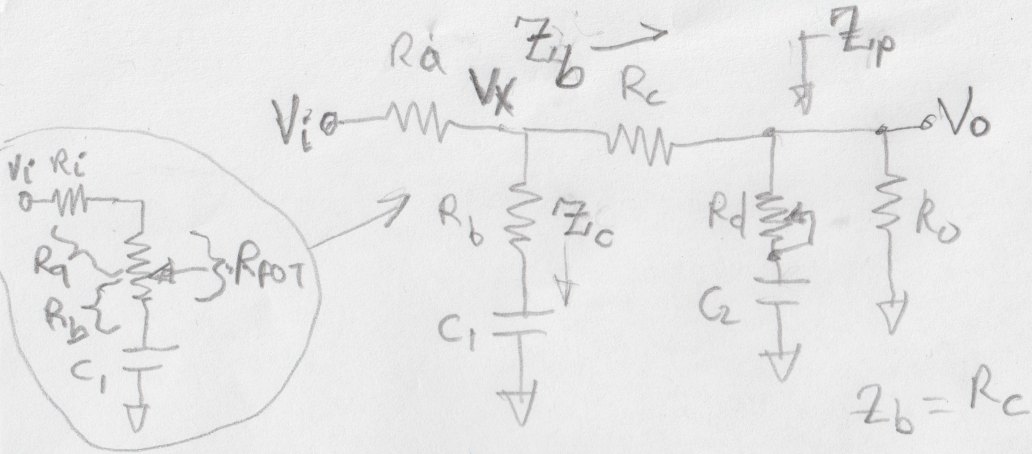

$$Z_p = \frac{R_o R_d + R_o \frac{1}{sC_2}}{R_o + R_d + \frac{1}{sC_2}} = \frac{sC_2 R_o R_d + R_o}{sC_2(R_o + R_d) + 1}$$

$$Z_b = R_c + Z_p =$$

$$Z_b = \frac{(sR_cC_2(R_o+R_d) + R_c + sC_2R_oR_d + R_o)}{sC_2(R_o+R_d)+1}$$

$$Z_b = \frac{sC_2[R_c(R_o+R_d) + R_oR_d] + (R_o+R_c)}{sC_2(R_o+R_d)+1}$$

(a_1 = $sC_2[R_c(R_o+R_d)+R_oR_d]$, a_0 = (R_o+R_c), b_1 = $sC_2(R_o+R_d)$, b_0 = 1)

SUB-1

$a_0 = R_o + R_c$

$a_1 = (R_o+R_d)R_cC_2 + R_oR_dC_2$

$b_0 = 1$

$b_1 = (R_o+R_d)C_2$

$$Z_b = \frac{a_1 s + a_0}{b_1 s + b_0}$$

STRATEGY:

① Derive $\frac{V_x}{V_i}$

② Derive $\frac{V_o}{V_x}$

③ $\frac{V_o}{V_i} = \frac{V_x}{V_i}\,(①) \cdot \frac{V_o}{V_x}\,(②)$ ← Expressed as two separate cascaded filter stages

$$Z_C = R_b + \frac{1}{sC_1} = \frac{sR_bC_1}{sC_1} + \frac{1}{sC_1} = \frac{sR_bC_1 + 1}{sC_1}$$

(B)

$$Z_x = Z_b \parallel Z_c = \left(\frac{b_1 s + 1}{a_1 s + a_0} + \frac{sC_1}{sR_bC_1 + 1} \right)^{-1}$$

$$= \frac{(sR_bC_1 + 1)(a_1 s + a_0)}{(b_1 s + 1)(sR_bC_1 + 1) + s^2 C_1 a_1 + sC_1 a_0}$$

$$= \frac{s^2 R_bC_1 a_1 + sR_bC_1 a_0 + a_1 s + a_0}{s^2 R_bC_1 b_1 + b_1 s + sR_bC_1 + 1 + s^2 C_1 a_1 + sC_1 a_0}$$

$$= \frac{s^2 R_bC_1 a_1 + s(R_bC_1 a_0 + a_1) + a_0}{s^2(R_bC_1 b_1 + C_1 a_1) + s(R_bC_1 + C_1 a_0 + b_1) + 1}$$

$$V_i = V_x \cdot \frac{1}{Z_x} \cdot R_a + V_x$$

$$V_i = V_x \left(\frac{R_a}{Z_x} + 1 \right)$$

$$\frac{V_x}{V_i} = \frac{1}{\frac{R_a}{Z_x} + 1}$$

SUB-2

$N_0 = a_0$

$N_1 = R_bC_1 a_0 + a_1$

$N_2 = R_bC_1 a_1$

$D_0 = 1$

$D_1 = R_bC_1 + C_1 a_0 + b_1$

$D_2 = R_bC_1 b_1 + C_1 a_1$

$$= \frac{s^2 N_2 + sN_1 + N_0}{s^2 D_2 + sD_1 + D_0} = Z_x$$

(C)

$$Z_x = \frac{s^2 N_2 + s N_1 + N_0}{s^2 D_2 + s D_1 + D_0}$$

$$\frac{V_x}{V_i} = \frac{Z_x}{R_a + Z_x} = \frac{\frac{N}{D}}{R_a + \frac{N}{D}} = \frac{N}{D R_a + N} = \frac{s^2 N_2 + s N_1 + N_0}{s^2 R_a D_2 + s R_a D_1 + R_a D_0 + s^2 N_2 + s N_1 + N_0}$$

STAGE-1

$$\frac{V_x}{V_i} = \frac{s^2 N_2 + s N_1 + N_0}{s^2 (N_2 + R_a D_2) + s (N_1 + R_a D_1) + N_0 + R_a D_0}$$

STRATEGY
STEP (1)

$$\frac{V_o}{V_x} = \frac{sC_2R_oR_d + R_o}{s(R_o+R_d)C_2 + 1 + R_c} = \frac{sR_oR_dC_2 + R_o}{s(R_o+R_d)R_cC_2 + R_c + sR_oR_dC_2 + R_o}$$

NUM / DEN $+ R_c$

$$= \frac{sC_2R_oR_d + R_o}{s\left[C_2R_oR_d + (R_o+R_d)R_cC_2\right] + R_c + R_o}$$

NOTES:

STAGE-1 is second-order

STAGE-2 is first order

cascading order 2 into order 1 filter would imply 3rd order.

By inspection the network is second-order. More algebra would expose this but since it is easy to bilinear transform both, computer can do the work for us.

STRATEGY

STEP ②

STAGE-2

$$\frac{V_o}{V_x} = \frac{R_oR_d}{C_2R_oR_d + (R_o+R_d)R_cC_2} \cdot \frac{s + \frac{1}{R_dC_2}}{s + \frac{R_c+R_o}{C_2R_oR_d + (R_o+R_d)R_cC_2}}$$