IOWA

CS3640

Network Layer (4): Routing Algorithms

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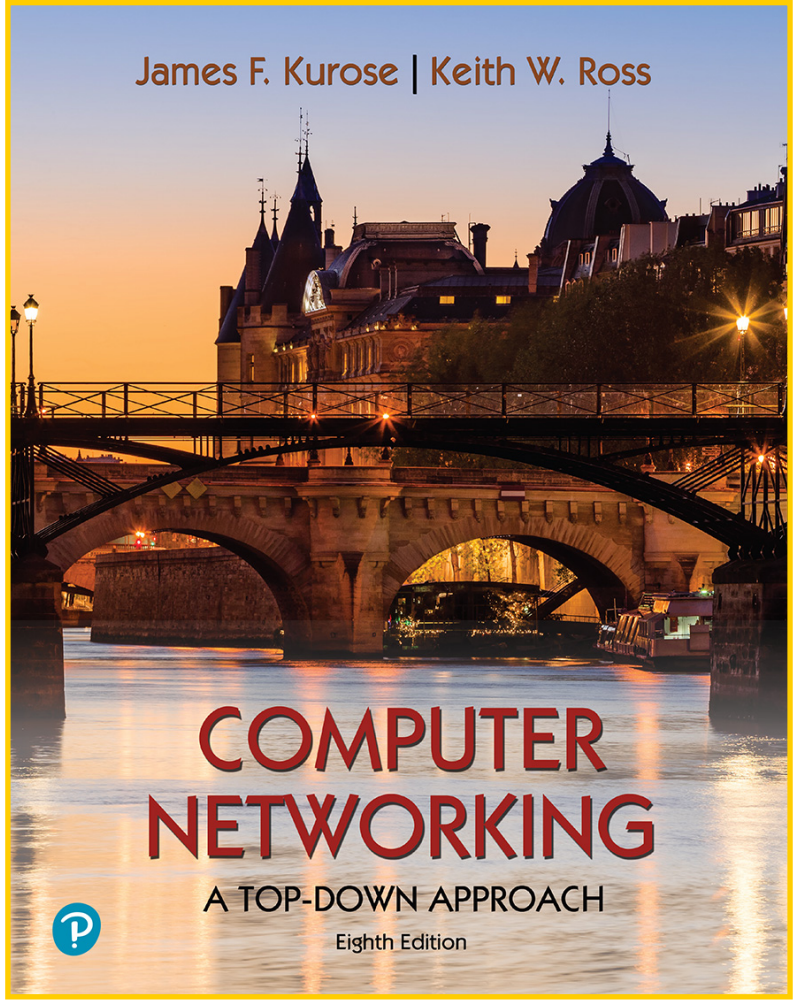
Lecture goals

a technical deep-dive into two classes of routing algorithms used in the Internet

- *Link-State algorithm*
- *Distance Vector algorithm*



Chapters 5.1 - 5.2

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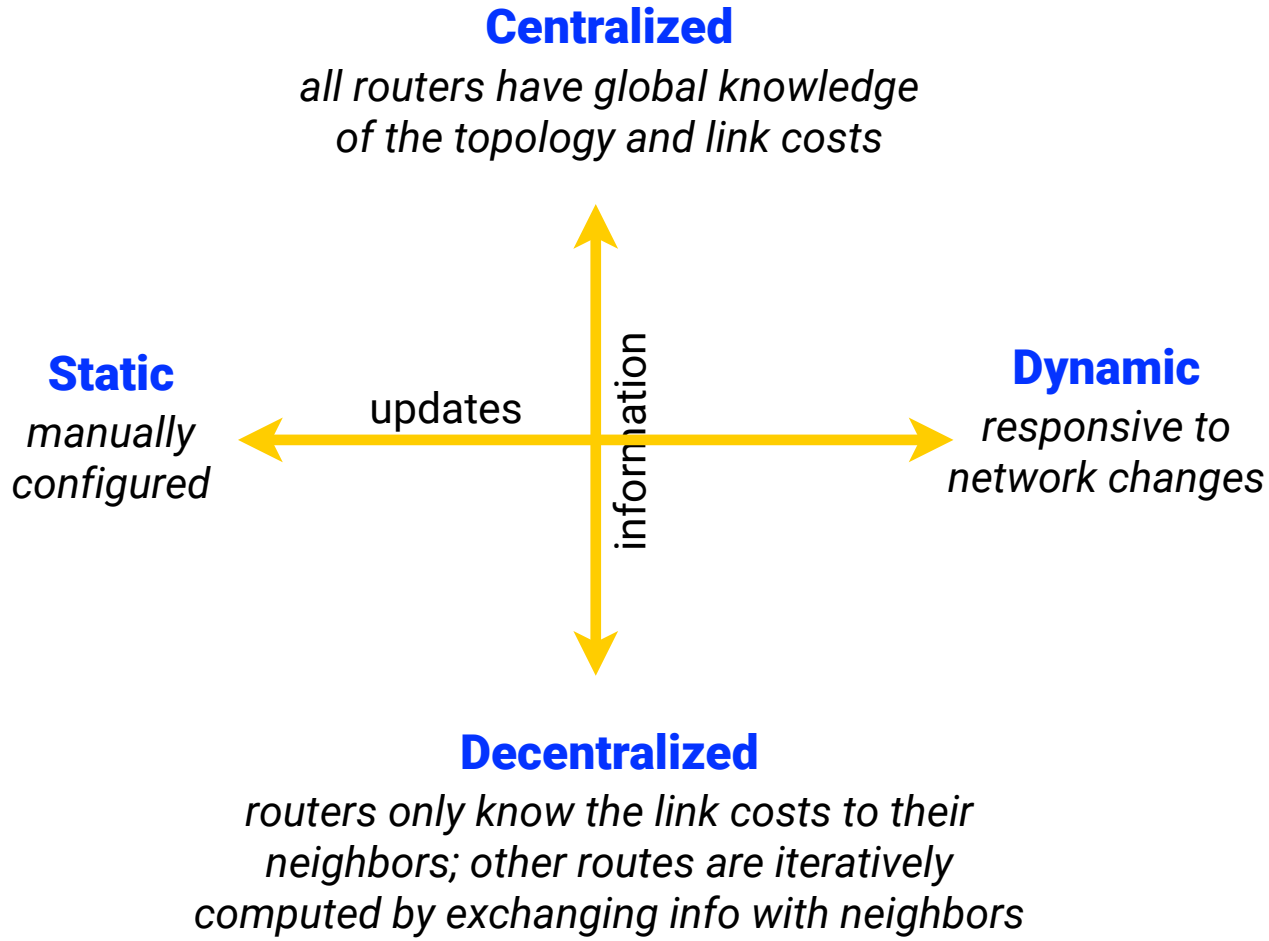
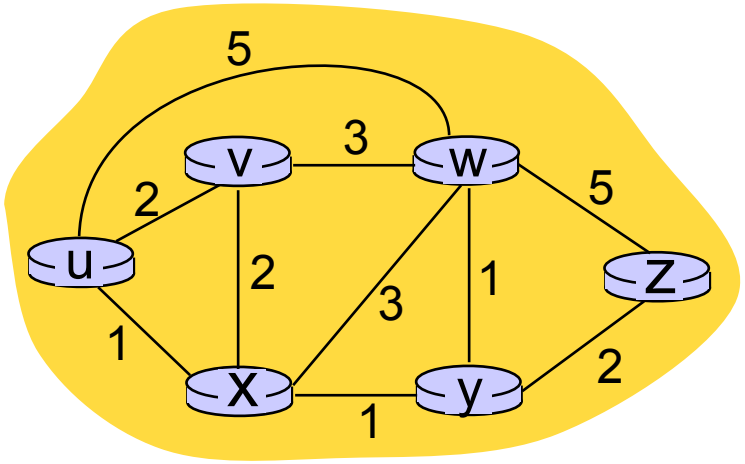
Routing Algorithms

Goal: determine “good” paths from sending hosts to receiving host, through network of routers

- **path:** sequence of routers packets traverse from given initial source host to final destination host
- **good:** least “cost”, “fastest”, “least congested”, and so on!



Representing Routing via Graph Abstraction



$C_{a,b}$: cost of *direct* link connecting *a* and *b*

e.g., $c_{w,z} = 5$, $c_{u,z} = \infty$

cost is defined by network operators: they could all be set to 1, or set to reflect a network metric such as bandwidth or congestion

Graph: *G = (N,E)*

N: set of routers = { *u, v, w, x, y, z* }

E: set of links ={ *(u,v), (u,x), (v,x), (v,w), (x,w), (x,y), (w,y), (w,z), (y,z)* }

Dijkstra's Algorithm

(a link-state routing algorithm)

Dijkstra's Link-State Routing Algorithm

- **Centralized**: network topology, link costs known to *all* nodes (which is accomplished via a *link state broadcast* such that all nodes have same info)
- computes least cost paths from one node ("source") to all other nodes, which generates the *forwarding table* for that node
- **Iterative**: after *k* iterations, we know least cost path to *k* destinations

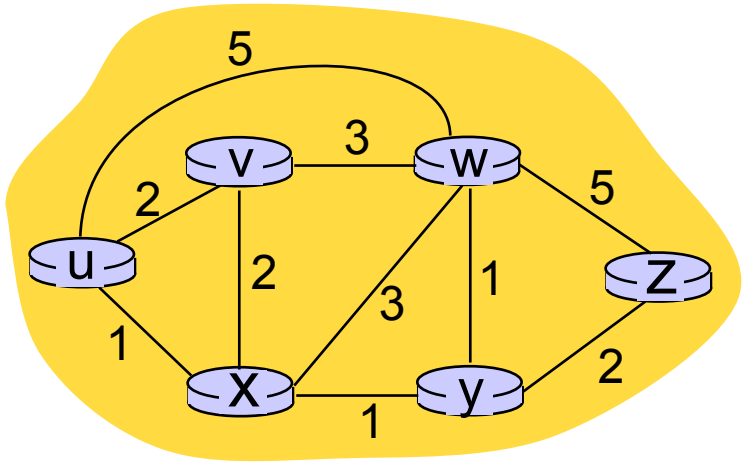
notation

- $C_{a,b}$: direct link cost from node *a* to *b*; = ∞ if not direct neighbors
- *D(a): current* estimate of cost of least-cost-path from source to destination *a*
- *p(a):* predecessor node along path from source to *a*
- *N':* set of nodes whose least-cost-path *definitively* known

```
1  Initialization:
2    N' = {u}                        /* compute least cost path from u to all other nodes */
3    for all nodes a
4      if a adjacent to u            /* u initially knows direct-path-cost only to  direct neighbors */
5          then D(a) = c_u,a         /* but may not be minimum cost! */
6      else D(a) = ∞
7
8    Loop
9        find a not in N' such that D(a) is a minimum
10       add a to N'
11       update D(b) for all b adjacent to a and not in N' :
12           D(b) = min ( D(b),  D(a) + c_a,b )
13       /* new least-path-cost to b is either old least-cost-path to b or known
14       least-cost-path to a plus direct-cost from a to b */
15   until all nodes in N'
```

Dijkstra's algorithm: an example

Step	N'	v D(v),p(v)	w D(w),p(w)	x D(x),p(x)	y D(y),p(y)	z D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1						
2						
3						
4						
5						

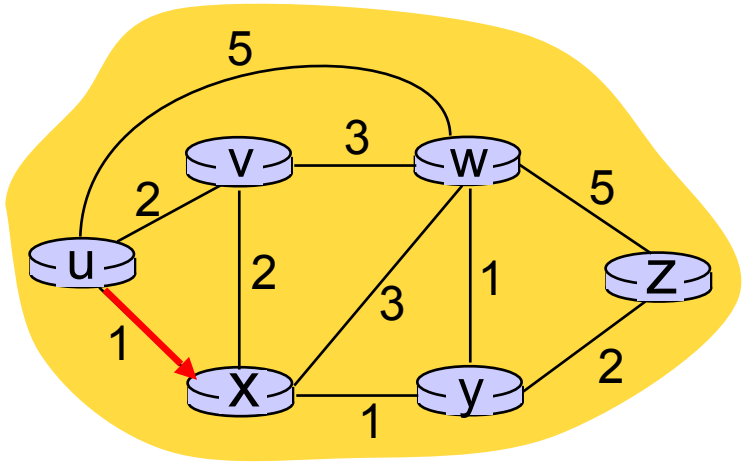


Initialization (step 0):

For all *a*: if *a* adjacent to *u* then $D(a) = c_{u,a}$

Dijkstra's algorithm: an example

Step	N'	v D(v),p(v)	w D(w),p(w)	x D(x),p(x)	y D(y),p(y)	z D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux					
2						
3						
4						
5						



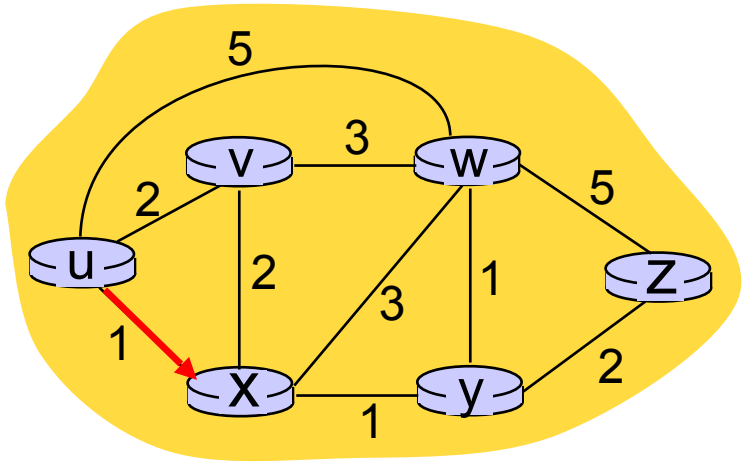
8 *Loop*

9 find *a* not in *N'* such that *D(a)* is a minimum

10 add *a* to *N'*

Dijkstra's algorithm: an example

		v	w	x	y	z
Step	N'	D(v),p(v)	D(w),p(w)	D(x),p(x)	D(y),p(y)	D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux	2,u	4,x		2,x	∞
2						
3						
4						
5						



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10 add *a* to *N'*

11 update *D(b)* for all *b* adjacent to *a* and not in *N'* :

$D(b) = \min (D(b), D(a) + c_{a,b})$

$D(v) = \min (D(v), D(x) + c_{x,v}) = \min(2, 1+2) = 2$

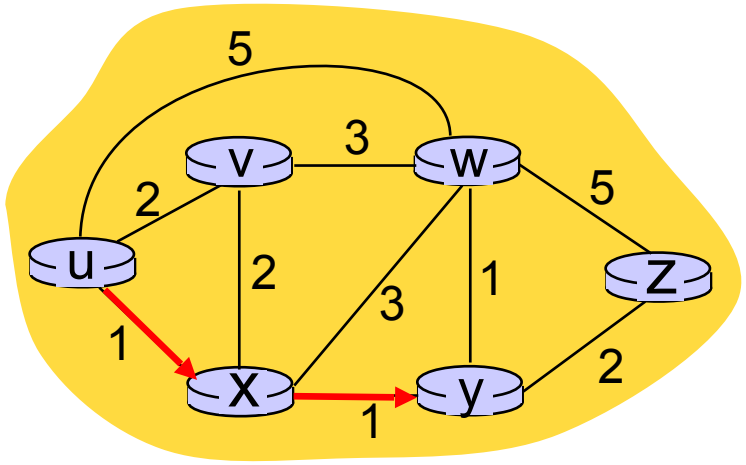
$D(w) = \min (D(w), D(x) + c_{x,w}) = \min (5, 1+3) = 4$

$D(y) = \min (D(y), D(x) + c_{x,y}) = \min(\inf, 1+1) = 2$

NEW! NEW!

Dijkstra's algorithm: an example

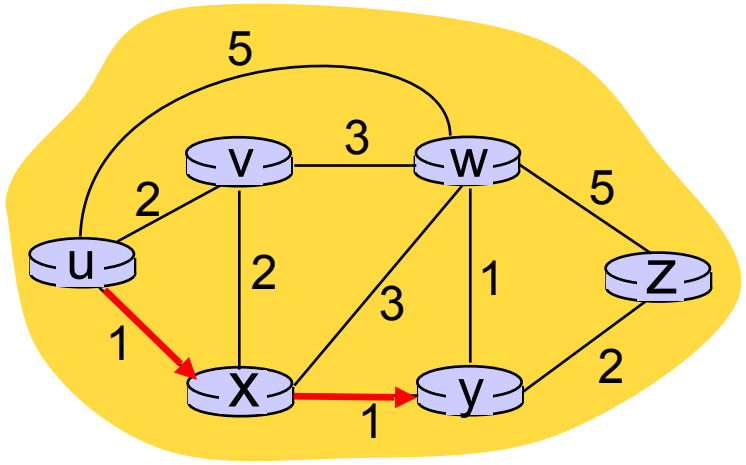
Step	N'	v D(v),p(v)	w D(w),p(w)	x D(x),p(x)	y D(y),p(y)	z D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux	2,u	4,x		2,x	∞
2	uxy					
3						
4						
5						




```
8  Loop
9      find a not in N' such that D(a) is a minimum
10     add a to N'
```

Dijkstra's algorithm: an example

		v	w	x	y	z
Step	N'	D(v),p(v)	D(w),p(w)	D(x),p(x)	D(y),p(y)	D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux	2,u	4,x		2,x	∞
2	uxy	2,u	3,y			4,y
3						
4						
5						



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$$D(b) = \min (D(b), D(a) + c_{a,b})$$

$D(w) = min (D(w), D(y) + c_{x,w}) = min (4, 2+1) = 3$

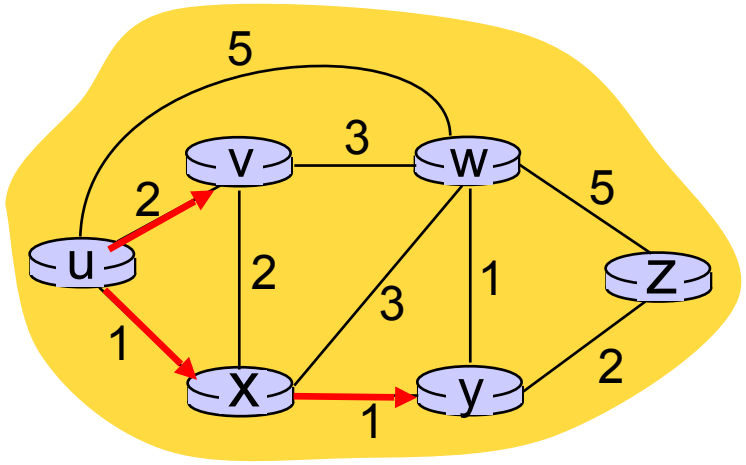
$D(z) = min (D(z), D(y) + c_{y,x}) = min(inf,2+2) = 4$

NEW!

NEW!

Dijkstra's algorithm: an example

Step	N'	v D(v),p(v)	w D(w),p(w)	x D(x),p(x)	y D(y),p(y)	z D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux	2,u	4,x		2,x	∞
2	uxy	2,u	3,y			4,y
3	uxyv					
4						
5						



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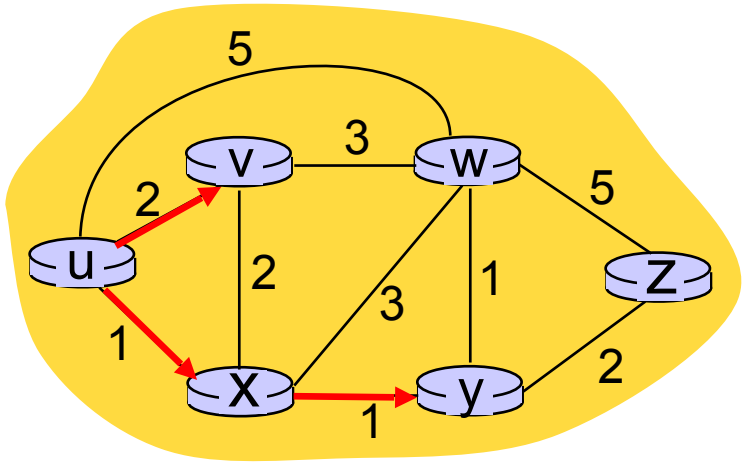
Step	N'	v D(v),p(v)	w D(w),p(w)	x D(x),p(x)	y D(y),p(y)	z D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux	2,u	4,x		2,x	∞
2	uxy	2,u	3,y			4,y
3	uxyv		3,y			4,y
4						
5						



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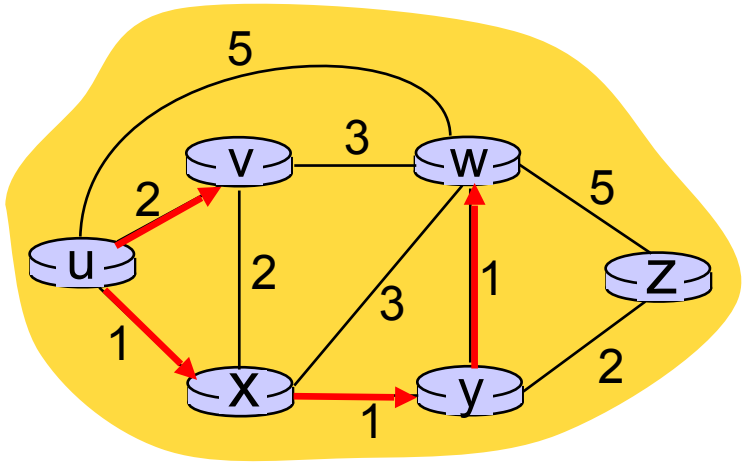
2	uxy	2,u	3,y			4,y
3	uxyv		3,y			4,y
4	uxyvw					
5						



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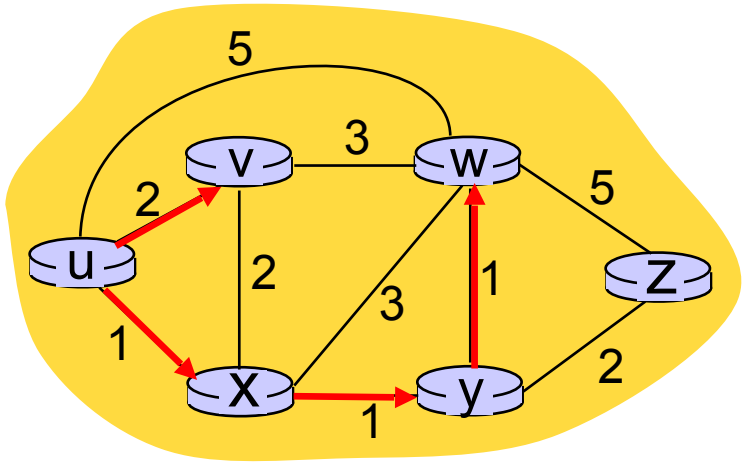

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Dijkstra's algorithm: an example

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1	ux	2,u	4,x		2,x	∞
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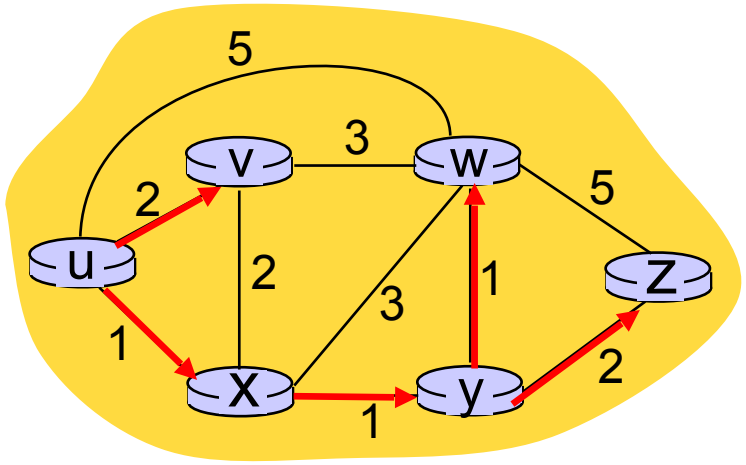
3	uxyv		3,y			4,y
4	uxyvw					4,y
5	uxyvwz					



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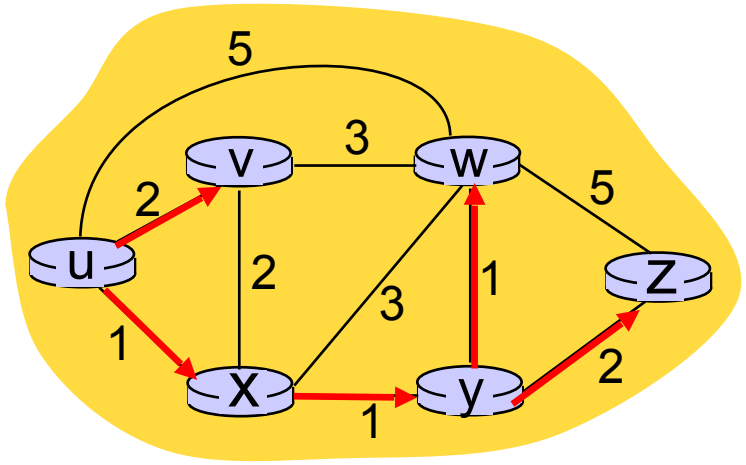
4	uxyvw					4,y
5	uxyvwz					



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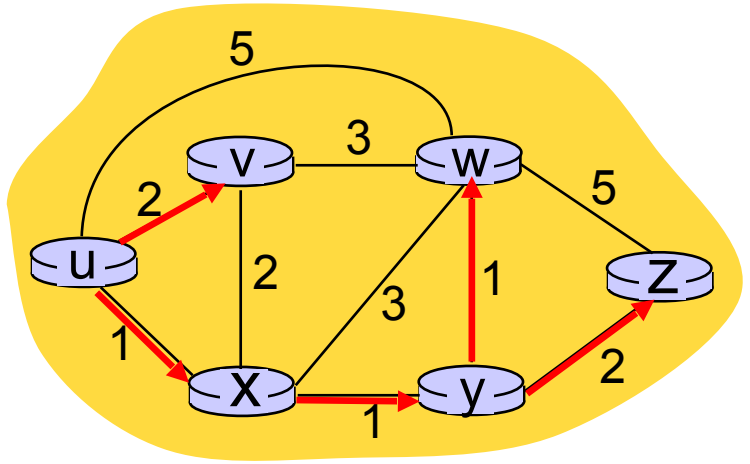
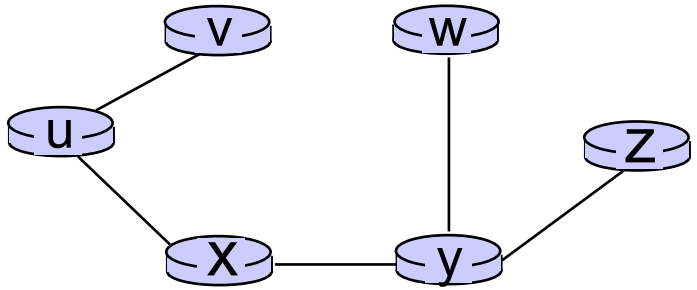
Dijkstra's algorithm: an example



resulting least-cost-path tree from u:



resulting forwarding table in u:

destination	outgoing link
v	(u,v)
x	(u,x)
y	(u,x)
w	(u,x)
z	(u,x)

route from *u* to *v* directly

route from u to all other destinations via *x*

Comparing Dijkstra and Bellman-Ford Algorithms

	Dijkstra (LS)	Bellman-Ford (DV)
Algorithm structure	Centralized	
Speed of convergence	$O(N^2)$	
Application	Routing within autonomous systems	
Robustness	Route oscillations	

Bellman-Ford Algorithm

(a distance-vector routing algorithm)

Bellman-Ford equation

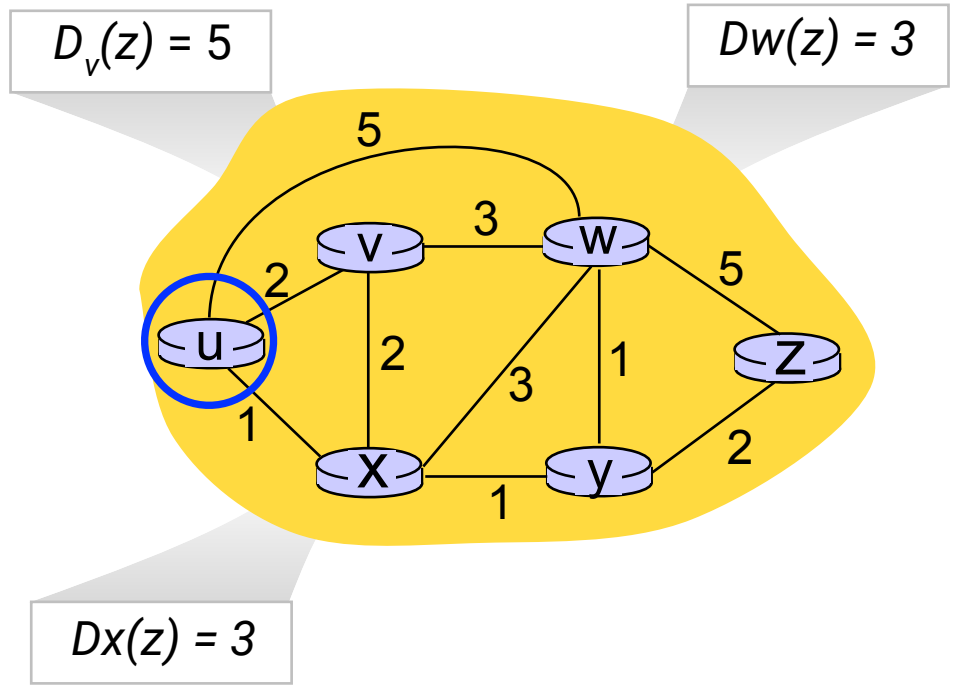
Let $D_x(y)$: cost of least-cost path from x to y.

Then, $D_x(y) = \min_v \{c_{x,v} + D_v(y)\}$

min taken over all neighbors *v* of *x*

v's estimated least-cost-path cost to *y*

direct cost of link from x to v



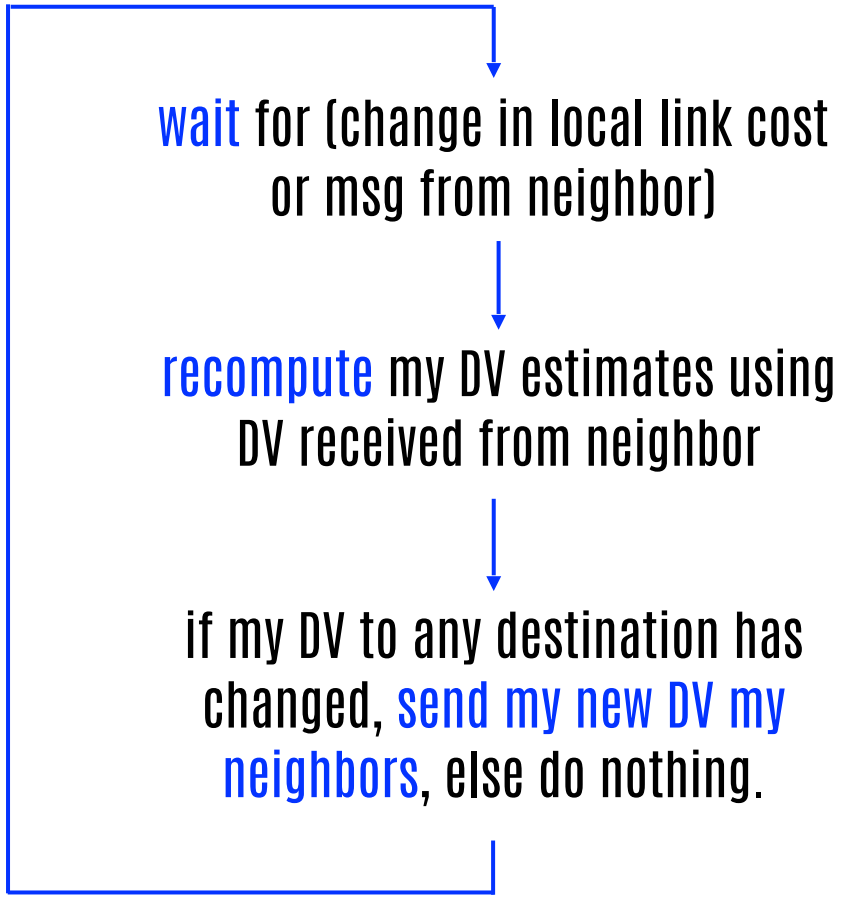
Suppose that *u*'s neighboring nodes *(x,v, and w)* know their cost for destination *z*:

Bellman-Ford equation says:

$$D_u(z) = \min \{ c_{u,v} + D_v(z), \; c_{u,x} + D_x(z), \; c_{u,w} + D_w(z) \} = \min \{2 + 5, \; 1 + 3, \; 5 + 3\} = 4$$

Bellman-Ford Distance Vector Algorithm

Each node:



Key Characteristics

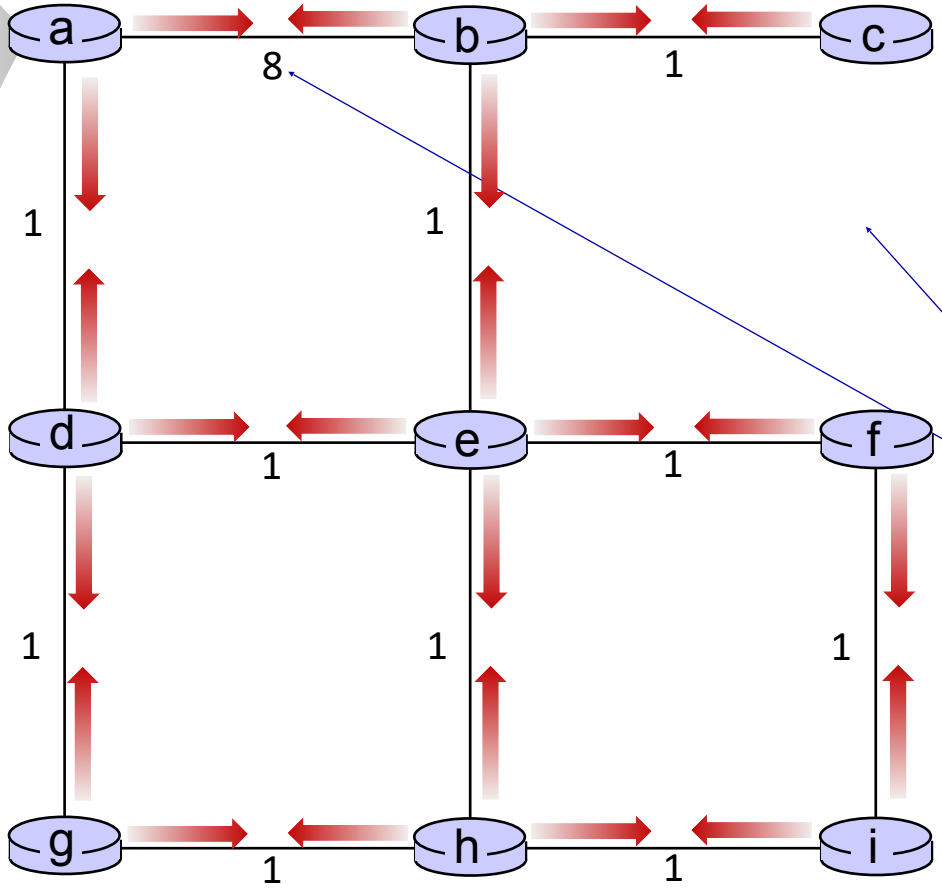
- **Distributed/Decentralized**: routers do not need global knowledge of network topology
- **Iterative**: routes are computed iteratively in response to link cost change or DV updates from neighbors
- **Asynchronous**: routers do not need to synchronize on their route computations, or DV announcements
- **Self-stopping**: neighbors communicate only if necessary, and stop when no notifications are received

Bellman-Ford Algorithm: an example

t=0

- All nodes have distance estimates to nearest neighbors (only)
- All nodes send their local distance vector to their neighbors

DV in a:
$D_a(a)=0$
$D_a(b) = 8$
$D_a(c) = \infty$
$D_a(d) = 1$
$D_a(e) = \infty$
$D_a(f) = \infty$
$D_a(g) = \infty$
$D_a(h) = \infty$
$D_a(i) = \infty$



A few asymmetries:

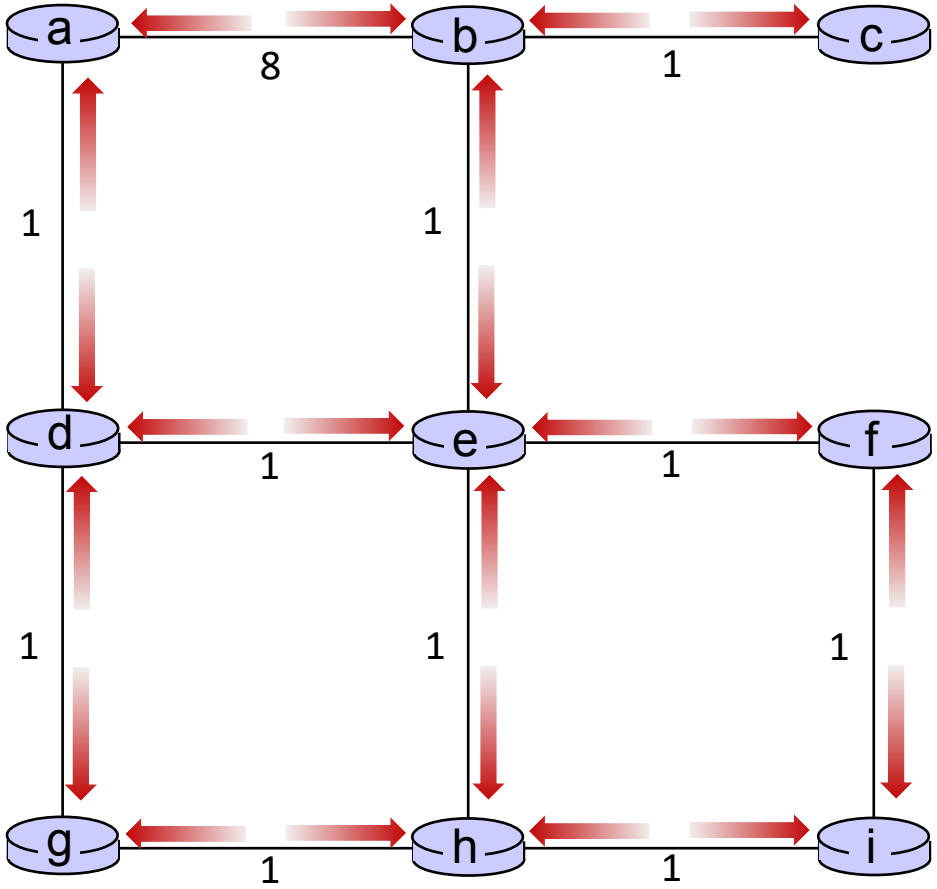
- missing link
- larger cost

Bellman-Ford Algorithm: how iterations work

t=1

All nodes:

- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors



Bellman-Ford Algorithm: how iterations work

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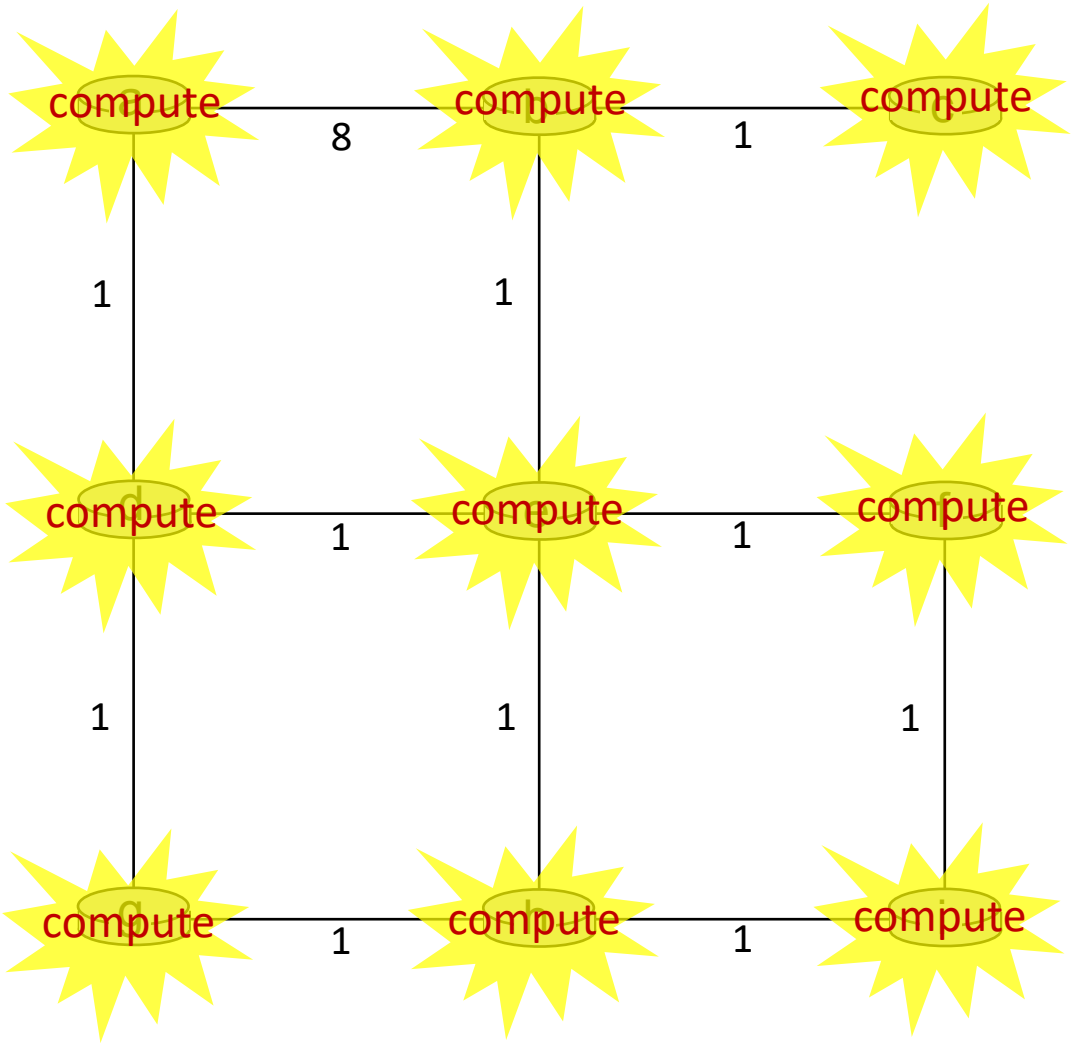

Bellman-Ford Algorithm: how iterations work

t=1

All nodes:

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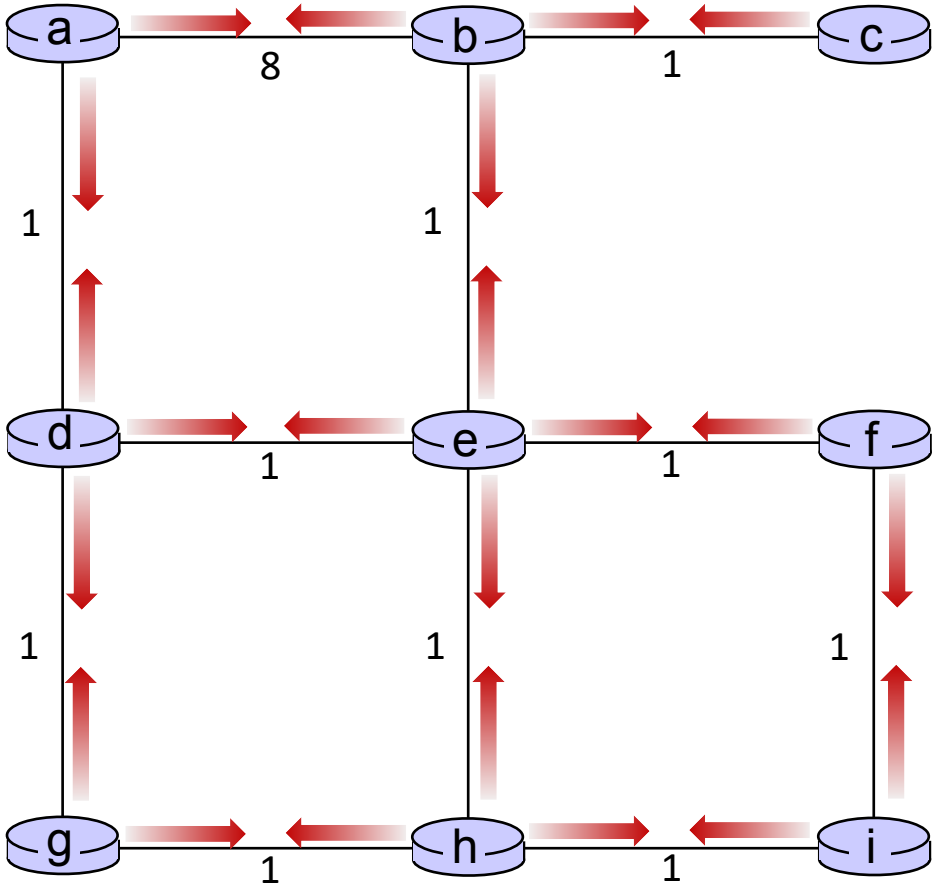
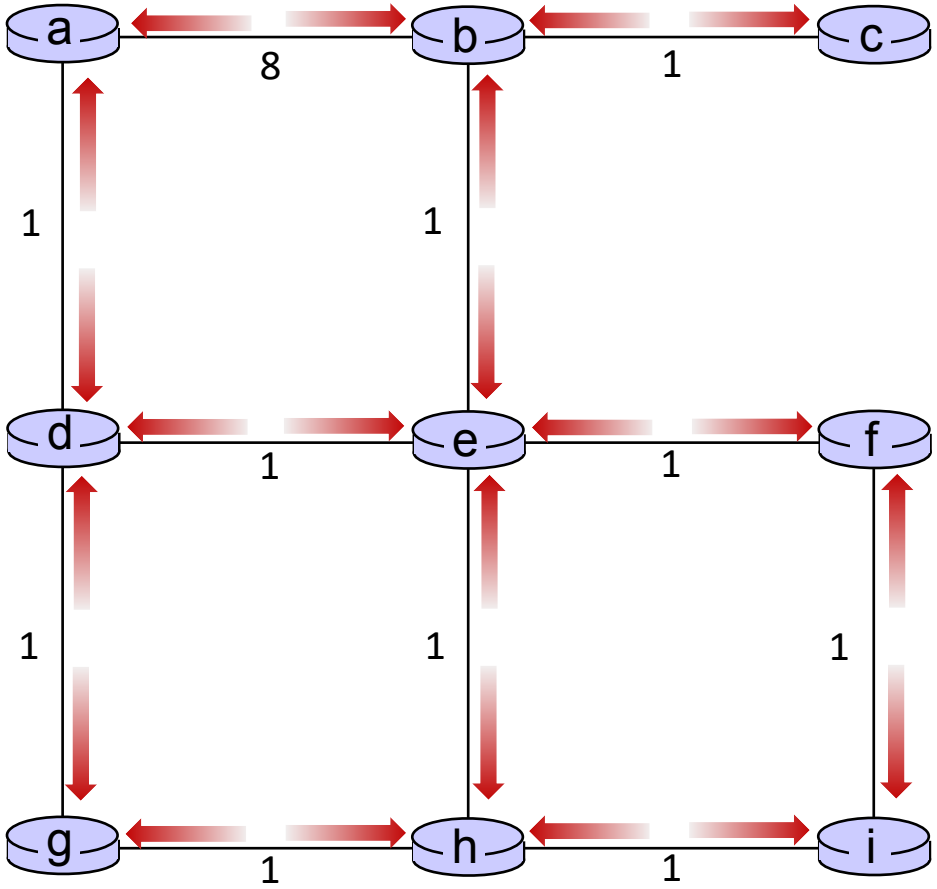


Bellman-Ford Algorithm: how iterations work

t=2

All nodes:

- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors



Bellman-Ford Algorithm: how iterations work

All nodes:

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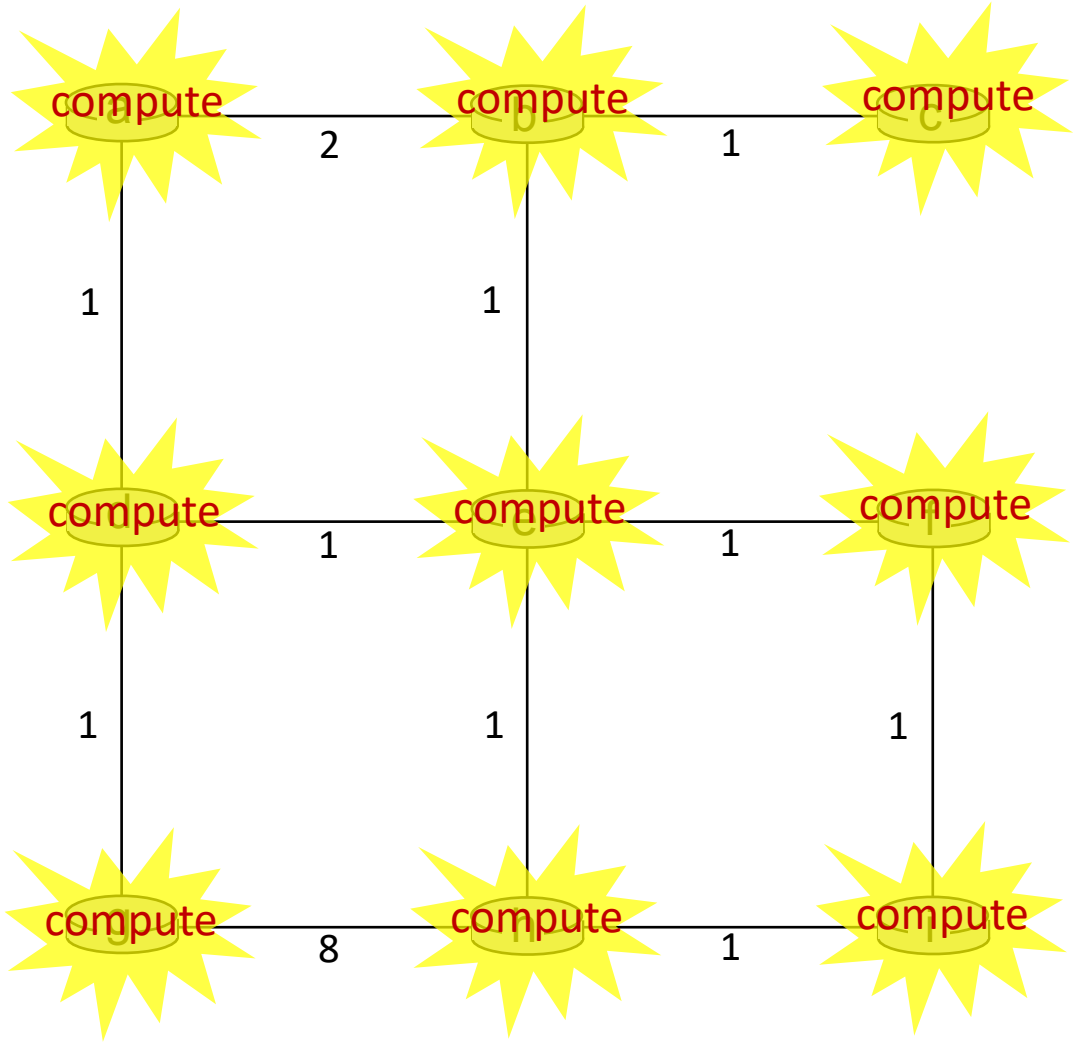

Bellman-Ford Algorithm: how iterations work

t=2

All nodes:

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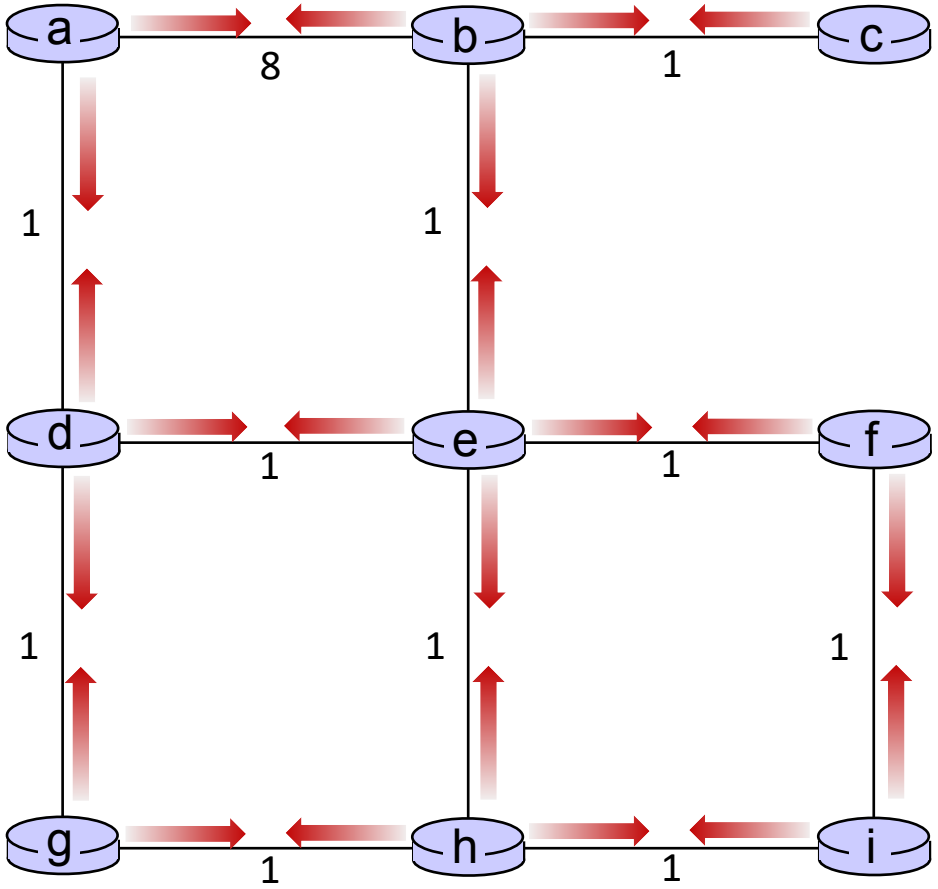
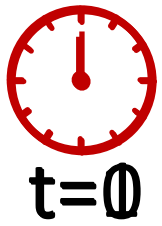
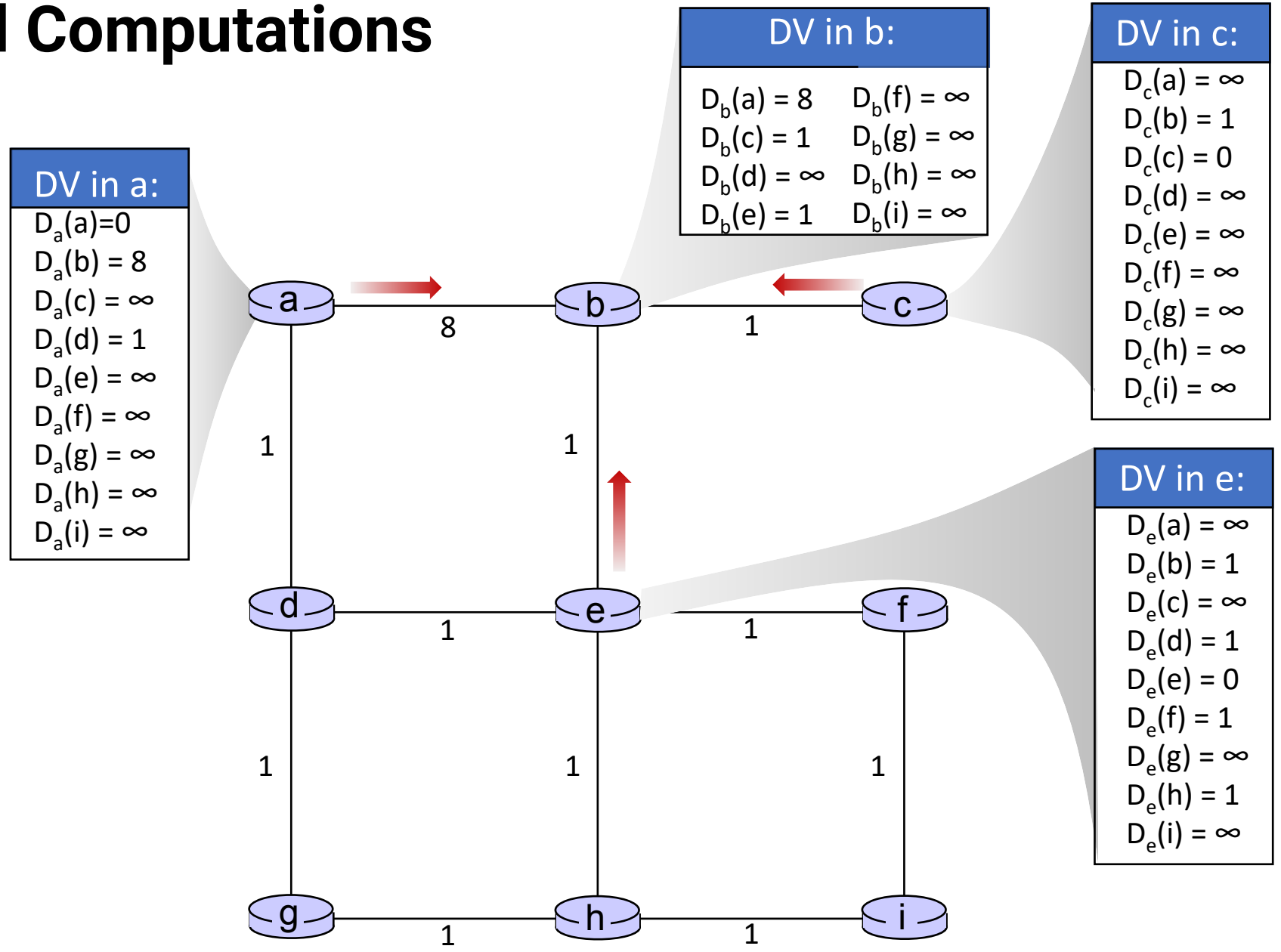
- compute their new local distance vector
- send their new local distance vector to neighbors



Bellman-Ford Computations



- b receives DVs from a, c, e

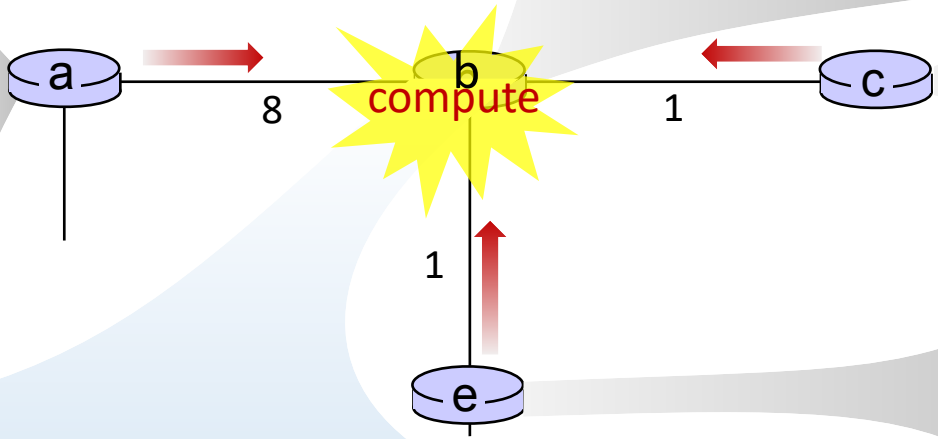


Bellman-Ford Computations

- b receives DVs from a, c, e, computes:

DV in a:

$D_a(a)=0$
$D_a(b) = 8$
$D_a(c) = \infty$
$D_a(d) = 1$
$D_a(e) = \infty$
$D_a(f) = \infty$
$D_a(g) = \infty$
$D_a(h) = \infty$
$D_a(i) = \infty$



DV in b:

$D_b(a) = 8$ $D_b(f) = \infty$
$D_b(c) = 1$ $D_b(g) = \infty$
$D_b(d) = \infty$ $D_b(h) = \infty$
$D_b(e) = 1$ $D_b(i) = \infty$

DV in c:

$D_c(a) = \infty$
$D_c(b) = 1$
$D_c(c) = 0$
$D_c(d) = \infty$
$D_c(e) = \infty$
$D_c(f) = \infty$
$D_c(g) = \infty$
$D_c(h) = \infty$
$D_c(i) = \infty$

DV in e:

$D_e(a) = \infty$
$D_e(b) = 1$
$D_e(c) = \infty$
$D_e(d) = 1$
$D_e(e) = 0$
$D_e(f) = 1$
$D_e(g) = \infty$
$D_e(h) = 1$
$D_e(i) = \infty$

$D_b(a) = \min\{c_{b,a}+D_a(a), c_{b,c}+D_c(a), c_{b,e}+D_e(a)\} = \min\{8,\infty,\infty\} = 8$

$D_b(c) = \min\{c_{b,a}+D_a(c), c_{b,c}+D_c(c), c_{b,e}+D_e(c)\} = \min\{\infty,1,\infty\} = 1$

$D_b(d) = \min\{c_{b,a}+D_a(d), c_{b,c}+D_c(d), c_{b,e}+D_e(d)\} = \min\{9,2,\infty\} = 2$

$D_b(e) = \min\{c_{b,a}+D_a(e), c_{b,c}+D_c(e), c_{b,e}+D_e(e)\} = \min\{\infty,\infty,1\} = 1$

$D_b(f) = \min\{c_{b,a}+D_a(f), c_{b,c}+D_c(f), c_{b,e}+D_e(f)\} = \min\{\infty,\infty,2\} = 2$

$D_b(g) = \min\{c_{b,a}+D_a(g), c_{b,c}+D_c(g), c_{b,e}+D_e(g)\} = \min\{\infty, \infty, \infty\} = \infty$

$D_b(h) = \min\{c_{b,a}+D_a(h), c_{b,c}+D_c(h), c_{b,e}+D_e(h)\} = \min\{\infty, \infty, 2\} = 2$

$D_b(i) = \min\{c_{b,a}+D_a(i), c_{b,c}+D_c(i), c_{b,e}+D_e(i)\} = \min\{\infty, \infty, \infty\} = \infty$

New DV in b

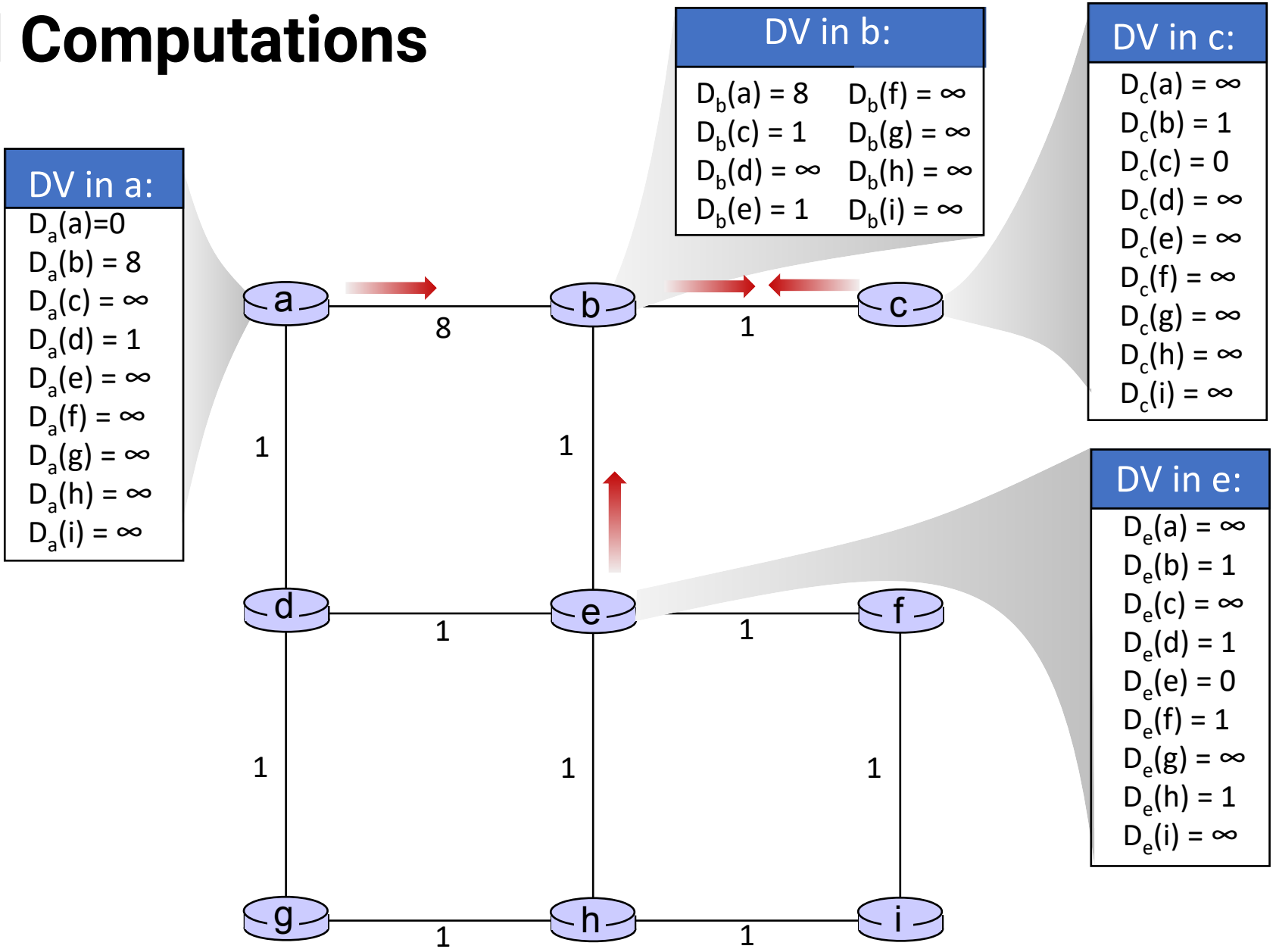
DV in b:

$D_b(a) = 8$ $D_b(f) = 2$
$D_b(c) = 1$ $D_b(g) = \infty$
$D_b(d) = 2$ $D_b(h) = 2$
$D_b(e) = 1$ $D_b(i) = \infty$

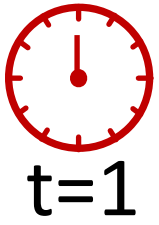
Bellman-Ford Computations

t=1

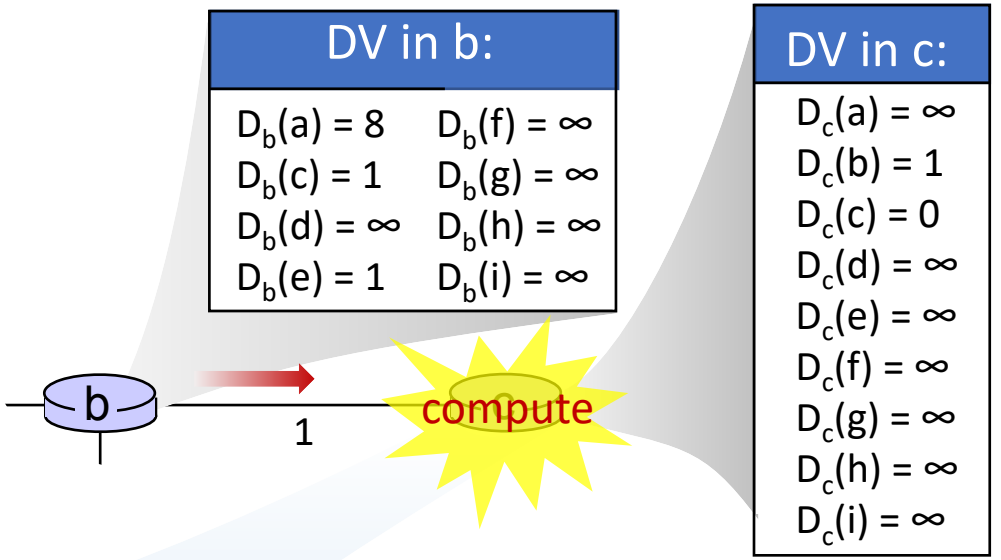
- c receives DVs from b



Bellman-Ford Computations



- c receives DVs from b computes:



$D_c(a) = \min\{c_{c,b}+D_b(a)\} = 1 + 8 = 9$

$D_c(b) = \min\{c_{c,b}+D_b(b)\} = 1 + 0 = 1$

$D_c(d) = \min\{c_{c,b}+D_b(d)\} = 1+ \infty = \infty$

$D_c(e) = \min\{c_{c,b}+D_b(e)\} = 1 + 1 = 2$

$D_c(f) = \min\{c_{c,b}+D_b(f)\} = 1+ \infty = \infty$

$D_c(g) = \min\{c_{c,b}+D_b(g)\} = 1+ \infty = \infty$

$D_c(h) = \min\{c_{bc,b}+D_b(h)\} = 1+ \infty = \infty$

$D_c(i) = \min\{c_{c,b}+D_b(i)\} = 1+ \infty = \infty$

New DV in c

DV in c:

$D_c(a) = 9$

$D_c(b) = 1$

$D_c(c) = 0$

$D_c(d) = \infty$

$D_c(e) = 2$

$D_c(f) = \infty$

$D_c(g) = \infty$

$D_c(h) = \infty$

$D_c(i) = \infty$

Bellman-Ford: Iterative Information Propagation

Iterative communication, computation steps diffuses information through network:

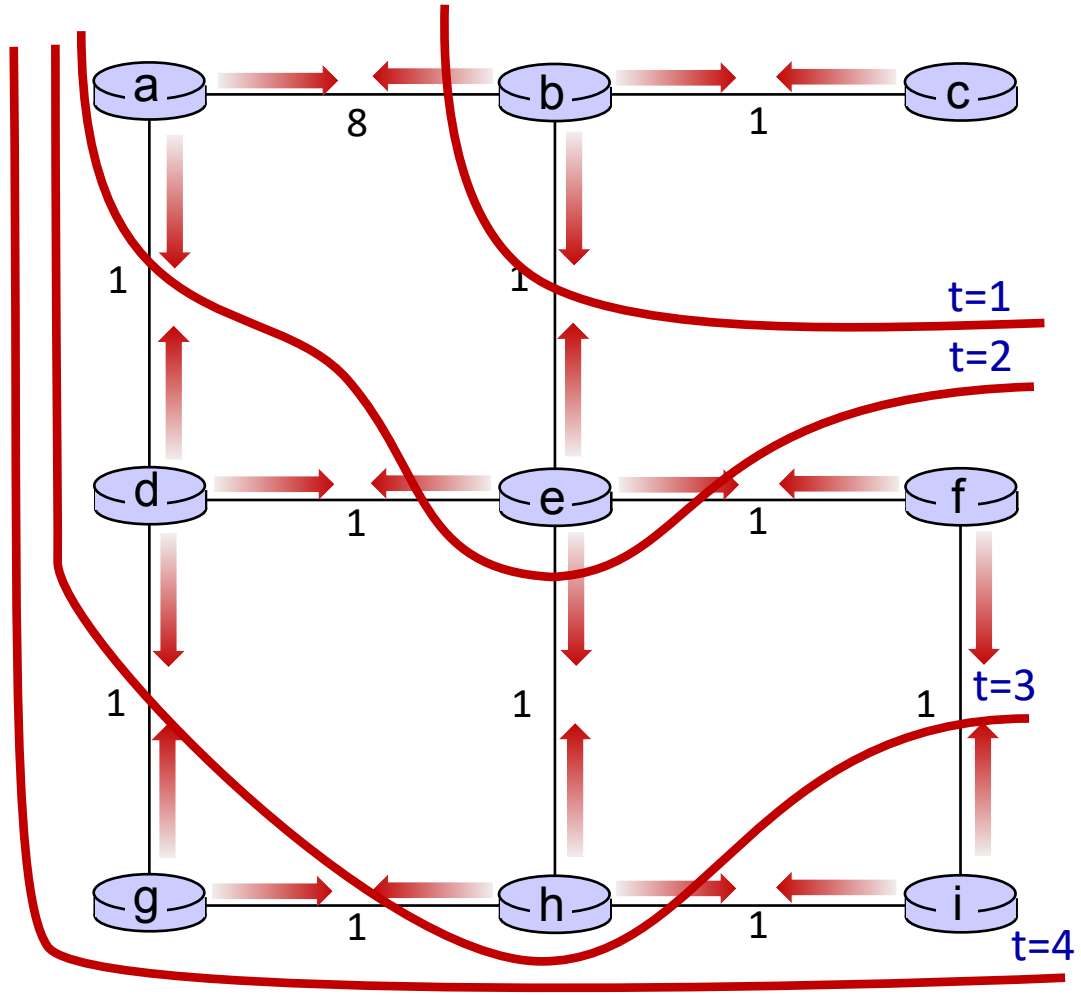
t=0 c's state at t=0 is at c only

t=1 c's state at t=0 has propagated to b, and may influence distance vector computations up to **1** hop away, i.e., at b

t=2 c's state at t=0 may now influence distance vector computations up to **2** hops away, i.e., at b and now at a, e as well

t=3 c's state at t=0 may influence distance vector computations up to **3** hops away, i.e., at d, f, h

t=4 c's state at t=0 may influence distance vector computations up to **4** hops away, i.e., at g, i



Comparing Dijkstra and Bellman-Ford Algorithms

	Dijkstra (LS)	Bellman-Ford (DV)
Algorithm structure	Centralized	Decentralized
Speed of convergence	$O(N^2)$	slower than Dijkstra; O(N*E) in worst
Application	Routing within autonomous systems	Routing across autonomous systems
Robustness	Route oscillations	Routing loops; Count-to-infinity

Exercise for you: read about these three routing problems from the textbook

Spot Quiz (ICON)