ are 1 and the rest are all 0.

2) Sparse Matrix $\boldsymbol{\Psi}$

We define $\boldsymbol{\Psi}$ as an $N\times N$ sparse matrix. If a target in the j($1\leq j\leq N$) grid is detected by the sensor node located in the i ($1\leq i\leq N$) grid, then $\Psi_{ij}=1$, otherwise $\Psi_{ij}=0$. So, the matrix $\boldsymbol{\Psi}$ can be presented as formula (1):

$$\boldsymbol{\Psi}=\begin{pmatrix} \Psi_{11} & \Psi_{12} & \cdots & \Psi_{1N} \\ \Psi_{21} & \Psi_{22} & \cdots & \Psi_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \Psi_{N1} & \Psi_{N2} & \cdots & \Psi_{NN} \end{pmatrix} \tag{1}$$

Taking $\boldsymbol{X}=\{x_1, x_2, \cdots, x_N\}^{\mathrm{T}}$, then formula (2) shows the targets number sensed by each node, which we place at every grid at the all monitored area .

$$\boldsymbol{X}=\boldsymbol{\Psi s} \tag{2}$$

3) Measurement Matrix $\boldsymbol{\Phi}$

If $M(M=O(K\log(N/K)\ \&\ M<<N)$ sensor nodes are randomly deployed in N grid, the measurement matrix $\boldsymbol{\Phi}$ is shown as formula (3).

$$\boldsymbol{\Phi}=\begin{pmatrix} 0\cdots & 1 & 0 & \cdots\cdots \\ 0\cdots & 0 & 1 & 0\cdots \\ \vdots\ \vdots & \vdots & \vdots & \vdots\ \vdots \\ 0\cdots & \cdots & 0 & 1\ 0 \end{pmatrix}_{M\times N} \tag{3}$$

.

If sensor node i ($1\leq i\leq M$) is located in the j ($1\leq i\leq N$) grid, $\Phi_{ij}=1$,otherwise $\Phi_{ij}=0$. According to the network model, only one element is 1 in each row of the matrix $\boldsymbol{\Phi}$ and the rest are 0. So there are M non-zero values in $\boldsymbol{\Phi}$. After deployment, sensor node positions are determined, the measurement matrix is also determined.

4) Measurement Vector $\boldsymbol{y}$

$\boldsymbol{y}=\{y_1, y_2, \cdots, y_M\}^{\mathrm{T}}$ is measurement matrix. When the i sensor node detected there were k targets within its sensing range, $y_i= k$ ($k\leq K$). The measurement vector $\boldsymbol{y}$ can be used to record the detected target information of M sensor nodes.

5) Sensing Matrix $\mathbf{A}$

According to definitions and physical meaning of each matrix above, the measurement vector $\boldsymbol{y}$ satisfies the following equation:

$$\boldsymbol{y}=\boldsymbol{\Phi X}=\boldsymbol{\Phi\Psi s}=\boldsymbol{As} \tag{4}$$

The equation builds a relationship between the measurement vector and the target position information vector, then $\boldsymbol{A}_{M\times N}$ is defined as sensing matrix.

Considering the measurement noise, the above formula $\boldsymbol{y}=\{y_1, y_2, \cdots, y_M\}^{\mathrm{T}}$ should be expressed as:

$$\boldsymbol{y}=\boldsymbol{\Phi\Psi s}+\boldsymbol{\varepsilon} \tag{5}$$

Where $\varepsilon=\{\varepsilon_1, \varepsilon_2,\cdots,\varepsilon_M\}^{\mathrm{T}}$ is the additive Gaussian white noise.

III. Multiple Target Localization Algorithm Using Compressive Sensing Theory

A. Algorithm Description

According to compressive sensing theory [12], if recovering signals are sparse or compressible under a certain basis, we can acquire the sparse signal or its unique sparse representation with fewer noisy measurements through a recovery algorithm. Targets number sensed by each sensor node can easily be obtained. Considering the static wireless sensor monitoring network, there are only K non-zero values in the target position information vector $\boldsymbol{s}$, so $\boldsymbol{s}$ is a sparse vector by the definition of sparsity [13]. Furthermore, the vector can be recovered by taking a recovery algorithm.

According to above statement, we can obtain the network localization information without the matrix $\boldsymbol{X}$ by placing a sensor node at each grid. Only through $\boldsymbol{X}$ multiplied by the matrix $\boldsymbol{\Phi}$, the measurement vector $\boldsymbol{y}$ detected by M sensor nodes can be obtained. Then the sparse vector $\boldsymbol{s}$ representing targets position information can be recovered by using the recovery algorithm. In the centralized localization, M nodes will deliver the compressive measurements vector $\boldsymbol{y}$ and sensing matrix $\boldsymbol{A}$ to the sink node, and the sparse vector $\boldsymbol{s}$ can be reconstructed through the compressive sensing theory. Eventually multi-target localization is finished.

B. Orthogonalization of Sensing Matrix

As stated in CS theory, the successful recovery of a signal by CS has a great relativity with the characteristics of measurement matrix and sparse matrix. The matrix $\boldsymbol{A}=\boldsymbol{\Phi\Psi}$ obeys RIP(Restricted Isometry Property) with parameters (K, δ) for $\delta \in (0,1)$ if function (6) holds for all K-sparse vector $\boldsymbol{x}$, then the sparse signal can be recovered with high precision.

$$1-\delta \le \frac{\| Ax \|_2^2}{\| x \|_2^2} \le 1+\delta \tag{6}$$

By the definition in section 2, the sparse matrix $\boldsymbol{\Psi}$ and the measurement matrix $\boldsymbol{\Phi}$ are obtained according to the sensor network topology structure. They are coherence in spatial domain, namely $\boldsymbol{A}=\boldsymbol{\Phi\Psi}$ does not satisfy the RIP , the CS theory cannot be directly applied. To solve this problem, a data pre-processing on measurement vector $\boldsymbol{y}$ is introduced. Considering the sensing matrix $\boldsymbol{A}=\boldsymbol{\Phi\Psi}$, $\mathbf{T}=\mathrm{orth}(\mathbf{A}^{\mathrm{T}})^{\mathrm{T}}$ where $\boldsymbol{T}$ is an orthogonal basis for the range of $\boldsymbol{A}$, and $\mathbf{A}^{+}$ returns the generalized inverse of matrix $\boldsymbol{A}$. Then,

$$\boldsymbol{Y}=\boldsymbol{TA}^{+}\boldsymbol{y}=\boldsymbol{TA}^{+}\boldsymbol{As}=\boldsymbol{Ts} \tag{7}$$

In the case of noise:

$$\boldsymbol{Y}=\boldsymbol{TA}^{+}\boldsymbol{y}=\boldsymbol{TA}^{+}\boldsymbol{As}+\boldsymbol{TA}^{+}\boldsymbol{n}=\boldsymbol{Ts}+\boldsymbol{n}' \tag{8}$$

This procedure has the same effect as orthogonalizing the two matrixes [8]. Since $\boldsymbol{T}$ is an orthogonal matrix, $\boldsymbol{s}$ can be well recovered from $\boldsymbol{Y}$ via recovery algorithm based on the CS theory after the above process is performed at the sink node.

C. Localization Recovery Algorithm

According to the above analysis, multi-targets localization in WSN can be properly solved by the sparse vector recovery algorithm of the CS. Among the existing recovery algorithms, ℓ_1 minimization and greedy algorithm are two major approaches. ℓ_1 minimization methods, such as Basis Pursuit (BP), solve a convex minimization problem instead of the combinatorial problem. The methods work correctly for all sparse signals and provide theoretical performance guarantees. Most ℓ_1 minimization methods are sensitive to noise and often suffer from heavy computational complexity. Greedy algorithms, such as Orthogonal Matching Pursuit (OMP), iteratively the supports of the targets signal and construct an approximation on the set of the chosen supports, until a halting condition is met. They can solve large-scale recovery problems more efficiently. The computational complexity of greedy algorithms is significantly lower than that of ℓ_1 minimization. In this paper, we applied both BP and OMP to derive non-zero elements in the unknown sparse vector $\boldsymbol{s}$, which exactly indicate the targets position. The performance of the two recovery algorithm is analyzed respectively.

IV. SIMULATION RESULTS AND PERFORMANCE EVALUATION

Simulation is done to evaluate the performance of the algorithm proposed in the paper. The monitored network is divided into N(N=n×n) grids. K(K<M) targets without position information are randomly placed in K grids. There is no more than one target in each grid. Detected targets number are collected at M(M<<N) arbitrary sensor nodes , but there is no more than one sensor node in each grid. In addition, if MTLCS reports a target at a grid, the center of the grid is used as the estimated position of the reported target. The node's sensing radius is r. When the distance between nodes and target is shorter than r, the sensor nodes can detect the target. Considering the reliability and robustness of the proposed method, we intentionally add the Gaussian white noise to the measurements. Due to the specificity of the model, the Gaussian white noise is added to the distance between nodes and targets. Each presented result is the average value with 200 randomly runs. Finally, the localization performance is studied under different sensing radius, targets quantity and sensor quantity.

A. Localization Error

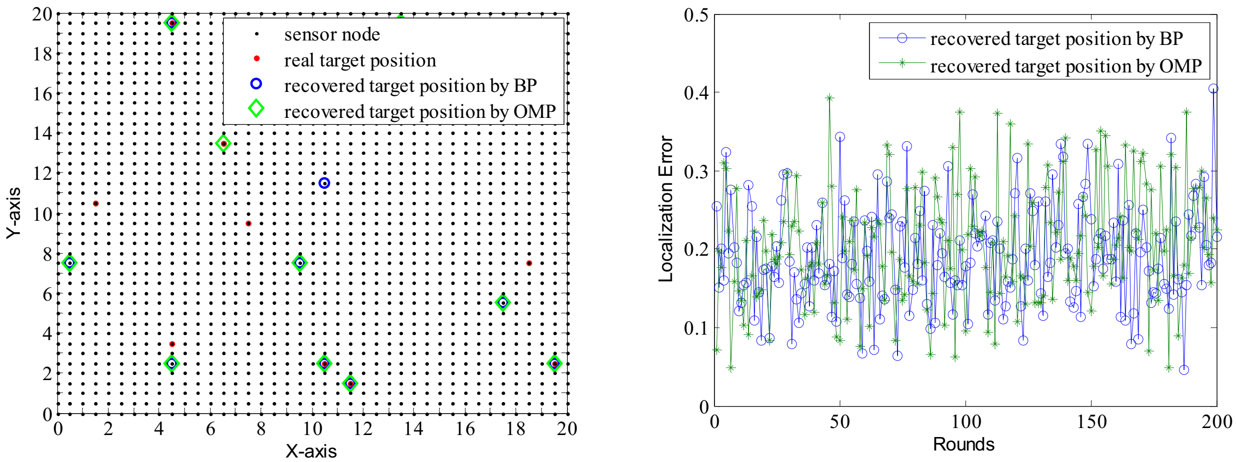
In this paper, localization error is defined as the average Euclidean distance between the real positions and the recovered positions of the K targets. It is shown as following:

$$e=\frac{\frac{1}{K}\sum_{i=1}^{K}\sqrt{\left(x_i-x_i^{'}\right)^2+\left(y_i-y_i^{'}\right)^2}}{r} \tag{9}$$

In (9), K is the total number of the targets with real positions (x_1,y_1) , (x_2,y_2) · · · (x_K,y_K) , respectively. (x_1,y_1') (x_2,y_2') ·· ·(x_K', y_K') are the corresponding estimated target positions. r is the sensing radius of nodes.

In the simulation, there are M=40 sensor nodes randomly deployed in the monitoring area, which is divided into a 20×20(N=20×20) grid. The sensing radius of nodes is 14(r=14). There are 10(K=10) targets located in the monitoring area. Fig. 2 (a) shows the recovered targets position and real target position, while Fig. 2 (b) shows the localization error without noise. The average localization error with BP and OMP are 19.02% and 20.50% respectively. BP does better than OMP as expected. Both of them satisfy the localization requirement in WSN [14].

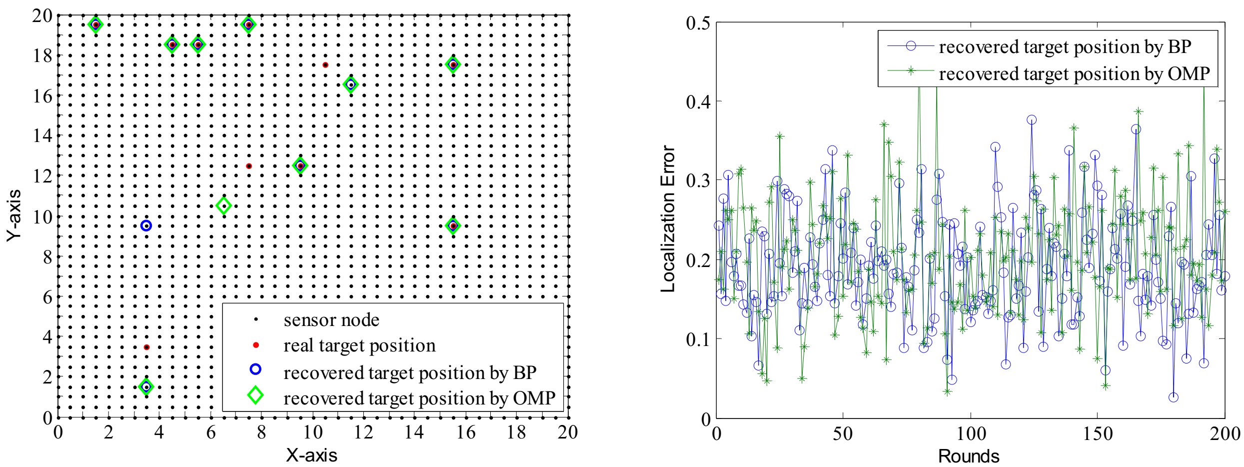
MTLCS performances with noise measurements are also studied. Fig. 3 and Fig. 4 show the localization error under SNR15dB and 25dB. The average localization error with BP and OMP are 19.07% and 20.62% under SNR 15 dB respectively, and the average localization error with BP and OMP are 19.22% and 21.99% under SNR 25 dB respectively. Localizing same number of targets with MTLCS, the bigger the measurement noise is, the bigger the error is, which is consistent with the reality. Furthermore, the localization error is very close for the case with measurement noise and without noise. It indicates that MTLCS can tolerate a certain level of measurement noise.



(a) Distribution of nodes, targets and recovered targets

(b) Localization error v.s. rounds

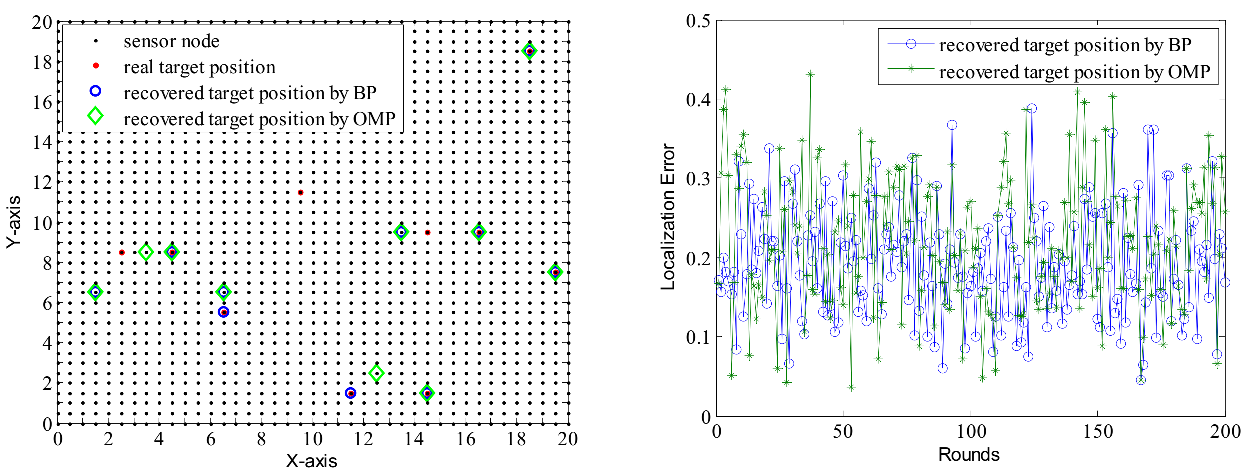
Figure 2. K = 10 targets localization results without noise



(a) Distribution of nodes, targets and recovered targets

(b) Localization error v.s. rounds

Figure 3. K = 10 target localization results under SNR=15dB.



(a) Distribution of nodes, targets and recovered targets

(b) Localization error v.s. rounds

Figure 4. K=10 targets localization results under SNR=25dB.

B. Localization Error v.s. Sensing Radius

The bigger the node's sensing radius is, the higher probability the node detect more targets. Different sensing radius will lead to different measures matrix $\boldsymbol{y}$. The performance will be affected by different sensing radius. Localization error varies with the ratio r/n is studied. In cases $K = 10$, $M = 40$, simulations are done without noise, SNR = 25dB and SNR =15dB respectively. In Fig. 5, MTLCS using BP recovering algorithm has a lower localization error than MTLCS using OMP recovering algorithm. Localization error shows same trend varying with r/n under different noise environments. Localization error decreases with the increasing of the ratio if $r/n<0.7$, and the error increases when the ratio continues to increase. The error reaches the minimum when $r/n = 0.7$. When $r/n <0.7$, r is smaller compared with the fixed n .The absolute distance between real positions and recovered positions of target changes less with the radius r. Localization error is defined as the ratio of the absolute error and sensing radius shown in for(9) above. So for the fixed absolute distance error, when r is smaller, the localization error is bigger. When $r/n > 0.7$, for the sensing radius is bigger, each sensor node can almost detect all targets in the monitored area, which make the measurement matrix does not match the parameter characteristic of the reconstruction algorithm in CS. So the localization error becomes bigger as r/n increasing.

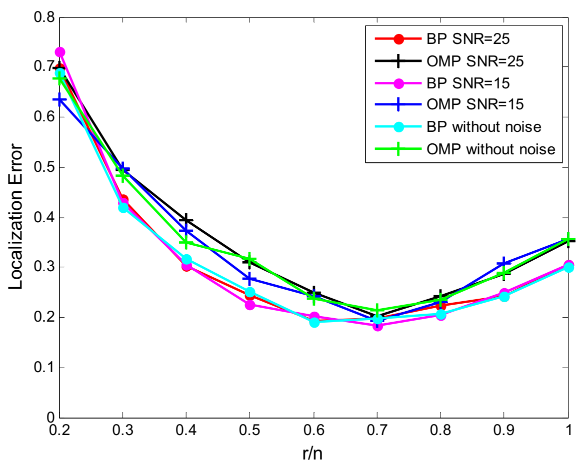


Figure 5. Localization error v.s. sensing radius

C. Localization Error v.s. Number of Targets

Under the parameter $M = 40$, N=20×20, and r/n=0.7, the performance of localization algorithm is investigated with the number of targets changing from 2 to 25. Three cases SNR = 25dB, SNR =5dB and without noise are studied. As shown in Fig. 6, MTLCS using BP and OMP show the same trend. When K<15, under different noisy level, the less the number of targets is, the lower the localization accuracy is. It is consistent with the fact in CS theory that the bigger the sparsity is, the more accurate the recovery sparse vector is. When the targets number K = 15, the localization error with BP is close to 19%, while the localization error with OMP is nearly 25%. With the targets number increasing when K>15, the localization error changes less and maintains at around 19% and 25% for MTLCS using BP and OMP respectively. On the other hand, when K<8, MTLCS with OMP does better than with BP.

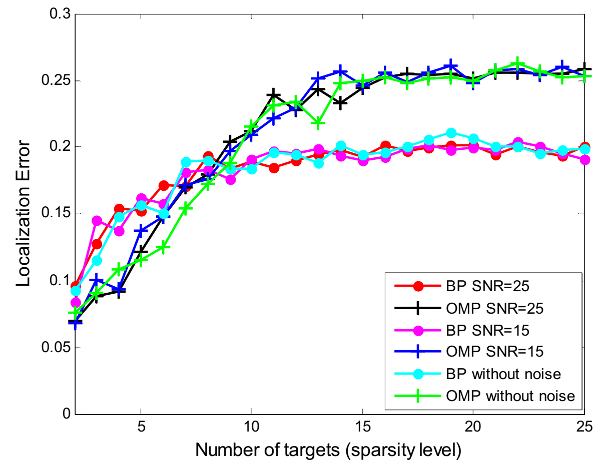


Figure 6. Localization error v.s. number of targets

D. Localization Error v.s. Number of Sensor Nodes

Under the parameter $N = 20 \times 20$, $r = 14$, localization error is studied with different number of sensor nodes in SNR = 25dB, SNR = 15dB and without noise cases. Simulation is conducted when the number of sensor nodes M varies from 20 to 100 at a step size of 5. As shown in Fig. 7, for both BP and OMP recovered method, the larger the M is, the smaller the error for all numbers of targets will be. Localization error range is from 35% to 5%. It is rational in the theoretic analysis. In CS theory, more measurements mean more information obtained about the network. The sparse vector can be recovered more precisely and the localization error is smaller.

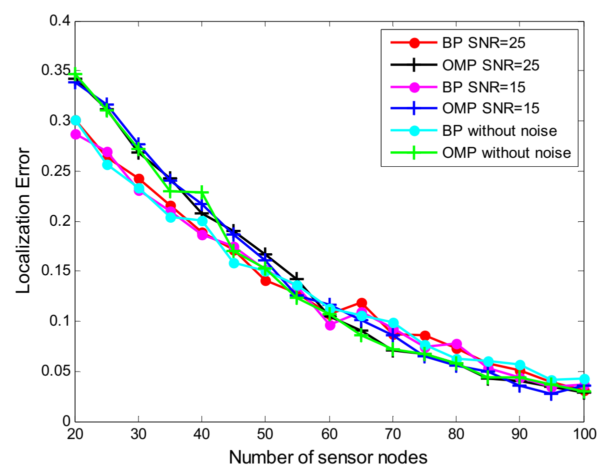


Figure 7. Localization error v.s. number of sensor nodes

V. CONCLUSION

In this paper, considering hardware measurement cost in wireless sensor networks localization, we use targets number sensed by each node to induce the targets position. It is a range-free algorithm. On the other hand, the localization information shows sparisity for the network characteristic and environment. We applied compressive sensing theory to the localization mechanism MTLCS. The positions of target are represented as a sparse vector, and the number of detected targets by sensor nodes is expressed as the product of measurement matrix, sparse matrix and sparse vector in compressive sensing theory. The sparse vector of target positions is reconstructed by Basis Pursuit (BP) and Orthogonal Matching Pursuit (OMP). MTLCS performance under different measurement noise, sensor radius, targets number and sensor nodes number is investigated. Simulation results validate that MTLCS can satisfy the multi-target localization accuracy requirement in the case of incomplete information. When r/n = 0.7, the localization error of MTLCS reaches the minimum. In the situation stated in the paper, the localization error increases with the number of targets increasing, and maintains at about 19% with BP and 25% with OMP eventually. With the number of sensor nodes increasing, the localization accuracy will be improved.

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