CHAPTER 3: LOGIC PUZZLES

Are you a "cat person" or a "dog person"? How irrelevant a question, I say! But there are people who will ask such a question, trying to label individuals as belonging to simple, black-and-white contrasting groups, as if it is impossible to like both kinds of animal. In the world of puzzles there is a similar question one hears all the time, using the archetypal puzzles featured in newspapers: "Are you a crossword person or a sudoku person?" In some crowds, you might see some rolled eyes if you give the "wrong" answer, as if there is such a thing.

If you forced me to self-identify, I'd have to say I'm a logic guy. I'm good with numbers and abstract reasoning. My skill at solving puzzles has taken me around the globe to World Sudoku Championships and World Puzzle Championships, and I've won my share of individual and team titles at those events. So when Mike asked for a coauthor to help him write the Puzzlecraft feature in *Games*, I knew I was being asked to be the logic person as a counterpart to Mike's incredible word expertise. I'll warn you up front that I don't write logic puzzles the way many people make such puzzles. Most people program a solver and then output lots of similar puzzles by computer-generation. If you are looking for some basic routines in C++ that will make countless sudoku, you're reading the wrong book. But hand-constructing logic puzzles can achieve elegance in a way no other approach can. Would you let a computer generate a crossword? Perhaps if you didn't care about the quality of the words, the cleverness of the theme, or the originality of the clues!

For the foreseeable future, computers will simply not have the creativity to stretch the limits of a puzzle type and explore thrilling new places, whether in a word puzzle or a logic puzzle. If you are reading a book about how to construct puzzles, you probably want to make really great puzzles that stand out. So I'm here to offer a human perspective on constructing logic puzzles, allowing you to experiment with existing types and to create whole new ones. I hope the simple steps I offer here can teach you a thing or two about how to write (and solve) the logic puzzles you'll find out there.

So what are the characteristics of a logic puzzle lover? It starts at an early age with a fascination with numbers. If you've ever made a conscious note of when you were at a "perfect" age (like 28),[32] or in "a prime of your life," you probably have the bug. If you find the number 1729 fascinating,[33] you're likely to call this chapter home. There is a classic numbers game, the "24 Game," where you are given four integers and must use all of these numbers in some order and any of the four basic operations of addition, subtraction, multiplication, and division to equal 24. One challenging example, from my good friend and four-time world puzzle champion Wei-Hwa Huang, is the set {3,3,8,8}.[34] I often find myself playing the 24 Game with the numbers found inside a fortune cookie. What can I say? I've got the bug.

But logic puzzlers aren't just about playing with numbers; the joys of mathematics include more than just arithmetic. For many of us, Martin Gardner's columns in *Scientific American*, which explored the beauty of recreational mathematics, showed the way to lots of interesting mental sandboxes (hexaflexagons, Conway's Game of Life, Penrose tiling …). Playing in artificial sandboxes is what defines logic puzzling, at its core, for me. You might not enter a puzzle knowing what to expect or how to solve it, but you'll find yourself performing some mental gymnastics on the way to finding out. What at first seemed obtuse and impossible becomes more transparent and obvious as you learn how to determine what must be true. You'll leave the exercise feeling smarter for having gone through it, and want to find more puzzles to test these newly discovered rules.

In this section we'll look at puzzles that deal with numbers, paths, and more abstract things. In each case we'll be chasing after the "truth," the single solution that must follow from understanding how to obey the puzzle's short set of rules. I've found this chase fun for many, many years; hopefully you will too.

CHAPTER 3A: NUMBER LOGIC PUZZLES

It's a bit ironic that the iconic number logic puzzle isn't really that much about the numbers, at least compared to many other number puzzle types. Sudoku, the puzzle that swept the world starting in 2005, can be solved with any set of nine unique symbols, but the digits 1 to 9 are a convenient set that works in almost any language. Sudoku is more of a logic puzzle than a numbers puzzle. Still, I bet some of the joy people experience when solving sudoku is the feeling of conquering the numbers in the 81-cell grid, when other tasks that involve arithmetic, like making correct change, seem much harder.

[32] 28 is a perfect number, equal to the sum of all of its smaller factors ($1+2+4+7+14$).

[33] In a famous anecdote, mathematicians G.H. Hardy and Srinivasa Ramanujan were riding in taxi number 1729 when Hardy made the comment that 1729 was an uninteresting number to him. Ramanujan responded straightaway that it was actually a very interesting number, being the smallest natural number that can be represented as the sum of two cubes in two different ways. Such numbers are now known as "taxicab numbers."

[34] The solution is $8 \div (3 - (8 \div 3))$.

It's also a bit ironic that sudoku is considered a "Japanese puzzle" when its origins lie much closer to home. The story of sudoku begins with an Indiana architect, Howard Garns, who created a numbers game that involved placing digits into boxes so that no digit repeated in any row, column, or 3×3 region. Garns sent some of his puzzles to Dell Magazines, which printed the first "Number Place" puzzles in 1979. Number Place eventually made its way over to Japan in the mid-1980s, when traveling puzzlemakers saw Garns's idea in *Dell Pencil Puzzles and Word Games* magazine. While the rules never changed, the presentation of the puzzle evolved in Japan. The number of clues was reduced, leading to much harder (and more interesting) puzzles than had been published before. Also, with an eye toward the beauty of the grid, rotational symmetry became the norm, and themes in the layout of the digits became commonplace.

While most Japanese publishers used the transliteration "nanpure" (from Number Place), one publisher, Nikoli, came up with its own name: *Sūji wa dokushin ni kagiru* ("the digits must be single"), which was quite a mouthful and was quickly shortened to "sudoku." Nikoli's puzzles attracted the attention of a retired New Zealander judge, Wayne Gould, who spent several years trying to replicate Nikoli's sudoku by computer generation. Gould took his creations to the *Times* of London, which picked up the puzzle as "Su Doku." Within weeks, most other British newspapers had also started running their own forms of the instantly popular numbers puzzle. Many newspapers took to advertising just how many sudoku their pages contained—*above* the main headline of the day. Within months, the puzzle had returned to America with a new Japanese name and a popularity that Howard Garns may never have expected. Unfortunately, Garns did not live long enough to see his puzzle idea conquer the planet, but he would likely be proud that his simple creation has led to a recent boom time for logic puzzles of all sorts.

In this chapter I'll explore how to create sudoku puzzles, and then use the cornerstone of that advice—basically, "write logic puzzles as if you are solving them"—to show you how to make many other number puzzles, including some that really use arithmetic. I'll give you the basic construction rules in each case, but I'll leave it as an exercise for the reader to find new ways to stretch the rules to achieve amazing things. While nothing changed in the rules from the time Dell published Number Place to what we see now in sudoku from Japan, the focus on the elegance of the puzzle design, its appearance, and capturing "beautiful" steps of logic, is what turned sudoku into an art form. The same care of design is what I strive to capture in every puzzle I make, and what I hope to teach you as we go along.

SUDOKU

Enter a single digit from 1 to 9 into each cell so that no digit repeats in any row, column, or bold 3×3 region.

Easy as 1-2-3

		5	6		4	7		
			5		3			
1				8				2
4	2						1	3
		6				8		
5	1						4	7
6				7				5
			3		1			
		7	8		2	3		

Four-Leaf Clover

		5			9	6		
	2	9	5		7	4	8	
	4	3	7		5	1		
				3				
		1	2		6	3	5	
	9	2	4		3	5	6	
		6	9			7		

Window Frame

7	6		2		8		9	3
8			5		6			2
2	3						6	4
				4				
4	7						5	8
9			6		5			1
1	8		7		9		3	6

CRAFTING A SUDOKU

There is no more ubiquitous number puzzle these days, with the sudoku sharing the puzzle page with the crossword in almost every U.S. newspaper. While you've probably spent many hours learning solving tricks that go by weird names like "naked single" and "X-Wing" (though you may not have known their names), you probably haven't spent anywhere near as much time thinking about how you'd actually go about making a sudoku. Learning the ins and outs of crafting these puzzles is a prerequisite for any serious puzzlesmith, and if you follow these steps you too will be able to make elegant sudoku on par with those from Japan.

1. Choose a pattern of givens

While the first Number Place puzzles had 36 or more givens, randomly arranged, the Japanese puzzlemakers that turned this idea into sudoku learned that fewer givens leads to more challenging and interesting puzzles. So follow their lead and use fewer. While the minimum possible in a classic sudoku puzzle is 17, anything from 24 to 32 is a manageable number, with more givens often meaning an easier puzzle.

Lay out a pattern of positions for the givens—not the numbers, just the positions—that contains some symmetry (rotational, vertical, or horizontal). Consider the density of positions in each row, column, and region. Any row or column where you are placing six or more digits is going to be an easy starting point most of the time. On the other hand, it is harder to construct a puzzle that has several completely empty rows or columns or regions—and some patterns, such as having no clues in the first two rows, will have no possible unique puzzles. Find a good balance between dense and sparse rows and you are set.

For these puzzles, I chose some different visually appealing patterns with 28 or 29 givens. In "Easy as 1-2-3," the entire center region is empty, but four of the regions have five givens and will be easy starting points to set up logical deductions.

Certain geometries can have very critical givens put in by design; in Window Frame, that central position is the only given in the middle row and column. But it can eliminate all but two empty cells in the adjacent regions, depending on what numbers go in the other positions.

2. Identify several digits to set up a first break-in point

Your paper copy should now have a bunch of shaded or X'd out cells but no digits inside them. Your goal is to specify these cells to give just one possible solution.

Make two copies of the grid. Use one to mark just the clues and clue positions, and the other to try to "solve" the puzzle as you put new givens into the shaded cells. Enter a handful of digits so that some digits outside of the shaded cells are now forced to contain a single digit themselves. If there is a good spot to put in an interesting set of digits (like a diagonal with 1–9 in it) that can start a theme, slot that in at the beginning of the process.

One of the first construction steps to learn is how to choose a few positions to contain the same digit, forcing all the other instances of that digit. When choosing given locations to contain a digit, you can view all the other possible locations as X's, or "not your number." You will eventually control all the digits that go into the X's, so you can force them to not be a particular digit as need be. Once you've done this, look at the resulting placements. Do any filled squares offer new opportunities for forced deductions? Is a row, column, or region almost complete? Build off your progress.

In "Easy as 1-2-3," I tried to accomplish two different goals. First, I wanted a puzzle that solved from 1 to 9, meaning a solver could look at the 1s first, place all nine of them, then move onto the 2s, place all nine of them, and so on. I also wanted a grid with no 9s as givens, to highlight the impossibility of starting off at 9 as opposed to 1.

In the grid here you can see some of the early steps in constructing this puzzle. To force a 1 in row 7, column 3 (R7C3), for example, I could specify the 1s in R3C1, R6C2, and R8C6. Placing a fourth given 1 at R4C8 forced the other four 1s. So however the rest of the puzzle turned out, those four given 1s would allow the first step to be writing the other five 1s. Then I started with the 2s. Having filled in the cell in R1C5 with a 1 in the previous step, there was an easy clue spot to put a 2 in to force a 2 in R2C5, and I worked from there.

2		X	X	1	X	X		
			X	2	X	1		
1				X				2
X	2						1	X
		X	1			X	2	
X	1		2				X	X
X		1		X		2		X
		2	X		1			
		X	X		2	X		1

3. Plot out more complicated steps

Instead of just using "singles," as we did in the last step, to place an individual digit into one cell, you can also experiment with forcing groups of more digits into sets of cells, such as pairs into two cells or triples into three cells. While these won't specify exact placements yet, the blocked out cells may force something else in the surrounding areas.

Or consider other deductions you've discovered while solving. The hardest logical step you embed in the puzzle will be the hardest step required in the solution path, although it may be possible for it to be circumvented.

In the grid below you'll see the initial step I planned for "Four-Leaf Clover." Triples are sometimes pretty tricky to spot, so I thought I'd embed a triple right off the bat, as you can see with the digits 2, 5, and 6 in row 6 and column 3. This gave me a few options to force a digit in that same region, either in R5C3 (using the given in R6C7), or in R6C1 using two givens in columns 2 and 3. Since filling something into the empty first column felt more important, I went with the two 9s as shown. The hard part was not hiding this triple, but making sure looking at this region to place a 9 was the required first step.

		5			X	X		
	X	9	X		X	X	X	
256	X	X	X		X	X		
256	256			X				
9		X	2		6	X	5	
	9	2	X		X	X	X	
		6	X			X		

In "Window Frame," the hard steps were nested within each other. The first placements are in the center of rows 1 and 9, where there is only a single choice left given all the other digits put into the top and bottom rows. Once the two naked singles are found, lots of pairs result (all marked in my "solved" copy as shown). That a 3 must be in R3C4 or R3C6 is a direct result of several other pairs, and now clears out a crafty placement in the upper left of the puzzle.

7	6	45	2	1	8	45	9	3
X		3	5	79	6			X
			34	79	34			
X	3			56			X	X
				4				
X	X			56			X	X
			14	38	14			
X			6	38	5			X
1	8	45	7	2	9	45	3	6

4. Finish off the puzzle

As you start to fill the solution cells in your grid, you'll soon reach points where you have defined all but one cell in a row, column, or region. This means that even without specifying a given there, you can specifically make that spot a 4 or 6 or 9 or whatever. You should shade or X out these cells once you find them, figuring out what regions of the grid are least specified and need to be filled. If you don't get digits into these tricky places sooner rather than later, you'll never get to just one solution. If you back yourself into a corner with no possible answer, undo a few steps and try something different. If you are really in trouble, you can always add a given or two into your puzzle as well, but if this ruins your pattern then backtracking and persevering may be a better choice.

Once you've gotten all the way to the end, check your work to make sure the puzzle is what you intended. Check if there are any "shortcuts" that don't follow your intended path. There are some good online resources to help with this step. (I'm partial to the solver at sudokuwiki.org, which lists the steps needed to go through your puzzle, and will tell you if there is just one solution or more than one.) Or have a friend test the puzzle out for you.

With "Easy as 1-2-3," it was after I'd placed the first six numbers that I had to backtrack a few times to make sure I'd specified enough digits in the center to avoid a uniqueness problem. The most common issue is with pairs of two cells in different boxes with an either/or choice. The first time through, I had this exact situation, as I'd left four cells empty where rows 4 and 6 and columns 3 and 6 intersect. This gave two answers and no easy way to specify just one, so I went back and got a 6 into R4C6 earlier since I knew this was a problem spot.

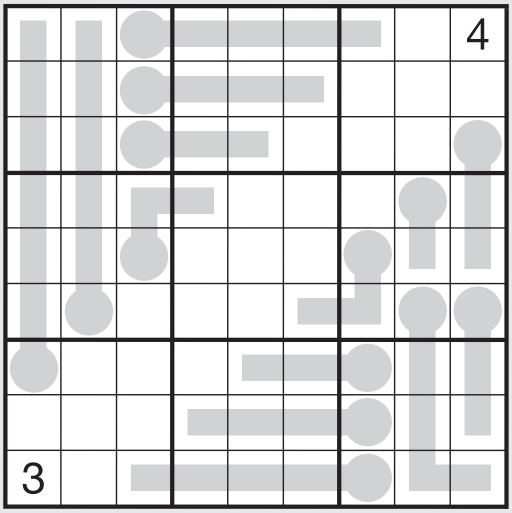
With "Four-Leaf Clover," every placement after the first was a very careful choice to make sure I did not give an alternate entry point to the puzzle besides the triple. Keeping a mostly blank (unsolved) copy next to my work space helped me make sure those extra givens at the end didn't give anything new. I could keep viewing the puzzle "fresh" without all my following deductions in place. There's a single 5 you can place in the upper right corner, but it doesn't build to anything else until you spot that intended 9 in the middle left.

CRAFTING SUDOKU VARIATIONS

In some of my other work (such as in *Mutant Sudoku*, which I cowrote with Wei-Hwa Huang), I've twisted sudoku in all sorts of ways: squeezing and stretching the grid, moving the givens from the inside of the grid to the

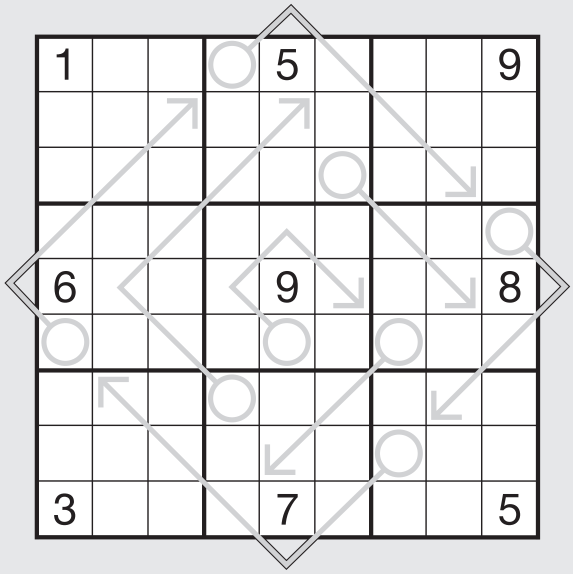
THERMO-SUDOKU

Follow standard sudoku rules. In each thermometer shape, the digits must strictly increase (not necessarily consecutively) from the round reservoir to the flat end.



ARROW SUDOKU

Follow standard sudoku rules. In each arrow shape, the digit in the circled cell must equal the sum of the digits along the path of the arrow.



outside, and many other variations. These create a wide range of challenges with new logical deductions to be made. Here I focus on different ways to vary a sudoku, although this general advice can apply to many other puzzle types.

1. Try new number constraints

While the defining constraint of a sudoku should always be the uniqueness of digit entry in a row, column, or region—this is what makes a sudoku a sudoku, after all—you can put in new constraints on where numbers go to enrich the experience. Extra regions, diagonal constraints, and the like won't do too much to discomfit the solver but do add new solving complexities.

I particularly like new constraints that guide the placement of numbers. In a nonconsecutive sudoku, for example, consecutive digits (like 1 and 2) cannot be placed in adjacent cells. This leads to a lot of new deductions. Many types of number placement constraints exist, from positional ones (no diagonal touching, no identical numbers a "knight's move" away) to relational ones (no two numbers that sum up to 10 can be in adjacent cells).

One simple constraint I like is the concept of number order, with greater-than or less-than constraints. I created a type, Thermo-Sudoku, where the smallest digits in a series must be placed near the "bulb" shape of a thermometer and the largest digits placed at the exterior. To construct such a puzzle, I drew in a few such shapes and marked all the candidates by ranges that could go into each cell of a thermometer shape. A 9-cell thermometer is clearly 123456789, and a 7-cell thermometer contains one of (123) in its first cell and one of (789) in its last cell. Because 1s and 9s can only be at the start and end of a thermometer shape, I could use the presence of middle sections in a region to guide the initial placements. In the middle left, there is only one spot the 9 can go, for example. In the bottom middle, there is only one spot the 1 can go. My first version of the puzzle actually had givens all along the lower-left-to-upper-right diagonal. But in testing it I found it was unique with just the two givens I left behind because the thermometer shapes constrained so much.

2. Try major changes in logic

Sudoku are symbol placement puzzles that happen to use numbers, but do not strictly need to use numbers. Yet the use of numbers suggests the possibility of adding mathematical layers onto the puzzle for additional challenge.

In one common variation called Killer Sudoku, all the givens in the grid are removed and instead a set of outlined cages is provided, with the sum of the digits that go into each cage given in the corner. A 2-cell cage with a 4-sum must be 1+3 in some order, for example (to give you some idea how to get started on such a puzzle). The geometry of cages can often be used to hide additional discoveries. If a large set of cages covers all but one of the cells in a set of two rows and no other cells, then you can sum up the values of all those cages and subtract the result from 90 (the sum of two rows of 1–9) to get the value of the missing cell. Not everyone likes doing this much arithmetic when solving puzzles, which may be why the puzzle got named “Killer Sudoku.”

I like adding some, but not a lot, of arithmetic into sudoku. The style Arrow Sudoku does this by putting a few mathematical groupings into the puzzle. The digit in the circled cell must equal the sum of all the digits along the path of the arrow. Letting the arrows jump outside and then bend back into the grid is half the fun of the design here. Especially long arrows need large numbers (like 8 and 9) in the circled cells and small numbers (1 and 2) in the arrow bits.

A good first step is to try to draw an arrow pattern and see if any possible sudoku solution can match the set of sums you’ve defined. If so, then set the arrow pattern in place and think about where you can add in a few given numbers to the grid, or more arrows, to force just one solution. In the puzzle on page 119, I started with the theme of nested boxes of arrows. The very large arrows on the left sides of the middle-sized and largest boxes are the starting points for the puzzle, with the choice of the given 1 in the upper left corner made precisely to deny this small number from the arrow in that region. While I was not sure what would happen on the right side of the grid initially, I did know that some answers were possible. Careful selection of the remaining givens and arrow breaks finished off the puzzle.

3. Try varying the input data

Most sudoku start as a grid with numbers in the cells. But finding ways to give the solver less or different information is another good way to vary the sudoku formula. There is one variation, for example, that starts with all the numbers on the borders of the grid, outside of specific rows and columns, telling you that those numbers must belong in the first three cells of that row or column but not specifying where.

Other variations use what I’ll call “limited information.” For example, Pencilmark Sudoku doesn’t give sure digits for any cell but instead gives a limited number of choices for what digits can go into a cell. A typical pencilmark clue might be “1348,” which tells you immediately that 2, 5, 6, 7, and 9 don’t go in there. But surrounding information may tell you even more. Its name comes from the notation system many solvers use to indicate what digits can go into an empty cell of a regular sudoku puzzle. Applying such notes to the whole puzzle (without giving any sure numbers) is the twisted concept of the puzzle.

Digital Sudoku is another type that works with limited information. In that variation all the cells are presented like the segments of an LED digit on a calculator. If a bit is dark, the number that goes in that cell must also use that LED segment. So, the lower left bit being dark could mean a 2, 6, or 8, which have that segment shaded, but could not mean a 1 or 3 or 4 or 5 or 7 or 9, which do not.

Wordoku

While most sudoku use the numbers 1 to 9, the rules of sudoku can work with any set of nine elements. This led to the quick invention of “wordoku,” with letters replacing the numbers, offering some crossover appeal to word puzzle solvers. In wordoku puzzles, some row or column (sometimes shaded or indicated by an arrow, but sometimes not) spells out a word or phrase using nine different letters. Or you can use one of the main diagonals, which lets you potentially repeat some letters in a likely harder-to-guess answer. If solvers know where the word entry will be, then at any time during the puzzle, but particularly after a few letters are placed, they can solve a word puzzle of identifying the anagram to fill out those squares quickly.

Designing a wordoku puzzle consists of two independent steps. The first consists of the same steps for making a valid sudoku using just the numbers 1 to 9 as usual. Leave at least one row or column completely blank. Second, choose a set of nine unique letters that form an interesting word or phrase when arranged in a certain order. This will be the “payoff” of the wordoku puzzle, so choose something appropriate for the setting.

For example, I used “I THANK YOU” as a phrase in a wordoku made to give out to donors to a friend’s “Miles for Miracles” fundraising effort. The givens in that puzzle were arranged like a big 26, the number of miles in the marathon he was raising money for, with the top row left blank where I THANK YOU would appear after solving.

Once you have your concept and puzzle set, put the two elements together. Convert whatever numbers are in the target row to the appropriate letters. If this row contains 564981723, then all 5s now become the first letter in the anagram, all 6s now become the second letter, and so on. Be sure all nine letters appear as givens in the puzzle. If not, you’ll have to provide all nine letters under the grid to be sure solvers know what must appear in the puzzle. It sometimes helps to provide a nonsense anagram so solvers can keep the letters in their heads while solving. “HAIKU TONY” worked for the I THANK YOU puzzle, with no part of HAIKU or TONY resembling a part of my actual intended answer.

S AS IN SUDOKU

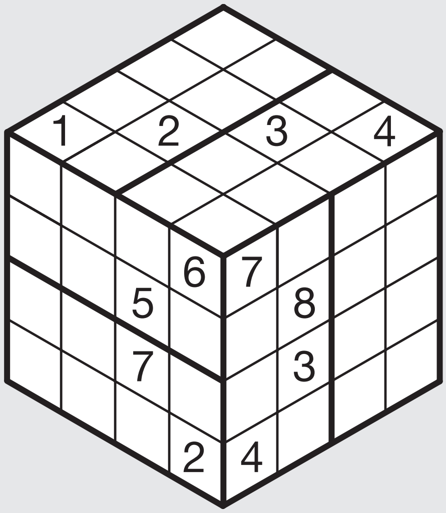
Follow standard sudoku rules. In this puzzle some clues are represented by letters. The digit that goes into one of these cells must have that letter in the English spelling of the number. So an S can represent a 6 or 7, as SIX and SEVEN contain an S, but not a 4, as FOUR has no S.

1 - ONE
2 - TWO
3 - THREE
4 - FOUR
5 - FIVE
6 - SIX
7 - SEVEN
8 - EIGHT
9 - NINE

O	N	E						
		T	W	O				
				T	H	R	E	E
					F	O	U	R
F	I	V	E					
		S	I	X				
				S	E	V	E	N
E	I	G	H	T				
N	I	N	E				9	

3-D SUDOKU

Follow standard sudoku rules, only use just the digits 1 to 8. In this puzzle the "rows" are formed by paths that join opposite parallel sides of the quadrilateral cells.



I like the limited-information type S as in Sudoku as it allows some letter-based themes, which are uncommon in logic puzzle types. The basic concept is that the letter in a cell must be in the spelled-out form of that number to go in that cell. If you find the English forms of the numbers don't contain enough letters in the alphabet, you can always choose another language. If it weren't for such unusual sudoku puzzles, I wouldn't have any good reason to know that Lima is not just a city in Ohio or Peru, but also the Samoan word for "five."

With S as in Sudoku, I chose a theme that breaks the most basic of sudoku rules in an unusual way. Notice how there are two NINEs in that last row of the grid. How is that possible!?! Because of the uniqueness of letters like G, U, W, and X in the written English numbers, and the rarity of letters like F, H, R, S, and V, the puzzle had some natural starting points that required careful placement of each word in each row. EIGHT has to be (1579)(569)832, for example. I didn't know this going in but thinking about the limitations of the theme here guided the construction. The cool discoveries you make before even starting construction will become the cool discoveries you leave behind for your solvers as they start solving.

4. Try varying the grid

A more difficult but rewarding way to vary a puzzle is to use a different grid. This can mean anything from using irregular shapes in place of the 3×3 regions to using a complex pattern of triangles or hexagons instead of squares that still essentially breaks apart into "rows" and "regions." I get inspired all the time by geometric patterns around me to see if they would work well for a puzzle. If one of my puzzles ever makes you think, "hey, that looks like the tiles on my kitchen floor," now you know why.

When experimenting with a new grid design, determine whether a solution is even possible for that grid. Before you start constructing, try to find an answer by placing values in the cells. Use a pencil so you can erase and swap numbers if you can't fulfill a constraint. In addition to getting a valid solution, you may discover less obvious grid properties that are useful in solving (or writing) a puzzle on that grid.

Using an isometric projection of a 3-D shape onto a sheet of paper is one way to make a more interesting-looking grid puzzle. The "3-D" sudoku here is actually a fairly easy puzzle compared to the rest in this section, especially after you realize from geometric constraints that on each face you can color 2×2 squares like a checkerboard, with the shaded squares containing the same four digits, and the unshaded squares containing the other set of four digits. I uncovered this fact as I tried to fill in any valid solution grid, and it's easy enough to prove based on how the "rows" wrap around. So I could use it to construct a harder puzzle with fewer givens, forcing the solver to *discover the same fact. The 1234 on the top of this puzzle, for example, identifies 1234 and 5678 subgroups that can then be filled in from a few of the givens below to get started.*

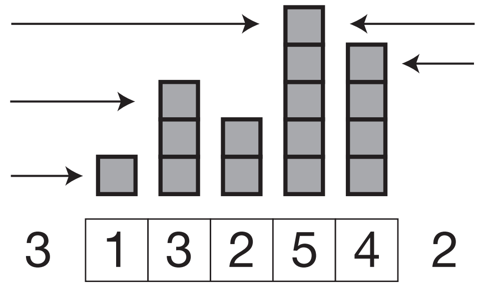
5. Test your puzzles out on some friends

However you've varied the sudoku recipe, it's worth seeing how approachable the new challenges are for other people. The leaps of logic that seemed obvious to you when you went about constructing the puzzle might not be as easy for your solvers to grasp, and some test-solving will help you gauge if you need to make your puzzles easier or harder.

While I don't use the computer to construct these variations, I've found in making many of these types that it is helpful (if you know how to code such a thing) to write a solver that can check that there is only one solution to the puzzle. It's easy on a "new" type to overlook something, or to make a mistake, so having an automated check, or a check from a trusted solving friend, is an absolute must when you venture into uncharted waters.

CRAFTING A SKYSCRAPER PUZZLE

While most paper puzzles are confined to just the two dimensions of the page, skyscraper puzzles can evoke a third dimension as solvers visualize the buildings sticking up out of the page. With a mental image like the one to the right for a single row of a city, solvers can recognize why only three buildings (the 1-, 3-, and 5-story buildings) are seen when looking from the left side and two are seen from the right, for example. The following tips will hone your architectural skills.



1. Work backward from an answer

Make a valid N×N Latin square that can be a puzzle answer; I found 5×5 and 6×6 to be good sizes to start exploring. Then, make a blank grid and put all 4N clues on the outside exactly as in the answer. Try to solve this puzzle and see if it has just one solution; if so, remove some of the clues to leave a harder puzzle and repeat this process until satisfied. While this method works for small puzzles, you will often run into invalid grids with multiple solutions for larger sizes, which generally means you should start over.

Doing this exercise, you can learn a lot about the importance of different clues in a skyscraper puzzle. 2s and 3s don't immediately give a lot of information. On the other hand, 1s give instant placements on the edges (and there must always be a 1 on each edge). In most cases you won't want to give too many 1 clues, but erasing only the 1 clue on an edge won't work either, as the single missing clue can still be easily identified as a 1. Try to erase at least two or three digits from each side to leave a valid 5×5 puzzle with eight to ten clues.

2. Work backward from a more interesting answer

Repeat step one but now, when filling in a solution, place numbers that leave opportunities for good clues. You can fill a row or two where all N buildings are observable (leaving an easy N clue); it's also incredibly valuable to set up N−1 clues or N−2 clues where possible. Having two N−1 clues on the same edge is very constraining, as is three N−2 clues. Another interesting situation logically is any row or column where *all* buildings are observable from one of the two directions (such as having a 3 on each side of a row in a 5×5 puzzle). In this situation, having ascending heights on both sides places additional limits on where the small or large numbers go, and often allows one placement to chain deductively to the opposite side. By setting up good skyscraper clues in the answer, working backward to get a good puzzle is much easier.

This is the approach I took with the first 5×5 puzzle. I wanted a grid that had a lot of 4 clues, so I positioned

Latin Squares

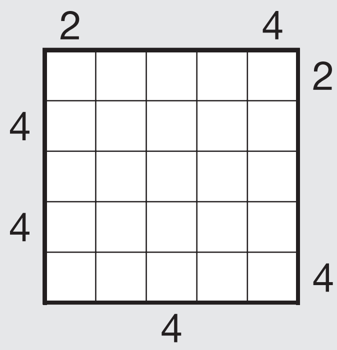
A Latin square is an N×N grid of boxes, with each box filled by one of N different symbols so that no symbol repeats in any row of column. The name was motivated by the work of the mathematician Leonhard Euler, who did some of the initial studies on the combinatorics of Latin squares using Latin characters as the symbols.

Numbers can also be used as the symbols in a Latin square, with the digits 1 to N being most common. We've seen this exact situation in sudoku, although a sudoku solution is a special subset of Latin squares, as the 3×3 regions place additional constraints on the placement of the symbols. For those counting at home, there are approximately 6.67×10^{21} valid 9×9 sudoku solutions, but there are many more (5.52×10^{27}) different 9×9 Latin squares. In neither case do we run the risk of running out of different puzzles in our lifetime. Latin squares form the basis for many different puzzle types, including the ones described on the next few pages.

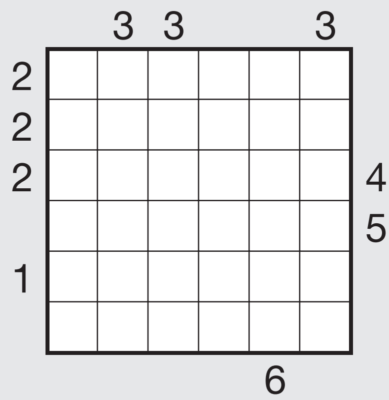
SKYSCRAPERS

For each N×N grid, place a number from 1 to N into each cell so that no digit repeats in any row or column. Each number represents the height of a skyscraper. The clues on the outside of the grid indicate how many skyscrapers can be seen when looking at that row/column of the city from the outside. Taller buildings block the view of any smaller buildings behind them. In the last puzzle, instead of the number of buildings seen, the outside clues indicate the sum of the buildings seen.

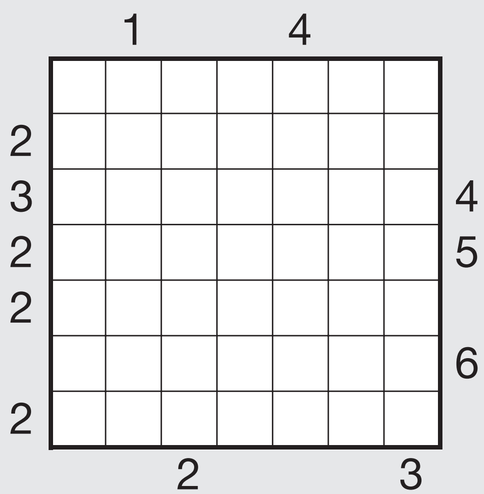
5×5 skyscraper



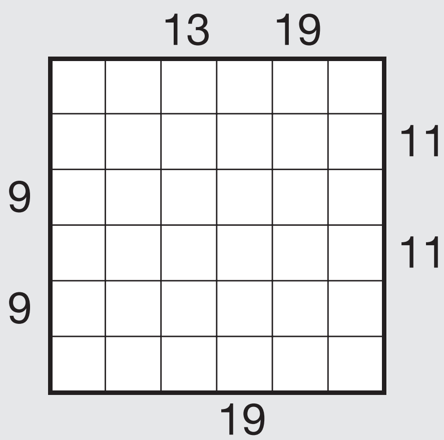
6×6 skyscraper



7×7 skyscraper



6×6 additive skyscraper



the first several numbers to leave behind a lot of rows and columns where four buildings were seen. In choosing the clues I started with all the 4s I could give in the grid and found adding in just a few (pseudo-symmetric) 2s would complete a good puzzle.

3. Work forward with interesting clues

The first two steps have given you a bit of practice in figuring out what clues to start from and what clues can be removed entirely. Now you can work forward from a blank grid. You'll set up some of the same "interesting" situations mentioned in step two by placing the relevant clues on the outside of an unsolved grid and pretending to solve the puzzle as you are constructing. The first step is often placing all the large buildings, and you can consider where clues appear as a strategy for accomplishing this. Marking some clues as ≥2 or ≥3, for example, can help, as this removes the largest and second largest buildings from the nearby cells. Add clues until you force a single answer, then replace the ≥ clues with the actual numbers to finish the puzzle.

In the 6×6 puzzle, I tried to showcase every possible clue value (from 1 to 6) going clockwise around the grid from the bottom. Both the 1 and 6 are very forcing clues, so I started with those. This didn't leave a lot of places where the 5 could be on the right side, and I found I needed several 2s and several 3s (which don't tell as much) to finish off the puzzle.

In the 7×7 puzzle, instead of trying to make a visual theme, I incorporated a very hard solving theme. One *thing you will learn about 2s is that when the largest building size is completely separated across the grid, then the second-largest building size must immediately touch the 2 clue. This puzzle would take that concept to an extreme. You'll likely find you can enter about five digits really easily (four 7s and a 6) but then get stuck. There are three rows left (all with 2 clues on the left side) that must take a 7 in the fifth, sixth, or seventh columns. One of these must be 6XXXXX7 as described above, but the next one must be 5XXXX76 and the last 4XXX765 because big buildings must be hidden behind the 7 to continue to satisfy the 2 clues. The 3 on the bottom right is the clue that forces an order for these three rows, and puts in enough digits to force most of the rest of the grid. I had to add one or two more column clues to specify just one answer, but that hard initial step is the real meat of the puzzle.*

4. Add some variety

For variety, add some additional constraints to these puzzles, such as allowing "gaps" in the city, where one square per row/column does not contain any building whatsoever. Other region constraints (such as sudoku-like regions or things like domino sets) can be used. The outside clues can also be changed to involve arithmetic like the "sum of visible buildings" or "product of visible buildings."

Here, I tried an arithmetic variation with a few different features. Specific arithmetic values now can specify a particular set of visible buildings. A 9 clue, for example, can mean seeing 1, 2, and 6, or 3 and 6, but in either case the 4 and 5 must be hidden at the other end of the row or column. In this puzzle I chose some different numbers with different possible subsets of buildings, and then balanced their positions to get different deductions chaining together, one into the next.

CRAFTING A CALCU-DOKU

In the mid-2000s, calcu-doku puzzles began to populate the shelves of bookstores with several names, including KenKen (or, when hand-crafted by me, TomTom), but always with the buzz of being the "next sudoku!" While I'm skeptical of the merit of this marketing, calcu-doku, a mathematical puzzle with fairly simple rules, does share its Latin square ancestry with sudoku. The addition of arithmetic to a number-based puzzle separates calcu-doku from sudoku, and makes this a good choice for exploring the construction of puzzles involving mathematics.

1. Choose a grid size

The size of a calcu-doku grid helps determine its difficulty, because the range of numbers opens up more possibilities for the operations. For example, the clue 7+ in two cells (indicating that the two digits within that region add up to 7) can be $2+5$ or $3+4$ in a 5×5 puzzle, but in a 6×6 puzzle can be $1+6$ or $2+5$ or $3+4$. Larger sizes also add complexity to operations like multiplication and division where the first time a 2-cell product can be expressed in two ways is $1\times 6 = 2\times 3 = 6$.

I picked a range of sizes from 5×5 to 8×8, with the smaller sizes demonstrating easier techniques (such as showing off the basics of addition in the 5×5 puzzle), and the larger puzzles offering more types of themes and challenges.

2. Choose some region shapes

Your puzzle will need bold regions, often called "cages," to uniquely determine the solution. Most puzzles currently on the market are computer-generated and have randomly oriented regions. If you are hand-writing these puzzles, you might choose some symmetric and interesting shapes as one route to making a more aesthetically pleasing puzzle.

Single-cell regions can be used to instantly give the solver a number and a place to get started. Two-cell regions are most common with several values for a given operation that may uniquely define a pair of numbers that goes in that region. As the sizes of the regions increase, the puzzle gets harder. Use more 2-cell regions if you want an easier puzzle; try more 3-cell, 4-cell, or larger regions if you want a harder puzzle. Because a number can repeat in a bold region if it spreads across multiple rows and columns, having bendy region shapes for multi-cell regions is also a way to make the puzzle harder. If you use 2-cell regions, make sure some are horizontal and some are vertical, or else you won't constrain the puzzle enough along the rows and the columns.

For the 5×5 puzzle, I started with a plus sign–shaped region in the middle to match the addition-only puzzle. For the remaining regions, I chose a mix of 2- and 3-cell regions with the top and bottom row having regions that extend just one cell into the adjacent row. These would end up being the first digits in our solving path based on geometry constraints.

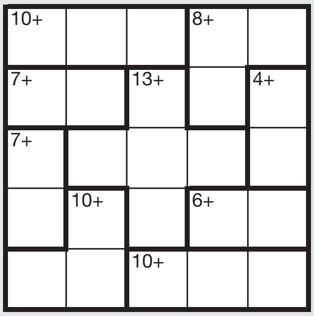
For the 6×6 puzzle, I chose a mix of horizontal and vertical cages that split the grid into quadrants. One operation would be used exclusively in each quadrant, and the larger cages were selected for the addition and multiplication corners.

For the 7×7 puzzle, I did something unusual and only used horizontal cages, like in a brick wall. This type of design is very challenging to construct, with only the offset of adjacent rows allowing digits to get into the middle of the grid from left to right.

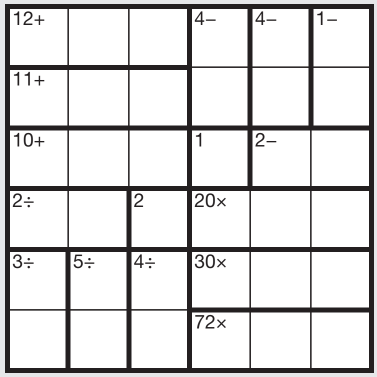
CALCU-DOKU

Fill each N×N grid with the numbers from 1 to N, one per square, so that each row and column contains each number exactly once. In the bold regions, clues indicate the value of a mathematical operation (+, –, ×, ÷). A 2-cell bold region labeled with 6+, for example, could be 1+5 or 2+4. A bold region can contain the same number more than once, as long as it only appears once per row or column. In the 7×7 puzzle, just the value of each expression is given, so you will need to determine which operation is appropriate for each bold region.

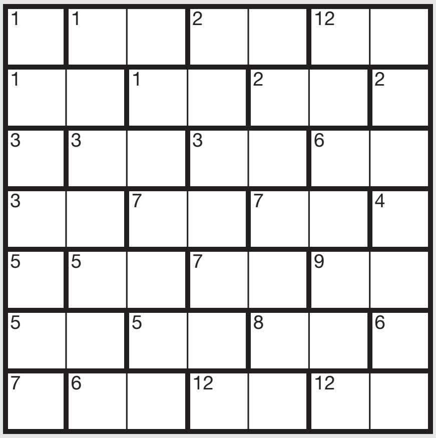
5×5 calcu-doku



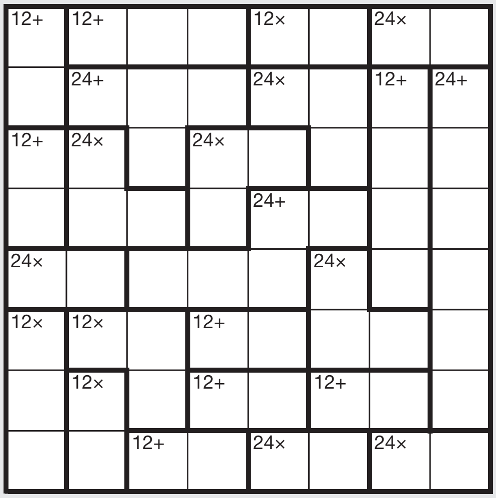
6×6 calcu-doku



7×7 operationless calcu-doku



8×8 calcu-doku



3. Fill in some numbers, slowly

Unlike a sudoku, where one can easily put in too many numbers so that a row/column/region cannot be filled, it is fairly simple in a calcu-doku, with just row and column constraints, to reach a valid solution grid. The best approach is to clue some of the regions to determine some of the digits, and proceed until the entire grid is filled. You may only need to put clues in about 60% to 75% of the regions before the grid is solvable. This means that harder or less meaningful clues (like a 1– in two cells) can be used for the remainder of the puzzle so that solvers are more likely to encounter your intended solving path.

For each operation (+, –, ×, ÷), there are often "unique" assignments of values for the regions that are easy starting points. Learning these assignments, and which ones are trickier, will help you make more difficult puzzles. You can also choose to use just a single operation in your puzzles, like addition, or a mixture of any of the four possibilities. Different kinds of puzzles will result from the types of mathematical operations you use.

For addition, as in the 5×5 puzzle, a simple region can be something like 4+ where 1 and 3 must go in some order. But the rest of that puzzle used more difficult sums with lots of possible values. This ensured that the geometric work-in I set up above, where the sum of two clues in the top row gives a value in the second row, would be most critical. The actual values of the top two clues were only set after the whole grid was full, but that they would be 15+3 was defined at the beginning, to put a 3 next to the (1,3) pair and start fixing digits in the grid.

In the 6×6 puzzle, I set up long additions and multiplications that would have defined triples, but not defined locations. The subtraction and division regions set the rest. Large divisions and subtractions have many fewer choices than smaller ones. 4÷ is uniquely 4 and 1, for example, while 4– can be 1 and 5 or 2 and 6, but cannot contain a 3 or a 4. While I only use two cells for subtraction and division in these puzzles, you can make larger subtraction and division clues if you specify the operation always starts from the largest value. A cage with 4, 2, and 1 would be 1– = 4 – 2 – 1, for example.

For the 8×8 puzzle, I first made a mental list of all the ways different combinations of 12 and 24 could be made, as I wanted a puzzle where only these two clue values appeared. The sum of all the digits in 1 to 8 is 36 (12 + 24) which makes this a good choice of values. So I thought about ways to get to 12 and 24. In two cells, 12 can be 3 × 4 or 2 × 6 or 5 + 7 or 4 + 8; similarly, 24 can be 4 × 6 and 3 × 8. With more cells, more options appear. The sense of "rhyming clues," like the two 24×'s in the bottom row, or the two 12+'s above them, is an interesting thing to put into your puzzles. Here, they specify a complete set of four digits that cannot appear anywhere else in the row. There are only two ways to get a sum of 12+ with four different numbers, and recognizing this fact is important to break into the right side.

4. Clue the puzzle—or don't

If you want a really tough puzzle, you can try to remove all of the operations from it and see if it is still solvable. Often, a puzzle designed as a normal calcu-doku can have the operations removed without making an invalid puzzle, but it does help to have unusual values like, say, 11, which can only be addition and, in a 6×6 puzzle, can only be 6 + 5. For any "operationless" puzzle, you have multiple choices for any region for how you define its value. A 5 and a 3 can be clued as an 8 (for 5 + 3) or 15 (for 5 × 3) or 2 (for 5 – 3)—division is not possible for this example as calcu-doku traditionally do not use fractions. Notice that a 15 is a very easy clue as it can only be multiplication and can only be 3 × 5, while 2 and 8 have several choices. Picking the right numbers to leave behind can make the operationless puzzle a fair but fitting challenge.

Magic Squares

A magic square is an arrangement of N^2 different numbers, usually distinct integers, into an N×N square so that every row, column, and both diagonals add up to the same value. Magic squares are works of symmetry, wherein every number placed affects the placement of every other. They can be used in puzzles as well, and are easy to create once you understand some perhaps surprising rules. While I'll describe here the steps to make a 5×5 magic square, retired anesthesiologist Alan Grogono maintains a thorough and user-friendly website at www.grogono.com/magic that can teach you how to make many larger squares up to 97×97.

The big trick to constructing magic squares is to view each box as the sum of several numbers laid out in a predetermined pattern. In the odd sizes 3×3 and 5×5, as shown here, you will be adding three different numbers together.

AEG	CDG	BFG
CFG	BEG	ADG
BDG	AFG	CEG

For a 3×3 square, pick three numbers called A, B, and C; three numbers called D, E, and F; and one number called G. You can pick any numbers you want. A standard set for a square that uses one each of the numbers 1 to 9 are A 6, B 3, and C 0; D 2, E 1, and F 0; and G 1 (or some transposition of those numbers).

AJK	CFK	EGK	BHK	DIK
EHK	BIK	DJK	AFK	CGK
DFK	AGK	CHK	EIK	BJK
CIK	EJK	BFK	DGK	AHK
BGK	DHK	AIK	CJK	EFK

For a 5×5 grid, pick five numbers called A, B, C, D, and E; five more called F, G, H, I, and J; and one more called K. For a standard square of 1–25, you'd use an arrangement like A 20, B 15, C 10, D 5, and E 0; F 4, G 3, H 2, I 1, and J 0; and K 1.

Note that the last variable is always 1. If you just raise or lower that last variable, which sits in all boxes, you will end up with a string of consecutive numbers in the grid that start at some other value than 1.

To set up the array, place these letters (A–F or A–I) into the boxes so that *no letter* appears more than once in any row or column of the square. In the 5×5 square, no letter should repeat on the two diagonals either. Then pick values for your letters and you have your magic square.

If you just want your solver to try to fill in the grid, then just give him or her the blank grid. Put in a couple numbers (9 or 10 for a 5×5 is reasonable) to steer the solver to the exact arrangement you want. Or you can clue the numbers and make it a math/trivia hybrid. The solver will go back and forth between solving for answers he or she knows, and adding together rows, columns, and diagonals to narrow down the possibilities.

With the 7×7 puzzle, I specifically constructed the grid with operationless presentation in mind. While I could obscure a lot of the clues with small and common values, I needed to make some clear starting points. This is what some of the 12s offer. The singletons on the end of each row (1 to 7 in order from top to bottom) also serve to eliminate options from the specific row they belong in. The 7 in the bottom eliminates 7 + 5 as a 12, for example, so the 12s must both be products (2 × 6 or 3 × 4), making the bottom right corner likely the first place you'll write a digit after the lone cells when solving.

5. Check your work

As with any puzzle, give your construction to some friends and see if they can solve it and arrive at the same solution. Particularly for puzzles without any operations, it is easy to construct something that is harder than you thought it was, and that may not be your goal.

Just as with sudoku, you can find a few calcu-doku utilities online to check an input grid as well. Knowing you have just one solution is much easier, particularly on an operationless puzzle, with an exhaustive computational search.

CRAFTING A CROSS SUMS

While sudoku was conquering America, another American innovation got rebranded by the Japanese with the goal of taking over our puzzle bookshelves. "Kakuro," the Japanese name for the venerable cross sums, is now in newspapers everywhere too. While some of the arithmetical clue breakdowns will feel familiar from puzzles like calcu-doku, the use of intersecting clues, just as in a crossword, introduces a unique feel to solving a cross sums puzzle, where you really have to search across and down clues at the same time to find sure digits.

1. Lay out an empty grid

Just as with its word puzzle cousin the crossword, certain grid design rules are always followed with a cross sums. First, the grid should have symmetry, typically rotational symmetry. Second, all the white squares need to be connected to each other. Lastly, all across and down entries must be at least two cells and no more than nine cells long (a sum of 45 is the largest possible sum under the rules).

So, start with a grid that is probably no smaller than 8×8 and no larger than 18×18. Square grids work fine, but you can use rectangular grids as well. Just as a crossword usually has an odd number of rows and columns to allow for a center entry, a cross sums usually has an even number of squares because it has an added row and column put on the top and left of the grid to accommodate some clues but containing no white squares. Ignoring this extra row and column, the grid will appear symmetrical around the middle square. Put in a bunch of black squares to divide the grid into entries, and place a diagonal line in boxes containing clues to distinguish the across totals from the down totals.

I wanted a grid that showed off a wide range of digit combinations, so I made sure there was a central nine-box entry, some eight-box entries, and so on. The grid was set up to have a difficult middle (where almost all entries are in long clues of five or more boxes) and easier corners, such as the upper right and lower left where several 2- and 3-cell clues intersect.

2. Lay in some "absolute numbers"

Anybody can slap the numbers 1 through 9 into a cross sums grid. The trick is to put in a set of numbers that can't be any other set of numbers. Making sure you have a unique solution is crucial, and it's harder than it sounds.

Fortunately, some clues are your friends. Mike calls these "absolute numbers," as they're guaranteed to have one unique set of digits that, in some order, will make the sum work. Each length of boxes from 2 to 7 has four such absolute numbers: the lowest possible set, the lowest plus one, the highest minus one, and the highest set. The totals of all nine 8-box entries are unique (each one is 45 minus the missing digit, so the solver can always tell which digit is missing by subtracting the clue from 45), and there's only one 9-box entry. Of course, the digits can be in any order, but it's hugely helpful to know what numbers you need. (These numbers are referred to by fans as "X-in-Y," such as a 3-in-2 or a 45-in-9.)

The more of these absolute numbers you put in, the easier your puzzle will be (especially when they cross each other). Similarly, the fewer you put in, the greater the difficulty—and the chance that your cross sums does not have a unique solution.

Absolute numbers

digits	low	low + 1	high – 1	high
2	3	4	16	17
3	6	7	23	24
4	10	11	29	30
5	15	16	34	35
6	21	22	38	39
7	28	29	41	42
8	all values (36–44) are absolute			
9	all digits (summing to 45) must appear			

Several absolute numbers are in this grid, but the clearest place they are used is in the upper right, where several absolute numbers intersect. The 3-in-2 and 4-in-2 small clues are the first spot many solvers will see, but there are others nearby as well, such as the 6-in-3 that will be filled in next and then the 7-in-3 that intersects that.

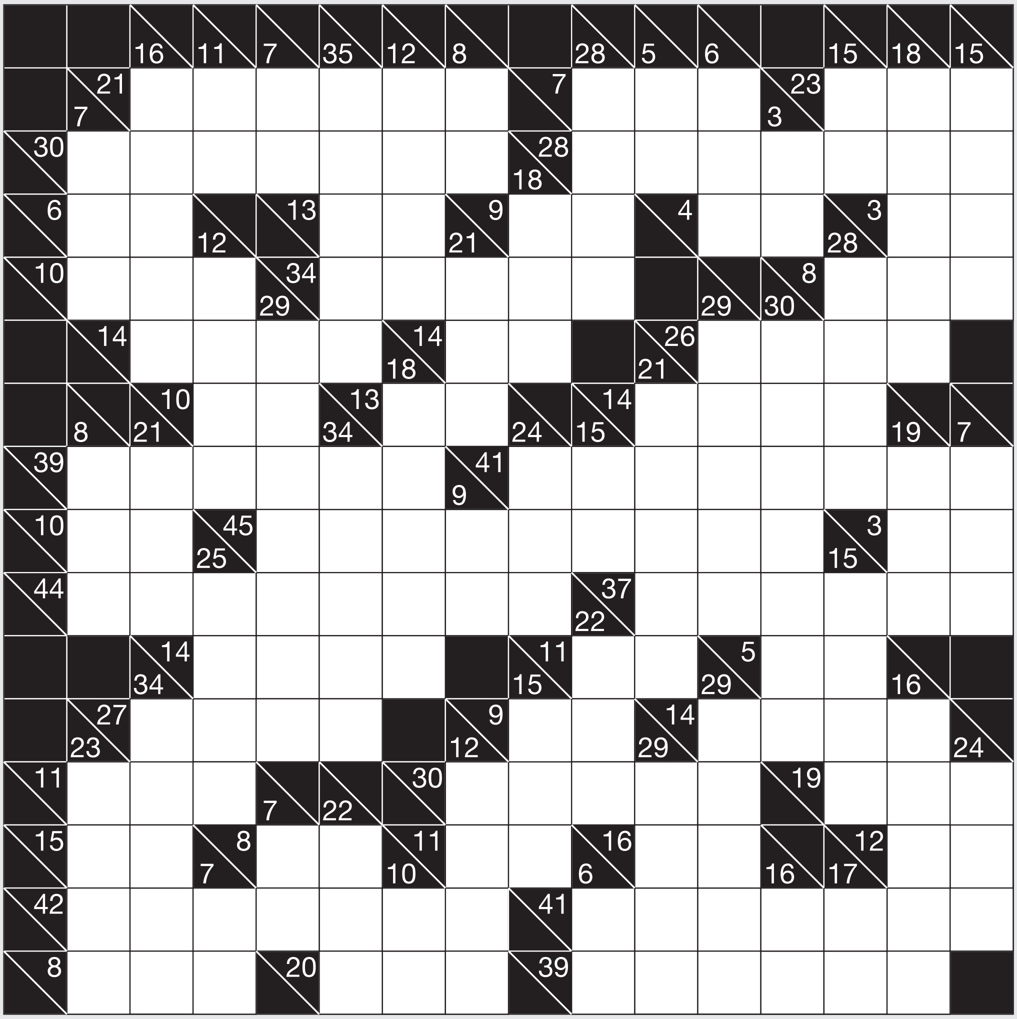
Even with the large entries, particularly the 6- and 7-box entries, I tried to use helpful absolute sets. The 42-in-7 and 41-in-7 in the penultimate row, for example, are groups that are missing the pairs (1, 2) and (1, 3) respectively, while the 39-in-6 in the bottom row only contains digits from 4–9. With these two absolute groups working together in the lower right, there aren't many small digits left for the down clue that is 6-in-2 or for the 16-in-5 that is nearby, too.

3. Lay in some "almost absolute numbers" and other forcing moves

One step removed from the absolute numbers are the sets that have just two options, and learning how to use them will lead to more challenging deductions. Following from above, the values that are low + 2 and high − 2 will always have just two options. 8-in-3, for example, can be 1 + 2 + 5 or 1 + 3 + 4. Notice that whichever set is used, 1 is always in the sum. As more cells are used, more numbers get pinned. A 12-in-4 must be 1 + 2 + 3 + 6 or 1 + 2 + 4 + 5, forcing the solver to place one 1 and one 2 regardless of which of the two possible groups is used. (An equivalent argument shows a 9 must be in a 3-box 22 sum and a 9 and 8 must be in a 4-box 28 sum.) One

CROSS SUMS

Place a single digit from 1 to 9 into each white square so that the sum of each across and down entry is correct. The number above a diagonal line refers to the sum of the across boxes to its right, while the number below a diagonal line refers to the sum of the down boxes below it. No digit is used more than once in any sum.



way to make use of this knowledge is to intersect big and small clue sets together. If an 8-in-3 needs a 1, but only one of the three intersecting clues is small enough to allow for a 1, then you have a forced placement.

One interesting example of "almost absolute sets" occurs in the upper left. The 12-in-4 coming down from the top row must take a 1 and a 2, but neither can go in the bottom two boxes. This leaves just a 3-and-6 or 4-and-5 for the remaining two cells, and you'll find only one possibility works.

A few intersections of large and small occur in the lower left. A 34-in-5 coming down intersects an 8-in-3 across. The 8 can take any number from 1 to 5, so it must take the 4 in the intersecting clue because that is the only overlap of the big and small clues. A much harder deduction follows immediately to the left of that. Once the 4 is in, the remaining digits in that clue are 1 and 3. If you think about using the 1, you'll find you cannot make the remaining boxes add up to 22 (the remainder of the 23-in-4), specifically because there is also a small 11-in-3 going across that hits the 34-in-5 that now only has (6,7,8,9) left unplaced. Having many small acrosses and large downs leads to a set of deductions that solves the entire lower left.

4. Fill the grid

For each section of the grid, toggle back and forth between entries until you have a single answer. This is usually accomplished by leading the solver around the grid through series of singular conclusions. Hiding absolute sets within partial progress is the best approach to lead solvers on a particular path. If, for example, some other clues have forced a 1 into the top of a 3-cell entry, you can now make the whole clue an 18-in-3 (meaning the remainder is a 17-in-2) to force a 9 and an 8 into the other boxes. Until the 1 is found, the clue will seem innocuous; once the 1 is placed, the clue will scream for attention. Continue identifying new clues and forcing sure digits until you have forced a single solution to the puzzle.

You'll find difficult clues that become easier clues once some digits are entered all over my grid if you look hard enough. In the top left there is a somewhat free 30-in-7 that becomes much easier once you place a 9 in it to form an absolute 21-in-6. In the bottom there is a 30-in-5 that is even more open-ended until the moment the lower left falls and you place a 1 in that entry, making it an easy 29-in-4.

The middle of this grid is particularly tough, as it showcases long intersecting entries that are initially very unconstrained. But after making partial progress there becomes just one choice for some boxes. The 29-in-7 that intersects the 39-in-6 on the left side is one such place. Initially, either 4, 5, 6, or 8 could belong in that intersecting box, but after placing a 4 and 8 in the same column and a 6 in the same row, you are left with only a 5 choice. The middle will eventually fall by a chain of these "one value left" type cells, which are very tricky to spot in long intersecting entries.

5. Prepare the puzzle

Make a blank grid, and consider providing a pocket calculator to the arithmetically challenged.

And to answer the question everyone asks: No zeroes. Ever! (Okay, never say never. There is a variant that uses 0 to 9, but it's much more brain-busting. Save it for special occasions, when you really want to vex someone.)

CRAFTING A WORD DIVISION

While we have covered many grid-based number puzzles to this point, several number puzzles don't use a grid at all. Those that take the form of mathematical equations are called cryptarithms, so named by Maurice Vatriquant in a 1931 issue of the Belgian magazine *Sphinx*. The types are word addition, word subtraction, word multiplication, and the most interesting of the group, word division. It appeals to both math and word puzzle fans because neither the math nor the wordplay is that hard. If you don't like math, you can attack it as an anagram. If you don't like wordplay, you can stay away from the answer blanks until you're done with the arithmetic. If you like both, you can work on them simultaneously.

1. Choose some 10-letter words

Well, not just any 10-letter words.[35] Your 10-letter words and phrases must be "heterograms"—that is, made up solely of unique letters—to ensure that each digit from 0 to 9 gets represented by a unique letter. If you use a phrase like WORD PUZZLE, your solver doesn't know whether multiplying by Z is 6 times a number or 7.

It turns out there are a lot fewer 10-letter heterograms than you might think. You could look at a comprehensive list of a particular subject, and not find more than a handful. For example, you can look at every car make and model ever produced, and might only find the Buick Regal, the VW Microbus (or just Volkswagen), the Reliant Fox, the Opel Signum, and—get this—the MG XPower SV. And boy, there are a lot of cars out there.

If you want to use a themed set of heterogrammatic words and phrases for your puzzle, it'll take a lot of time searching through word lists to find enough to use. Instead, you might consider using a computer to find heterograms for you. The Qat word finder at http://www.quinapalus.com/qat.html has a few options. If you give it the search string "10:*/abcdefghijklmnopqrstuvwxyz"

[35]This all presumes you want words at all. There's a type of division problem called digititis, which replaces all but a few numbers with blanks. It's very hard to make these unique, and they have a narrower appeal (involving as they do much more arithmetical trial and error), so we have opted not to go into them here. They share some characteristics with word division, though, so if you want to try one, more power to you. The main thing to remember is: make *sure* it's unique!

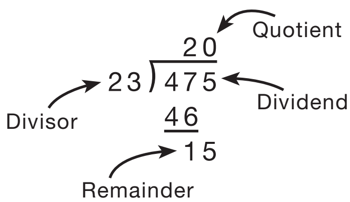
then it will give you a list of 10-letter entries in its database that only use letters from the exact set of letters after the slash. You can find nondictionary phrases by picking a likely starting word and deleting its letters from the search string. If I want a heterogram that starts with the word "right," I would then search using the string "5:*/abcdefjklmnopqsuvwxyz"; scanning the results for things that go with "right," I find phrases like "right ankle," "right elbow," and "right flank."

Once you've got a set of answers, put each valid target word on blanks from 0 to 9, one letter at a time.

For these puzzles, I used OneLook dictionary search (http://www.onelook.com) to find some heterograms. As I was working on a book myself, I considered the theme of authors and writing. I found a few valid words like JOURNALISM and MANUSCRIPT and PLAYWRIGHT as well as some phrases. I soon got determined to find names of authors, and settled on a nice set of four famous names from different areas of literature.

2. Hypothesize a dividend

In long division, the dividend is the number underneath the line; it's the number you're going to divide *into*. For word division, you need a dividend of five to eight letters, made up solely of the letters in the target word you're using. So if you had OPEL SIGNUM as your target word, GUESSING would be a valid dividend. You can either come up with this on your own, or use an anagram program to find possibilities. (You should type the word more than once into the field so that you can use letters more than once—you don't need heterograms for this step.)



Convert that dividend word into numbers, using the numbered blanks. So GUESSING would be 68244576. Write that number sequence below the line in the word division. (You may wish to have a few backup dividends, just in case this one doesn't work out.)

For these puzzles, I searched for some nice 7- or 8-letter anagrams made up of each letter set. In each case there were some really good choices, like RIDDLES, that immediately caught my eye. However not all of them could work in the puzzles. HOMEWORK, for example, caught my eye, as long division problems always remind me of math homework from elementary school, but the H in that answer was the value 0 and couldn't be used as the first digit of the dividend.

3. Solve for a hypothetical divisor

The divisor is the number to the left of the dividend. Pick a 3- to 5-letter word made up of some of the letters in the target word. You'll do yourself a favor if it contains at least one letter that isn't in your dividend. Sometimes you can get all your target's letters into a dividend and a divisor.

Convert the divisor into numbers. Then get out a calculator, and divide the dividend by the divisor. You'll get a number, which will be your quotient, the number above the line in the division. Ignoring everything to the right of the decimal point, convert that number to letters and see if it makes a real word. It probably doesn't, so try again with a new divisor. And again, and again, until you get a quotient that is a real word. Voilà, you have a word division puzzle.

Okay, not quite. You still have to solve for the entire long division. This means placing your dividend, divisor, and quotient in the right places, and then multiplying each digit of the quotient by the divisor, subtracting that amount from the numbers above it, and then bringing down the next digit to add to what's left over. Continue to do so until you create the remainder, the last number in the long division. Here's the most important step: Make sure you have used all ten digits from 0 to 9. If you leave out even one, you have to start over.

By the way, if your division includes a 1 in your quotient, you've made the puzzle a lot easier for your solver, as any number times 1 is the same number.

I tried dozens of words until I got the right divisions. Using an anagram program I got a long list of the possible 4- and 5-letter words and then got greedy and tried some words that were (sort of) thematically related to the divisor. THINK and OPINION went well together, and amazingly enough formed a valid word for the quotient. Other times I wasn't so lucky. With the last puzzle I completely struck out with my first quotient (NOWHERE) and had to go back to choose a new one and try again. When I was done, I had four numerical long divisions, which I checked thoroughly before moving on to make sure all the numbers from 0 to 9 appeared.

4. Convert the divisions back into letters

Using your target word letter by letter, search and replace for each digit from 0 to 9. Confirm that you've ended up with the dividend, divisor, and quotient you wanted. Place blanks from 0 to 9 above the lettered division, and check everything again to make sure you have no math or conversion errors.

I converted everything, then checked to make sure I liked all my answers. At this stage I chose an order for the puzzles, considering the approximate difficulty of anagramming the author's names as well as the difficulty of identifying simple digits like the 1s and 0s in the long divisions. The second line of the division in puzzle 1, for example, struck me as a very easy place to determine the values of 0, 1, and 9 based on a 6-digit number being subtracted by a 5-digit number to leave a 4-digit result.

WORD DIVISION

In each of these long division problems, the numbers have been replaced by letters. Each letter stands for the same digit throughout a problem, but substitutions are different from one problem to the next. Solve each puzzle using logic and basic arithmetic; when each puzzle is completed, the letters representing the digits 0 to 9, written in order, will spell a famous author. A sample puzzle and solution is shown at right..

```
      X L A M X              1 2 0 8 1
E ) X A M P L E         9 ) 1 0 8 7 2 9
    E                       9
    -                       -
    X M                     1 8
    X M                     1 8
    ---                     ---
      P L      ------>        7 2
      P L                     7 2
      ---                     ---
          E                       9
          E                       9
          -                       -
          A                       0
```

Word Division #1

```
_ _ _ _ _ _ _ _ _ _
0 1 2 3 4 5 6 7 8 9

                N I P
ONION ) P O U N D E D
        U O O N E
        ---------
        O J H E I E
          E P E E O
          ---------
            D E K K D
            D K P D P
            ---------
              O N H O
```

Word Division #2

```
_ _ _ _ _ _ _ _ _ _
0 1 2 3 4 5 6 7 8 9

                T O T
THINK ) O P I N I O N
        H A P I K
        ---------
        A O A A I O
        A H H E C K
        -----------
          H I C O N
          H A P I K
          ---------
            O O I N
```

Word Division #3

```
_ _ _ _ _ _ _ _ _ _
0 1 2 3 4 5 6 7 8 9

                R A W
SOLID ) R I D D L E S
        R A O L C
        ---------
          A D S W E
          A C A O R
          ---------
            W D W W S
            W A D R O
            ---------
              R L S S
```

Word Division #4

```
_ _ _ _ _ _ _ _ _ _
0 1 2 3 4 5 6 7 8 9

                E A R
WHERE ) U N K N O W N
        W H E R E
        ---------
        R N U M W W
        R A H A U A
        -----------
          E O U U R N
          E R H R A R
          -----------
            N U N U M
```

CHAPTER 3B: ABSTRACT LOGIC PUZZLES

The "abstract" in abstract logic puzzles is a bit of a paradox. There's nothing abstract about what you as the solver are doing. The procedural path to solving is as concrete as a house foundation, with logical deductions arising from simple consideration of the rules of the puzzles. Discovering these deductions for the first time is half the fun of solving abstract logic puzzles. But the goal of identifying mines or placing battleships or building an ocean around some islands is certainly abstract in the sense that you will never do these tasks in real life—could never do these tasks in real life—so the on-paper goals are a bit fantastical as they take you from laying out atoms to moving whole galaxies and everything in between.

The cornerstone event for this type of puzzle is the World Puzzle Championship, which I've been fortunate to attend on many occasions, in far-off places like Minsk, Belarus; Rio de Janeiro, Brazil; and Antalya, Turkey. The key element of an international competition like the WPC is that the puzzles must be language-neutral and culture-neutral. So while crosswords are certainly out, abstract logic puzzles like battleships and paint by numbers, which somehow make sense in any language, are in. Abstract logic puzzles can sometimes be explained with just a picture; showing how the curious symbols in a grid work can be clearer than a text description. There have even been rounds at the WPC where no instructions were given out at all. In these rounds, figuring out the rules from a pictured example was the first step in solving each puzzle. There is a language of logic puzzles—the common goals, the common rules—that transcends the many other spoken languages at a WPC as solvers race to uncover the fundamental truths hidden within these abstract forms.

The challenges at a WPC go way beyond the basic puzzle types in this section—though they are not always blatant about it. You might walk into a round expecting a normal puzzle on a square grid and be given hexagonal or 3-D puzzles instead. (Somehow the rules survive intact despite the stretching of the foundational geometry.) At the 2000 WPC in Connecticut, Mike's Hasbro-based team created a puzzle round centering on the construction of a Mr. Potato Head. One puzzle was a paint by numbers designed by Paul Peterson. Tetsuya Nishio, the creator of paint by numbers, found himself vexed by the puzzle and politely alerted Mike to the fact that the puzzle seemed unsolvable. Just as politely, Mike encouraged Nishio to keep at it, which he did with some suspicion. Then, several minutes later, Nishio's eyes lit up. The puzzle was indeed solvable, assuming you deduced that the numbers on the outside of the grid were in *base five*. That might seem like quite a leap of logic, but it was one that competitors at the WPC were eventually able to make from understanding how that puzzle type normally works and thinking outside the box to find a new frame that made more sense.

In this chapter I won't stretch the rules too much, but I will show you a lot of the common "abstract" logic types that I've encountered. These puzzles were created in many different countries, got shared at the WPC, and have now spread around the world. They fit into a few different subgenres based on what you have to do to complete the puzzle. In this section, there are **shading puzzles**, where you have to shade some white cells black to paint a picture or mark off an ocean or fulfill other goals. There are **object placement puzzles**, where you place different objects into the grid to obey some constraints; a frequent rule in this subgenre is that the objects do not touch each other in adjacent cells. Finally, there are **region division puzzles** where you must split up the grid in different ways. The boundaries between these subgenres can be fuzzy. Is LITS a shading puzzle, an object (tetromino) placement puzzle, or both? Much of the fun comes from combining different rules in different ways to unlock new puzzle styles. Perhaps some of these puzzles will inspire you to make your own new challenges, which you too can share with your friends around the world.

CRAFTING A PAINT BY NUMBERS

Paint by numbers puzzles go by many different names (nonograms, Pic-a-Pix, Picross, Paint It Black, Hanjie, Edel, and Oekaki, to name just a few). But regardless of the name, the joy of slowly revealing a picture is a nice reward for working through the logic of the puzzle. Paint by numbers puzzles were created in the late 1980s independently by two different Japanese designers, Non Ishida and Tetsuya Nishio. I've had the privilege of meeting and writing puzzles for Nishio—who is also legendary in the sudoku world—and must admit paint by numbers was one of the first puzzles I truly loved as a kid.

1. Come up with a picture idea

All good paint by numbers puzzles feature pictures you'd be happy to see when you're done solving. So give some thought to the one you're going to put into grid form. You're limited in color and detail, so don't try to replicate the Mona Lisa in just 100 pixels (a 10×10 puzzle). Pick something that's distinct enough with the tools you've got.

While most paint by numbers use just black cells, you can use other colors as well. Using a mixture of colored clues and black clues can simplify a hard puzzle and make a prettier (and more distinct) picture. But even if you're using black and white, you can do the American flag because no other flag has that pattern. On the other hand, you couldn't distinguish the flags of France and Italy without a blue bar for France and a green bar for Italy.[36]

I decided, since these would appear in a book of puzzles, to make two paint by numbers where the solutions themselves were puzzles. One would be a rebus, with smaller images indicating a longer word. The other would end up looking like another kind of logic puzzle. Since neither of these ideas really needed color, I stuck with black and white.

2. Lay out the image

Sketch your picture on a normal sheet of paper, without worrying about how it's going to fit into a grid. Settle the basic shapes, paying special attention to big bands of color. Don't make a puzzle with nothing but thin broken lines, unless you want to frustrate your solvers immensely.

Drop your sketch into a grid or a program that allows you to color in squares. If you do the latter, make sure you're using squares instead of rectangles. Most puzzles divide the squares into 5×5 boxes to help the solver with counting, so pick a number of squares that's divisible by five in both directions.

Long bars of color will make your puzzle easier. If a bar exceeds half the length of an area it can fit in (either an entire row or column, or a section bounded by some other block), the solver can place some information in the grid. There may be other ways to orient your image that will increase the number of these long bands. If you're drawing something with a long, straight border, realize that making it completely vertical or horizontal (with a big clue like 15) will be much easier than a 45 degree diagonal orientation (with only 1 clues in a large number of rows and columns). Something closer to vertical or horizontal (without being completely in one row or column) may be the right compromise.

[36]This, of course, would be a terrible idea for a paint by numbers.

PAINT BY NUMBERS

Paint a picture by shading some of the cells in the grid black. The numbers on the outside of the grid indicate the lengths of the consecutive shaded segments in each row (from left to right) and in each column (from top to bottom). There must be at least one unshaded cell between each of these segments. And here's some bonus fun: When you're done with the painting step, you'll discover that each of these puzzles has a second puzzling step.

Paint by Numbers #1

Column clues (left to right):

2 | 1 4 | 7 | 3 1 | 1 5 | 1 1 1 1 | 1 1 4 | 1 1 1 1 | 1 1 4 | 1 1 1 1 | 1 2 3 | 1 1 | 7 | 1 2 | 1 | 1 2 | 7 | 1 | 4 | 1 7 | 2 5 | 1 6 | 3 4 | 1 1 | 3

Row clues (top to bottom):

3 3 1 1
1 4 1 1 1 1
1 1 1 1 1 1 1
2 1 4 1 1 1 1
4 1 1 1 1 1 1
5 1 1 1 7
9 2 2 5 1
1 1 1 1 1 3 7
1 1 1 1 2 5
8 3

Paint by Numbers #2

Column clues (left to right):

1 | 1 3 | 1 1 | 1 5 | 1 | 17 | 1 1 1 | 5 1 1 | 1 1 | 6 10 | 1 1 | 2 1 1 1 | 2 1 1 1 | 6 1 | 1 12 | 1 1 1 | 1 1 1 | 1 1 1 | 1 1 1 | 10 6 | 1 | 3 1 1 1 3 | 1 1 1 1 1 1 1 | 1 3 1 3 1 | 1

Row clues (top to bottom):

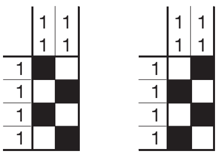
1 1
2 1
1 1
1 1
6 1 11
1 1 3 1 3
1 1 1 1 1
1 1 3 1 3
1 1 1 1 1
5 9 3
1 1
1 1
1 1
1 1
1 1
1 1 1 6 6
1 1 1 1 1 1
3 1 1 1 1
1 1 1 1 1
1 1 1 1 1
5 6 3
1 1
1 3
1 1
1 3

With large puzzles, rectangular grids are less tedious than square ones (since one of the dimensions is "easier" to fill in), so I settled on a long rectangle for my first rebus puzzle, with 25×10 being a size that got everything in. The second puzzle needed to be square, and 25×25 felt natural given the final image I was hoping to embed.

With the first puzzle, I started by figuring out a good way to draw the first and last part of my rebus answer (the middle letter was not as hard). The left part, an insect, needed some clear wings as well as a recognizable striped body. I found orienting the body sideways was the best way to capture what I wanted. For the right image, the most common picture in my mind was a cup and saucer. Including the saucer seemed like it would be a real problem so I went with the iconic cup and handle, and some steam lines to further highlight what it was.

3. Test the image and clean up unsolvable intersections
While you may have a perfectly recognizable picture now, this doesn't mean you have a valid puzzle when giving just the outside clues. So get a fresh grid and try to solve your puzzle (and while your solvers might eventually skip some of the logical steps because they think they know what the picture is, you must be much more rigorous).

Even before you strictly test your puzzle, you can look out for certain kinds of trouble areas. Single isolated pixels can be problematic (and interchangeable), such as in the situation seen at right, where there are clearly two options.



If you have spots where there aren't many connected pixels, think of small tweaks to better resolve the situation.

One of my first considerations was how to draw the steam trails off of the cup. I could have used something like the bad pattern above, but instead decided to both connect the steam directly to the top of the cup, and to use 1×2 and 1×3 rectangles along the trail to constrain the image to just one solution.

A different set of considerations was made in the second puzzle. While a lot of long lines were being shaded, there was also a whole lot of empty space throughout the grid. My first thought was "I bet there is no way this can be solved" and I was absolutely right. So I thought of how to adjust line positions up/down or left/right to build "very full rows." I was planning to give five different number clues, and had flexibility in what numbers went where, so I chose to put the ones with long horizontal bars in the most critical rows (particularly the 10th row) so that I had a chance—cross my fingers—to have just one solution.

4. Make a solving grid
Count each block of color in each row, putting their numbers in left-to-right order in a line next to the row. Then do the same with each column, putting the numbers above the column. This is where most mistakes in paint by numbers happen, so do this very carefully.

I made my paint by numbers grid in Excel, since it allows for multi-colored shading and variable width border lines already. Making my solution out of 1s and 0s, I could even use some of the default counting functions to make sure that my outside clues had the right number of spaces shaded in each row and column. Even then, I printed another copy on paper and solved from scratch to make sure everything was perfect.

CRAFTING A BATTLESHIPS PUZZLE

Battleships puzzles are based on an old pencil-and-paper game invented by Clifford von Wickler in the 1900s, a game that later became a similarly named board game first published by Milton Bradley. The solitaire version shown here was invented in the 1980s by Argentine Jaime Poniachik, and is now a staple of the World Puzzle Championship. The goal is to locate a fleet of ships inside a grid, using a combination of outside number clues and inside ship and sea clues. The simple tips here should keep you from getting lost at sea when you try to make your own battleships puzzles.

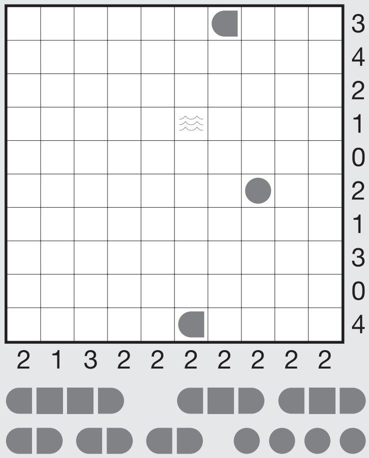
1. Start by filling in some spaces
There are two ways to approach a battleships grid, from the front or from the back. Working from the back here means writing down a possible solution, copying the relevant numbers on the sides of the grid, and then erasing all the ships. While this might seem to be the simplest approach, it is also risky. Without any extra clues inside the grid (either ship segments, seas, or both), you will almost certainly not have a unique solution to the grid although you will know you have at least one valid solution (the one you started with). With some practice, you can figure out what extra clues will make a particular puzzle solvable while not becoming too trivial.

I prefer to design from the front, which involves thinking of what the solver will place in the grid first. This means I'm not specifying all the row and column clues at the start, just a few that will force particular deductions. If you decide that a column has a "0" at the bottom, the solver is going to put all that water in first. So you can place things in the grid as if that water is already there, and try to add clues to force specific logical placements of ships as if you are also "solving" the puzzle. When you do this correctly, you will always have a valid puzzle with a single solution. Keep in mind that forcing ship segments can be valuable for two reasons. First, a ship can fill enough spaces in its row

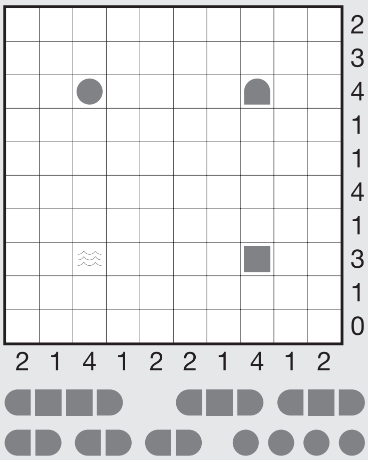
BATTLESHIPS

Each grid represents a section of ocean in which a fleet is hiding. This fleet consists of one battleship (four grid cells in length), two cruisers (three cells each), three destroyers (two cells each), and four submarines (one cell each). The ships may be oriented either horizontally or vertically, and no two ships can occupy adjacent grid cells, not even diagonally. The digits along the grid's perimeter indicate the number of cells in the corresponding rows and columns that are occupied by vessels. Some "shots" have been taken to start you off. These may show water (indicated by wavy lines), a complete sub (a circle), the bow or stern of a ship (a rounded-off square), or a midsection of a battleship or cruiser (a square). In the fourth puzzle, some of the perimeter digits have been deliberately omitted.

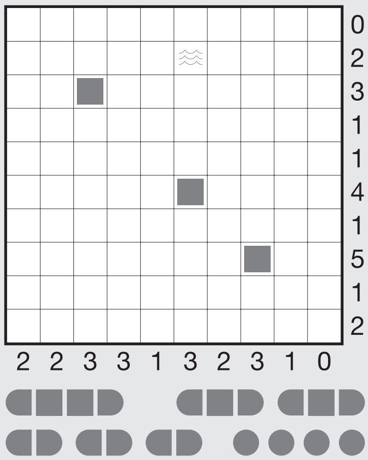
Battleships #1



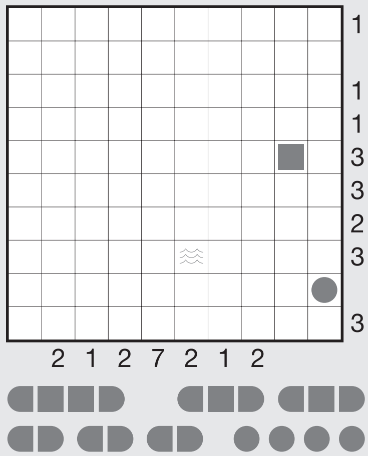
Battleships #2



Battleships #3



Battleships #4



or column to minimize options for any other ships in the same space until eventually all remaining cells must be water. Second, a ship can force unused cells in the adjacent rows and columns, and eventually the loss of unused space in those regions may force all remaining cells to be ships. I like to balance deductions with both kinds of reasoning, and keep meticulous track of the implied sea cells of any ship placement.

With either approach, think of what kinds of situations will help drive the logic of your puzzle. Most solvers go about these puzzles by trying to place the largest ships first, so you can make a grid that minimizes the number of places the 4-cell battleship can be. Having some empty rows and columns also helps, as they give a bunch of seas at the start while dividing the grid to minimize the options for larger ships. Think about it this way: If you can't figure out how solvers would break in to your puzzle, they probably won't be able to either.

In the first puzzle, I worked from the back and made a solution grid where none of the columns had very large numbers. (Originally, I had every column having two ship segments but this was too nasty and I changed it by moving a ship toward the end.) I ended up with only two rows that could have the 4-cell battleship, and then added three ship clues to effectively block one of the two rows from working. While these seeds did most of the work in forcing a single solution, I still ended up with some ambiguities. So I added the fourth clue, a single sea cell, in a place where only one of my solutions did not have a ship, leaving a valid puzzle.

2. Add inside clues as needed

Whether you are working from the front or from the back, having some clues inside the grid will be the key to getting your puzzle going. Clues are most valuable in this order: squares first, then rounded segments, then submarines, then seas. Squares are the easiest to build around, since they must contribute at least three ship segments to a row or column they belong in and often only have one possible orientation. Rounded segments always clue at least two ship segments (and as many as nine empty sea cells) while submarines clue exactly one ship segment and as many as eight empty sea cells. Sea clues are almost never important until the last few steps of a puzzle, and are almost always used to eliminate multiple solutions.

Inside clues can really set the challenge and fun of your puzzle if you think of unique ways to use them. For example, I've written at least one puzzle where I gave all four submarines at the start. Normally you fill in the small ships toward the end where you need them to fill in gaps; in a puzzle where you start with all four submarines, you can't leave any single cell gaps anywhere and must instead think about larger packing features of the whole grid.

In the second puzzle, I worked from the front but made a conscious effort to showcase all four types of clue in a nice symmetric pattern. Since the sea here was placed more for aesthetics than for logic, it is almost useless, but you will quickly see how the rounded segment (pointing toward a square clue) forces the first few steps of the puzzle. Leaving the solver limited options for the battleship was enough to complete the rest of the puzzle.

In the third puzzle, I gave three square clues on one of the main diagonals, meaning that the general locations of all three big ships are known at the start. By placing two of these squares near each other, the clues act like gears with each other. If the big ship is vertical in one, it must also be vertical in the other. The result is a fairly different kind of starting logic than you usually see in battleships puzzle, entirely set up by those first three clues I put in before considering any row or column numbers. I did make sure to place enough ship segments in the "unused" directions so that each square would see at least three ships in that row and column.

3. Vary the "usual" formula

Most battleships puzzles solve in the same way: try all the places you can fit the 4-cell battleship, then try all the places you can fit the 3-cell cruisers, and so on. To make memorable puzzles, think about varying this standard formula by considering some extreme cases. For example, make a grid where a whole lot of ships fill only two columns (the adjacent column clues "5-5" or "6-4" are very limiting and need mostly horizontal ships). Battleships puzzles rarely start with a lot of sea clues so consider writing a puzzle with "extra" sea information (say, 20 clues). You may be able to set up a pattern of seas where "negative" thinking dominates the puzzle, such as how you can eliminate a cell as putting a ship there would remove too many spaces from an adjacent row or column to leave a valid solution. Besides just extremes of logic, you can do a lot to play with the standard formula, such as varying the composition of the fleet (having more battleships and fewer submarines) or making all ship segments square so that the formerly rounded submarine clue is no longer distinguishable from other clues.

Most battleships puzzles give you all of the outside clues, but a few can be removed to increase the challenge level. While 0s are usually gimmes, you can remove a few 0s and another small number or two, so the empty column steps only reveal themselves midway through the puzzle.

In the fourth puzzle, I explored two "extremes" to make a really challenging puzzle. First, I removed some clues to hide a 0 column and a few other small rows and columns. While the blank rows take longer to tease apart, the first step of this puzzle is to recognize there are three unidentified ship cells in the blank columns and as a result the square clue must be for a horizontal cruiser since there aren't enough unused spaces (with the submarine clue) to have a vertical ship. It's not phenomenally hard to see what to do here, but it does take a little more work in collective column thinking than if I'd just put a 1 at the bottom of the column with the square.

The other "extreme," which is the more challenging part of this puzzle, was to pack a lot of ships into three columns. The 2-7-2 pattern here (with one cruiser already identified) actually requires a vertical battleship and three horizontal cruisers and destroyers. This may not be immediately obvious to solvers, particularly if they are thinking about individual ships, but once they think about whole sets of ships they may catch on to the trick here.

4. Check and re-check

Finishing the puzzle requires checking all the possibilities, so you cannot just assume "the battleship is here," find one valid solution, and stop. Most battleships puzzles are computer-generated to ensure uniqueness, but any hand-setting of a battleships puzzle will need this extra checking step. You'll find that having multiple grids, and doing branching—"the battleship must be *here*, *here*, or *here*"—will let you prove that there is just one solution. If you find more than one answer, add a sea or other clue to remove the extra answers.

If you write from the back by drawing a solution state and then using those numbers as the seeds for your puzzle, checking for uniqueness is the biggest requirement of your work. I really prefer writing "from the front," and embedding single logical steps (even very hard ones) one at a time, which makes checking easier. While both approaches work (and computers certainly generate these puzzles "from the back"), I've found that learning how to write any abstract logic puzzle, particularly battleships, going forward is tremendously helpful in figuring out how to solve them better and faster. Once you've learned some new tricks, you can put them into future puzzles to further challenge your solvers.

Here, I double-checked my work, especially on the first puzzle, which I wrote from the back and where my first few drafts had multiple solutions that I resolved by adding that extra sea clue. I had Mike check things over too. What I learned is that the fourth puzzle with missing clues is REALLY hard. I expected it to be really hard, but the capitalization of "really" was a little unexpected. I decided not to edit that puzzle to make it easier, but you can only learn where something that seems obvious to you is not obvious to your solver by having someone else test your puzzles.

CRAFTING A STAR BATTLE

The name star battle may evoke images of aliens shooting lasers in spaceships, but the "battle" in these puzzles is really against gravity. Stars need to be placed so that none are too close to each other, or share the same space, lest their gravity cause them to collide and destroy the fabric of space. While the puzzle shares elements with sudoku (the focus on rows, columns, and regions) and battleships (placing objects in a grid that do not touch), the logical steps can be entirely out of this world. Here we explore the far reaches of space to teach you how to lay out your own star battles.

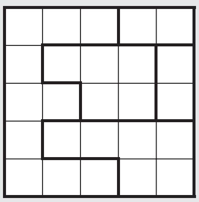
1. Choose your galaxy size

Star battle puzzles come in various sizes, and with variable numbers of stars. Since two stars cannot be in adjacent squares, each star effectively takes up about four squares of space (think about tiling a plane with stars—25% is the maximum density), which can help you estimate a reasonable "galaxy size" for your grid. What this means is that a 4×4 puzzle with four total stars (16 squares worth) is as small as a star battle can be, but most puzzles will be a bit larger. Similarly, an 8×8 puzzle with 2×8 stars (about 64 squares' worth) is pushing the lower limit, and there are only two possible solutions to such a puzzle before you even divide the grid into regions, so starting slightly larger than this is best.

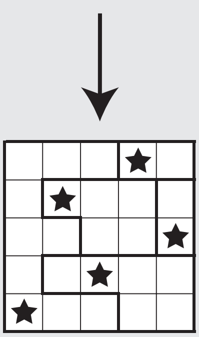
I decided to make a 6×6 and 8×8 galaxy with one star per area, and two 10×10 galaxies with two stars per area,

STAR BATTLE

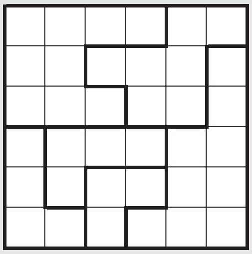
Place stars into some squares of each grid so that each row, column, and bold region contains the indicated number of stars (1 in the smaller puzzles, 2 in the larger puzzles). Stars cannot touch each other, not even diagonally. A sample puzzle and solution is shown below.



1★



Star Battle #1



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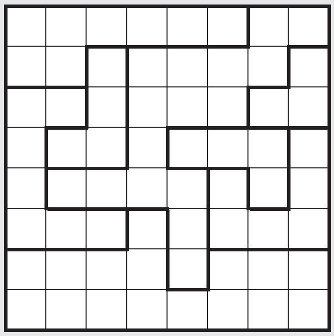
Star Battle #2



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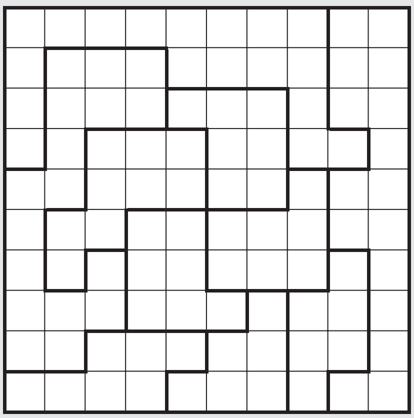
Star Battle #3



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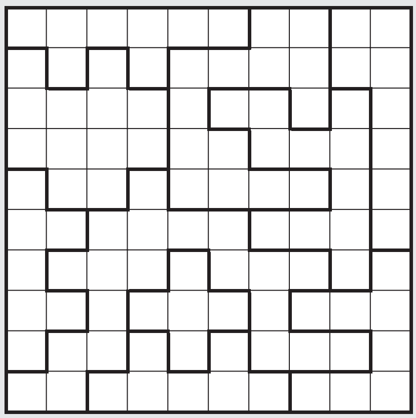
Star Battle #4



2★

each just slightly above the smallest possible sizes to leave some room for all the stars, but not so large that the grids would be very unconstrained and hard to construct.

2. Start out small

When beginning, start with some single-star puzzles to learn the basics. First, try to draw some region shapes that immediately define squares that can or, more often, cannot contain a star. One trick is to draw a region that only has squares in one row or column, eliminating all other squares in that row or column. Another approach is to draw a region that covers all the squares in one row or column, but also has squares in other rows or columns. Since the fully covered row or column must contain a star, all other squares can be eliminated. You can extend these basic rules into larger sets, such as having two or three regions that are only contained within two or three rows collectively eliminating all other squares in those rows from containing stars.

Your goal in defining these regions, and in eliminating some squares, is to eventually leave just one spot in a row, column, or region for a star to go for sure. You can often get there by constraining stars into smaller spaces, like 2×2 boxes or, even better, 2×1 boxes, which will eliminate the four adjacent squares on the long edges as well as any others in the row or column the box belongs in. After a few regions are drawn in, you should be able to delineate a region with almost all squares already eliminated, and one square left to put a star into. This first placement will often force a lot of the solution, or at the very least set up further opportunities to set up easy placements.

For my two single-star puzzles, I used several small regions to immediately constrain some stars. In the first puzzle, the nested L shapes lead to a lot of immediate eliminations as there are some cells in one region that "see" all the cells in the other region. Obviously, these cells cannot contain stars. You'll also notice that there are only three regions that exist in the top three rows, which means the last three rows of the bottom right region are all empty.

In the 8×8 puzzle I again nested L-type shapes together. In the middle right there is an L-shaped region where the short end of the L cannot be filled as it sees all the cells in another region. This leaves a 3×1 rectangle, all in one row, which cancels every other cell in that row. Now, on the left side, that leaves a 1×2 rectangle all in one column, which eliminates all the other cells in that column (and the four cells immediately to the left and right of the 1×2 rectangle). Chaining together eliminations like this is the key to building these puzzles.

3. Double the stars, double the fun

Adding more stars per region complicates the construction challenges, but will lead to more interesting puzzles once you master the steps. The biggest key to working with more than one star per region is to remember that a star effectively takes four squares and that a 2×2 square can only have one star. Joining together two small regions (like two 3-cell L's) lets you draw region shapes with two mostly constrained star placements, but without seeming as blatantly obvious as a 3×1 rectangle with its two stars in the outer squares. Interlocking small regions is particularly effective in getting a quick start, as spots that touch many squares in another region likely cannot contain a star, as that would not leave enough space for two stars in the touching region.

While these "almost" placements aren't yet fully set, they do start to fill the star quota for individual rows and columns, and you should frequently notate what is left in any group of rows or columns. For example, if seven possible stars are marked in the first four columns, then only one star is left in any of those columns, and you can use this fact to drive the next logical step.

In the first 10×10 puzzle, some small regions near the bottom (like the 6-cell region in the lower right) break into two-star placements quickly and eliminate nearby stars. Another technique was to set up a region that must have at least one star in a given row or column. Consider the bottommost row. The triangle-shaped region must have at least one star in this row (you can show it is exactly one star, actually), which means the region to its left can only have one star in the bottom row and must have at least one star in the next row above that one.

In the second 10×10 puzzle, I chose my favorite region shape as the starting point, namely the 5-cell plus sign in the bottom. It has two possible ways to be filled, but both eliminate five common cells. This forces another region to have its stars in just one row, which influences an adjoining region to finally affect the plus sign and force just one of the two orientations. The three-way gearing of stars at the start is made possible by carefully planning the position of these early regions. The larger, more nebulous region above the plus sign was formed out of mostly eliminated cells and a few leftovers to force the next step.

4. Apply some finishing touches

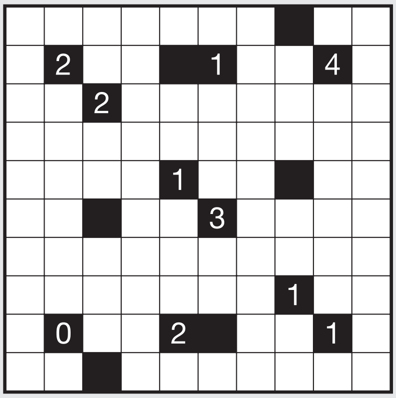
While laying out a puzzle, imagine what would happen when an either/or star placement goes a certain way, as this can trigger a chain reaction of other placements that can lead to a contradiction like having too many stars in a row or column. You may even discover that you've entered a dead end where no solution is possible. Small tweaks of the puzzle are almost always possible to fix these problems. Moving a couple squares from one region to another can open up the space you need.

You may also discover that you have too many either/or choices and no way of specifying just one solution. Again,

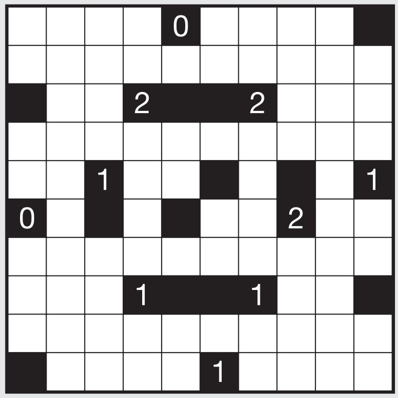
AKARI

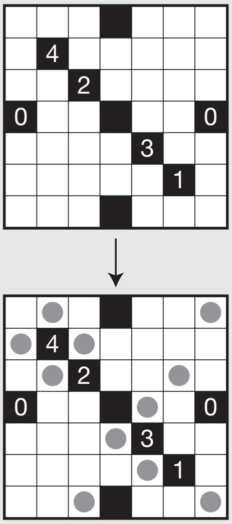
Light bulbs shine light on all horizontally and vertically adjacent white squares until the light reaches a wall or a black square. Place some bulbs into the white squares so that all white squares are lit by at least one light bulb. No two light bulbs can shine light on each other. Numbers on a black square indicate how many of the edge-adjacent white squares contain light bulbs. If a black square does not contain a number, it may be surrounded by any number of bulbs. A sample puzzle and solution is shown below.

Akari #1

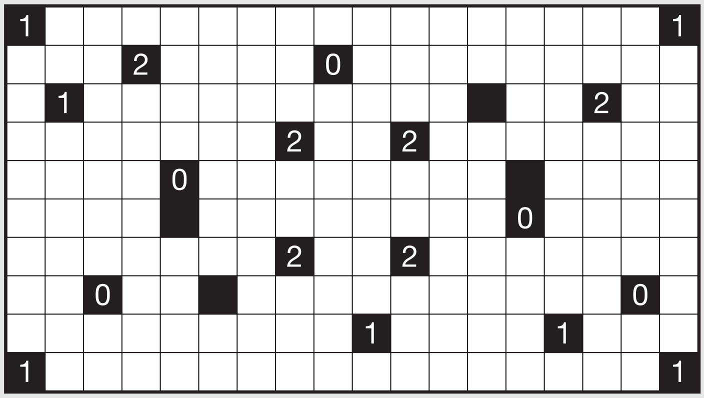


Akari #2





Akari #3



any either/or choice can become a sure choice by simply removing one of the cells from that region and regrouping it into another. Tweaking region shapes will refine your puzzles into their final form. However, any late changes you make must be retested from the beginning, to be sure you did not interrupt any of the logical steps you intended.

In the second 10×10 puzzle, after the first several placements forced by the logic described above, I played around with different region shapes at the top. I ran into some problems getting stars in, and realized I'd created an odd parity in the leftmost columns. The bottom center region put one star in column 3 in the bottom row, but the other all the way over in column 7. I needed another region on the left side that stretched to the right side that would only contain one star in the first few columns. It may be challenging to see why a star must go in row 1, column 6, but this was a necessary step when considering the star counts in the columns in different parts of the puzzle. You'll notice that shape is almost the same as two identically shaped regions elsewhere in the grid. Why? Well, it started out like the others without the cell in row 1, column 6, but when I saw I needed to get a star over there I added one cell on to my original rough sketch to both get out of a corner and embed a tricky logical step.

CRAFTING AN AKARI

Akari, sometimes called Light Up, is a logic puzzle from Japan. Akari translates roughly to "light" or "glimmer," and so the puzzle involves making sure all squares are illuminated according to certain rules. But I sometimes imagine the puzzle as organizing security at a museum. You have to make sure every square foot of the museum is monitored, but the docents are shy and don't like seeing each other. However you view the puzzle, I hope these construction tips prove illuminating.

1. Sketch out a grid with some black cells

Just as constructing a crossword puzzle often starts with thinking about the black cells, so too will constructing an akari. And as with crosswords, there are rules about the black cells worth keeping in mind. First, they should have some form of symmetry; second, however the black cells are placed, all the white cells must be connected.

So pick a grid size (10×10 is a good place to start) and sketch out a pattern of black cells that can form the basis for your puzzle. You may go for a more open grid with fewer black cells, or add lots of black cells and get more enclosed spaces. Don't feel locked down by early choices; you can always move things later.

I started with a lot of diagonally adjacent clues in the first puzzle and some longer rectangles in the second puzzle. For the largest challenge I wanted to keep some very long open rows and columns, like the outer border. In all cases, a few black cells were added or moved later.

2. Add single clues to begin to place bulbs

The easiest starting clue is a 4 where all adjacent cells will get a light bulb, but depending on the position of the border or surrounding black cells there could be similar situations with 3, 2, or 1 clues. When you add these "instant" clues to your grid, also sketch in the bulbs they force and draw vertical and horizontal lines from them to indicate all the cells now lit up. The lit cells may now allow new "instant" clues in other spots where the options have become limited.

High numbers are valuable clues, but 0s can be crucial clues too. Since all cells must be lit up, eliminating certain cells from being lit can force a light bulb somewhere else in that row or column, particularly in more enclosed spaces.

I put two starting clues into the first puzzle in symmetric spots. The 4 in the upper right is the first thing solvers will fill in by instinct, and those bulbs then force the 1 in the top center. But did you notice how the 0 in the lower left corner also comes with an easy placement right at the start? The corner cell must contain a light bulb or the cell to its right cannot be lit at all.

3. Add interacting clues for more involved logic

The key to interesting akari puzzles is not found in placing single clues, however, but in placing interacting clues that work together to provide new deductions. Such situations arise when light bulb positions from one clue either share or "see" potential light bulb positions from another clue. A 1 clue diagonally adjacent to a 3, for example, forces two empty positions and two light bulbs in the white cells around those two clues (with one light bulb and one empty cell in some order in the spaces shared by the two clues).

As a more complicated example, consider a 1×2 "domino" of black cells alone in the middle of the grid, with both black cells labeled with 2s. There are three white cells around each 2, but two of these three spots "see" the white cells for the other clue. To leave enough room for 4 bulbs around the domino, on each side there must be a light bulb in the spot on the thin edge of the domino that does not "see" any cells from the other clue.

In the second puzzle, the long rectangle at the top is an extended version of the 1×2 domino described above. In order for the 2s to be completed, the two white cells in row 3 touching these clues must be used, and then row 2 and row 4 are completely seen by one of two possible orientations of the remaining lights. Cancelling the possibility for a light in row 4 column 10 sets up another clue interaction, between the 1 below that cell and the 2 nearby it in row 6.

In the largest puzzle are two tightly linked sets of clues, one more complicated to see than the other. You probably noticed that the 1s in the corners form a cycle with two options; these two options completely light the border rows and columns which helps to eliminate choices around the 1 and 2 in the upper left corner. It is harder to notice that the middle set of four 2s is linked together and highly constrained; the bulbs around these clues will have to occupy every row and column around the central square area, which lights a lot of the cells in row 3 in the left-to-middle stretch as one critical deduction.

4. Add more black cells or clues to finish the puzzle

The last steps of building an akari are dealing with a handful of cells that are not yet lit but must get lit. If no other cell remains in that row or column, then the cell itself contains a light bulb. But when there are more choices, you often need to add a black cell or add a new clue or both to eliminate multiple possibilities and leave one solution.

The first puzzle was initially too easy, so I added the black cell in row 6 column 3 to set up a harder "how do some cells get illuminated" finish. In the second puzzle, the center of the grid had two unlit white cells and therefore two possible solutions until I added in two black cells. I also added the 0 and 1 clues in the top and bottom rows to better set-up some of the intermediate steps. It may take some practice to tidy up the last few placements, but any decent docent will appreciate the work you did to get a perfectly lit museum.

CRAFTING A MINESWEEPER PUZZLE

Minesweeper is one of the more compelling games included with your computer's operating system. You've probably spent many hours clicking on a square grid of cells, revealing numbers and planting flags, hoping

MINESWEEPER

There are some mines hidden in many of the blank cells of the grid (but no more than one mine in any cell, and no mines on any of the cells with numbers). Using the given numbers, which indicate how many of the adjacent cells including diagonally adjacent cells contain mines, determine the position of all of the hidden mines.

Minesweeper #1

									3
	6		4		6		4		
	5						2		
	4		1			1		0	
	3				1			1	
	2			1				2	
	1		3			5		3	
		2						4	
		1		4		2		5	
0									

Minesweeper #2

1								2	
		5		3		1			2
1					4		1		
		2						2	
	3	2		4	4		2		
		2		4	4		3	3	
	2						2		
		2		2					2
1			2		3		3		
	3								2

to avoid any mines.[37] I certainly have. While the paper puzzles based on the game may not have the same "game over" feel when you make a mistake, the same compelling abstract logic can be captured if you follow these steps.

1. Choose a grid and some clue locations

Just as the computer game offers different grid sizes and mine counts to vary difficulty, choose a grid size and a rough mine count commensurate with the challenge level you want. Then, think of a clue pattern and circle the spots on the grid where you intend to place numbers. Make sure each empty cell sees (that is, lies adjacent to) at least one number clue, preferably two or more. Don't feel locked down to these locations; you can always move some or add new ones in if you get stuck without a single solution. If you are going to add a thematic element to the placement or identity of some numbers, do this upfront as well.

Another choice you should make now is whether to give the overall mine count, such as saying there are 43 mines. Giving the mine count allows "meta-logic" when the count is very high or very low, such that steps like "these mines must be placed where all these numbers intersect as I don't have enough available mines otherwise" become most critical.

I chose to make 10×10 puzzles for this section, which is a large enough size to give a moderate challenge without feeling as tedious or intimidating as a much larger grid. I don't find counting all of the mines very fun on paper—it's much simpler when a computer automatically does it—so I went with puzzles that would solve cleanly without knowing the total number of mines. I chose a symmetric arrangement of clues since it makes the puzzles look nicer and isn't too hard to pull off. I had 28 and 32 clues respectively in the first set of puzzles, averaging 30% of the total cells in the grid, which meant most empty cells saw at least two clues and often more.

2. Identify some clues to place the first few mines

There are several ways to instantly place mines or empty cells in a grid. An 8 in the middle with no surrounding clues does the former, and a 0 does the latter. Your solver will certainly recognize the 8s and 0s rather quickly. But as fun as clearing a large section in the game Minesweeper might be when you click on a 0, it is far less interesting in the paper puzzle.

It is important to focus on shared logical deductions, therefore, and the most interesting parts of minesweeper puzzles are the clues that "communicate" with each other. What do I mean by this? Well, consider a 1 touching a 3 on the edge of the grid. There are four empty cells around the 3, but two of them are also touched by the 1 clue. So you can place two sure mines on the side of the 3, one "either/or" mine placement in the spaces seen by both clues, and two empty cells in the other spaces by the 1. This is a simple example, but you'll find many others if you try.

Having large clues near small clues will often do this. You can even relay multiple clues with each other, such as having two clues "sandwich" a third in the middle by having similar large or small values that don't match with the value in the middle. Your goal should be to force both sure mines as well as some either/or mines into the grid.

[37]Or maybe you've avoided flowers instead. In 2001, the International Campaign to Ban Winmine protested that Minesweeper for Windows trivialized the plight of landmine victims and those who risked their lives clearing them. So Microsoft included a version in subsequent iterations of Windows that changed the mines to flowers. Bully for them. Perhaps an organization for victims of submarine warfare will soon prompt similar changes to battleships.

After you have a lot of either/or mines in the grid, use clues that either see multiple either/or situations, or just one of the two cells in the either/or situation, to further chain the logic and eventually force just one answer.

In the first puzzle, I started with two very long chains of communicating clues, the 654321 column on the left and the 012345 column on the right. The latter is particularly interesting at the start as the 0 eliminates all but two cells from the 1, forming one either/or placement. The 2 then has two other cells that take one more either/or placement. The 3 then sees these two either/ors and has two other cells that take one more mine somewhere until finally the 4 forces two sure mines. Looking at just the 4 wouldn't give you much, but following the chain from top to bottom gives a work-in to the puzzle.

In the second puzzle, I wanted the center to be a place to do some work-ins, so I chose a square of all 4s as the centerpiece. Notice, on this square's lower left corner, that there are sandwiching clues that are all 2s. To get four mines around this corner, you cannot use the cell to the lower left of the 4. This would share one mine with the two 2s, and leave only two more mines to place, one fewer than needed. The right side of the center square is harder to resolve with this logic, and ends up being a bit different in structure.

3. Experiment with the puzzle format

Once you get good at the basics, vary the rules themselves. Some puzzles let mines occur on the cells that have numbers. Others go further and say that a numbered cell with a mine does not tell the truth while others must be exactly true. Perhaps exactly one number in each row and column must be covered with a mine in such a case.

The general rule of minesweeper (numbers give the count of surrounding cells containing some type of object) is a really interesting one in general and works well with other puzzles and shapes. So consider using something other than 1-cell mines in the puzzle. Putting minesweeper clues in the middle of a battleships grid, for example, is a nice variation I've seen of those two puzzles, which both involve object-placement in an abstract logic sense. You can also use other fixed sets of pieces, such as tetrominoes (see sidebar).

For the second set of puzzles, I went with tetromino minesweeper grids. In these puzzles, you have seven 4-cell shapes to place into the grid (which cannot touch each other, even diagonally). That the mines are part of connected shapes, and that these shapes themselves cannot touch, opens up a whole new set of logical deductions within the minesweeper rules. In the second puzzle, for example, consider the upper left corner. Because of the 0 in the corner, the 3 has only four cells that can be mines. But two of these touch at corners (and cannot be in the same tetromino), so the corner cells must be an either/or placement and the other two outer cells sure placements. This either/or placement then gives three more sure filled cells in the diagonally adjacent 4 clue. As with most shape

TETROMINO MINESWEEPER

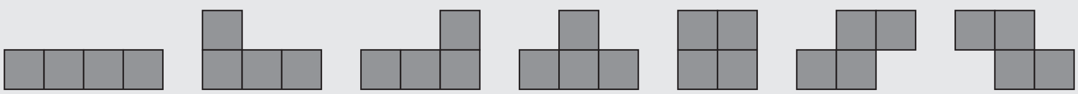
Place the full set of seven tetrominoes into the grid so that each given number indicates the number of surrounding cells (including diagonally adjacent cells) filled by tetromino segments. No tetrominoes can touch each other, not even diagonally, and no pieces will occupy the same cells as the given numbers. Tetrominoes may be rotated, but not reflected.

Tetromino Minesweeper #1

		2				1	3		
	5		2					2	
				1					
	4		3			1		1	
	2		2			3		3	
					1				
	3					2		4	
		4	3				3		

Tetromino Minesweeper #2

0								1	
	3								3
		4	3			4	4		
		4					3		
				4	3				
				4	3				
		3					2		
		5	3			3	1		
2								0	
	2								0



placement puzzles, once some of the shapes are in for sure you can leverage the identity of the remaining shapes to force other deductions. In the first puzzle you might find you still need to place the long I tetromino, for example, but not have many places it can go.

CRAFTING A LITS

As introduced in the last section and in the sidebar below, tetrominoes are an interesting set of shapes to use in abstract logic puzzles. Here I'll cover LITS, a puzzle type from the publishing house Nikoli in Japan, whose very name comes from the four basic tetromino shapes, L, I, T, and S, allowed in the puzzle. (J and Z are treated as equivalent to L and S in this puzzle. The last unique shape, O, is not allowed to appear anywhere in the grid.) While LITS puzzles can have a great deal of interesting logic behind them, they can be challenging to construct without understanding some basic steps.

1. Mark out some small, simple regions

LITS offers a free-form canvas on which to paint a logic puzzle, with virtually no restrictions on a region's size and shape except that it must contain at least four cells that form a valid tetromino. Since the larger regions will have many more ways to be shaded, the simplest way to start specifying some cells is to draw a 4-cell region shaped like an L, I, T, or S tetromino with only one possible shading. The next simplest thing to do is to make a 5-cell region, preferably a snaking region shape with only a single path from one end to the other. With regions like this, the middle three cells must be shaded. This may be enough to influence adjacent regions, particularly if those three cells are all in a 2×2 square forcing the fourth cell to be unshaded. By extension, you can also make 6-cell regions that must have two central cells shaded, although these are less useful as starting points.

In the first puzzle, I spelled out the puzzle name LITS at the top with 4-cell regions. These are certainly the first areas that will be shaded, but by surrounding them with a much larger region and also leaving the S separated from the rest, I set up an interesting future step to connect these "freebies" to the rest of the puzzle.

The upper left corner of the second puzzle showcases how you can use small cages and eliminated cells to get started. Shading the three forced cells in the corner region eliminates one of the five possible cells in the adjacent region. Filling in that now-forced S tetromino eliminates two more cells directly (and three others indirectly by isolating them from the rest of the region below the 5-cell ones). The geometry chosen for the regions in this corner quickly force an L, S, and I placement, with each region affecting the others to force all of the placements.

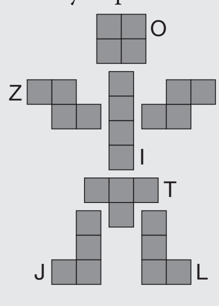
2. Outline adjacent regions that limit possibilities

With some starting spots, build off of these known locations. Try to take advantage of forced unshaded cells. These will occur whenever three out of four cells in a 2×2 square are shaded. One cell outside each L tetromino, and two cells outside a T or S, must remain unshaded. Consider a region that extends through an unshaded

Tetrominoes

Tetrominoes—shapes formed from four 1×1 squares joined at the edges—are probably most familiar as objects falling from the top of a screen while playing Alexey Pajitnov's game Tetris. After hours of frantically rotating and sliding them to clear lines of squares, you might even see tetrominoes in your nightmares. But don't despair! There's nothing to fear from tetrominoes, an interesting set of shapes that can be used in a variety of puzzles.

Tetrominoes are the seven unique tilings of four 1×1 squares. The different shapes are commonly referred to by the single letters they resemble in one of their orientations (I, J, L, O, T, S, Z) as you can see in the "tetromano" figure I made to the right. There are two pairs of mirror images represented in this set; J/L and S/Z may look similar but they cannot be rotated to superimpose on each other. So, depending on how you see the world, there could be either five or seven different tetrominoes. When you work with a full set of seven, take extra care with the J/L and S/Z pairs to be sure you don't accidentally put two of the same shape into your grid. It's easier to do accidentally than you might think, since they'll likely be rotated in different ways anyway.



When working with tetromino puzzles, you'll need to consider the proper grid size and shape to accommodate them all. There are 28 squares in a single set of tetrominoes, but it is impossible to pack them into a 4×7 rectangle so you'll need a bigger space than that, certainly with some holes. Most 9×9 or 10×10 grids work perfectly for tetromino placement puzzles, especially if you require empty space around each piece. Of course, you can also use a rectangular grid or another irregular shape as well.

cell. If there are only two or three cells on one side of that unshaded cell, then those cells must also be unshaded. You'll also find that regions that "hug" close to a shaded region often force you to leave adjacent squares unshaded to avoid violating the rule about 2×2 shaded squares.

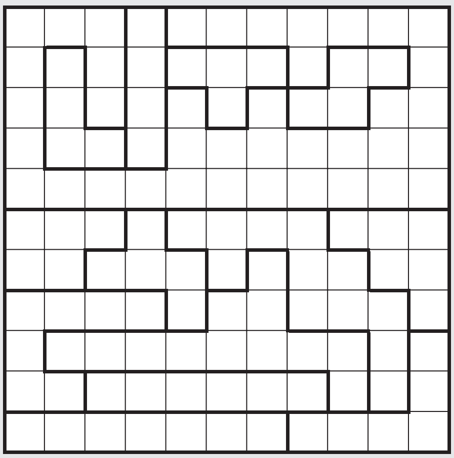
Another strategy to embed into your LITS puzzles involves the rule that no like-shaped tetrominoes can touch. There are ways to draw regions that, regardless of where cells are shaded, must have a particular kind of tetromino. A quick-bending snake must always contain an S, while a slower-bending snake can be formed to always contain an L. A 5-cell plus sign must contain a T, while a long narrow rectangle must always be an I. When you have already shaded a particular tetromino in an area, you can draw region shapes nearby that would force the same type of tetromino to touch, marking out a lot of unshaded cells.

LITS

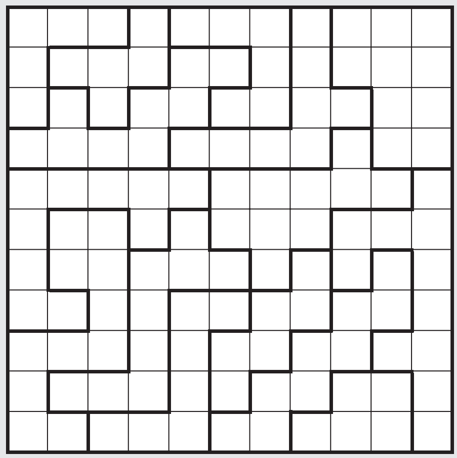
Shade exactly four cells in each outlined region to form an L, I, T, or S tetromino (shown below right). Tetrominoes may be rotated and/or reflected. When all regions have been shaded, the following conditions must be true:

- All shaded cells will be connected through a network of horizontally and vertically adjacent shaded cells;
- No 2×2 square will contain entirely shaded squares;
- When two tetrominoes share an edge in adjacent regions, they must not be the same type (L, I, T, or S), regardless of potential rotations or reflections.

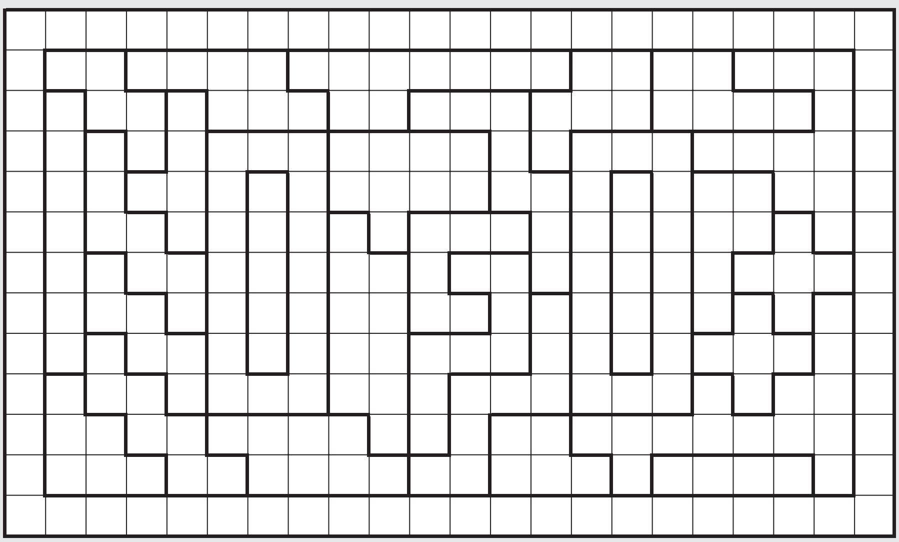
LITS #1

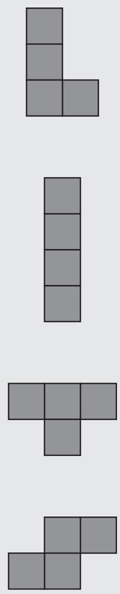


LITS #2



LITS #3





Forcing unshaded cells can be as valuable for constructing these puzzles as forcing shaded cells, particularly if you can isolate tetrominoes from the rest of the grid.

Each puzzle has a few areas where region shapes were chosen carefully to force "same tetrominoes cannot touch" thinking. In the first puzzle, the two long rectangles at the bottom, which must contain I's, only allow for one shading that keeps them from touching each other. In the bottom right of the second puzzle, and on the left side of the larger third puzzle, you'll see some nested snaky regions that must contain S's that cannot touch. The two touching "plus sign" regions on the right of the third puzzle form another such situation. The hardest groups to recognize as possessing just one type of piece are L-only regions, such as those in the middle of the third puzzle. Yet considering how to place four L's in adjacent regions in the middle is key to breaking into that puzzle.

While these are obvious groupings of similar region shapes at the start, I always try to keep this type of logic in mind in the middle of a puzzle too. After I've forced, say, an S in some region by whatever means, I then try to shape some of the adjacent regions that touch the S to exclude some cells by making only S-tetrominoes possible if that cell is shaded. The left side of the third puzzle, for example, has a region above the nested S shapes that can contain either an S or an L. But the S-only cells near the bottom touch a sure S tetromino, and all get eliminated to leave the L.

3. Connect everything together

The hardest part of any LITS puzzle is always the conclusion. While you may have set up some very interesting deductions to approach the finish, the last regions are basically the leftover cells. You have almost no control left over their shapes, which can leave non-unique solutions. Sometimes you'll just have to back up a step or two, to a time before painting yourself into a corner, to try to reshape the endgame.

One way to limit possibilities for this last step is to have left isolated groups of shaded cells so the "all must connect" restriction comes into play. You may need to add/subtract cells from defined regions to finish the puzzle. This kind of adjustment is often okay, provided those cells were unshaded or would become unshaded. You'll still need to recheck the puzzle afterward to make sure the changes didn't open up new possibilities.

In each of these puzzles I left a few locations where global connection thinking was required. The top of the LITS rectangle in the first puzzle is set up so that the solver must eventually figure out how to place one tetromino to connect the LIT group, the S, and the bottom of the puzzle together.

The upper right corner of the second puzzle highlights a fairly common finishing move you should learn. Because I tetrominoes are pretty rare in most region shapes, the most common situation for the last piece is needing to use an L, T, or S to join two disconnected groups. Try to set up a three-piece "sandwich" where two different tetromino shapes surround the empty region. Here, the last placement is an S in the upper right corner, forced by having an L and T that must be connected. If the two pieces of "bread" in that sandwich were both L's or both T's, then there would probably be several solutions using different shapes, since the last piece was left unconstrained to be just one type.

The last puzzle, while especially tricky, highlights how creative region shapes and connectivity can be the dominant logic in a puzzle. The big outside frame offers one relief point to connect an orphaned set of tetrominoes to the rest of the grid, but after it gets filled the remaining regions (which will initially be in three or four different clusters) will now have to be filled almost exclusively to meet the goal of joining everything. The two rectangular loops with an internal 1×5 rectangle are interesting groups (with just four possible shadings with an I and L) that for the most part are stoppers for doing the connections. The most challenging step involves the middle and the left rectangular loop. I set up a situation where three links are needed in this area (to the bottom, to an L piece on the right, and somewhere to the top) to get everything connected. While it might seem there is a lot of open space and an abundance of possibilities, anytime you need to join three things in a small space there actually aren't too many routes, since certain cells must be touched. Bridging into and out of the rectangular loop (which hasn't seemed possible before in the puzzle) is the only way to reach the conclusion.

CRAFTING A TAPA

Created by Turkish puzzle designer Serkan Yürekli, tapa has taken the international logic puzzle scene by storm in the last few years. Combining elements of paint by numbers, nurikabe, and minesweeper, tapa has its own unique charm. Once you've had your first taste of some tapas, you'll be left wanting more.

1. Think about basic starting clues

Many of our logic puzzle sections start by telling you reasonable grid sizes; tapa is special in that it works with almost any size grid. A good but short puzzle can fit in a 7×7 space. A longer puzzle can fit in a much larger space. And the clues affect the difficulty much more than the size. So pick any size you are comfortable with and then focus primarily on the clues—what they are and where they go.

There are some clear "gimme" clues, like an 8 in the middle of the grid that is fully surrounded by black cells.

A 5 on an edge or a 3 in the corner acts the same. But clues with multiple numbers can be close to a gimme too. The most numbers you can have in a clue is four, with a [1111] clue. There are two possibilities for how to shade this clue, as it has to have four white cells separating each of the black cells. It is easy to see that a [1111] clue cannot exist on an edge of the grid, but realize it also cannot sit "near the corner," one cell diagonally from the corner. Further, for a [1111] "near an edge," only one of the two possible options leaves escape paths for the black cells. Other pretty full clues, like [33] or [24], will have similar properties and behave differently when near a corner, near an edge, or isolated in the middle. Placing clues near big clues can often force a starting point because clues in the same neighborhood have to agree with each other. A clue that wants a lot of surrounding black cells can't easily sit near a clue that wants a lot of surrounding white cells.

The first puzzle shows a lot of basic "forced" clues. The [111] and [122] may not on their own look like forced clues, but by sitting near a corner they don't have a lot of options. There are other clues, like the [13] on an edge, where three cells can be instantly blackened and then exactly one of the two remaining cells must be black.

Adjacent clues can also work together to mark some cells quickly. The [33] in the center of the first puzzle would not be forced on its own, but due to the diagonally adjacent [22] clue being a forced white cell it can be solved quickly. Clues don't need to be touching for these effects to happen, just close.

2. Think about connecting small groups locally

With some seeds that define some shaded cells and some unshaded cells, now is the time to think about how to connect these groups. At the end of the construction there will be one wall, but looking locally you might see two or three or more separate bits of wall that must join together somewhere.

Connecting little groups is most easily done by adding new clues. Clues can act to grab the paths, or to deflect the paths. A clue with just 1s serves to deflect the path around it, either to an edge or to the middle of the grid. A clue with a 3 lets the path pass by straight through an edge; a clue with a 5 might even let it pass by a full corner. So decide whether you want to bring some groups together by adding clues that build off your deductions.

This is also the time to consider the "no 2×2 shaded square" rule. When you have a small section of path shaded, a new clue can lead to a logical deduction simply by needing to avoid shading too many cells.

In the second puzzle, I chose a clue geometry with a clear middle and four isolated outside corners. The puzzle started in the upper left corner, where I placed a big clue on the left side but small clues on the top side so that the wall could not cross along the top side (it takes three cells to cross into these corners). That corner ultimately sets up three different wall strands, and one of them needs to go around the upper right corner clues. I set very small clues there to constrain the wall really quickly.

One particularly interesting clue is [33], which we use many times in the third puzzle. It is a great connection clue, and has up to four shading possibilities. But it is also like an on/off switch. Sometimes it can be shaded so two paths both go vertically. Sometimes it can be shaded so two paths both go horizontally. Or it can be shaded to allow cornering. When one cell in the [33] is shaded, the opposite cell is shaded too. This has a large impact on adjacent clues, which is observed a few times in the third puzzle, including with the [13] sandwiched between two [33] clues.

3. Think about connecting the whole black wall globally

The final solution must have exactly one wall. So once you get closer to having most of the grid shaded, figure out if you have one wall yet. If not, perhaps because you were "deflecting" wall segments away from each other in step 2, bring everything together. You can do this by introducing new clues, or if you are building a symmetric puzzle, finally specifying clues in the opposite side of the grid where you haven't set anything up yet.

Thankfully, tapa is a very "local" puzzle, meaning you can often change exactly one clue to work out a problem spot and not affect anything else. So if you have two walls, find a good spot to make them instead become one. For example, if a [11] can be turned into a [3] to force a connection, do so.

In the second puzzle, the placement of clues happened before any numbers were written for them. This set up a geometry that gave only a few choices for how everything would connect together. Should the wall connect into the center here or should it continue to wrap around this corner? Identifying a few clues let me deflect in some places and unify in others, to ultimately get just one wall that passed through all these narrow chambers.

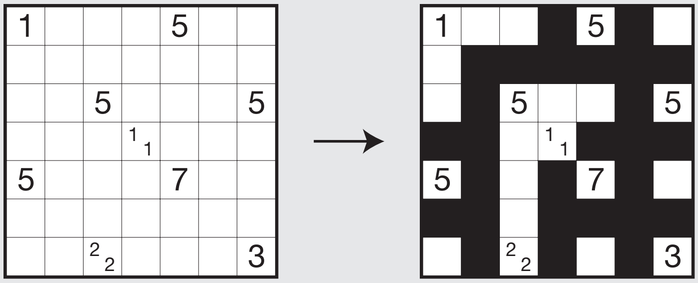
In the third puzzle, the [12] clues at the top act to separate the wall with one piece coming out of column 3 and another out of column 5 but no easy way through the center to connect them. The wall has to go all the way around the left and bottom of the grid to ultimately reconnect. I added in clues along those edges to limit the choices. The [13] in row 4 column 4 was originally a [15] and then a [14] before shaving another cell off to get to a clue that made the separation more extreme.

4. Check your work

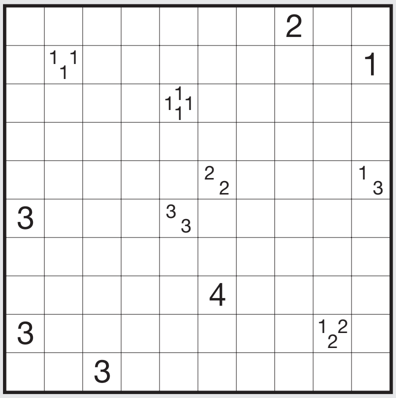
The most common mistake in making a tapa is to think, once a lot of black squares are specified to give one wall,

TAPA

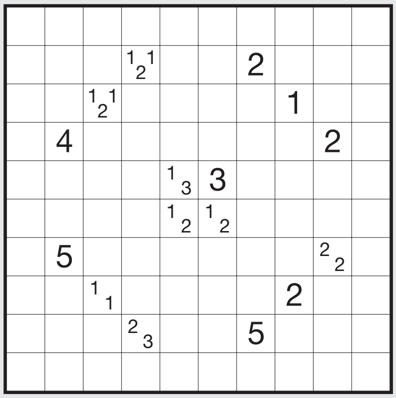
Paint some cells black to create a single connected wall of black squares. A number in a cell indicates a length of consecutive shaded blocks in the neighboring cells. If there is more than one number in a cell, then there must be at least one white cell between the black cell groups. Cells with numbers cannot be painted, and the painted cells cannot form a 2×2 square anywhere in the grid. A sample puzzle and solution is shown at right.



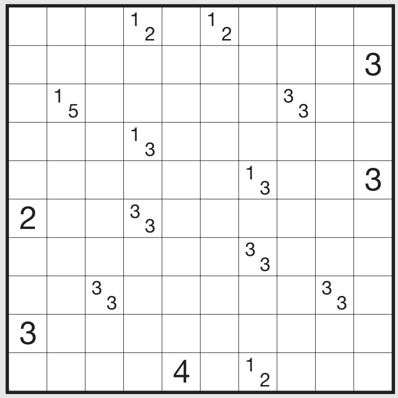
Tapa #1



Tapa #2



Tapa #3



that you are done. A tapa is not finished until every cell is either a number, a black cell, or a white cell. Your first check should always be that every white cell is logically forced to be white (from clues, the no 2×2 shaded rule, or the impossibility from other forced white cells of the shaded wall ever reaching there). Once you've done that, particularly if you tweaked some clues towards the end, resolve the puzzle from a clean copy. Check again that all blank cells are clearly black or white. Then serve it to your friends.

The most common white cells to miss are in corners or edges where no clues touch them. Notice the upper left and lower right corners in the third puzzle have no clues touching them. While the upper left corner must have the wall passing through it for connectivity, the lower right corner is mostly empty and could have more problems. With some clues, I found I got multiple solutions using extra cells in that corner (change the [12] to a [11] and you'll see what I mean). I figured out that I needed to identify clues that would shade three quarters of a 2×2 block to mark row 10 column 9 (and consequently row 10 column 10) as unusable, finishing the puzzle.

CRAFTING A NURIKABE

The name of the Japanese logic puzzle nurikabe comes from a spirit in Japanese folklore that manifests itself as a wall and blocks travel. As a puzzle, the nurikabe acts like a vast ocean, which separates a set of islands from touching each other. While the rules take longer to state than a puzzle like sudoku, the underlying logic of this puzzle type can be tremendously enjoyable to discover once you've learned the requirements for the ocean and the islands.

1. Select a grid size and position a few islands
Nurikabe constructors have a lot of freedom, as there are no standard requirements for a nurikabe grid such as clue symmetry. Select a grid size, often 10×10 for easier puzzles or 12×12 for harder ones, then plan out where some of the island clues will go—not what numbers will be necessary, just where they will go. Groups of islands that are one space apart on the edges of the grid must contain several ocean cells between them, and these cells can be shaded immediately. Clues that start diagonally connected also indicate starting points, and you can chain

several diagonally connected clues together to determine many ocean and island cells quickly. Regardless of the values of the numbers in these clues, their location will start seeding some points of progress in the grid.

I started with two 10×10 puzzles for easier solves. I then used slightly larger grids for more difficult challenges.

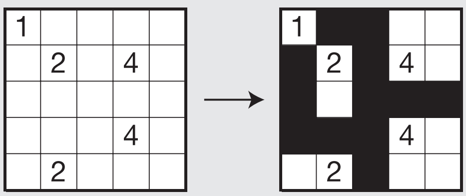
The first puzzle shows several clue patterns that lead to some quick initial progress. A diagonal of connected clues in the lower left gives several immediate water placements. Because of surrounding clues, the two ends of that diagonal have only one direction to extend the first island cell, regardless of how large that clue ends up. A slightly more complicated setup is in the upper right. There, a similar diagonal combination of clues is used along the edge. Note, with a number in the upper right corner, that the ocean must flow between the 2 and the 5. These entire islands end up getting placed because of the initial clue placement and needing the ocean to fully connect.

2. Start labeling the islands

With some starting points now marked in the grid, put in some island sizes. The main rules to keep in mind are the ones that deal with not isolating bits of the ocean and not forming 2×2 squares of ocean. Having too large of an island can often lead to isolating the ocean while having very small islands, or not enough islands, will lead to forming ocean regions that are 2×2 squares. Sometimes making an island that is just large enough to

NURIKABE

Shade some of the cells in the grid to leave behind white areas—“islands”—that must contain a single number and be of a size equal to that number. The shaded cells will form an “ocean” that must form a continuous stream and all be connected by other ocean cells to each other, but the ocean cannot contain any areas with 4 shaded cells that form a 2×2 square. The islands can touch diagonally but cannot touch each other horizontally or vertically. All of the numbers that appear in islands are given in each grid and cells containing numbers cannot be shaded. A sample puzzle and solution is shown below.



Nurikabe #1

4									5
								2	
									7
9		3							
			1						
				2					
	9								5
				5					

Nurikabe #2

2			2			2			2
				2					
2									2
		2							
						2			2
	2								
2			2						22

Nurikabe #3

		2			2						
									4		
2							4				
										4	
				2			4				
2									4		
	6			6			8				
					6						
						8					

fill a space, but which cannot be any larger, is a good way to get started in directing your puzzle's solution.

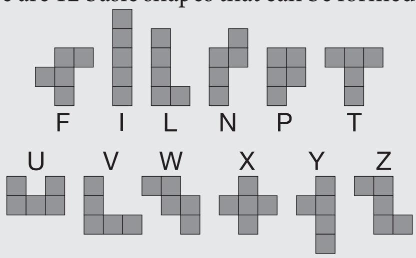
When you run into trouble defining a specific island arrangement, add some new island clues into the grid. Since islands must avoid touching on their edges, you can put in a small island clue like a 1 or a 2 right near a partially identified island as a means to "deflect" that island's path and force it to expand to fill a different space. You'll quickly realize, when thinking about islands, that you should also think about a 1-cell wide border of ocean that must travel alongside it most of the time, especially when the ocean would otherwise be isolated.

If you find your original choice of island sizes too big or too small, its very easy to change their values to be one larger or one smaller. Small adjustments will almost certainly be needed on every puzzle to get things just right.

The most important observation in the first puzzle is that the 9 in the lower left exactly fills that corner as a 3×3 square and has no other options. This now starts two separate pieces of ocean that must flow up and out and eventually reconnect. Notice how the 4 and 9 on the left edge eventually come together and then push to the right. This kind of deflection is easily set up when there is simply nowhere else for an island to go.

Pentominoes

Since bigger is sometimes better, and since I've already covered tetrominoes, it's time to introduce the pentominoes, another great set of shapes to use in puzzles. With pentominoes, there are 12 basic shapes that can be formed out of joining five 1×1 squares at their edges. Each shape is assigned a name based on a letter of the alphabet it closely resembles in one of its orientations, as shown at right. While there are six pentominoes with different mirror images (F, L, N, P, Y, and Z), usually only the set of 12 is used in a puzzle, not the larger set of 18, unlike tetrominoes. However, you will sometimes see rules in pentomino puzzles that allow "rotation but not reflection" which means whichever F is being shown is the one that must be used, not its reflected doppelgänger.



The simplest pentomino puzzles involve just a 5×12 or 6×10 rectangle, with a few letter clues indicating specific cells taken by a specific pentomino, that must then be divided into exactly a set of 12 pentominoes using each cell just once. Trying to make valid tilings, you'll find that only certain pentominoes can be in the corners of a rectangle, and others tend to get packed together all the time (like the U + X, which fit rather well together to then give nice edges). Irregular shapes can be used in place of large rectangles, but still with 60 total cells and still with the main goal of dissecting a tiling back into the component pentominoes.

The next most common pentomino puzzles are battleships-like, with the whole set of 12 shapes in a grid, but with no touching allowed, and outside clues giving the total count of cells shaded in that row or column. With 60 total cells of objects to place, an 11×11 or 12×12 grid size is by far the most common to account for the necessary empty space between pieces. Letter clues are used on the inside to isolate a few cells of some of the pentominoes, as well as striking out some internal cells to further limit choices. But pentominoes also can be used with other puzzle types naturally, as I have done here with nurikabe.

The logic behind pentomino puzzles most often exploits the variety of shapes that are seen within the set. To get used to the variety of the shapes, take a blank piece of paper and simply try to draw the 12 pentominoes from scratch. You'll really come to appreciate how many different ways they get connected to fill spaces. Only one piece, the I, can be 5-long. Three other pentominoes (N, L, and Y) are 4 cells long, but only two of these can touch the opposite corners of a 2×4 rectangle. The rest of the pentominoes all fit within a 3×3 or 3×2 box, with three pentominoes (V, W, and Z) capable of touching opposite corners of a 3×3 box. The U and P are the most compact, fitting within a 3×2 space, and the P is the only pentomino with a 2×2 connected set of cells.

In almost every pentomino puzzle, after four to six pieces are placed, you must now very carefully consider how the remaining pieces can satisfy certain kinds of requirements; if you need a piece that reaches between two particular squares, there may be only one option remaining. The pentomino nurikabe puzzle on the facing page works exactly this way. Standard nurikabe steps will place the L and the I quickly, but then you have to consider how to shade some other long islands that can no longer be the L or I (and end up being the N and the Y). Midway through, to avoid a 2×2 ocean in the lower right corner, you need a pentomino that can reach opposite corners of a 3×3 square. Only the V will remain at this time. The end of the puzzle becomes simple once you realize the X still must be placed, but forced island cells elsewhere that can't be in an X shape leave only one spot for it.

3. Start isolating some ocean cells

Every cell in the solution will be an island cell or an ocean cell. An alternate strategy to consider when laying out islands and assigning numbers is to isolate some cells that cannot be reached by any island clue (note that all islands must be numbered). This means these cells must be shaded. If you use this approach to shade in three of four cells in a 2×2 square, you now also make an isolated "must be an island" cell.

In the second 10×10 puzzle, there are a lot of small clues and one big clue. The result, particularly in the lower left of the puzzle, is a set of cells that cannot be reached by any island. The large 12×12 puzzle, which has a 2/4/6/8 quadrant theme, similarly has a lot of isolated ocean cells in the upper left because of all the small clues and the wide-open space. You can identify two 2×2 blocks where only one cell can be an island (to connect to a 4, and a 6), and these form the starting point for that puzzle.

4. Connect together small starting points using more global strategies

The early stages of construction should have left you some areas with defined ocean and island cells; now is the time to start to build in some more "global" thinking which will bring the puzzle to a satisfying conclusion.

Global strategies take a few forms. First, because the ocean must be one large stream, blocking a potential ocean connection point on one side of the puzzle may force the rest of the ocean to wrap the other way around the grid. You can certainly consider ways to arrange clues so the number of options for how to get the ocean all connected is rather limited, so that thinking about the ocean more than the islands is key.

Second, filling in islands and water often creates unattached "must be an island" cells as the last cells of 2×2 blocks where the other three cells are ocean cells. These must eventually connect somewhere, since all islands have a number, but you can sometimes limit the size of nearby island clues so that there is only one potential number to connect to. Actually forming this connection may require a very specific arrangement of island and ocean cells. Setting up some limited paths (possibly by adding in new small 1- or 2-islands) to force these connections is an acquired but necessary skill.

Balancing between "ocean must connect" and "must be an island" steps should complete your puzzle. You may need to slightly change an intended island size plus or minus one cell, or even add a clue, depending on how the last connection is made. If you do make such an alteration, recheck the puzzle from the start to be sure the rest of your logical approach is intact.

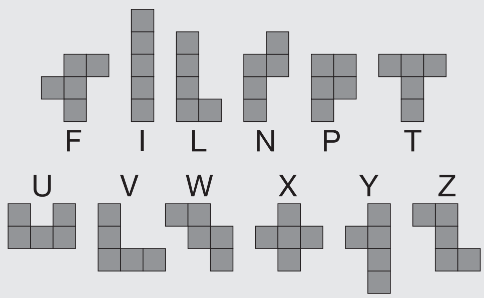
The second 10×10 puzzle is almost exclusively a "global thinking" puzzle, with the need to (1) fill in a connected ocean and (2) fill in a large 22-cell island. The 22-cell island must snake between the smaller islands to eventually reach the top left and top right corners, but there must also be a path for all the ocean cells to be connected. As the 22 starts in a corner, the ocean cannot go around it in the bottom right corner, and must instead connect around the other three corners of the grid.

The third puzzle has a lot of gradually revealed tricks that require global thinking as well. My favorite is in the lower left corner. Needing to not form a 2×2 ocean will force the lower 6 island to go across five cells. It must fill one more cell, but several would form 2×2 oceans and others would fully isolate the ocean in that corner. Figuring out how to balance the 6, the ocean, and the nearby 8 island must be

PENTOMINO NURIKABE

This puzzle obeys the standard rules of nurikabe. Also, there are exactly 12 islands, all 5 cells large, and these islands must form a complete set of the 12 pentominoes shown below the grid (rotation and reflection of these shapes is allowed).

			5								
		5									
			5		5		5				
											5
						5					5
			5								
								5			
				5							
			5								



FILLOMINO

Divide the grid along the dotted lines into regions called polyominoes so that no two polyominoes with the same area share an edge. Inside some cells are numbers; each number must represent the area of the polyomino it belongs to. A polyomino may contain zero, one, or more of the given numbers. (It is possible for a "hidden" polyomino—a polyomino without any of the given number—to contain a value that is not present in the starting grid, such as a 6 in a puzzle in which no clue higher than 5 is given.) A sample puzzle and solution is shown at right.

1	2	2	3
2	4	3	3
2	4	2	1
4	4	2	3
2	2	3	3
1	6	6	1
6	6	3	3
6	6	1	3

→

	2		3
2			
			1
4		2	
	2		3
1			
			3
6		1	

Fillomino #1

	1	2	3	4			2	3	
	8	7	6	5			9	4	
							8	5	
							7	6	
	7	6							
	7	6							
	2	1			8	8	7	6	
	1	2			2	4	7	1	

Fillomino #2

		1		3	7		8		
	3	4					4	3	
			3	7	3	2			
	4		5			4		3	
	4		1			2		8	
			4	5	5	6			
	8	3					9	2	
		4		4	2		3		

Fillomino #3

		6						6	
2			8				3		
	2			2		1			2
		8			2			2	
			4				7		
		6				2			
	4			7			2		
2			5		3			1	
		3				3			7
	1						7		

considered together to get the right solution. The lower right is then pretty hard to finish, but is assisted by realizing none of the islands can touch the right edge as this would fully isolate the ocean from itself. Once you shade in the right column, and some of the cells that must be islands to avoid 2×2 oceans, the end should become clear.

CRAFTING A FILLOMINO

Fillomino might remind you of packing for a trip. Sure, when you first head out everything fits easily in your luggage. But the return trip is a whole different story. Maybe it's the new souvenirs, but your bag just won't shut without sitting on it. Without learning the secrets of tightly fitting objects together, you'll be leaving some items at the hotel. When constructing fillomino puzzles, you too must learn how to tightly pack some polyominoes just right or else start all over.

1. Consider geometry

Even before identifying any numbers, you should recognize the importance of "location, location, location" in fillomino puzzles. A 2 clue in the middle of the grid with nothing around it has four different ways to expand to grab that second square. Larger clues will have even more choices then for the third square and higher. Contrast this situation with a clue surrounded by other clues or by edges of the grid. If there is just one open edge for another cell, then there is a certain next cell in that polyomino whatever its size.

Decide on a general layout for some clues that will create a few situations with constrained polyominoes to build off of. You can add additional spots later, but the start to any fillomino puzzle is usually a compact pattern.

In the first puzzle, I used some compact 2×4 rectangles for the clues which could pin four different positions depending on the exact clues placed into these rectangles.

In the second puzzle, I did something less common that still started from just the geometry. I made a few touching "corners" that were part of different polyominoes. Consider the 3 and 4 in the upper left that touch diagonally. The 4 can either go down or right for its next cell. Whichever cell it takes, the 3 will take the other cell. This crowding sets off some unique deductions.

2. Identify some clues

With a geometry chosen, identify some clues. With a constrained geometry, an easy way to start is to put in some very large numbers. One or more polyominoes might need to expand in a very limited space, which will eventually block off options that other spots started with.

As polyominoes start to grow, another way to "deflect" them is to put the same number in other spots where the growing polyomino cannot reach without getting too large. The "same size polyominoes cannot touch" rule is very easy to use with small clues like 2s, where diagonally adjacent 2s block off the cells that are edge adjacent to both clues. But you should get familiar with using it when a 5 polyomino has four cells filled and cannot reach a particular square without picking up two more cells because of an additional 5 clue nearby.

In the first puzzle, I sketched out some possible sizes of polyominoes where each of the clue numbers was different. I found I could place 23456789 in clockwise order in one way where the 4 and 5 have just enough space to be put in, giving a quick start to the puzzle. The remaining steps use large polyomino deflection a lot.

The second puzzle takes advantage of the geometry described above with the 3 and 4 that touch at a corner. By adding two 4s in the second column in the middle of the grid, I could block off one of the two options for that 4; it cannot touch the pair of 4s without growing to be too large of a polyomino. The same applies to the 4 that diagonally touches the 3 in the lower left. So that pair of 4s forces both of those clues to move away from the middle left.

The third puzzle has a large number of 2 clues, some of which cannot point at each other. Even 2 clues that don't diagonally touch, like those in columns 6 and 7, still interact with each other. You can use the "cornering" mentioned above, as well as additional clues, to get started.

3. Hide some unclued polyominoes

After tackling the basics of getting polyominoes in the grid, now is the time to figure out if you can hide extra, unclued polyominoes. Such spots almost always start near "1" clues which act like polyomino vacuums. A 1 cannot touch another 1, so some larger polyomino must fill the space around the 1. If you can place a 2 polyomino not one but two cells away, then that larger polyomino must be 3 or larger. This can continue to grow large hidden polyominoes.

The third puzzle was built to demonstrate this concept, with a large polyomino hidden in a sneaky way in the lower left corner. While its exact size is not known at the start, it is at least 8 large and blocks off a lot of the options for the left side of the grid.

I had a few different options in the first puzzle to finish the bottom left corner of the grid. Rather than using large clues, I placed several 1 and 2 clues there and was able to hide two hidden 3 polyominoes nearby.

4. Fill the last cells

As you get close to the end, there are often just a handful of clues left to give but a lot of space to fill. One way to complete the puzzle is to take the number of empty cells plus clue cells and find some breakdown of numbers that adds up to all the remaining cells. Make sure that if, say, 9 cells remain and you make the last clues a 4 and 5 (4+5 = 9), there is only one possible 4-omino that can fit into the grid.

The last clues in the third puzzle went along the top side of the grid. I originally had a 4 and an 8 in the upper left, but that allowed two options which I missed the first time through (the 4 could form a 2×2 square by taking cells to the left and down, or take three squares to the right). To fix this, I turned the 4 into a 6, and then also tweaked the upper right to have another 6 that made things fit perfectly. Which I was thankful for, as I was about to throw the whole puzzle out and start packing again from the start.

CRAFTING A SHIKAKU

The "packing puzzle" is a puzzle type where a blank or mostly blank area must be stuffed with objects of varying sizes. Of all the packing puzzles out there, shikaku (sometimes called rectangles or divide by box, a translation of the Japanese "*shikaku ni kire*") is one of the easiest to understand: simply divide the grid into rectangles so that all the indicated areas fit perfectly into the space provided. Despite the simple rules, shikaku have a good depth of logic to explore.

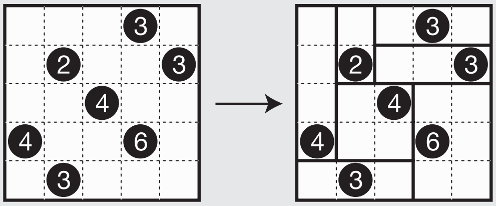
1. Decide on a clue pattern

Your first instinct for how to construct a shikaku might be to simply take a square grid, say 10×10, divide it randomly into some rectangles, and then place clues somewhere in the boxes so that all areas are correctly marked. While this will certainly lead to a valid-looking solution, it is unlikely to lead to an acceptable puzzle with a unique answer. Getting a good puzzle will require a bit more careful planning.

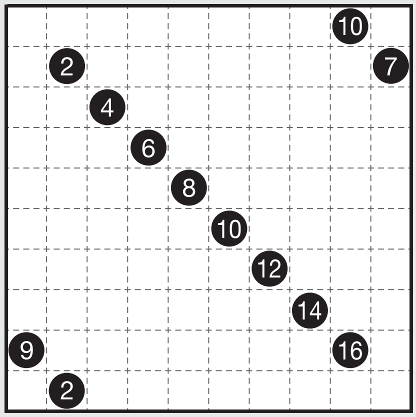
The first step in constructing a shikaku is to choose a clue pattern, preferably a symmetric one. When laying out a pattern, consider how many clues can contain particular unmarked cells. You can lay out a pattern where some cells must belong to a particular clue; harder puzzles will leave a lot more options for each cell, making it more difficult to know where to start. The density of clues is also important to consider. Any clue surrounded by several others will be more constrained in its reach than a single clue surrounded by white space.

SHIKAKU

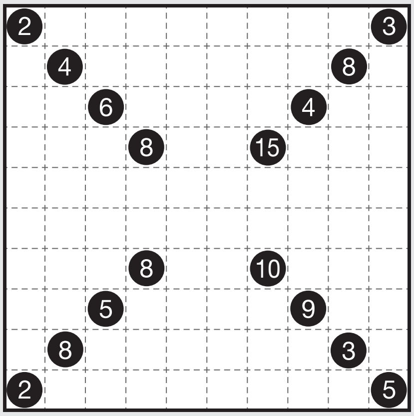
Divide the grid along the dashed lines into a set of rectangles so that every cell is part of exactly one rectangle. Each rectangle will contain just one number, which must indicate the total area (in cells) of that rectangle. A sample puzzle and solution is shown below.



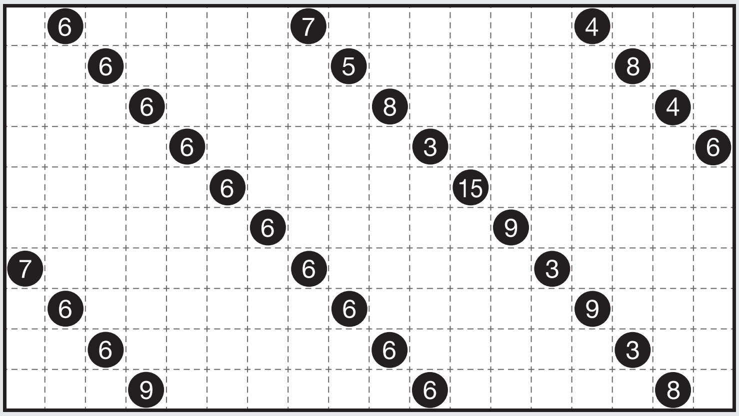
Shikaku #1



Shikaku #2



Shikaku #3



One common clue style that works very effectively in shikaku is diagonally adjacent clues. The first and third puzzle both use long diagonal stripes; these stripes limit the orientations of each rectangle, typically projecting outward from the diagonal, either to the upper right or lower left. When one rectangle goes in one direction, it tends to force the next rectangle to project in the opposite direction. This effect is showcased very well with the packing of rectangles along the main diagonal in the first puzzle.

Even with diagonal clues, the respective orientation of the pattern is important. In the second puzzle, which forms an X, the clues in the corners each have only two possibilities because they are being crowded out by the borders of the grid and the adjacent clue. Compare this to the middle of the grid (where I left out clues that could have gone in the middle four cells to make a full X). By having a lot more white space in all directions, the central clues have many more options and are a more challenging place to start.

2. Fill in some areas to define starting points

Identify some (not all, and not even most) of the values to define some starting points. Harder puzzles can result if you isolate a cell in an otherwise open area, so possibly pick a cell to try to cut off. By setting some numbers to be too small to reach a cell, or too large to fit and reach a cell, you can leave just one option, which will then form the first sure rectangle (or part of one).

The identities of digits can also be used in theming a puzzle. The most interesting numbers are those with the most factors (and therefore the largest number of possible rectangle shapes and orientations). An area of size 6, 8, or 12 is going to be much more interesting than an area of size 5, 7, or 11 since the latter prime numbers must be in 1×5 or 1×7 or 1×11 rectangles.

In the first puzzle, the main diagonal shows a progression of even numbers, each with a lot of placement options and most with several possible shapes. By putting the small 2 in the bottom row, there are a bunch of cells that *can only be reached by that 16 clue. This should position the first rectangle for sure, and the rest will follow quickly afterward.*

In the third puzzle, my desired theme was to have an entire diagonal made out of 6s (which would have a lot of 1×6 and 2×3 options). It is unlikely anyone can get started on that side of the grid. A more conspicuous clue, likely the 8 at the bottom right (which only has one possible rectangle), will be step one.

3. Build out from the packed regions to reach the end

Once you've placed a starting point, the solution will propagate out from it. Identified rectangles will isolate new spaces of the grid, and thinking again about making nearby clues either too small or too large to reach them may leave one choice. Adding in larger clue areas can quickly constrain a grid; an area like a 12 may have several choices, but these choices often overlap in a small number of squares that can therefore be immediately assigned to the clue.

As you near the finish, you may find that you've made a grid where your clues cannot reach some of the cells. You can either backtrack and undo some of the deductions which caused this problem or, more often and more easily, you can add in new clues (maintaining symmetry) to reach the orphaned cells. Shikaku are reasonably tweakable, but make sure just one unique solution remains after you make any changes that could impact your initial starting path. Again, it's easy to form a valid-looking solution, but unless you go back through the steps of identifying each rectangle from the start, it is hard to know it is the only solution.

In the second puzzle, the two key numbers in the puzzle are the two largest, the 10 and the 15. Both seem to have several options (for 2×5 and 3×5 rectangles respectively), but you can position 2×2 rectangles that must be a part of any 2×5 or 3×5 choice that contains those numbers. Once you place that partial information, the 15 now has only one choice. When constructing your puzzles, don't just focus on "sure" placements, but also consider how knowing a few cells that belong to a rectangle can be enough to block the rest.

The third puzzle, with all the 6s, was almost too big a challenge to pull off. I had several versions that were almost valid except for a few non-unique solutions where two adjacent 6 clues could fill the same space in multiple ways. Instead of abandoning my ambitious theme, I focused on where an isolated cell might be particularly valuable to forcing a particular arrangement of the remaining rectangles. The 7 in the top row was one key clue. Once placed, it leaves just four cells in the top left, but the third and fourth cells cannot be reached by the 6 in the top row. Putting in the two vertical 1×6 rectangles eliminated a lot of the options for the rest, and eventually finished off this puzzle. When the 7 clue was one shorter, and left five cells in the top row, there were many more options to reach these cells.

CRAFTING A CAVE

Solving a tough logic puzzle can feel like being trapped in a cave. You're in the dark, slowly feeling around for walls but you don't know if you'll ever get out. And then the first sign of progress—a beam of light—leads to more light, and then even more light, and finally escape! Solving cave puzzles (sometimes called "corral" puzzles) is much like this, with tough initial steps but eventually enough ahas to get out. Follow these tips and you'll be able to leave spelunking challenges for all your friends.

1. Think big and small

Choose a grid size (anything from 8×8 to 12×12 is reasonable to start) and place a few numbers. Really big numbers, like 15 in an 8×8 grid, make every cell in that row and column part of the cave. But "almost" really big numbers, like anything 10 and above, are good starting points too. A 10 in the corner of an 8×8 puzzle must extend at least two cells in each direction, but possibly more considering context.

On the other extreme, really small numbers, particularly 2s, don't give a lot of flexibility. A 2 that is almost adjacent to another number (like the 2 and 5 in the sixth row of the example) cannot connect to that number, so the intervening cells must be outside the cave. Once you've identified several cave cells, placing small numbers near them that cannot connect is a good way to advance the puzzle. One useful approach is to place big and small numbers together in the same row or column to interact.

I thought both big and small in our first "odd" puzzle, with one largest possible 15 clue and several 3s. After filling in the 15, many of the options around the 3s are no longer possible because of the cells already used around the clue. Large and small clues close together in the same row/column (like the 3 and 9 in column 2) can also force the cave to move in the perpendicular direction. In this case, the 9 must connect several cells to the right.

The third puzzle uses big digits as well at the start, with an outer frame forming from the 23 and 21 clues that has only two options for one unused cell.

2. Avoid the checkerboard pattern

One unwritten rule in cave puzzles that arises from the "no enclosed cells" constraint is that you cannot have a 2×2 checkerboard-like coloring of cave and not-cave cells. In other words, if the opposite corners of a 2×2 box are both inside (or outside) the cave, then at least one more of the cells in that box must be inside (or outside) the cave. No matter what you do, if you create a checkerboard, you will cut your cave in half or isolate non-cave cells. Try it and see.

The most immediate consequence of this rule is that diagonally adjacent numbers, which form such opposite corners within the cave, must always connect in at least one of the two possible unnumbered squares in the same 2×2 box. So a 2-clue diagonally adjacent to another lets you mark a lot of not-cave cells immediately—namely those around the outside of that 2×2 box near the 2 clue. You'll run into other "checkerboard" situations as you go along and you should figure out how best to incorporate these for non-trivial deductions.

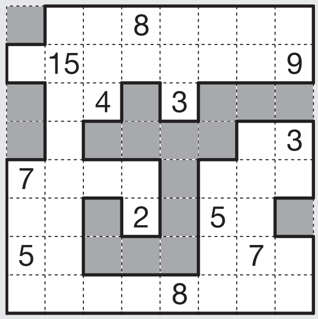
In the second puzzle, which uses just even numbers as clues, I used a few diagonally adjacent clues with 2s to get started. After marking off all the not-cave cells around the 2 in row 2, column 5, a set of steps quickly chain together. First, the 8 in the upper left corner now just has only enough space to fit. Then the 4 in the lower left corner has just enough space to fit.

CAVE

Shade some cells to leave behind a single connected group—the cave—with no enclosed, shaded cells. In other words, all shaded cells must be connected by other shaded cells to an edge of the grid. All numbered cells must be a part of the cave, with each number indicating the total count of cells connected vertically and horizontally to the numbered cell *including the cell itself*. A sample puzzle and solution is shown below.

			8				
	15						9
		4		3			
							3
7							
			2		5		
5						7	
				8			

→



Cave #1

				9			5
	3					7	
			15				
5					7		
		3					5
				9			
	9					9	
3			11				

Cave #2

8							4
				2			
		4			8		
						2	
	6						
		8			12		
			8				
4							2

Cave #3

23											
					4						
		3							4		
			5								
				10			5				
	3					6					
					7					15	
				8			10				
								10			
		4							11		
						8					
											21

This checkerboard thinking can happen in the middle of a solve with larger clues after other cave cells are identified. In the third puzzle, can row 2, column 3 or row 3, column 2 be part of the cave? If they are, then the 3 in row 3, column 3 will not have any cells left for the diagonally touching 5 clue. So you can mark three not-cave cells around this 3 clue near the start as an extension of the checkerboard pattern rules. A critical step in this puzzle is observing that the 15 clue cannot connect all the way to the 7 as a consequence of this rule.

3. Think about connecting everything together

After the first few moves are set, you'll probably find yourself in one of two situations: A) you have a few separated cave regions and they need to come together, or B) you have a well-connected cave but you have some shaded cells outside the cave that are trapped and still need to reach an edge. In either case, you need to figure out how to get the cave connected, and all the cells outside the cave an escape route. Adding an extra cell or two to the cave, or an extra cell or two outside the cave, can be incredibly valuable. Isolated cells that cannot easily escape being trapped are a kind of "global constraint" you'll want to exploit to force just one solution.

In the second puzzle, the 4 in the upper right corner has to connect with the rest of the cave, which should give an easy placement. Another example of the connectivity rules can be seen in a clue like the 2 in the lower right corner which has to connect to the rest of the cave via the cell in row 7 and column 7, as whichever way it goes, the next cell it adjoins must be that diagonally adjacent one.

The larger puzzle is primarily about finding how to get the shaded cells out to an edge of the grid, as there is only one exit at the grid's border. After dealing with the large 15 clue, you'll identify this exit and will be able to start wrapping a narrow path along the top and left of the grid. The 7 clue in row 7 column 6 becomes very important as if it goes too far to the left or too far to the top, it will trap some not-cave cells inside the cave.

4. Tweak until finished

The last steps in a cave construction are to make sure every cell is forced to be inside the cave or outside the cave. Do this by double-checking all your counts and X'ing out only those cells that cannot be part of the cave because of these counts or because of needing to be a part of an escape route. You'll typically find one or two cells that can be either inside or outside the cave. Add additional clues to constrain these cells, or move or change one or more clues to do the same.

I initially had a lot more 3s in the first puzzle, such as at row 4, column 1, but those clues did not specify just one answer. Tweaking that clue to be two larger forced a unique solution.

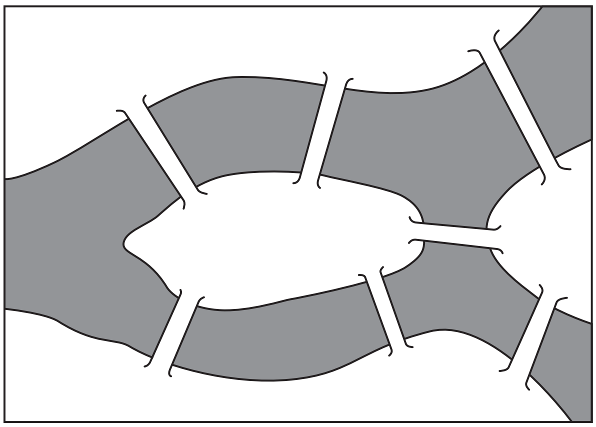
The end of the third puzzle also took some careful adjustments to give a unique answer. Initially I had a 2 in row 6, column 2 as well as multiple options between the clues below it in the lower left corner. I had a few ideas for how to constrain the grid, including moving the 2 clue down a cell (and the 15 in column 11 up a cell to preserve symmetry). I ended up increasing the size of that clue instead since a 3 hides the intermediate logic better than a 2 and limits the escape path of the cave at the same time.

Remember, anytime you tweak your puzzle, please be sure to recheck the puzzle from the beginning. No use leaving a cave-in for the next visitor. They'd like to escape too!

CHAPTER 3C: PATH LOGIC PUZZLES

One of the most interesting aspects (at least to a mathematically minded individual) of the former East Prussian capital of Königsberg is how the Pregel river runs right through the center of town and splits the city into sections, as shown opposite. In addition to the two riverbanks, there are two islands that can only be reached by bridges from the other parts of Königsberg.

The unique topology of the city led to one of the classic path puzzles in mathematics: Can you chart a course starting in some part of the city and ending in another part that crosses each of the seven bridges of Königsberg exactly once? Without needing to swim, or heading way off course to avoid the Pregel river entirely? Take some time to trace with your finger how that might go. I bet on your first few tries you'll get six bridges in before you realize you've got a problem. It's a very tricky puzzle. And maybe a little unfair. Since it has no answer.



In 1741, mathematician Leonhard Euler published a paper called "*Solutio problematis ad geometriam situs pertinentis*"[38] that stated exactly this, and showed why.

[38]Roughly, "The solution of a problem relating to the geometry of position."

Basically, you can treat each individual island or bank of the city as a vertex and the bridges as edges that connect those vertices. Every time you visit a vertex, you must come in by an edge and leave by another edge, so you spend an even number of bridges at each vertex. Only the first and last points, where you exit one time or enter one time, allow for an odd number of bridges to be passed. Since all four vertices (areas of the city) in Königsberg have an odd number of bridges, no path exists that uses all seven bridges exactly once. Euler's paper laid the foundation for the modern field of graph theory, and the concepts of an Eulerian path and an Eulerian circuit remain in use to this day.

While it won't be very often that a puzzle you create will launch a whole new discipline, you can't escape the touch of mathematics as we travel through this chapter. We'll encounter the properties of paths and loops throughout, in both obvious and unobvious ways. While finding your way from point A to point B might seem a rather mundane task, this chapter will introduce you to the many wonderful ways you can entertain your solvers along that course. Because with path puzzles, it is all about the journey.

CRAFTING A MAZE

Mazes have existed since the earliest civilizations, with the Labyrinth at Knossos being among the most famous. There, as legend has it, master architect Daedalus created a complicated maze for King Minos. Every year, fourteen young people would be let loose as tribute, only to get lost and be eaten by the Minotaur who lived inside the labyrinth. This worked until Theseus, son of King Aegeus, slew the Minotaur and escaped with his life (he did benefit from having a trail of twine to follow back to the entrance). While your mazes won't hide any legendary beasts, if you follow these steps, you too will be able to design your own classic puzzles.

1. Lay down the boundaries

Get a sheet of graph paper, or, if your maze is not based on a grid, an appropriate surface to draw upon. Outline your maze's boundaries. The bigger the maze, the harder it will be for you and the solver.

Decide upon your starting and ending choices. Set a "goal," either getting out, across, or into the center. Once you have that, start at the center or at one of the borders. You can also have multiple starts or exits.

I decided that a guitar was a natural shape for a visually interesting maze, with three of the tuning pegs serving as possible entrances and the opposite three as possible exits. While these start close to each other, and the paths travel down the neck together with borders looking like strings, the solution to the maze travels all throughout the body before getting back out.

2. Draw a true path

Sketch a path through the outline. The "true path" winds toward the goal, never crossing itself unless you have multiple levels or teleportation (jumping from one part to another). A good true path wends through many portions of the maze, leading the solver in unexpected directions.

Shade the true path so you don't accidentally erase it. Put in walls only where you need to stop the true path from contacting itself, such as when it turns back toward the same direction from which it came.

My true path started at the middle peg on the top, and then wound its way around the top side of the body before getting back to the bottom of the body and connecting to one of the exits. I made sure the path doubled back a few times, seeming to get farther from the most likely spaces a path would take, and also left some areas completely unused, like the region in the middle of the guitar near the soundhole, so that I could hide false paths in that space. If you split the guitar body into quadrants, you'll see that I made sure the path entered all four quadrants.

3. Put in false starts and endings

Make offshoots from the true path that sucker the solver away from the goal, even if they seem to lead toward it. Concern yourself with directions from the start only; you'll fill in the details later. These can be from the beginning, end, center, or anywhere along the true path. Put borders around the paths that don't lead to the goal, as they can't again intersect the true path. False paths can contact each other, though.

Now do the same but work your way outward from the end point, as solvers frequently start from the end and work backward. Make false paths from the end, or multiple end points. Have these intersect with other false endings, but they can't link up with either the true path or the false starts. Otherwise, they're alternate solutions.

Since my maze had a couple of false entrances and false exits, I tried to make some of those false paths travel rather long routes before appearing to be impossible. The low E string's peg (the lowermost peg on the left side), for example, leads you on a path that swings all the way around the body of the guitar, only to smack into a wall just as it gets close to the exits.

I chose two other paths (one from a false entrance and one from a false exit) and had them spiral around each other in the center of the body, but never touch. Following either path will suggest it might meet the other, but eventually they too run straight into walls.

A FRETFUL SITUATION

Find a path through the guitar that starts at one of the three tuning pegs on the left and comes out of one of the three tuning pegs on the right.

4. Fill in the rest of the maze

Wherever unused space can be found, build false termini for your true and false paths. These shouldn't link with anything else. Check the whole maze for unexpected answers.

Every path shorter than a half dozen spaces was filled in after the more interesting paths were laid down.

5. Prepare a clean copy

Any maze you draw will have erasures and other blemishes. Make a fresh, unshaded copy, and you've got a solvable maze.

While I originally drew my guitar sideways, with a regular square grid with vertical/horizontal lines within the rounded shapes, I decided to rotate the shape to better fit the page at the end. I also made a few borders in the final maze bolder than the rest so that the guitar shape would instantly pop.

CRAFTING A BRIDGES PUZZLE

Bridges puzzles, also known as hashiwokakero ("build bridges!" in Japanese, often shortened to just "hashi"), share many common features with the classic Seven Bridges of Königsberg puzzle, but hopefully without the defining trait of having no solution. Connecting all the islands without crossing any bridges will really test your wits if you don't consider these basic rules.

1. Lay down an invisible grid

Lay out a grid (say, 10×10). While the islands in bridges puzzles seem to float on air, they actually sit on a grid of squares. A good puzzle will have about 30–45% of the squares filled by islands. Too many islands gives you a hard time getting a unique set of paths of islands in the grid, as bridges will tend to be shorter and will therefore block other islands' connections less frequently. Too few islands doesn't give you enough choices of linkages to make for an interesting puzzle.

Circle some of the squares to place where islands will go. To get everything connected you might later move or add in a few more circles, but this is why you should be using a pencil.

Now, put numbers in a few islands to place a logical starting point on the grid that lets a solver draw in a set of connections. Draw in these connections on the grid as if you were solving the puzzle. The easiest kinds of numbers to force connections in a bridges puzzle are "big" ones. In the center of a grid, an 8 means all directions must have two bridges. On an edge or in a corner, a 6 or 4 can mean the same thing. Depending on the density of islands in your grid, you can even have an edge-like island in the middle (one that can only see three other spaces).

Connect the Dots

Among the first puzzles you solved as a child was almost certainly a connect the dots puzzle. Before too long, you likely outgrew the style to move onto bigger challenges.

A connect the dots consists of a group of numbered dots arranged so that drawing a continuous line from 1 to 2 to 3 to who-knows-how-high will complete a picture. With a nice visual reward at the end of a disguised lesson in counting, children around the world have learned the order of the numbers. But connect the dots puzzles can use letters or other symbols too, provided there is a natural order for all of the members of the set. While I haven't seen a connect the dots puzzle with the names of all the U.S. presidents, I could certainly imagine making one as a trivia challenge (provided I figured out how to handle Grover Cleveland).

The key concept to making a connect the dots is to practice drawing images without lifting your pencil, because your solvers certainly won't. If you were drawing a face in some other context, you might naturally draw an outer oval, then lift the pencil to start on the eyes, then lift the pencil again to start a nose. In a connect the dots face, you need to figure out how to draw an eye and connect back to the nose to connect to the mouth to connect to the outside of the face without breaking the path of the drawn line. Minimizing the number of times you need to double back, or have dots really close together, is another worthwhile goal. These constraints normally mean your image is of a single object or thing, as opposed to a more complicated scene, but be sure you make something that will please your solver. After drawing the picture, label all the spots where your drawing changes direction with dots in order, and then give the dots out to your solver. While you can label some intermediate points on long straight lines, this is neither required nor terribly elegant. Fewer dots are almost always better.

While making a connect the dots puzzle may not seem any more challenging than solving one, some basic path constraints you run into when making one, particularly in drawing an image with a continuous line, will pop up again in other path puzzles in this chapter. So give constructing a connect the dots a try, and then outgrow the style once again to move onto bigger challenges.

Another easy way to force some starting connections is with a "small" number. Two 1s cannot connect to each other, as then no other bridges could touch those islands. Sometimes, by placing a lot of 1s together, you can force a starting path as well.

An unwritten rule followed by some designers is that no two islands should be just one cell apart, since then that bridge connection would be too short to block any other possible bridge connection. If you need to put two islands really close together, you can. Just try to minimize how often this happens.

I chose to make a 20×25 puzzle, large enough to hide a few different kinds of steps inside but small enough to fit comfortably on a sheet of paper. I wanted to make a harder puzzle so I didn't include any gimmes like 8s, but several of the large numbers are still good places to look. There is an edge-like 6-clue that only has three directions for bridges, but I obscured it by having it in the middle of the grid. There also aren't too many 1s in the puzzle, but there is at least one spot to break in with them: the 3 in the second-to-last row that's surrounded by two 1s and only one other clue gives a quick vertical bridge. Having a lot more 1s at the start would offer several more of these spots.

2. Lock in your starting state

This step is where you draw circles and lines. There are a few tricks to make the sections unique.

Most bridges puzzles don't have too many big numbers like 8, but many use numbers one less than the largest possible number, allowing the solver to draw in some bridges. A 7 must have at least one bridge in all of the four directions (two bridges in three directions, and one bridge in the last). However, being two less than a maximum number is less useful, as a 6 in the middle of the grid might not have a bridge in all directions. The most common of these "one less than a big number" situations is a 3 in a corner, with just two directions to go. You can take advantage of this when starting your puzzle by placing 3s in some places where a 4 is also very constrained. (A "corner" doesn't have to be on the actual corner of the grid, if you can force areas where bridges cut off central possibilities as if there were walls there.)

You can also take advantage of situations where, if you did not extend a bridge to an island, you could not connect them all. One of the first such rules I discovered was a 1/3/2 on an edge. If the 3 only connected to the islands along that edge, then the 1–3=2 would be a group to itself. So, you can draw at least one link from the 3 toward the center of the puzzle to another island. Now, the 1/3/2 doesn't need to be on an edge. It can be anywhere in a grid, but the logic will be the same.

The corners of this puzzle highlight several different starting styles. The upper right has a 1/3/2 situation in the second row, where you can draw a single down and right bridge from the 3 to ensure everything gets connected. The upper left corner has another simple rule that arises when a corner has three 2s. If the corner 2 had a double bridge in either direction, it would isolate the two islands. So it must have two single bridges in both possible directions.

I also used a few clues that are one smaller than maximum in spots throughout this puzzle as starting points, like the 7 and some of the edge 5s. A trickier way to hide this kind of deduction is to put some 1s around a large number to effectively reduce the maximum number of bridges. For example, consider the lowest of the three 6 clues. Directly above this clue is a 1, so there are only seven potential bridges that can be placed around this clue (one above, and two in each of the other directions). You can immediately draw in one, but not yet two, bridges in the three other directions. In the same row as this clue is a 5 that has two ones in two of the directions, again making it one off maximal. So you can draw in two single bridges going down and right. Spotting these two clues is critical to getting started in the puzzle, and is certainly better disguised than if either were an 8 or 7.

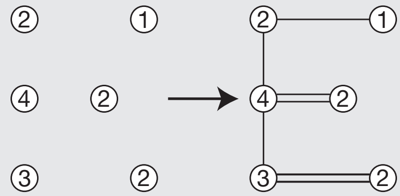
3. Finish connecting the islands

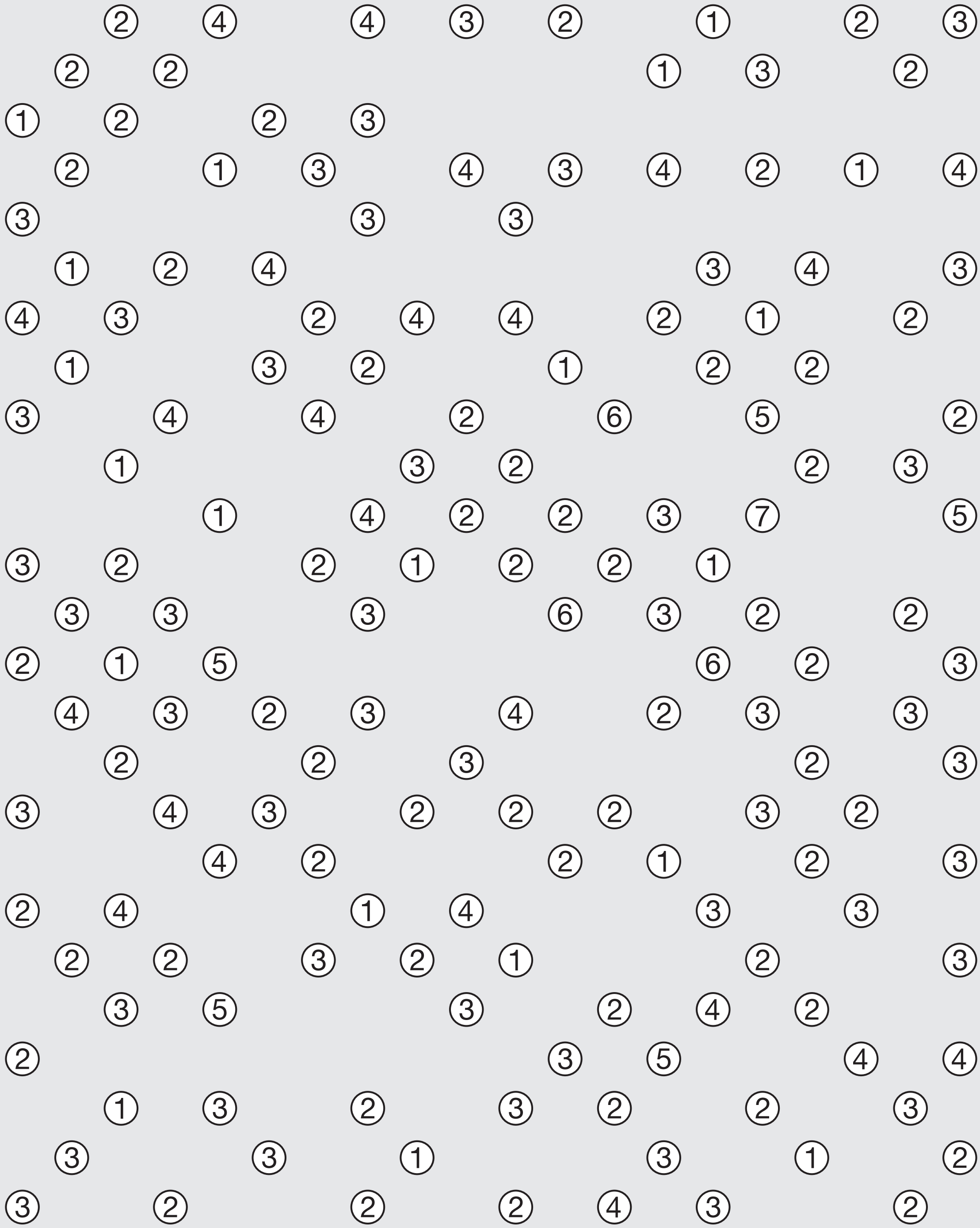
Now that you've embedded a starting state, connect all your islands. Use some positioned bridges (which now block some islands from connecting to others) to turn previously unimportant clue locations into good progress points. In particular, consider how long bridges from other clues have turned inside clues, with four possible directions, into edge-like or corner-like clues with only three or two. This is quite easy, for example, around big clues like 7s and 8s; bridges must extend from them in all four directions, offering many opportunities for corner-like clues nearby.

Continue this process until you have almost every island connected. At the end you can vary some island values to force single or double bridges in spots where the exact count is not yet fixed. Setting values closer to 4 will give a harder puzzle, while values that are smaller or larger will be easier to figure out. For example, consider an island that makes four connections. At one extreme you can choose to set this to an 8 with four double bridges, but this will now introduce an easy work-in to your puzzle. On the other extreme, you can set that island to a 4, with four single bridges, which will be much harder to resolve until the end. Since you've been laying out a logical path to get to this point, choosing harder, more obscured values is the best way to maintain your intended solution and difficulty. It can be particularly valuable to set any isolated islands to be 2s instead of 1s, provided doing so doesn't interfere with any earlier logic, so recheck any big adjustments you make.

BRIDGES

The numbered cells are islands in the water. Your goal is to connect all of the islands into a single connected group by drawing bridges between them. Bridges must begin and end at distinct islands, traveling a straight line in between. Bridges may run only horizontally or vertically and must not cross any other bridges or islands. At most two bridges connect any pair of islands. Finally, the number of bridges connected to each island must match the number on that island. A sample puzzle and solution is shown at right.





It's likely that the first set of circles you drew will not let all the islands join up; you tend to have two big groups. Find a good spot where the groups can come together and put in a new island to make this happen.

I used bridges to make edge-like and corner-like clues a lot in this puzzle. The long horizontal bridge between the 5 and 6 described in the last step, for example, isolates a 6 with only three directions. Putting in those bridges now makes a corner-like 2 with a 1 and 2 in the other directions (which is instantly identifiable). This creates another corner-like 2 with a 1 and a 4 (forcing a single bridge joining the 2 and 4).

When I got to the end, I had a few large groups that needed to connect at the top. I decided to be a bit tricky and force a square as the finishing step. You'll see this in the 3/2 and 4/3 clues in the middle of the top. If any of these connections used double bridges, you'd have two isolated groups of islands. If they instead use single bridges to form a square, you get the correct answer.

CRAFTING A MASYU

Masyu is an elegant loop-forming puzzle from Japan that consists of just white and black circles, similar to stones in the game of go. The circles have a natural balance: white implies a straight path going through it with an immediate turn afterward, black implies a turned path going through it with straight segments afterward. Even with just these two main rules, masyu can be surprisingly deep and challenging.

1. Choose a grid size

10×10 puzzles are a reasonable starting point to make a simple design, but larger grids allow for the most interesting ones. Because masyu use border and edge constraints a lot, particularly as early, isolated work-ins, use rectangular grids instead of square grids when you make larger puzzles to give yourself much more easily usable grid space.

I chose to make two 18×10 puzzles, highlighting different properties of the circles in each.

2. Master the white circles

When masyu was first introduced in Nikoli's main puzzle magazine, the puzzle involved just the white circles and their single rule.[39] While nowadays a proper masyu would not have just one color, it is worth experimenting with single-color puzzles to learn what the white circles can do.

Isolated white circles in the middle of the grid do not give a lot of information to get started, but circles on the edges of the grid do. Touching white circles are also important; the path can go through two such circles in a row, turning afterward on each side. But it cannot pass through three or more in a row, which lets you draw line segments through them in the other direction.

The left side of the first puzzle is made primarily out of white circles and demonstrates many of the basic possibilities with just white circles. The four vertically adjacent circles, for example, give a quick start to the puzzle. Notice that at the top of that set, the horizontal line drawn through two circles cannot extend to the third one in the same row, forcing a vertical line segment. You'll see some other white work-ins on the bottom of the second puzzle, showing how having two white circles on the border that are either connected or just one cell apart forces an edge segment and two turns.

3. Master the black circles

Just as with the white circles, constructing a puzzle (or a sub-region of a puzzle) with just black circles can be instructive.

Isolated black circles in the middle also don't give a lot of starting information, but black circles near the edges or near other black circles do. This is because the "must go straight" constraint after the turn in a black circle means that there must be at least two cells free in the direction the loop goes. A black circle on an edge, or even one cell away from an edge, cannot travel toward that edge and must have one of its segments going immediately away from it. If two black circles are touching (or if a path ever gets one away from a black circle), you can similarly draw a 2-cell segment that points away.

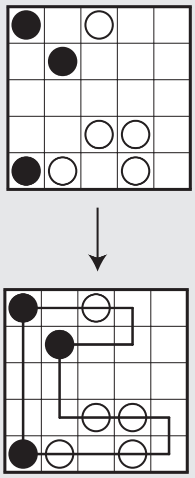
When dealing with black circles, you will also encounter some masyu "meta-strategy." Because you are making a single closed loop, you cannot prematurely close the loop by making a connection. The 2-cell segments you tend to draw at black circles are the most likely to lead to problems like early loop closures. You'll also see that some options for paths from black circles will strand a single connection without any way to make a loop. Indeed, you can never isolate an odd number of unconnected ends in any region, as these cannot close to form a loop.

On the right side of the first puzzle, a few different black cell tricks are highlighted. Adjacent black circles such as in the 2×2 black circle group must all extend outward from each other. Diagonally adjacent black circles often force a chain of segments due to the 2-cell straight path requirement, and both in the lower right corner and to the upper right of the 2×2 black circle group, you'll see where some (trivial) chains of segments spread from diagonally adjacent black circles. Long rows or columns of black circles spaced two cells apart can also chain into each other. Notice in the middle that once the vertical segment from the black circle closest to the edge is drawn to the second above it, another black edge can be drawn upward from the third black circle in the same column.

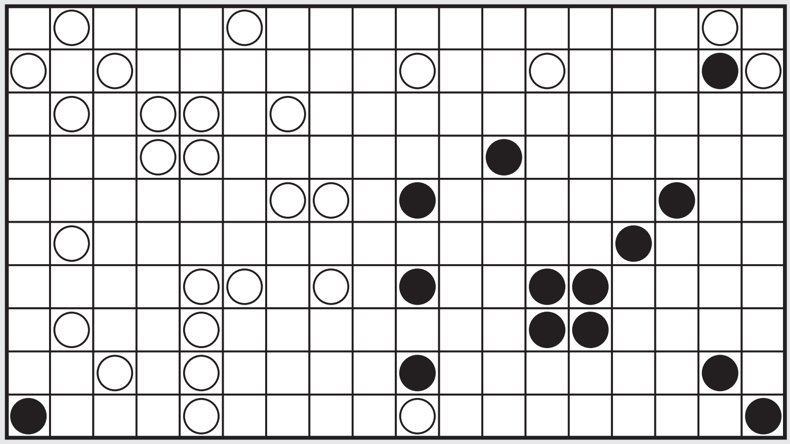
[39] Masyu's original Nikoli title was *shinju no kubikazari*, meaning "pearl necklace." When the black circles were added, the puzzle was renamed *shiroshinju kuroshinju* ("white and black pearls"). The current title came from the Nikoli president's misreading of the kanji for *shinju*, which changed the meaning quite a bit—since *masyu* is the Japanese term for "evil influence."

MASYU

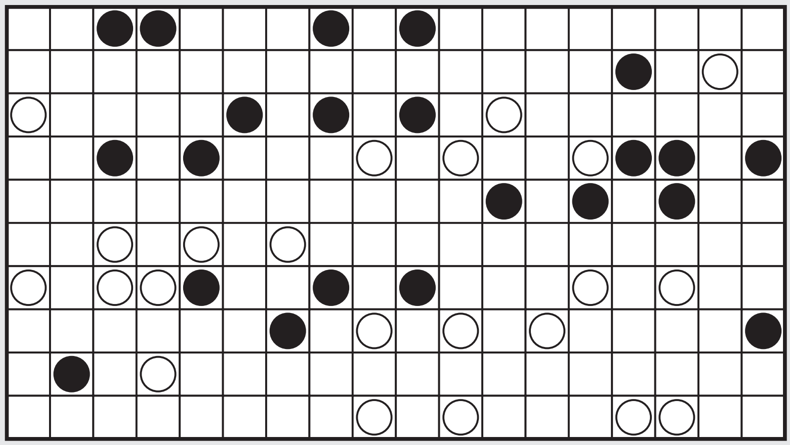
Draw a single closed loop traveling horizontally and vertically between adjacent squares that passes through all the white and black circles. When the loop passes through a white circle, it must proceed straight through the circle but turns immediately in at least one of the two adjacent squares. When the loop passes through a black circle, it must make a 90-degree turn, but it cannot turn in the first square immediately before or after passing through the black circle. A sample puzzle and solution is shown below.



Masyu #1



Masyu #2



4. Put it all together

Place a couple white and black circles that will force path segments. Like most logic puzzles, you construct these puzzles as if you are solving them, and keep adding circle after circle until the whole loop is built. You can do this by adding clues in one spot—from which the whole solution will grow—or putting in different starting points all over the grid. Many patterns of white and/or black circles have particular deductions (you likely encountered some in steps two and three); use them here when making a real puzzle. Repeating the same “trick” a few times around a grid will give that puzzle a theme.

On the lines of meta-strategy, as you are placing circles in the grid and laying out a path, you will see occasions where one option will prematurely close a loop. Use this constraint while constructing and you can set up a “chase” around the grid where the same two ends that cannot meet will come close together again and again, but always force the same dodge.

If you look carefully at the four corners of the first puzzle, you’ll see that each corner allows the instant introduction of two L shapes and 10 sure line segments. One corner does this with just white circles, another with just black circles, and the others a mix of both. While this is an easy example, often patterns with white, or black, or both kinds of circles will force particular kinds of logic. Playing with different arrangements can lead to variety in the solving path.

The second puzzle—which has “inverse symmetry” where every black circle has a white circle on the opposite symmetric spot in the puzzle—contains one less common work-in involving the interplay of black and white circles. Look at the arrangement of black and white circles near the top center and bottom center of the grid. If a black circle has two diagonally adjacent white circles on one side (in this case below to the left and right), it cannot extend in that direction between the white circles as it leaves no path for its other line segment without putting a white circle *in a corner. While this may not be the most common of moves to get started, by the middle of the puzzle the solver will deduce this or get completely stuck. Even when the starting state of a grid might not allow this situation, you may find after drawing some of the path that diagonally adjacent black and white circles interact with each other in a way that excludes one of the two vertical directions, or one of the two horizontal directions, for the lines from the black circle.*

Both puzzles also feature different kinds of meta-approaches where avoiding the premature closing of the loop is necessary. In the first puzzle, the most common white-circle steps on the left side of the grid will form a small loop in the lower left corner that has a few opportunities to close on itself that you have to avoid. Similarly, the most common black-circle steps on the right side of the grid will form some loops that only have open ends at the top. But you'll need to be extra careful in directing the rest of the path so that the loop doesn't close upon itself too soon, which is quite easy on that side. In the second puzzle, there are also a few occasions where a path traveling a particular way through a black circle or white circle would close a loop or leave an isolated strand. The easiest to see is the white circle in the upper right corner, but many others were hidden too.

5. Check your work

Go over your completed puzzle again to be sure there is just one answer or, better yet, have a friend check it for you. If you do encounter a problematic region with a couple answers, add in another circle or two. There is usually no requirement for symmetry in a masyu grid, so most often you can erase and rework any portion of the grid to get it just right. But if you have been trying to make a symmetric grid for a more elegant appearance, you'll need to simultaneously think about where you can also add circles to the existing loop on the other side while you resolve your ambiguity.

With the second puzzle, where I wanted to mirror each circle with its color opposite, I didn't have the flexibility to add any kind of circle anywhere I wanted. But I did find it harder to place black circles near almost filled spaces compared to white circles. So I worked hard to get important black circles into the grid, assuming the mirrored white circle could line up with the other side's path without too many tweaks.

CRAFTING A FENCES PUZZLE

Another loop puzzle with surprisingly simple rules but great complexity is fences, often called slitherlink. In a fences puzzle, adjacent clues interact with each other in many different ways, some quite obvious and others quite devious. Learning the basic forced patterns is only the first step in a long journey to constructing these puzzles. Follow these guidelines to jump-start that quest.

1. Learn the value of each type of number

Here is a surprisingly simple exercise to get started with crafting fences. On a grid, plot a 10×10 layout of squares and draw a large loop on it with lots of bends, basically any path you want. Then fill each cell with the number of edges used around it and effectively make a 100-clue Fences puzzle. Now, copy those numbers over to a blank grid and try to solve this "easy" puzzle. It should have just one solution, but as you try to reconstruct it you'll find some numbers are more useful than others, and some pairs of numbers are more useful when next to each other. Take note of these seeds.

Then try to erase about half the clues (such as in a checkerboard pattern), and try to solve that puzzle. You might now run into situations where there are too many solutions. For example, one particularly problematic type of area is a corner that passes through a 2 cell. If none of the adjoining cells are filled, there are two ways to form a corner through that cell. This would give two solutions, so you would have to add more clues around such an area to avoid this problem. While you won't yet necessarily be making a memorable puzzle, you should learn some of what works and some of what doesn't in constraining a single loop, and picking out a few small patterns that you can use in the future.

For the first puzzle I made a large loop just as described and then checked if I could remove two rings of clues to leave three concentric rings. This almost worked, but was much better if I had the main diagonals filled in too. The puzzle has a few interesting seeds, such as the lower left corner where the 0 forces the start to a loop and the nearby set of 33 clues gets quickly specified. The lower right also has a nice starting orientation to remember. While the diagonally adjacent 3 clues force a few line segments, the 1 is very constrained since this constellation is on an edge. In the third puzzle, you'll see how I reused this same 331 group in the upper left, near a "hidden edge" formed by a long set of 3s. While I could draw the result out immediately in that third puzzle as I knew the pattern, my solver might not.

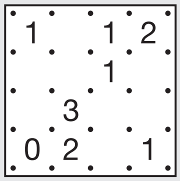
2. Plant some seeds in a new grid

The experimentation in step one should teach you a few of the basics. You've probably learned that the easiest clue to use is a 0, followed by the 1 and 3, with the 2 the hardest since it has the most potential options for how to use its edges. Try to construct from the front and lay numbers one at a time to form your desired loop. To do this best, you'll need to have a firm grasp on how clues work together.

Horizontally or vertically adjacent clues can be quite useful, such as a 0 and 3, or 3 and 3, or (along an edge) a 0 and 2. But you should focus your attention on diagonally adjacent clues. You'll want to think about paths that enter and exit each cell along the touching diagonal corners. When a 0 and 3 are diagonally adjacent, for example, there are two sure line segments to draw, but also one line segment that must exit the 3 into the next diagonally adjacent cell. Putting a 1 in that spot allows the two edges along the 1 that touch the opposite corner to be marked as unusable. Putting a 2 diagonally adjacent to that 3, on the other hand, forces another edge to leave the cell from its opposite corner, traveling to the next diagonally adjacent cell. You may recognize that a diagonally adjacent pair of 3s has four forced segments that are easy to draw; if you understand how diagonals chain together, you'll see that a long diagonal chain like 3222223 has the same four forced segments, as the 2s communicate with each other along that corner-to-corner chain.

FENCES

Draw a single loop made out of vertical and horizontal line segments between the dots. The loop should never cross itself. The numbers inside a cell indicate how many of the four possible line segments surrounding that cell are occupied by the loop. A sample puzzle and solution is shown at right.



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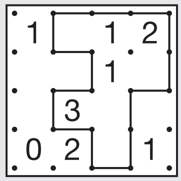
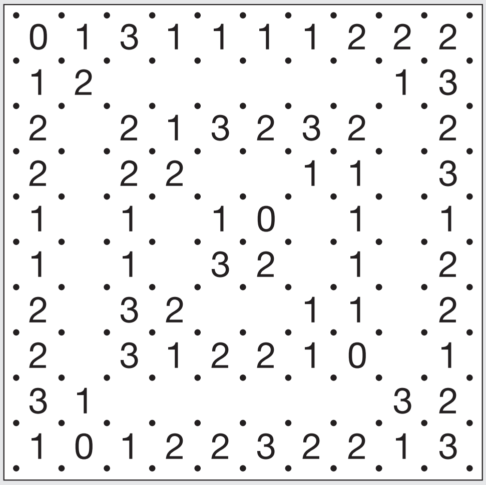
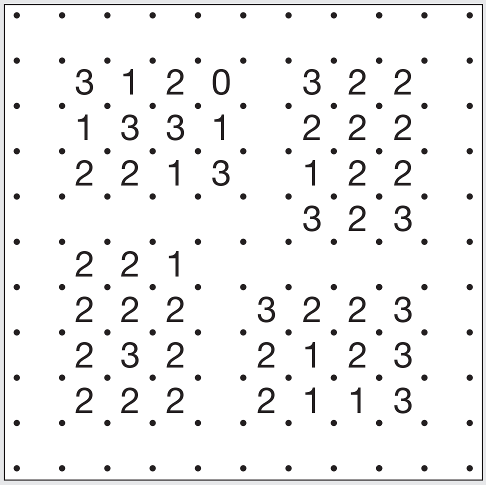


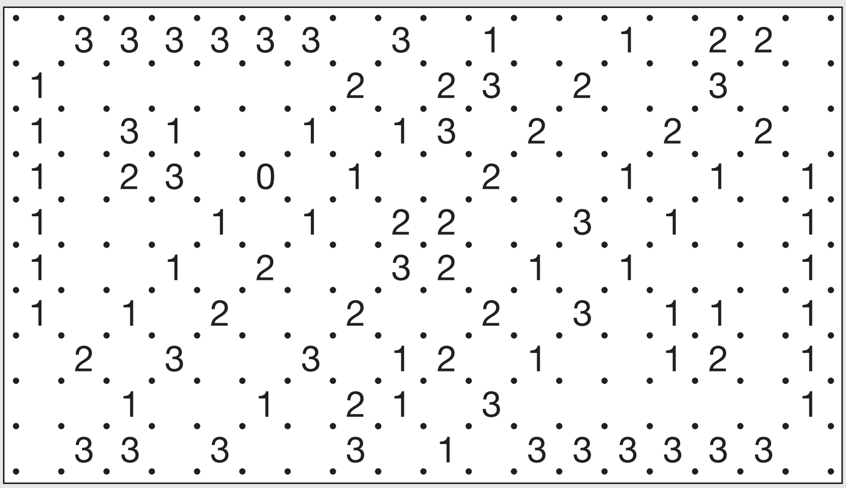
Fences #1



Fences #2



Fences #3



Even if you don't fully understand that last point, your next step is to choose a set of spots in a grid to put in some clues. Add numbers, mark forced path segments (and forced unused segments with X's), and continue this process for a while. You might need to add clues in more spots—it is typical to have rotationally symmetric clues in a fences puzzle, so this means adding clues in two spots—but try to limit the number of clues you use.

For the second puzzle, I defined four different 3×4 rectangles that would put clues in a visually pleasing pattern. I then put in some sure clues in the upper left and lower right rectangles to start the loop in two different spots. It might not immediately seem obvious, but the 2 adjacent to the three 3s in a column in the lower right forces a pattern with just one option, a fact that I'd discovered before from solving these puzzles.

For the third puzzle, I put in a lot of long diagonals that position 1s and 2s to transfer information along the diagonals or to eliminate certain edges entirely. You'll certainly be tested to track all of that information to finish the puzzle, and should learn a fair share of diagonal tricks if you get all the way through. I also wanted to put in some interesting long groups of cells with the same value. The 3s and 1s in two of the corners serve to define "almost walls," creating new internal corners that aren't immediately obvious.

3. Bring the ends of the loop together
As I just discussed for masyu, there are larger things to think about at the end stages of a puzzle. Specifically, you must eventually form one loop. Whenever two ends of a loop approach each other, if it is not the last connection in the puzzle, those ends must not connect together. Use this fact to set up a chase around the grid. For example, if one end connects to a 3 clue, the other end will have to run all the way around the 3 clue to avoid closing the loop.

The typical parity constraints for loops are important as well; namely, in any isolated space in the grid, you must have an even number of ends to connect. If drawing in a particular segment would result in isolating an odd number of ends, you can exclude that segment as a possibility. These situations are hard to set up, but are the kind of challenging logic that the better large puzzles use all the time.

In the second puzzle, after completing the two easy rectangles, the upper right corner of the grid will have three sure ends and just one way out. When nothing else seems obvious, you'll have to realize you need to get a fourth end into that corner, through the only possible path that remains. The rest of the solution grows out in both remaining corners from this one segment.

In the third puzzle, there are a few spots, particularly on the left side, where two ends come close to forming a closed loop and must be extended without connecting. This chase will start along the vertical 1 column, and then continue along the bottom and into the middle of the grid until it finally reaches the ends on the right side of the puzzle to form a single loop.

4. Draw out a clean grid
Now draw out a clean grid with just the dots and the numbers and print it to share with a friend. And, of course, it can't hurt to have someone else test your puzzles to be sure you didn't make up a rule that doesn't actually work.

Mike says that his number of erasures on the third puzzle was by far the most on any puzzle of mine in this book.

CRAFTING A YAJILIN

Yajilin is one of many kinds of space-filling loop puzzles, in which a loop has to visit every cell in a grid without crossing itself. It originated in Japan where the name breaks apart into words meaning "Arrow" and "Link." You'll find yourself chasing lots of arrows to link segments into one big loop.

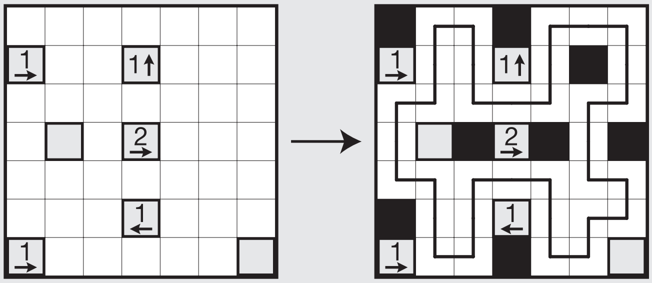
1. Experiment with clue location
The clues in a yajilin serve two distinct purposes. First, they block off cells that the loop cannot pass through. Second, they place constraints on some other cells that may be blackened or may be part of the loop. While the second feature gives yajilin its unique character in the genre of space-filling loop puzzles, I'll start with just the first concept in isolation.

Try to make a puzzle where the only clues are "unusable" squares. Consider how you can isolate sections of the grid with such clues. If a particular region of the grid only has two paths in and out, the loop must use both paths for sure. The space you leave for the loop is critical too. If you isolate a single cell on an edge, for example, it will have to be blackened as there is not enough room for the loop to enter and get back out.

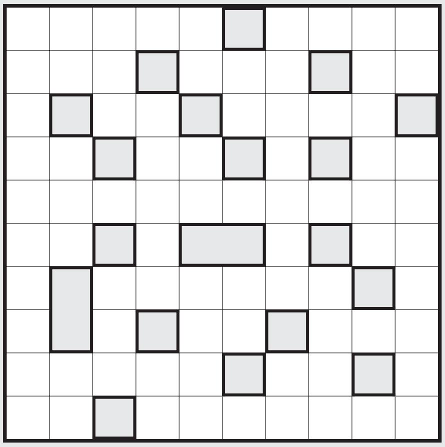
In the first puzzle, I played with just clue location to build a valid yajilin—even if it contains none of the arrows typically associated with these puzzles. Consider the three gray cells in the upper right corner. They leave that corner with just two paths for the loop to enter and exit and also no room for any black squares. In the upper left corner, there are three apparent entrances and exits but two of them have longer paths connected to them that must be used. The last is blackened off. At the bottom of the puzzle you'll find several places where I found I could leave exactly two extra cells, which always means the loop has to take both since you cannot shade connected cells.

YAJILIN

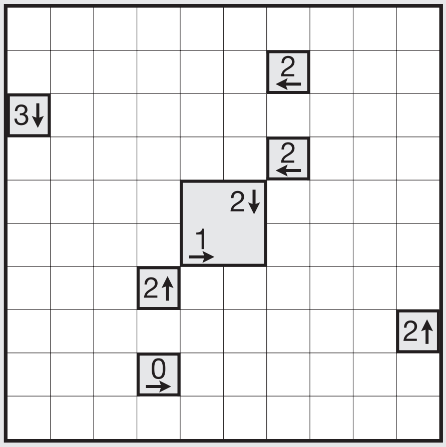
Blacken some white cells and then draw a single closed loop (without intersections or crossings) through all remaining white cells. Blackened cells cannot share an edge with each other. Some cells are outlined and in gray and cannot be part of the loop. Numbered arrows in such cells indicate the total number of blackened cells that exist in that direction in the grid. A sample puzzle and solution is shown at right.



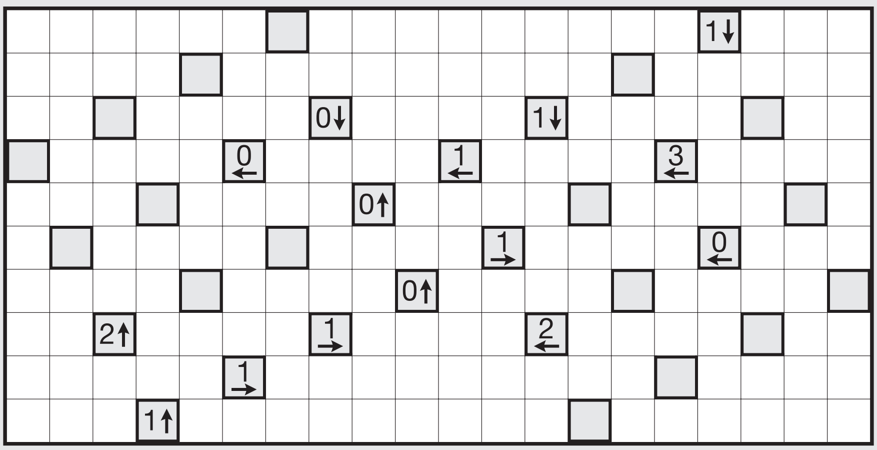
Yajilin #1



Yajilin #2



Yajilin #3



2. Experiment with numbered arrow clues

The simplest arrow clue is a 0, which identifies all non-gray cells it points at as unblackened and a part of the loop. You can never have enough of these when getting started. Most other arrow clues come down to spacing. As blackened cells cannot touch each other, the most basic clue is one for "N" black cells that is positioned "2N–1" spaces from the border of the grid and points towards that border. This forces an alternating pattern of black and white cells.

Clues along the borders follow different rules. The loop needs to get in and out of those spaces requiring more than one white cell between black cells. Consider placing a clue for two blackened cells along a border.

This clue cannot work if there were only three spaces available. But if there are either four or five spaces available, there is just one way to place the black cells and leave space for the loop.

In the second puzzle I tried to have symmetric clues. I started by putting several almost basic clues into the puzzle such as the 2-down in the sixth column (which due to the layout of nearby cells only has one option). The 3-down clue on the left edge is another simple forced clue because of the extra constraints when on a border. After sketching out the loop consequences of those clues, I could add other clues like the 1-right in the sixth row; of the four cells that might seem available to be shaded for that clue, only two were actually possible options at the start of the puzzle, and only one is left after the loop at the bottom is drawn in.

3. Add interacting clues for more involved logic

Synthesize the two different clue roles together in a fresh grid. First mark out some cells which will be gray (and may already define a lot of the loop shape) and then label only some of them with arrows to specify the rest of the puzzle.

Build in some interacting clues to add variety to the logic. The simplest interacting clues are two clues that point in the same direction. Subtracting the numbers in the two cells leaves information for just the space between the gray cells. More involved situations can arise from clues that point at each other, as now the number of blackened cells between them must still work for the extra cells left on each side.

In the third puzzle I started with a repeated "knight's move" square pattern with lots of small 2×2 areas that either get two ends or four loop ends coming into them. It is an open-ended geometry that gives a lot of possibilities. I then placed some interacting clues into the grid like the three left pointing clues in row 4. With some additional 0-up and 0-down clues nearby, the shaded squares referred to by those clues can be easily found to start.

Elsewhere in the grid I placed some clues that point at each other like the 1-right and 2-left clues that point at each other in row 8. The most complex interaction actually uses both the 1-right clues in rows 8 and 9 which end up having blackened cells within the same three to four cell area. Figuring out how to validly place these two blackened squares and still get one loop is the challenging conclusion of the puzzle set up by parallel clues in adjacent rows.

4. Label some more cells or add new clues to finish the puzzle

To complete your construction, connect all the individual sections of the loop together to form one single loop. You may find the loop has some sections with two or more possible paths. Find ways to force just one path by adding a clue cell that either blackens something in that section or forces a cell to be white and, by being part of the loop, requires some other cell to become black. You can either label an existing gray cell, or add in a new clue cell, to achieve this goal.

A lot of logic can be created around the concept of making just one loop, and this comes up particularly at the end. Label the ends of the segments with letters so you know which need to avoid each other and which need to come together, and then identify the clue locations that can make the last links happen.

The third puzzle showcases a tricky logical step based on loop end counting, which my choice of clue geometry helped create. In any isolated part of the grid, there must be an even number of loop ends so that when the ends come together they are no free ends left. The cell in row 4 column 10 is one place where the loop cannot go down as it would leave too many loop ends going into the top center of the grid.

Both the second and third puzzles have large sections where the logic involves not closing the loop prematurely. In the second puzzle I created this tension by adding the 0-right clue which makes the bottom of the grid come together very fast, bringing two ends together in the lower left corner. These ends have to avoid themselves while coming up the left side, connecting with two ends coming from the upper right side as well. The third puzzle has a "loop end chase" on both the right and left sides as well, as parts of the loop have to stay disconnected to form a single loop at the end. As this chase continues all the way to the last moments, I like to imagine that after so much buildup when the loop finally came together my solvers screamed "Yajilin" in victory!

CRAFTING A CHESS PROBLEM

Chess problems are more exercises than puzzles. The simple types of problems, called "directmates," give you a setup and require you to achieve checkmate in some number of steps against any defense. There are more types of problems—series movers, selfmates, retrograde analysis problems, et al.—that you can attempt if you get the gumption to tackle something brain-bending. For now, I'll focus on the directmates.

1. Get a chess set and set out your goals

Start with a chessboard. You can start by moving pieces around on a real chessboard, or you can use a virtual chessboard where you can enter pieces and delete them whenever you like.

Now get your head into the right space. Per the British Chess Problem Society, a good chess puzzle has these six elements:

a) A **position**: Your puzzle should be a legal situation that conceivably could exist. It's okay if one player must make some egregious errors to get into the position, but it must be possible.
b) A **stipulation**: Your puzzle should have a goal for the solver, such as "White to play and win in 2 moves."
c) A **solution**: Your puzzle should aim to have a single solution (or sometimes several) that can be achieved through clever play.
d) A level of **difficulty**: Your puzzle should know what kind of solver can crack it. This is hard to judge, since solvers have unique knowledge bases even at similar levels of experience.
e) A **theme**: Your puzzle should demonstrate something to the solver, at least at the higher levels of difficulty.
f) A level of **originality**: Your puzzle should be unique and show off something that will impress solvers.

For the first problem, I was content with just accomplishing goals a–d, as easier problems are rarely thematic or original. My initial plan was to set up a mate situation in the square surrounded by three black pawns, but first I needed to chase the black king into that square. For the second problem, I tried to create a theme that focused on unusual pawn advancement. When a lowly pawn finally finishes his journey to the top of the board, he almost always turns into a queen; in this problem, going for queens is actually a mistake.

2. Set out an endgame

Start by theorizing a position. In the early going, you may have trouble envisioning the end of a game without seeing all the moves that came before. To get used to this, just set up pieces randomly on the board, seeing what happens during an endgame. This can open up your mind to all sorts of strange setups.

When you have a theory of what your piece placement will look like, set them out in a possible arrangement. It won't work at first. It never does. Try running part of your simulation to see what pieces you might need to add, subtract, or move. Keep fiddling around with positions until what you are trying to accomplish actually works.

With the first puzzle, I decided to have a set of pawn moves force the black king to my intended mate square. I specifically positioned a rook in the upper left corner and a bishop in the lower right corner to limit the options for the black king rather quickly. The mate step was set up to be more interesting. While there are two pieces (another knight, and a pawn) that can capture the checking piece, in each case they open up another route of attack for a white piece (a bishop and a rook, respectively). Observing that those two pieces are on the verge of attacking the critical square, if relevant pieces are moved, is the main challenge of the problem.

With the second puzzle, I needed to place the black king in a problematic spot, one where an immediate pawn-to-queen step would not work (possibly by forcing a stalemate). My first several approaches didn't work out, and I abandoned using stalemates as a reason behind not selecting a queen. Instead, I set up the bottom of the board to put some momentum behind getting quick checks. That pinned white king, with a black pawn one square from causing disaster, places greater urgency on following the unusual mating path at the top of the board.

3. Polish it up

Once it works, try to make it more elegant. A couple of things to aim for:

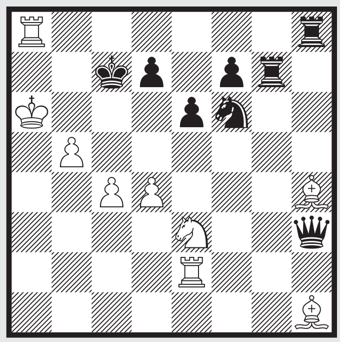
a) Make sure you have only one move that starts the process in motion. This "key move" should be unique, or the problem is considered "cooked" and thus inelegant.

CHESS PROBLEMS

Black is moving down the board in all cases.

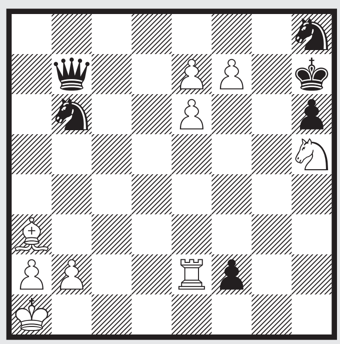
Problem #1

White to play and win in three moves.



Problem #2

White to play and win in three moves.



b) Try to require a unique defensive move every time the attacker moves. If the defender has a couple of options that are fundamentally identical, kill one off.
c) Avoid using promoted pawns at the start, and don't require a castling move unless it is clear that the rook and king could never have moved.
d) Use only the pieces you need to use. Every piece in the puzzle should have a reason to be there, even if it isn't used in the attack. You can have pawns facing off to paralyze a section of the board, or a wall of pieces that protect the attacker's king.

Make sure you have left no other options than the ones you want. It is easy to overlook a potential response to one of your moves, so using chess software to "test-solve" your directmate problem is a good idea. Then set the puzzle back to its original position, and give it to your solver.

With the second puzzle I broke rule c, but the puzzle breaks the standard promotion rules so I didn't mind. With that puzzle, I kept finding other options or unintended escape routes for the black king. I ended up adding the third pawn near the top of the board to check one square that was causing problems. Then I ran into problems where the final pawn promotion removed the defense for the first knight's square. So I added the bishop in the leftmost column to be sure that wasn't a problem as well, even though it might seem, at the start, to be a useless piece.

After setting up the initial problems, I triple-checked everything on my physical chess board to be sure no other moves made sense for either side. Then I used a chess program to test both of my puzzles as well. Only after all that was I absolutely sure I was done with the puzzles.

4. Finish the problem with a properly written solution

While the answer notation for chess problems may look like gibberish to newcomers, it makes sense if you take the time to learn it. The board's spaces are labeled by rank going up the board from 1 to 8, and by file going from left to right lettered a to h. So the upper left corner is space a8.

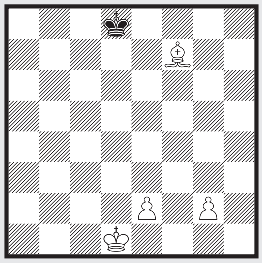
Pieces are abbreviated as letters: king = K, queen = Q, rook = R, bishop = B, and knight = N.[40] Pawns aren't given a letter; they're just noted by the absence of a letter.

Each pair of moves is given a number. A move is indicated by the piece and the space it moves to, such as "Bb4" (that is, "move the bishop to the space in file b, rank 4"). Pawn moves are just noted by the space ("h5"). When a piece captures, put an "x" between the piece abbreviation and the space ("Rxe4"), unless it's a pawn, in which case you'll start with the file the pawn attacked from ("dxe4").

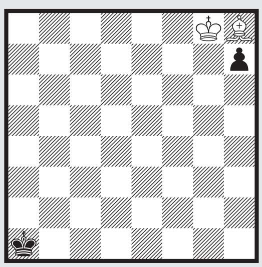
If a pawn gets promoted, add the piece it turns into at the end ("c8Q"). If a castling move happens, use

Chess Through the Looking Glass

Francis here again, interrupting to tell you more about "retrograde analysis," which Thomas mentioned in passing. It's a kind of chess problem I love, because it rewards puzzle-solving skill (which I have) more than it does chess-playing skill (which I don't so much), and which was introduced to me by two brilliant Raymond Smullyan books, *The Chess Mysteries of Sherlock Holmes* and *The Chess Mysteries of the Arabian Knights*. In retrograde analysis, you aren't trying to figure out what move to make next; rather, you're trying to deduce something about what's already happened.



A simple example would be a board with two white bishops, both on white squares. One must be promoted, since the two white bishops start on squares of opposite colors. But even though there is only one bishop in the position on the right, you can also prove it must be promoted. Why? The two white pawns have never moved, so the original white-square bishop can never have left its original square and must have been captured there.



Or consider the next position. Given that neither player's last move was with a king, what was the last move? The black king is in check, so white moved last. The white king didn't move last, so it wasn't discovered check. Any bishop move must have been from a square where it would have already been checking the black king, which is impossible. That bishop must have been a pawn, promoting by capturing a piece on h8. But what piece? Whatever it was, black's last move was with it (the king didn't move, and the pawn is in its original position). All three squares around h8 were occupied, so only a knight could get there. So the last move was a white pawn capturing a knight and promoting to a bishop.

Writing these is much more about logic than chess skill. Often the past moves to be deduced are ones that no reasonable chess player would ever make. And what you ask your solver is limited only to your imagination. A piece is missing from the board; what is it and where? Which of two queens is promoted? Once you start looking back, you may never solve forward again.

[40]It can't be K, obviously. Wei-Hwa Huang tells us that sometimes, in English publications even, "knight" is abbreviated as "S." In the early days of algebraic notation, "N" and "S" fought for contention, "S" being from "Springer," the German name for the piece. "S" still survives in some chess problem discussions, because "N" may be used for a fairy chess piece (a nonstandard piece used in chess variants), namely the "nightrider." This is the kind of stuff that fascinates us.

"0-0" for a king-side castle and "0-0-0" for a queen-side castle.

If the king gets checked, add a plus sign at the end ("Qh8+"); a double check (that is, a check by two pieces at once, e.g., by a knight both giving and revealing check) needs two plus signs ("Na8++"). Checkmate is noted by an octothorpe ("Rc7#"), and the game is sometimes ended with "1-0" if white wins, or "0-1" if black does.

So a short problem's answer might be written as:

1. g7 Kh7
2. gxh8Q#

(That is, the attacking pawn captured a piece, turned into a queen, and, presumably under the cover of a protective piece, mated the neighboring king.)

There was nothing particularly hard about writing up the solutions to these two puzzles, as I made sure there is only a single chain of reasoning to follow for either side after the first move. But I did enjoy seeing the three white moves f8N+, Nf6+, and e8N#, which set up a trio of deadly horsemen to finally mate the black king. Bet you've never run into that kind of mating situation before.

CRAFTING A DOMINO CASTLE

While there is definitely a line that separates games from puzzles, this line is often crossed. Dominoes are a perfect case study. The uniquely numbered set of 1×2 tiles is great for a variety of side-matching games. But what is strategy in a game setting can become logic in a puzzle setting. Let's look at how to build your own castles out of dominoes and then let your solvers knock them down.

1. Get a set of dominoes and try to build an interesting shape

Unlike a game of dominoes that might just have a lot of chains expanding from the center, a good domino castle puzzle will have a more complex shape. It may have more bends and linked loops, and ideally few free ends that don't touch other dominoes. Choose an interesting shape—like four interlinked rings or a heart—and try to build it out of a set of dominoes any way you can, making sure to match the values on all edges that touch.

Puzzles Based on Games

There is a perhaps surprising amount of overlap between board games and puzzles.[41] While chess and dominoes are two of the first examples that come to mind, other games lend themselves to path logic puzzles too. Since the first step of many of those descriptions would be "Get really good at playing X, so you can see Y moves ahead," I won't try to describe them all in detail. But you can see how the strategies discovered in gameplay can become the logical deductions encountered when solving a puzzle and vice versa.

Checkers puzzles are a mainstay. They invariably involve making a few sacrifices, losing pieces by forced jumps, to set up a very long kingmaking sequence that might then reverse itself and effectively clear the whole board. You can design these by starting from a long jump sequence, and slowly move back the opponent's pieces to situations that would have started out before a jump or two was made. Figuring out where and how to imagine the prior jumps to hide your answer takes some practice, but setting up blocked positions, opened up by moving one piece, is the most common approach I've seen.

Go is an inspiration for many Japanese puzzles, which often have an abundance of white and black circles as common symbols. One type, usually called goishi hiroi or hiroimono, considers some of the clustered stone situations often encountered in a go match. The challenge is to start at some stone and travel a path of vertical and horizontal moves to other stones, eventually capturing all of the remaining stones. Once a stone is captured, it is removed from the grid and can be "jumped over" in a future move. Both constructing and solving these puzzles involves considering the most isolated stones on the grid, which can't jump in too many directions, and figuring out when they must join the path. If you take just any arrangement of stones from a game of go you likely won't have a valid puzzle. But adding in another stone or two connecting the most isolated bits should resolve matters.

Black Box, created by Eric Solomon, is a game in which the clues to a puzzle are slowly revealed until the player can guess the single answer. In the game, players shoot rays into a grid containing hidden atoms and are told where the rays come out. The position of atoms allows for different situations including hits, deflections, and reflections. Having enough clues effectively makes a solvable logic puzzle to uniquely determine the atom location.

Ricochet Robot, designed by Alex Randolph, is another favorite with the World Puzzle Championship crowd. In a grid with many walls and four robots, you must find some path that bounces off walls and robots to eventually land on a target square. Minimizing the number of total moves to get the right robot to the target is the goal, and each round is effectively a new puzzle to find the global minimum before time runs out. The "puzzles" get harder when you are playing with more experienced players, as a half-good "answer" will no longer win out.

[41] I'm sure Mike will have no disagreement on this point, as much of his career is built around this overlap. I've often heard him introduce himself as "a game designer for people who like puzzles, and a puzzle designer for people who like games."

Having lots of dominoes close together will help make your puzzle more interesting to solve. Try to maximize connectivity where possible. Keep in mind that double dominoes can join other tiles together in unique ways and serve as a hub for lots of other dominoes. But even without doubles you can turn the tiles to get as many as three other dominoes to touch any one domino half.

While you may get close to your desired shape the first time, you will almost certainly have one or more dominoes left over. Try to fit these into the shape by breaking some links and making new ones, or adding or removing free ends where possible.

I pulled out the 0–9 set of dominoes I've played with for decades. After warming up with a game with friends, I set the largest dominoes (with 7, 8, and 9 on them) aside and got to building these puzzles with more reasonable sets of either 21 or 28 tiles.

I decided to make two thematic images. The first is a castle viewed from afar. I made use of some clear double-domino linkages on the top parapet. Initially, the center domino on the top pointed straight up, but I couldn't make the ground floor fit until I decided to turn that domino sideways. The bottom center dominoes started out connected but I knew I could flip them up (as in the final puzzle) if needed.

The second image is a tall tower. Here, I started by making several interlinked rectangles and many multi-direction junctions from bottom to top. With the leftover dominoes I decided to add a flag at the top to complete the picture.

2. Shift to paper and think about clues

While building a domino castle is easier with tiles in hand, finishing the puzzle will go better on graph paper. Sketch out your shape (with your eventual answer made out of dominoes beside you) and choose some rows and columns to label with clues. If you label every row and column your puzzle will be trivial, so the goal is to pick just enough clues to make the puzzle interesting without having multiple answers.

There are several good strategies to try, such as giving lots of clues that all include one number, or giving lots of clues that all ignore one number. The best rows and columns to clue are often ones where a number repeats more than once in different domino clusters. In addition to thinking about individual clues, think about their intersection as well. If a pair of row and column clues only have one number in common, then the solver has something they can clearly label.

With the first puzzle, I thought it would be unusual for the whole top row (including the two forced double dominoes) to only hold two numbers. I tried to make that work and found it led to some interesting logic. So I went with that choice and then started labeling clues in some of the columns beginning with the first column where I again tried to see if I could use only two clue numbers in total. The intersections of all of the column clues with the top row make an easy start to the puzzle.

With the second puzzle, I started by having five different row clues that include 0 in the arrows coming up the tower. I paired these 0s with smaller numbers at the bottom (e.g., 012, 02) and bigger numbers at the top (e.g., 034, 045) and marked that the fourth column would not contain a 0 or 5 to get the puzzle started. I was not sure yet which numbers this fourth column would contain, just that 0 and 5 were not going there.

3. Finish the puzzle off with a few "global" clues and try to solve it

While your first clues will position individual domino halves, the last clues are about challenging the solver to think "globally" about all the remaining dominoes and build the rest of the castle with what remains.

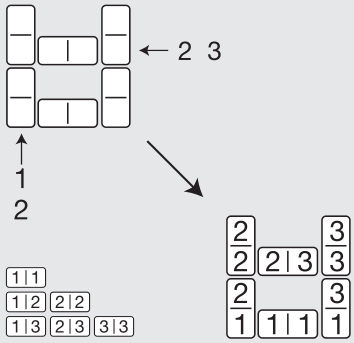
It helps to think about the size of unclued clusters at this point. A cluster is a set of dominoes that touch and share numbers. Clusters can require an even or odd number of dominoes and this often limits the choices dramatically, as a particular number may not have enough dominoes left. Figure out a cluster that can be disambiguated with just one more clue and add it, then move on to another cluster. See if this is enough to now specify every domino. If not, try to add another clue, or go back and add a clue elsewhere instead. When you think you have it set, give the blank grid and clues to a friend to test.

In the first puzzle, I noticed that the constraints on the 2- and 4-containing dominoes meant I could use a fairly small number of clues to force the entire top of the castle. In particular, after limiting column nine to only have 4 and 6 in it, the innocent looking 1256 in column six was enough. The bottom of the castle seemed trickier and I thought I might need another row clue. But with three dominoes containing 3 left to place including the double 3, this set of dominoes had to use one of the three-way junctions, which left only one choice given the rightmost column clue.

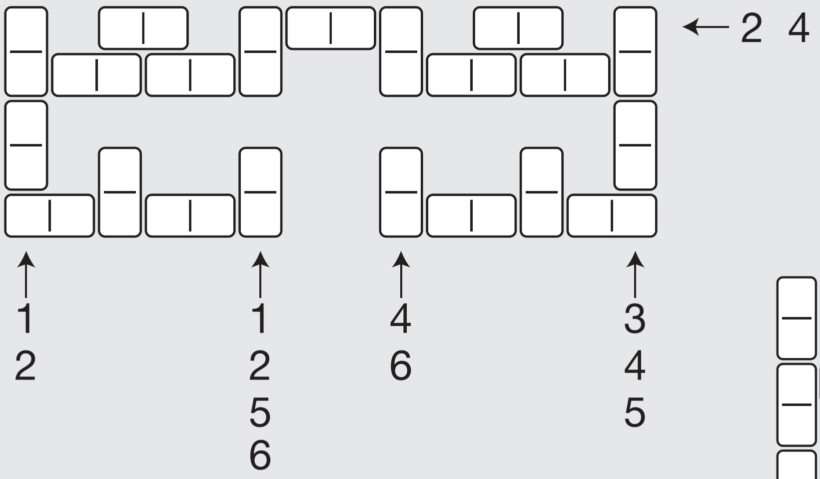
For the second puzzle, I found adding two more row clues (the 15 and the 16) placed most of the dominoes. A tricky observation then was that there were two numbers with just a single domino left (1 and 6); one of these would need to be the end of the flag and the other would end the domino below the flagpole. This meant that having a 4 clue on the far right (to now make the total column be 012346) forced a last set of placements to all come together. This is a sneaky final deduction, but I knew I'd found the right clues to set up a fitting domino rally for our solvers to find.

DOMINO CASTLE

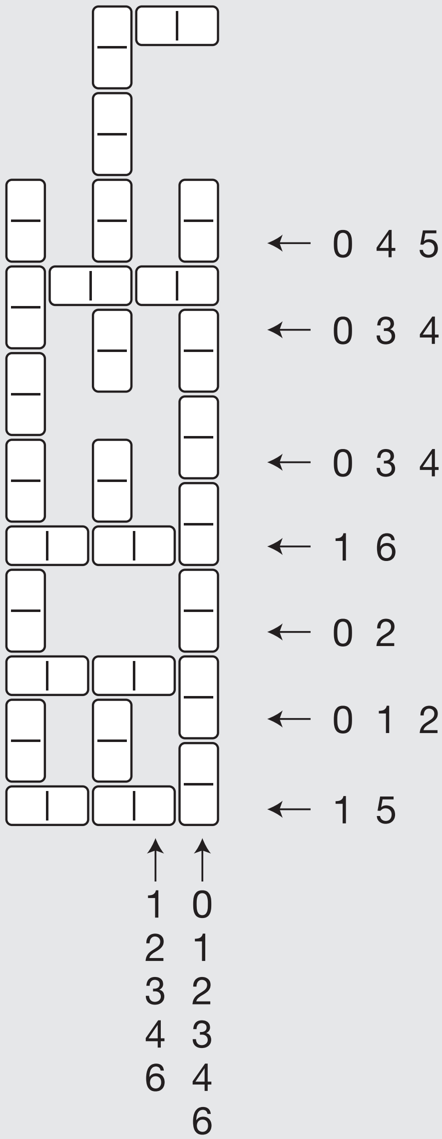
Label the dominoes with numbers so that the entire set of dominoes is present in the layout. When two dominoes share an edge, the numbers must match on those halves. The outside numbers indicate every number that appears in that row or column of the layout, each of them appearing one or more times. A sample puzzle and solution is shown at right.



Domino Castle #1



Domino Castle #2



Domino Set for Domino Castle #1

1|1
1|2 2|2
1|3 2|3 3|3
1|4 2|4 3|4 4|4
1|5 2|5 3|5 4|5 5|5
1|6 2|6 3|6 4|6 5|6 6|6

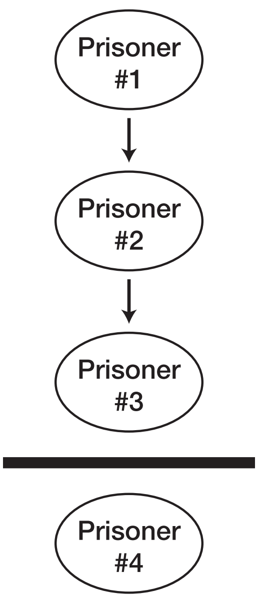
Domino Set for Domino Castle #2

0|0
0|1 1|1
0|2 1|2 2|2
0|3 1|3 2|3 3|3
0|4 1|4 2|4 3|4 4|4
0|5 1|5 2|5 3|5 4|5 5|5
0|6 1|6 2|6 3|6 4|6 5|6 6|6

CHAPTER 3D: LOGIC PROBLEMS

There is a nonzero possibility that, after attempting to master all the puzzles we've described so far, you will go crazy and murder Thomas and me. Thereafter, if there's any justice in the world, you'll be sentenced to the kind of prison that puts party hats on its inmates' heads and forces them to guess their hat color or be shot on sight.

No, really, those prisons exist. For fun, their unbalanced wardens routinely run the following experiment: A guard, knowing that the prison is full, lines up four inmates in a row, with the one in front behind a screen so that the others cannot see him, as depicted at right.



Instead of recommending parole, this guard gives each prisoner a party hat, two of them red and two blue. The prisoners can't see their own hats, but can see all the hats in front of them—except for that unfortunate Prisoner 4, whom no one can see, and who can see no one. The jailer says that the prisoners may not communicate with each other, but that if any one of them can say his own hat color, they all go free. Otherwise, it's the firing squad for all of them.

Now, in this prison, all prisoners are completely self-motivated, risk-averse, expert puzzle solvers. (See, that's why you're in there.) The good news is that they're gonna get sprung. Here's why: Prisoner 1 can see both Prisoner 2's and Prisoner 3's hats. If he sees either two red hats or two blue hats, he says his hat is the other color. Otherwise, he keeps his big yap shut. If that happens and he says nothing, Prisoner 2 looks at Prisoner 3's hat and says his hat is the other color. He knows his hat can't be the same as Prisoner 3's, or Prisoner 1 would have hooted it to the world.

This bit of psychological torture is known as "prisoners and hats," and is a cornerstone example of the induction puzzle. Induction is the process of testing a case that may or may not be true, and learning from what happens. This contrasts with deduction, in which one takes a case and determines conclusively through testing whether it is or isn't true.

Logic problems rely on the solver using both of these skills. There are several kinds of logic problem, and I'll take each in turn.

CRAFTING A TRUE-FALSE LOGIC PROBLEM

A "mathemagician" isn't just someone you'll find in the world of *The Phantom Tollbooth*. Like Martin Gardner, Raymond Smullyan embodied the term with compatible interests in both math and stage magic. He had a wizardly way with word logic, popularizing the concept of "knights and knaves," a type of logic problem where observation is as paramount as induction. They can be solved by using Boolean algebra and logic gate tables, or just by stabbing at possibilities. They rest on the principle that knowing why certain things must be true is only possible when you know what things are false.

1. Pick some characters

The simplest of these puzzles goes like this: "On Troofalz Island, one tribe always tells the truth, one tribe always lies, and one alternates between truth and lies. You meet an islander who says, 'I always lie.' Which tribe is he from?" Of course, the answer is the waffling tribe; a truth-teller can't be saying he lies, and a chronic liar can't come clean about being a liar.

You can do better than a guy on an island, naturally. Pick types of characters who can be delineated into true and false groups. You can choose kids, superbeings, debaters, used car salesmen ... okay, maybe not that last group.

You might also choose characters based on how many clues you want to write. Three to ten characters is a good number; more is probably unwieldy.

Writing the sidebar on Boolean algebra (see page 164) reminded me that the primary uses of digital logic are … well, digital. So robots seemed an excellent theme. I made my own insane version of Isaac Asimov's Three Laws of Robotics (essentially: don't hurt people, do what you're told, and try not to let the first two laws kill you). The unfortunate victim would be robotics megagenius Dr. Zero.[42]

2. Decide how many cases you'll have

A "case" is an on-off switch; either a character holds to the case or doesn't. The term "true-false puzzle" suggests a binary choice of true or false, but you aren't limited to just those two. Nothing wrong with two, of course, but more makes a richer puzzle.

[42]For years, I've wanted to do a puzzle called "The Death of Dr. Zero." That's the name of my team in the Microsoft Puzzle Hunt. The name was coined when my original team, the Staggering Geniuses, was planning a hunt about Las Vegas in a Microsoft conference room. As the room would be used the next day by many captains of puzzle teams, we outlined an entire fictional hunt on the whiteboard and pretended to forget to erase it. The fake hunt was a sci-fi epic involving crash-landing alien spacecraft, spleen removal (and reinsertion, and re-removal), and the epic conclusion, which I simply wrote, randomly, as "The Death of Dr. Zero." As the team captains arrived the next day, some panicked and started erasing furiously. So as the Geniuses staggered into oblivion, we took our new name to remind the other teams of our little prank at their expense. This self-indulgent footnote may do so as well.

THE DEATH OF DR. ZERO

Dr. Zero has been murdered! As a detective in the Future City Police Department, you hovercycle to Zero Industrial Products to investigate his demise. There you find five of Dr. Zero's DrOnes, a particularly quirky line of robotic servants, standing over his body! Your job is to determine which DrOne slew its creator. As you investigate, keep in mind:

DR. ZERO'S THREE LAWS OF ROBODYNAMICS

1. A DrOne must not kill any human being.
2. A DrOne must always tell the truth.
3. A DrOne must always do the opposite of any Law of Robodynamics it does not follow.

Sadly for both you and Dr. Zero (mostly him), not all of his models follow all of his laws. Each robot has a three-digit model number. For each law, if the robot follows that law, it has a 1 in that position of its number. If it doesn't, it has a 0 in that position. So a robot that follows all three laws is Model 111, while one which only follows the First Law is Model 100. If a robot doesn't follow a law, it can do whatever it wants in relation to that law, assuming that conflicts with no other law that robot follows.

There are five brand new DrOnes present, and exactly one of them killed Dr. Zero. They are one each of these model numbers, though you don't know which is which: 111, 101, 100, 001, and 000.

You've decided to interrogate them behind a protective field, because one of them kills all humans on sight. Stay safely shielded while you sort through their statements and figure out which DrOne is the murderer you're looking for. G00d 1uck!

ALPHA DRONE

- "I am not the DrOne you're looking for."
- "The middle digit of my model number is 0."
- "Omega DrOne has a 1 at the end of its model number."

BETA DRONE

- "I am not the DrOne you're looking for."
- "Gamma and Omega differ in their model numbers by only one digit."
- "Alpha DrOne has only one 1 in its model number."

GAMMA DRONE

- "I am not the DrOne you're looking for."
- "I never lie."
- "All of the digits in my model number are the same."

DELTA DRONE

- "I am the DrOne you're looking for."
- "I killed Dr. Zero."
- "Gamma DrOne's model number is 111."

OMEGA DRONE

- "I am the DrOne you're looking for."
- "I killed Dr. Zero."
- "I have a 1 at the end of my model number."

Decide how many characters fit into each group, assuming you want an absolute number. Add some color to the cases by giving them reasons for being in one category or another (e.g., superheroes always tell the truth, and villains always lie).

The laws of robodynamics created eight possible cases for my robots, some overlapping: 111 and 110 (can't kill, can't lie), 101 (can't kill, must lie), 100 (can't kill, might lie), 000 (might kill, might lie), 010 (might kill, can't lie), 011 (must kill, can't lie), and 001 (must kill, must lie). I picked a logical set of five of those.

3. Lay in a starting point for the truth

True-false puzzles often have many ways in for the solver, but you need to start with one. So figure out some statement(s) whose truth can incontrovertibly be determined, with the full intention of spreading it out or hiding it in a maze of less clear statements.

You have a number of choices for a starting point. Each character may make a similar statement, which means that if you limited the number of people who could be telling the truth, some must be lying about that statement. Two characters can directly contradict each other, meaning that one path out of those statements will conflict with the other. An impartial commentator can discuss the characters' statements, giving detail the characters aren't providing. And so on.

I started with three DrOnes claiming to be the murderer, and two not. Since only two of my robots could kill, that was a nice bit of duplicitous parallelism. The solver *could then start marking options from the viewpoint of each robot's statement of their own innocence or guilt, and go from there.*

4. Write and compare the characters' statements
Around your starting point, write statements that confirm or deny possibilities. These should interact with each other a lot. If a character says something is true, the case where he's telling the truth will confirm some possibilities, and the case where he's lying will deny others. The more uses you can get out of a statement, the better.

As you go, take notes on what happens when characters say something, whether they're telling the truth or lying. Note when a character's statements make any possibility absolutely true or absolutely false. Once a character does that, you never have to refer to that possibility again.

As I wrote statements, I'd note in shorthand which DrOne could or couldn't be a killer, and which could or couldn't be a liar. Staying in character, I used 1s and 0s for true and false, bracketed impossible cases, underscored critical information, and crossed out a statement if its entire possible path was disproven.

After a while, my notes had lines like this: ~~A111: ΩXX1 (Ω111, Ω101/001), ΩX00, A#3 F~~. Translated, that means: "For Alpha to be 111, Omega must have a number that ends in 1, and Omega can't be 111 since he'd be truthfully saying he killed Dr. Zero, and can't be 101 or 001 because he'd have to lie about ending in 1, so Omega must end in 0, so Alpha's third statement is false, so Alpha cannot be 111."

5. Triple-check your logic
Test-solve the puzzle a couple times to make sure you haven't let a possibility in that breaks the puzzle. Then try it on your friends.

My greatest flexibility was in what 111 said. In testing, I tweaked his second statement a few times to make it work. After that, I was sure I had the best logic problem ever created.

Of course, I could be lying.

CRAFTING A SEQUENCE LOGIC PROBLEM

In the previous section, I described the process of making a "true-false" logic problem. In those, each statement was binary; either it was true or it wasn't. Now we're going to enter the realm of more complex variables. In a sequence logic problem, the main variable is time, or, more precisely,

Boolean Logic

To understand your options with a true-false logic puzzle, it helps to know Boolean logic. In 1854, mathematician George Boole[43] wrote *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities*, which helped define elementary algebra as a series of operations dealing with the values 1 and 0.

In Boolean logic, 1 means "yes" or "true," and 0 means "no" or "false." The most basic of operations of these are when you define something that is true if something else is true (implication), or is false if something else is true (negative implication).

If you start with a concept like "There is a knight and a knave," and then make a statement like "The first man is not a knave," what you have implied is "The second man is a knave." But you didn't need to say it, because the negative implication did it for you.

You can describe your true-false choices for two cases in six Boolean operations:

- **OR**: The situation is true if either of the two cases are true.
- **AND**: The situation is true only if both cases are true.
- **NOR**: The situation is true only if both cases are false.
- **XOR**: The situation is true if only one of the two cases is true.
- **XNOR**: The situation is true if all cases are all true or all false.
- **NAND**: The situation is true except if both cases are true.

This is an extremely useful set of operations for logic problems. If you say, "The group is composed of all knaves or all knights," and you can find one knavish lie, the AND operation says that all the rest are knaves. But if you say "A person says that the group is composed of all knaves or all knights," then to determine whether he is a knight or a knave, you only need to find one lie to use that AND operator to identify the speaker as a knave. Once you do so, you also know that at least one is a knight. That's because of the XNOR operator which says that since one of the cases is false (the speaker is a knave, and his statement is false), the entire group cannot be all knaves or all knights. So at least one must be a knight.

Experiment with these operations, and your logic problems will get a lot clearer.

[43]Not to be confused with George Boolos, the MIT professor whose work in logical mathematics defined many of the relationships in Raymond Smullyan's puzzles. Math is funny that way.

HECK COMES TO DOOMSVILLE

The luckless residents of Doomsville have discovered a horde of Mutant Space Zombies—and it has discovered them. Eight residents—five men (the Fry Cook, the Ex-Marine, the Scientist, the Mechanic, and the Pacifist), two women (the Cheerleader and the Reporter), and one animal (the Trusty Pooch)—face the terrible terror of four Mutant Space Zombies. The residents have seven weapons at their disposal—a Pistol, a Flashlight, a Chainsaw, an Electrified Fence, a Can of Corn, and two Super Soakers—which any resident except the Pacifist and the Trusty Pooch can use. Once a weapon is used, it must be thrown away and cannot be used again.

The following catastrophic events occur in some order. Reconstruct the order of the tragic events, listing in order the residents, monsters, and weapons that are killed or used up.

1. The Mutant Space Zombies leap on the Pacifist, showing him too late the error of his ways.
2. The Reporter and the Fry Cook, now comprising half the residents, take the Cheerleader's advice and equip themselves with the remaining Super Soaker and the Flashlight.
3. Another Mutant Space Zombie misses the Fry Cook, who fumbles for the last Super Soaker. That weapon has the same effect as before.
4. The Fry Cook uses the Flashlight to blast the Mutant Space Zombie that killed the Reporter. Nothing happens.
5. The last two residents square off against the last monster. The Scientist plunges the Chainsaw into the Mutant Space Zombie, hardly scratching it.
6. The Ex-Marine, now alone, fires the Pistol. Nothing happens, and he is eaten.
7. Cruelly left behind, the Trusty Pooch is consumed by a Mutant Space Zombie.
8. The Fry Cook has little time to celebrate, though, as the last Mutant Space Zombie has him for a snack.
9. The Cheerleader stops running when she sees the Mechanic, who abandons his woodworking project to investigate. He is eaten alive.
10. The surviving resident watches the last other resident of Doomsville die, weapon still in hand. Weaponless, that survivor grabs for the final weapon and strikes the monster with it to no avail.
11. The Ex-Marine and the Pacifist, seeing the Fry Cook flee from his now useless Electrified Fence, are confronted by the foe.
12. The Cheerleader fires a Super Soaker, killing a Mutant Space Zombie, and then she flees.
13. That person, while standing over the Mutant Space Zombie they just killed, is consumed by another Mutant Space Zombie.
14. The Cheerleader, out walking the Trusty Pooch, discovers the Mutant Space Zombie invasion.
15. The duo is surprised by the Mutant Space Zombies, and the Reporter accidentally drops the unused Super Soaker. Still, she pummels one of the monsters to death with her kung fu fists.
16. Watching the Mechanic's death, the Fry Cook lures a Mutant Space Zombie to the Electrified Fence, which upon going off does not harm the monster.
17. The end comes for the last resident of Doomsville.

relationships between factors over time. This kind of logic problem involves balancing of multiple options for each possibility—but hey, at least they're all true.

1. Create a story concept

A sequential logic problem needs to be about a story. It should be one that people will enjoy. So pick a subject—a tea party, a baseball game, an epic of discovery in the New World, or whatever—that will resonate with your solver.

Your subject also needs to have lots of possibilities for plot points, characters, and/or settings. You might have people coming and going, or places that are visited, or the like. Make sure you can imagine lots of things happening in your story.

I started with a concept of a space zombie invasion. Since this was a horror story, I knew that I would have a lot of characters at the start, and not very many at the end. I also knew it would not be for the faint of heart. For my story to be captivating, I would need a town full of interesting horror movie archetypes, some monsters to beset them, and some weapons for the townsfolk to use against the monsters.

2. Storyboard your story

Unlike other types of logic problems, a sequence problem has a plot. It needs a beginning, a middle (or possibly multiple middles, each with a beginning, a middle, and an end), and an end. So come up with a storyboard for your logic problem.

The term "storyboard" comes from the bulletin boards that Walt Disney's animators used to lay out a cartoon's story. It's used primarily in TV and film writing, but it's also useful in puzzle design to track information.

Start by figuring out the essential story elements. Say you're writing a puzzle about Thanksgiving dinner. You'd need to know who's coming (your characters), what's being served (your plot points), and whose house it's at (your setting). Write all these down on index cards. If you can find some colored index cards, keep your characters in one color, your plot points in another, and so on. Or you can do this on a computer, either in Excel or in a flowchart program.

I wrote down some story elements in Word, using the flowchart function to sort what I created. I made a list of characters, a list of weapons, and some number (not yet finalized) of Mutant Space Zombies. I put the characters in square boxes, the weapons in diamonds, and the zombies in numbered circles.

I then realized I needed another set of cells. There's a hysterical board game designed by Tom Wham called The Awful Green Things From Outer Space. One of the key elements is that the terrified crew of the spaceship Znutar has no idea what its weapons will do against the titular monsters. A can of rocket fuel could blast the Green Things to bits, or make them spawn new monsters. For my puzzle, I needed results like that. So I created a new hexagon-shaped field type for weapon effects. (The electric fence in this puzzle is an homage to that game, which also contains one.)

3. Order your story

Move all your elements around your storyboard until you've accounted for everything that happens to all of them. You will undoubtedly find some plot holes, which you'll want to fill at this stage. Sometimes, you'll need duplicate versions of your cards, or whatever you're using to track info, because your elements may phase in and out at various points. It may be useful to draw lines between your elements, or to tape or group elements together so that you can move them as a unit.

When you have a sequence of events on your storyboard, transfer them to a series of numbered sentences. So if you're looking at "Grandma" "turkey" "dog snatches it" as a grouping, write "Grandma served the turkey, but the dog snatched it off the table and wolfed it down." Continue until you have everything in numbered sentence form, in chronological order.

I got all my characters, weapons, effects, and monsters into an order I liked. Then I wrote those groupings in sentences like "The last two survivors, the Cheerleader and the Scientist, square off against the last monster." But I knew I wasn't done with those sentences.

4. Disorder your story

Now the real fun begins. Looking at the chronological order, find some points of information you can obscure or delete, but still leave only one possible sequence for the story. For example, one of your characters may be the only character who can do some step in the sequence. You may be able to remove that person's name and still keep your sequence intact and unique.

Then scramble the sentence order. You can do this completely randomly, or place some sentences in relation to others so that false throughlines are implied. Once you're done with that, renumber the sentences so that the solver has no idea which order they were originally in.

River-Crossing Puzzles

River-crossing puzzles are sequence logic problems where all the sequential clues are dictated by a series of relationship statements between the characters. For example, there's this classic, notably found in the works of English puzzlemaker Henry Dudeney:

> *Three missionaries and three cannibals must cross a river in a boat that holds two people. If the missionaries are outnumbered by cannibals on either side of the river, the cannibals will eat the missionaries. How can all six cross the river?*

How indeed? The answer is represented as a series of moves: 3M+3C, 2C→, ←C, 2C→, ←C, 2M→, ←M+C, 2M→, ←C, 2C→, ←C, 2C→, 3M+3C. The puzzle works because the characters require you to think logically about their characteristics.

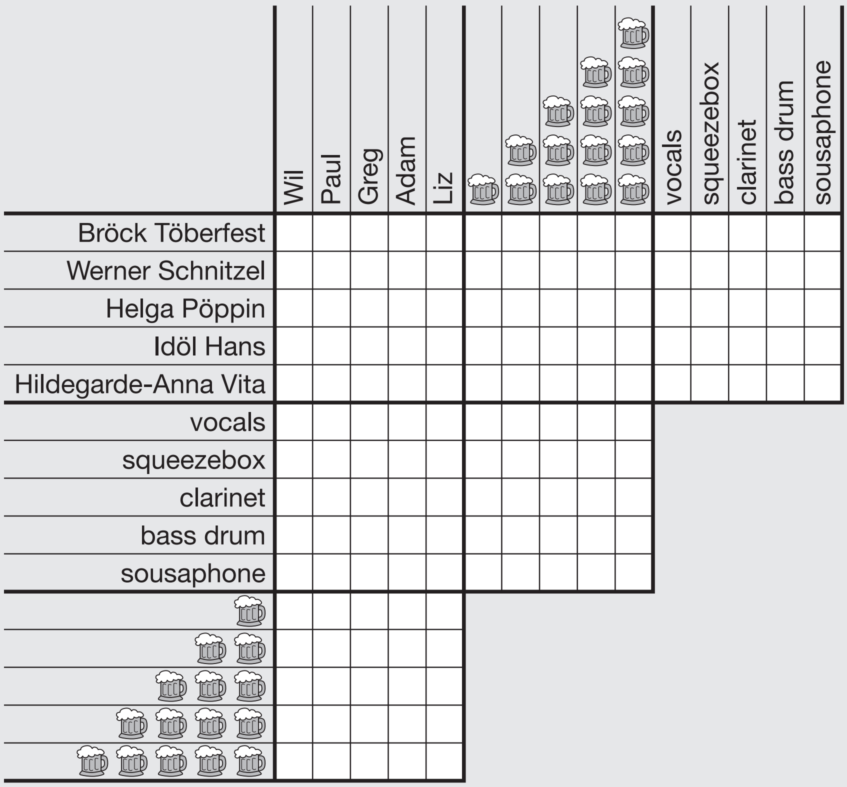
To make a river-crossing puzzle, define your characters and their unique quirks. For example, you might say that you have gnomes, men, and giants. Gnomes are small, and count as ¼ of the boat's capacity. Men are normal size, and count as ½ of the boat's capacity. Giants are tall, and count as ¾ of the boat's capacity. Then figure out a sequence where everyone can get from one side to the other.

But seriously, if you are ever presented with the real-life situation of being on a riverbank with three missionaries and three cannibals, put down the pencil and call 911.

THE WORLD SERIES OF POLKA

Welcome to v00tstok, a festival of polka-obsessed videogame nerds. Liz and her four guy friends have just finished playing the hot new videogame from Beermonix, Polka Band 3. Our quintet polkaed out to Poland Storm's classic "Kielbasa-Nova Baby." Each played a character and an instrument, scoring a different score from 1 stein to 5 steins. Who got 5 steins and won the World Series of Polka?

1. Liz did not pick one of the two female characters, Helga and Hildegarde-Anna.
2. Adam got a character whose name contained at least one umlaut; Wil's was umlaut-free.
3. The sousaphone and the squeezebox, neither played by Greg, had consecutive scores.
4. The player on vocals, who wasn't playing as Werner, scored lower than Adam did.
5. Greg scored an odd number of steins, but the clarinet didn't.
6. No character whose name contained double letters played an instrument with that trait.
7. Hans's score was three steins.
8. Wil scored more steins than the bass drum, which scored more than Greg did.
9. Each female character, neither played by Paul, scored an even number of steins.
10. Liz finished within three places of Greg.



I began obscuring what information I could. For example, I had combined the sentence "The last two survivors, the Cheerleader and the Scientist, square off against the last monster" with another sentence about the Scientist, so I didn't need to specify that he was one of the remaining survivors. And in an earlier sentence, the Cheerleader is listed as one of four remaining survivors—and the other two bite the dust afterward. So I no longer needed to refer to the Cheerleader as one of the last two either. So the entire clause of "the Cheerleader and the Scientist" could go. I continued until I was sure I didn't create more than one final possibility.

I renumbered the sentences mostly randomly, but I kept the last one in the last position. That's because I wanted to drive home the point that in the event of a Mutant Space Zombie invasion, the last place you want to be is a town called Doomsville.

CRAFTING A GRID LOGIC PROBLEM

We're going out in style. The final self-contained puzzle type in the book is one of the fundaments of puzzlemaking. The grid logic problem comes to us from Lewis Carroll, whose groundbreaking book *The Game of Logic* laid out all the principles of one of the most ubiquitous puzzle types of the modern era. A grid logic problem requires you to track and plot many types of data at once. These problems sometimes give the solver a grid like the one seen above, but whether or not the solver gets one, you'll need to make one.

1. Come up with a theme

Your first step in making a grid-based logic problem is to come up with a theme. Your puzzle could be about quintuplets' birthday presents or movie award shows or whatever else you can think of. If you find your theme entertaining, chances are your solvers will too.

After all the murder and mayhem in the last two puzzles, I wanted something a bit more relaxing for this puzzle. I started with the theme of a videogame-nerd concert similar to the w00tstock series, for which actor and author Wil Wheaton asked me to create some puzzles.[44] *I then ended up pretty far afield from that plan, mostly for humor purposes. But I kept the testament to my pals Wil, musicians Paul & Storm,* Mythbusters *madman Adam Savage, and stage manager Liz Smith just for kicks.*

2. Identify and plot the variables

Now decide on the variables that your puzzle will highlight. For example, you might have the names of some characters, their characteristics, and what happens to them. You'll want at least three of each variable, and probably no more than six.

Some variables are simple identifiers which you'll tell the solver; a character's name is just a name. Some variables are more complex, though. If you have a sequence of events, they might be identified by 1st, 2nd, 3rd, and so on. That creates a relationship between the variables, so that 1st comes before 2nd, and 2nd before 3rd. You can even make some variables derivable by the solver, such as times that events occur. The solver need not know all the times at the start, instead figuring them out as more pieces of the puzzle fall into place.

Put the variables into a solving grid such as the one on page 167. All the possibilities must meet each other. Rearrange the horizontal variables when you put in the vertical variables so that everything meets everything else.

My variables are: player name, character name, instrument, and score. This last variable's possibilities are sequential, and I made sure I specified that only one occurrence of each score could be attained.

3. Control the possibilities

Now you'll place the possibilities in the grid so that in each thick-bordered box, each row and column contains one true pairing (marked with an O), and all the rest false (marked with X's). Make sure the answers line up with each other; if a character has a characteristic, and that characteristic matches an event, that character must match that event.

My answer grid has one O and four X's per row or column in each section. Note that when a set of variables appears on both the vertical and horizontal axes, its order remains the same (reading top to bottom or left to right).

4. Write the clues

You want the solver to have to go back and forth between your clues to settle inconsistencies until, in Sherlock Holmes–like fashion, only the truth remains. You have a number of types of statements you can use:

- a **positive** clue ("A is B") allows the solver to place one or more O's in the grid. Each time an O is placed, a string of horizontal and vertical X's can be extended from the O.
- a **negative** clue ("A is not B") allows the solver only to place one or more X's in the grid. Only when all but one square in a line contains X's can an O be filled in. A negative statement eliminates far fewer possibilities than a positive one, so you should use more negative statements than positive ones.
- a **de facto** clue ("A can only be B") must be true given the conditions of the puzzle. For example, if all your characters are male except one, and you use the word "she," it can only apply to one possibility.
- a **hierarchical** clue ("A is greater than B") places the possibilities in some order, allowing the solver not only to place X's where possibilities are ruled out, but also to fill in mathematical details missing from the grid.

Try to use as few clues as possible. Clues that give multiple pieces of information are better than those that give only one.

Even though there are 150 pieces of information to unearth, I used only 10 clues. I used only one positive statement (clue #7's "Hans = 3 steins"), and several negative ones (clue #2 knocks out five options).

The hierarchical clue dynamic is used extensively in the puzzle. The stein ratings are often stated not as numbers but as relationships to other pieces of data like character names. For example, clue #8 positions Wil, the bass drum, and Greg in a hierarchical chain, though their exact positions on that chain must still be determined.

There's an unstated de facto clue, in that when Liz doesn't pick one of the two female characters, it follows that two of the four guys must pick female characters.

5. Create a blank grid

Double check that all your facts can be derived from the clues you wrote. Then prepare a blank grid and a copy of the clues for your solver. That is, if you're feeling nice. If not, conveniently leave the grid at home.

Apparently, I'm feeling nice. Must be the beer.

[44]For w00tstock, my co-designer James Ernest and I created a puzzling/juggling hybrid called "juzzling." I'm going to refrain from teaching you how to create a juzzle, mostly for insurance reasons.