Switch Mode Power Supply Current Modeling

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1.1 Geometry of the Problem

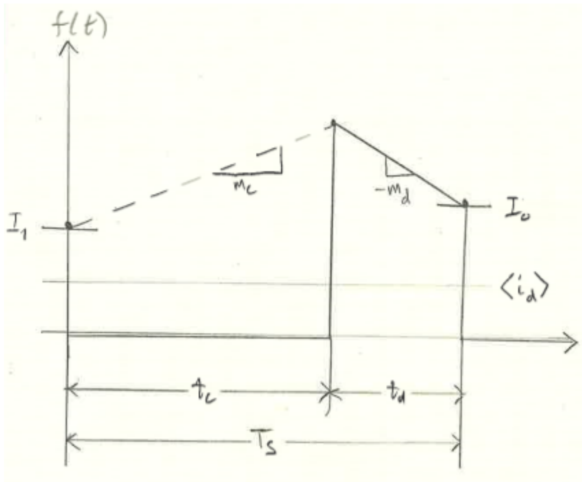

Figure 1: Geometry of the problem.

Above is a generalized plot of the charging and discharging current over one period where T_s is the period, I_1 is the input current, I_0 is the output current, m_c is the charging current slope, and $-m_d$ is the discharging current slope. T_s, I_1, I_0, m_c, and $-m_d$ are assumed to be known and the average currents, $< i_c >$ and $< i_d >$, for the period is what we are interested in solving for. The average currents over the period can be solved for by integrating the charging region for the charging current or the discharging region for the discharging current and averaging that over the entire period. Because of the simplicity of the structure of the problem, this integration can also be done geometrically by viewing the region as a triangle atop a rectangle.

1.2 Charging Current Solution

For my solution I choose an integral approach to the problem. Based on the geometry of the problem I came up with the following integral:

$$< i_c > = \frac{1}{T_s}\int_0^{t_c}(m_c t + I_1)dt \tag{1}$$

This integral yields the following equation:

$$< i_c > = I_1\frac{t_c}{T_s} + \frac{m_c t_c^2}{2T_s} \tag{2}$$

In the above equation every every variable is a known part of the problem exept for t_c. This can be solved for geometrically by finding the intersection of the two lines. The location of this intersection on the x-axis will be t_c. The charging and discharging slopes can be modeled as two lines using the equations:

$$f_c(t) = m_c t + I_1 \tag{3}$$
$$f_d(t) = -m_d(t - T_s) + I_0 \tag{4}$$

Where $f_c(t)$ is the value of the current from $t = 0$ to $t = t_c$ as a function of t and $f_d(t)$ is the value of the current from $t = 0$ to $t = T_s$. Setting these two equations equal to each other will give a solution for t where the lines intersect. As can be seen in Figure 1, the location of this intersection on the x-axis is t_c.

$$f_c(t_c) = f_d(t_c) \tag{5}$$
$$m_c t + I_1 = -m_d(t - T_s) + I_0 \tag{6}$$
$$t_c = \frac{I_0 - I_1 + T_s m_d}{m_c + m_d} \tag{7}$$

Substituting equation (7) into equation (2) provides the following solution for $< i_c >$:

$$\begin{aligned}
< i_c > &= cI_0^2 + dI_1^2 + eI_0I_1 + fI_0 + gI_1 + h \\
c &= \frac{m_c}{2T_s m_c^2 + 4T_s m_c m_d + 2T_s m_d^2} \\
d &= \frac{m_c}{2T_s m_c^2 + 4T_s m_c m_d + 2T_s m_d^2} - \frac{1}{T_s m_c + T_s m_d} \\
e &= \frac{1}{T_s m_c + T_s m_d} - \frac{m_c}{T_s m_c^2 + 2T_s m_c m_d + T_s m_d^2} \\
f &= \frac{m_c m_d}{m_c^2 + 2m_c m_d + m_d^2} \\
g &= \frac{m_d}{m_c + m_d} - \frac{m_c m_d}{m_c^2 + 2m_c m_d + m_d^2} \\
h &= \frac{T_s m_c m_d^2}{2m_c^2 + 4m_c m_d + 2m_d^2}
\end{aligned} \tag{8}$$

This equation is powerful because T_s, m_c, and m_d are known constants and I_0 and I_1 are either known or inputs into the function depending on how the equation is being used.

1.3 Discharging Current Solution

The solution for the average discharging current is of the same form as the solution for the average charging current; only the coefficients are different. The integral equation for the discharging portion is as follows:

$$< i_d > = \frac{1}{T_s}\int_{t_c}^{T_s}(-m_d(t - T_s) + I_0)dt \tag{9}$$

This integral yields the following equation:

$$< i_d > = I_0 - I_0 \frac{t_c}{Ts} + \frac{T_s m_d}{2} + \frac{m_d t_c^2}{2T_s} - m_d t_c + T_s m_d \tag{10}$$

Equation (10) is expressed in terms of t_c instead of t_d so that equation (7) can be reused in this calculation to find $< i_d >$. This is possible using the equivalency: $t_c = T_s - t_d$. The location of t_c can be found the same way as it was for the charging current, but because the problem has not changed, neither will t_c. Substituting equation (7) into equation (10) provides the following solution for $< i_d >$:

$$\begin{aligned} < i_d > &= cI_0^2 + dI_1^2 + eI_0I_1 + fI_0 + gI_1 + h \\ c &= \frac{m_d}{2T_s m_c^2 + 4T_s m_c m_d + 2T_s m_d^2} + \frac{1}{T_s m_c + T_s m_c} \\ d &= \frac{m_d}{2T_s m_c^2 + 4T_s m_c m_d + 2T_s m_d^2} \\ e &= \frac{1}{T_s m_c + T_s m_d} - \frac{m_d}{T_s m_c^2 + 2T_s m_c m_d + T_s m_d^2} \\ f &= \frac{m_d^2}{m_c^2 + 2m_c m_d + m_d^2} - \frac{1 - m_c}{m_c + m_d} \\ g &= \frac{1}{m_c + m_d} - \frac{m_d^2}{m_c^2 + 2m_c m_d + m_d^2} \\ h &= \frac{T_s m_d^3}{2m_c^2 + 4m_c m_d + 2m_d^2} + \frac{T_s m_d^2}{m_c + m_d} + \frac{T_s m_d}{2} \end{aligned} \tag{11}$$

Comparing equation (8) with equation (10), we can see that $< i_c >$ and $< i_d >$ are both quadratics of the same for with only the coefficients and constant being different between them.

2 Model Validation

These equations were able to be verified using a LTSpice model and a Python script. The data was exported from LTSpice and plotted in Python over the output from the Python simulation to show their correlation.

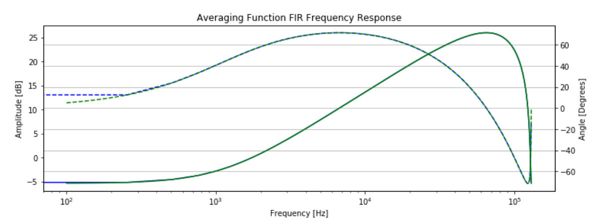

Figure 2: Python and LTSpice models.

3 Small Signal Model

In the small signal model, $i_0 = I_0 + \Delta_0$ and $i_1 = I_1 + \Delta_1$, where i_0 and i_1 refer to the small signal currents for time [t] and [t-1] and Δ_0 and Δ_1 refer to the small variation i has from I. To analyze the small signal analysis we can use this relationship and the following equations: