JET PROPULSION LABORATORY **ENGINEERING MEMORANDUM**

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From: David S. Bayard
Subject: First look analysis of IMU propagation errors for Mars Aerocapture Demonstration

SUMMARY

This memo presents a preliminary analysis of attitude and position error propagation while on IMU, as relevent to the Mars Aerocapture Demonstration proposal. The analysis is for a period of time beginning when the Mars aerocapture demonstrator vehicle has separated from the Mars orbiter, and ends at the time when it has completed aerocapture and established itself in a new orbit (roughly 30 hours later). This study is intentionally kept simple to support a first-look analysis. Only dominant gyro and accelerometer errors are considered, and central-force issues (i.e., Schuler dynamics) are ignored.

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First Look Analysis of IMU Propagation Errors
For Mars Aerocapture Demonstration

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1 Covariance Analysis

For all analysis in this section it is assumed that the Mars Orbiter has a star tracker, and that the Mars aeromaneuvering demonstration vehicle has access to the star tracker measurement up to the instant of separation. Also, it is assumed that the Mars demonstrator vehicle has been propagating attitude using the Orbiter's star tracker measurement in combination with its own gyros, while simultaneously calibrating its gyro biases up to the point of separation.

1.1 Attitude Covariance Analysis

The per-axis covariance p (scalar) of the attitude error t seconds after the last star tracker update (i.e., separation) is given by (neglecting gyro scale factor, frame alignment, and compliance error terms),

$$p(t) = Cov(e(t)) = \frac{q_2}{3}t^3 + p_{22}(0)t^2 + (2p_{12}(0) + q_1)t + p_{11}(0) + b^2 \tag{1.1}$$

where,

$$\ell = (q_1 + 2\sqrt{rq_2})^{\frac{1}{2}} \tag{1.2}$$

$$p_{11}(0) = r^{\frac{1}{2}}\ell \tag{1.3}$$

$$p_{12}(0) = \sqrt{rq_2} \tag{1.4}$$

$$p_{22}(0) = q_2^{\frac{1}{2}}\ell \tag{1.5}$$

$p_{11}(0)$ - Initial angle error covariance (rad^2)

$p_{22}(0)$ - Initial gyro rate bias error covariance (rad^2/sec^2)

$p_{12}(0)$ - Initial angle-bias cross-covariance (rad^2/sec)

q_1 - Gyro Angle Random Walk (ARW), (rad^2/sec)

q_2 - Gyro Bias Stability (rad^2/sec^3)

$r = \Delta\sigma^2$ - Star Tracker Covariance, per-axis ($rad^2 \cdot sec$)

Δ - Star Tracker sampling period (sec)

σ - Star Tracker NEA, per-axis (rad)

b - Star Tracker bias error, per-axis (rad)

1.2 Position/Velocity Covariance Analysis

The per-axis position error covariance $c_{11}(t)$ (scalar) and velocity error covariance $c_{22}(t)$, at t seconds after the last star tracker update (i.e., separation) is given by (neglecting accel scale factor, frame alignment, and g-sensitive terms, and central-force/Schuler dynamics),

$$\begin{aligned} c_{11}(t) &= \frac{6q_2}{5!}t^5 + 6\frac{c_{33}(0)}{4!}t^4 + \frac{(6c_{23}(0) + 2q_1)}{3!}t^3 + 2\frac{(c_{13}(0) + c_{22}(0))}{2!}t^2 \\ &+ (2c_{12}(0) + q_0)t + c_{11}(0) \end{aligned} \tag{1.6}$$

where the initial covariance at time $t = 0$ of the position/rate/accel error vector $u = [\delta x, \delta\dot{x}, \delta\ddot{x}]$ is defined as,

$$C(0) = Cov[u] = \begin{bmatrix} c_{11}(0) & c_{12}(0) & c_{13}(0) \\ \times & c_{22}(0) & c_{23}(0) \\ \times & \times & c_{33}(0) \end{bmatrix} \tag{1.7}$$

$c_{11}(0)$ - Initial position error covariance (m^2)

$c_{22}(0)$ - Initial velocity error covariance (m^2/sec^2)

$c_{33}(0)$ - Initial acceleration bias error covariance (m^2/sec^4)

$c_{ij}(0)$ - Initial cross-covariances ($m^2/sec^{(i-1)+(j-1)}$)

q_0 - Position Random Walk (m^2/sec)

q_1 - Velocity Random Walk (m^2/sec^3)

q_2 - Accel Random Walk (bias stability) (m^2/sec^5)

2 Assumptions

2.1 Hardware Assumptions

STAR TRACKER: (e.g., OCA, 25 deg FOV)

NEA: σ=333e-6 (rad), 1-sigma, worst-case axis

Bias: b=333e-6 (rad), 1-sigma, worst-case axis

Update rate: $\Delta = .5$ (sec)

GYRO: Honeywell MIMU (RLG-GG1320)

ARW: .025/3 ($deg/\sqrt{hr}$), 1-sigma, per-axis

Determines q_1=5.8761e-12 (rad^2/sec)

Bias Stability: .05/3 (deg/hr), 1-sigma, per-axis

Determines q_2=1.8138e-18 (rad^2/sec^3)

GYRO: Litton LN-100s (ZLG)

ARW: .0007 ($deg/\sqrt{hr}$), 1-sigma, per-axis

Determines q_1=4.1462e-14 (rad^2/sec)

Bias Stability: .003 (deg/hr), 1-sigma, per-axis

Determines q_2=5.8761e-20 (rad^2/sec^3)

ACCEL: Honeywell MIMU (QA-2000) and LN100S Accel

(For simplicity the hardware assumptions for the Honeywell and Litton accelerometers are taken to be identical).

Sampling Freq $f_s = 200\ (Hz)$

Position Random Walk: velocity read noise (.003/3) m/sec (1-sigma)

Determines $q_0 = (.003/3)^2/f_s\ (m^2/sec)$

Velocity Random Walk: $q_1 = 0$

Accel Random Walk: bias stability (.181/3) milli-g (1-sigma over 12 months)

Determines $q_2 = 1.124e - 14\ (m^2/sec^5)$

Velocity Quantization: .0027 m/sec/count

Determines 60 sec in-flight bias cal accuracy $c_{33}(0) = (.0027/60)^2\ (m^2/sec^4)$

2.2 Approach Navigation Assumptions

A first-look approach-Nav covariance matrix was received from Dan Burkhart. The covariance is resolved in the "Mars Mean Equator and Prime Meridian of Date" coordinate frame, and is given below,

$$P = Cov[u] \tag{2.1}$$

$$u = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T \tag{2.2}$$

x, y, z - position errors in km

$\dot{x}, \dot{y}, \dot{z}$ - velocity errors in km/sec

$$P = \begin{bmatrix} 3.9290e-001 & -5.0078e-002 & -1.0918e-001 & -1.6848e-004 & -1.9538e-004 & -2.7377e-005 \\ -5.0078e-002 & 6.7331e-001 & 1.0764e+000 & -2.2722e-004 & 7.8888e-004 & 2.3855e-004 \\ -1.0918e-001 & 1.0764e+000 & 1.7653e+000 & -3.5091e-004 & 1.2978e-003 & 3.7458e-004 \\ -1.6848e-004 & -2.2722e-004 & -3.5091e-004 & 1.6506e-007 & -2.0206e-007 & -7.5660e-008 \\ -1.9538e-004 & 7.8888e-004 & 1.2978e-003 & -2.0206e-007 & 9.8884e-007 & 2.7819e-007 \\ -2.7377e-005 & 2.3855e-004 & 3.7458e-004 & -7.5660e-008 & 2.7819e-007 & 8.6098e-008 \end{bmatrix} \tag{2.3}$$

2.3 General Assumptions

1. Honeywell MIMU (GG1320 gyro, QA2000 accel) on aerocapture demonstration vehicle
2. Accel bias= 4.5 micro-g (1-sigma, per-axis). This requires that the accelerometer bias is calibrated in-flight
3. No g-sensitive errors, scale factor or alignment errors have been included
4. The coupling between translational and rotational errors has been ignored.
5. Deep space covariance propagation for now (no central force Schuler dynamics)
6. Attitude from a 300 urad Star tracker @ 2 Hz (1-sigma, per-axis) available prior to separation

3 Results

3.1 Attitude Error Propagation

The propagation of the attitude error is shown in Figure 3.1.

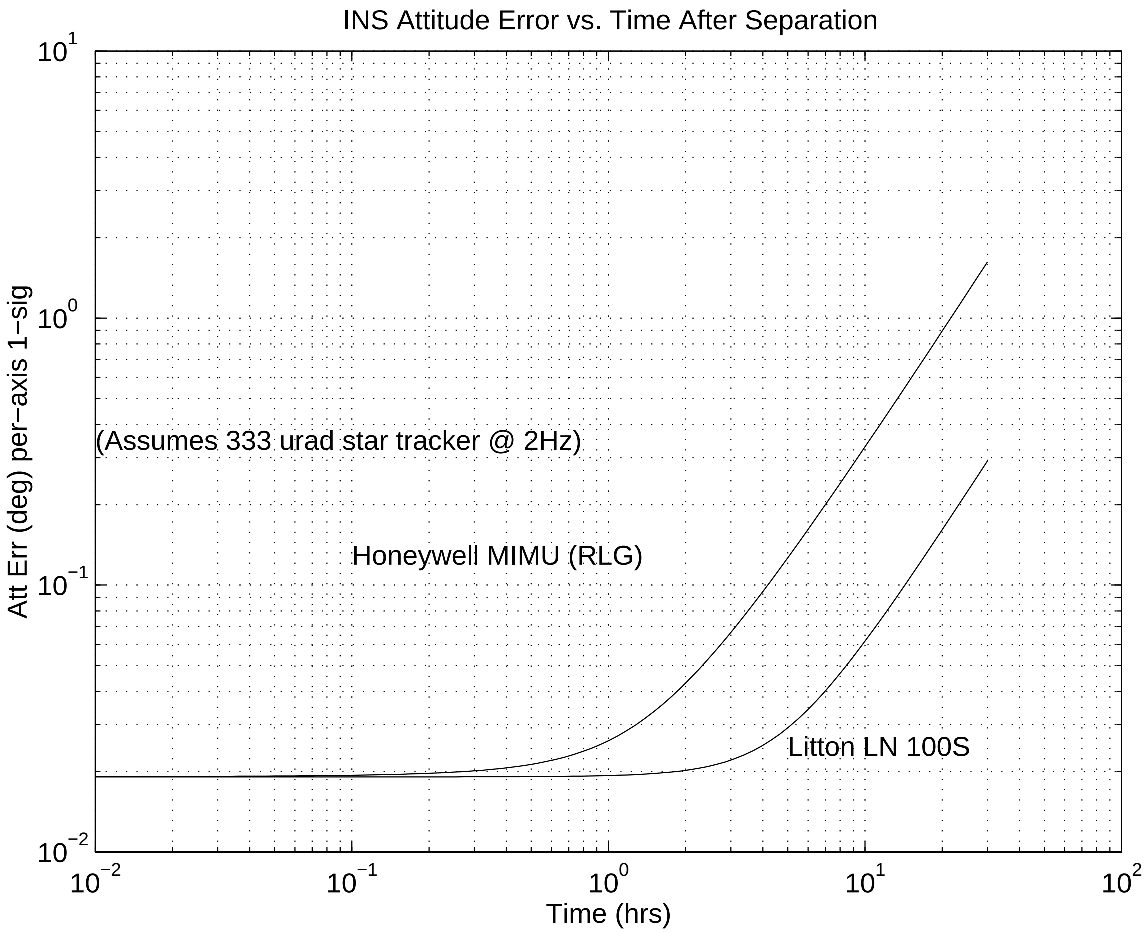

Figure 3.1: Attitude error of Mars aerocapture demo vehicle versus time since last star tracker measurement (i.e., separation)

3.2 Velocity Error Propagation

The propagation of the velocity error was calculated by diagonalizing the lower right covariance $P(3:6,3:6) = QDQ^T$ and propagating each of the diagonal entries of D using the scalar formula(1.6). The result is then mapped back to the original coordinates using Q and is shown in Figure 3.2.

Only one set of plots is shown since the hardware assumptions for the Honeywell and Litton accelerometers are taken to be identical.

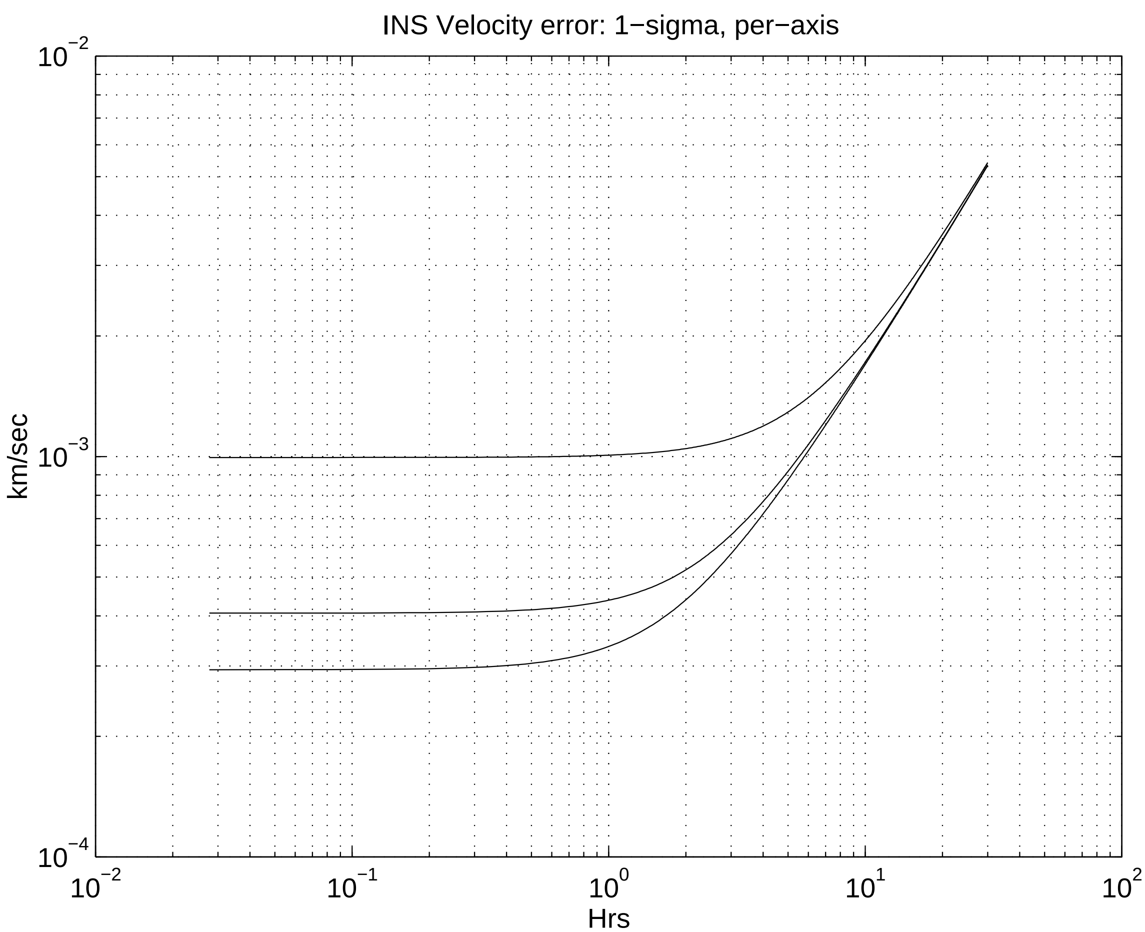

Figure 3.2: Velocity error of Mars aerocapture demo versus time since separation

3.3 Position Error Propagation

The propagation of the position error is shown in Figure 3.3.

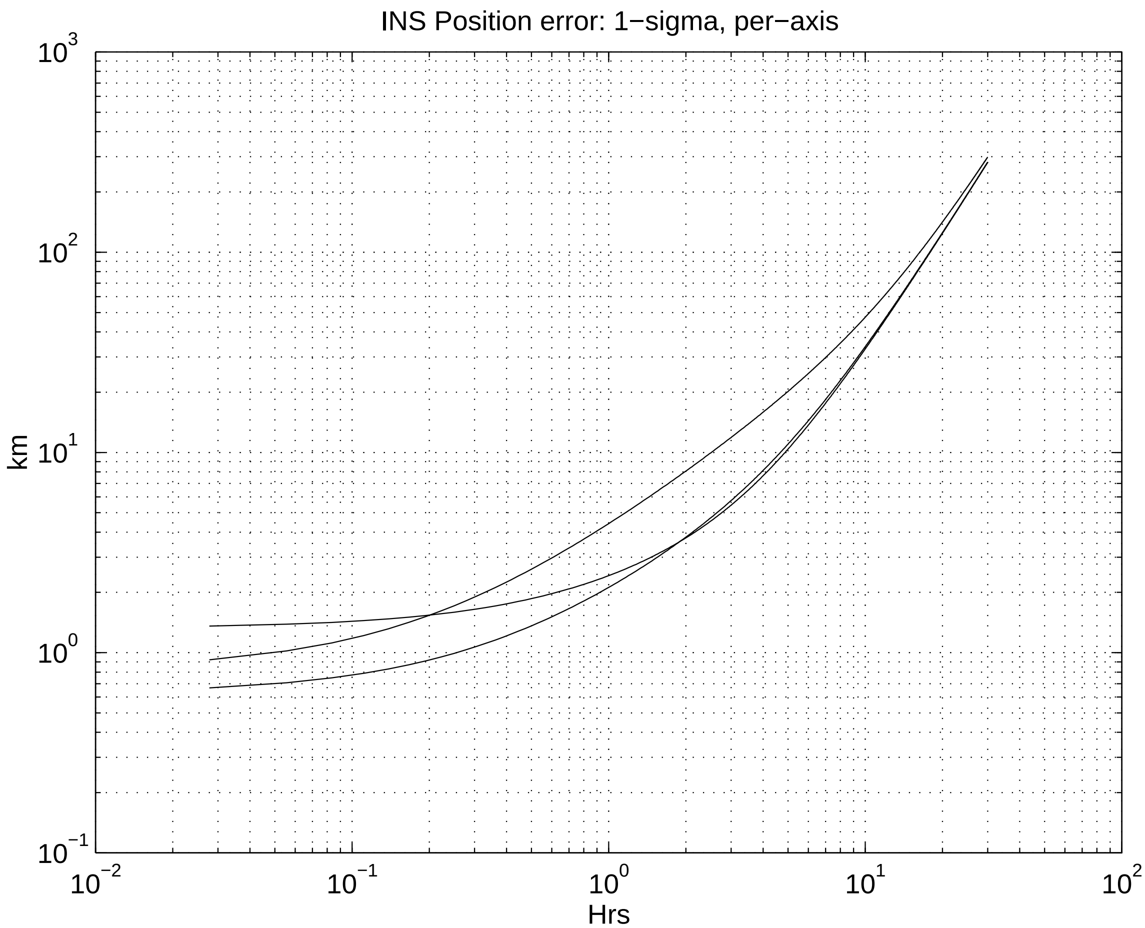

Figure 3.3: Position error of Mars aerocapture demo versus time since separation

For comparison with Figure 3.3, the position error due soley to the initial velocity error is shown in Figure 3.4. In principle, this level accuracy is not achievable since it would correspond to having a perfect IMU. However, this plots serves to show the large error contribution due soley to the uncertainty in the initial conditions.

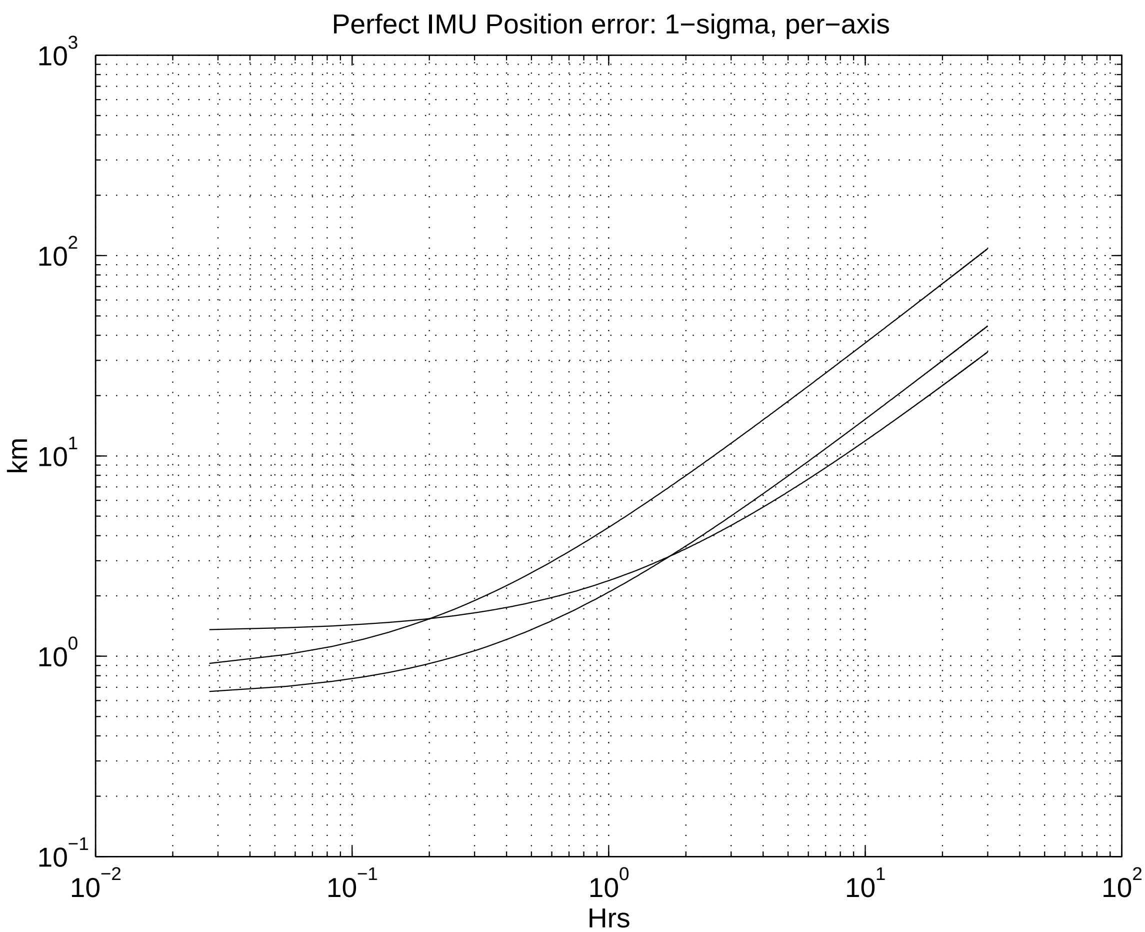

Figure 3.4: Position error due to initial position and velocity errors only - equivalent to state propagation with a perfect IMU

4 Comments

Scalar covariance analysis formulas were given for attitude and velocity/position error propagation, relevant to the time interval after the Mars aerocapture vehicle has separated from the orbiter.

Several comments are worth making related to this study.

1. All results are 1-sigma, per-axis
2. All results are based on optimal filtering theory and have ZERO MARGIN.
3. The addition of g-sensitive errors and scale factors will make results worse (in angle, and position/velocity along direction of maximum deceleration).
4. The addition of Schuler dynamics for INS will act to bound error growth and make the results better in horizontal position and velocity only. The altitude errors are not improved.
5. The use of an in-flight accelerometer calibration function has been assumed to give the bias to an accuracy 4.5 micro-g. This appears to be a reasonable goal given the accuracies which were obtainable on Cassini.