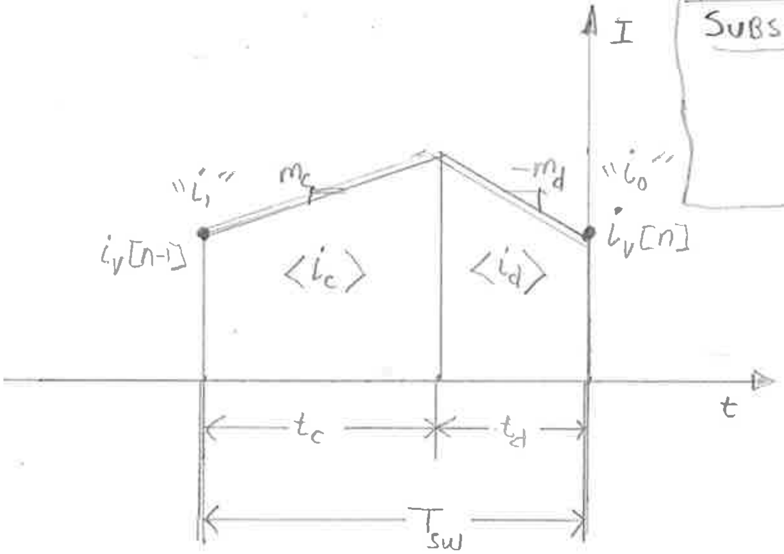
IMPULSE RESPONSE OF SWITCH-MODE INDUCTOR AS FIR FILTER



SUBSTITUTIONS

$$a = \frac{1}{m_c + m_d}$$

$$b = T_{sw}(1 - a m_d)$$

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(1) VALLEY CURRENT "$i_v[n]$" IS CONSIDERED AS AN IMPULSE TRAIN.

(2) (THIS DERIVATION) AVERAGE DISCHARGE CURRENT IS EXPRESSED AS AN FIR FILTER CONVOLVED WITH VALLEY CURRENTS, e.g.,

$$\langle i_d \rangle[n] = A_0 i_v[n] + A_1 i_v[n-1]$$

STEP 1: FIND AVERAGE DISCHARGE CURRENT $\langle i_d \rangle$ AS A FUNCTION OF VALLEY CURRENTS, "i_0" AND "i_1"

$$t_d = T_{sw} - t_c$$

$$i_0 = i_1 + m_c t_c - m_d t_d$$

$$= i_1 + m_c t_c - m_d T_{sw} + m_d t_c$$

$$i_0 - i_1 + m_d T_{sw} = t_c(m_d + m_c)$$

$$t_c = \underbrace{\left[\frac{1}{(m_c + m_d)}\right]}_{a} \left[i_0 - i_1 + m_d T_{sw}\right] = a i_0 - a i_1 + a m_d T_{sw}$$

$$t_d = T_{sw} - t_c = T_{sw} + a i_1 - a i_0 - a m_d T_{sw}$$

$$\underbrace{T_{sw}(1 - a m_d)}_{b} + a i_1 - a i_0 = a i_1 - a i_0 + b$$

$$\langle i_d \rangle = \left(\frac{1}{T}\right)\left(\frac{1}{2}\right)\left[i_0 + i_0 + m_d t_d\right] t_d = \frac{1}{2T}\left(2 t_d i_0 + m_d t_d^2\right)$$

$$= \frac{1}{T}\left(t_d i_0 + \frac{1}{2} m_d t_d^2\right)$$

$$t_d = a i_1 - a i_0 + b$$

$$t_d^2 = (a i_1 - a i_0 + b)(a i_1 - a i_0 + b)$$

$$= a^2 i_1^2 \boxed{- a^2 i_0 i_1} + a b i_1 \boxed{- a^2 i_0 i_1} + a^2 i_0^2 - a b i_0 + a b i_1 - a b i_0 + b^2$$

$$= a^2 i_0^2 + a^2 i_1^2 - 2 a^2 i_0 i_1 - 2 a b i_0 + 2 a b i_1 + b^2$$

$$t_d i_0 = i_0 (a i_1 - a i_0 + b) = a i_0 i_1 - a i_0^2 + b i_0$$

$$\tfrac{1}{2} m_d t_d^2 = \tfrac{1}{2} m_d a^2 i_0^2 + \tfrac{1}{2} m_d a^2 i_1^2 - m_d a^2 i_0 i_1 - m_d a b i_0 + m_d a b i_1 + \tfrac{1}{2} m_d b^2$$

$$+ \; t_d i_0 \qquad - a i_0^2 \qquad + a \, i_0 i_1 \qquad + b i_0$$

$$\frac{(t_d i_0 + \frac{1}{2} m_d t_d^2)}{T} = \underbrace{\frac{(\frac{1}{2} m_d a^2 - a)}{T}}_{c} i_0^2 + \underbrace{\frac{\frac{1}{2} m_d a^2}{T}}_{d} i_1^2 + \underbrace{\frac{(a - m_d a^2)}{T}}_{e} i_0 i_1 + \underbrace{\frac{(b - m_d a b)}{T}}_{f} i_0 + \underbrace{\frac{m_d a b}{T}}_{g} i_1 + \underbrace{\frac{\frac{1}{2} m_d b^2}{T}}_{h}$$

$$\langle i_d \rangle = c i_0^2 + d i_1^2 + e i_0 i_1 + f i_0 + g i_1 + h$$

STEP 2: PERTURB $\langle i_d \rangle$.

FIND $\Delta \langle i_d \rangle$ AS A FUNCTION OF Δi_0 and Δi_1 (SMALL-SIGNAL GAIN)

$$\langle i_d \rangle = \langle I_d \rangle + \langle \Delta_d \rangle$$

$$i_0 = I_0 + \Delta_0$$

$$i_1 = I_1 + \Delta_1$$

$$I_0 = I_1 = i_0 = i_1 \equiv i_\infty, \text{ steady-state}$$

FOR SMALL SIGNALS,

$$\langle I_d + \Delta_d \rangle \approx \frac{\partial}{\partial i_0} c\, i_0^2 \Big|_{i_0 = i_\infty} \times \Delta_0 = \Delta_0\, 2c\, i_0 \Big|_{i_\infty} = 2c\, i_\infty \Delta_0$$

$$+ \frac{\partial}{\partial i_1} d\, i_1^2 \Big|_{i_1 = i_\infty} \times \Delta_1 = \Delta_1\, 2d\, i_1 \Big|_{i_\infty} = 2d\, i_\infty \Delta_1$$

$$+ \frac{\partial}{\partial i_0} e\, i_0 i_1 \Big|_{i_1 = i_\infty} \times \Delta_0 + \frac{\partial}{\partial i_1} e\, i_0 i_1 \Big|_{i_1 = i_\infty} \times \Delta_1 = e i_1 \Big|_{i_\infty} \Delta_0 + e i_0 \Big|_{i_\infty} \Delta_1 = e i_\infty \Delta_0 + e i_\infty \Delta_1$$

$$+ \frac{\partial}{\partial i_0} f\, i_0 \Big|_{i_0 = i_\infty} \Delta_0 = f \Delta_0$$

$$+ \frac{\partial}{\partial i_1} g\, i_1 \Big|_{i_0 = i_\infty} \Delta_1 = g \Delta_1$$

$+ h$ CONST, NOT PART OF AC GAIN

$$\langle i_d \rangle = 2c\, i_\infty \Delta_0 + 2d\, i_\infty \Delta_1 + e i_\infty \Delta_0 + e i_\infty \Delta_1 + f \Delta_0 + g \Delta_1 + h$$

$$= \underbrace{(2c\, i_\infty + e i_\infty + f)}_{A_0} \Delta_0 + \underbrace{(2d\, i_\infty + e i_\infty + g)}_{A_1} \Delta_1 + h$$

RE-INTERPRET GEOMETRIC MODEL IN TERMS OF VALLEY-CURRENT IMPULSE TRAIN: $\Delta_0 = i_v[n]$, $\Delta_1 = i_v[n-1]$, $\langle i_d \rangle = \langle i_d \rangle [n]$

MAKING SUBSTITUTIONS,

$$\langle i_d \rangle [n] = A_0\, i_v[n] + A_1\, i_v[n-1]$$

STEP 3: FIND I_∞, OPERATING POINT GAIN

$$I_\infty = c\, i_\infty^2 + d\, i_\infty^2 + e\, i_\infty^2 + f\, i_\infty + g\, i_\infty + h \,, \quad i_\infty = i_0 = i_1 \text{ @ steady state}$$

$$I_\infty = (c+d+e)\, i_\infty^2 + (f+g)\, i_\infty + h$$

CAN BE RE-APPLIED AFTER [STEP 5] NORMALIZATION

STEP 4: Z-TRANSFORM

$$\langle i_d \rangle [n] = A_0 i_v [n] + A_1 i_v [n-1]$$

$$\langle I_d \rangle (z) = A_0 I_v(z) + A_1 I_v z^{-1}$$

$$H(z) = \frac{\langle I_d \rangle (z)}{I_v(z)} = A_0 + A_1 z^{-1}$$

STEP 5: NORMALIZE DC GAIN FOR EASY COMPARISON WITH CONVENTIONAL RHPZ

Observation: $z^{-1} = e^{-j\omega T} = 1$ @ $\omega = 0$

GAIN @ $\omega = 0$

$$H(z)\Big|_{\omega=0} = A_0 + A_1 e^0$$

A NORMALIZED TRANSFER FUNCTION IS AS FOLLOWS

$$H_N(z) = \frac{1}{A_0 + A_1}\left(A_0 + A_1 z^{-1}\right) = \frac{A_1}{A_0 + A_1}\left(\frac{A_0}{A_1} + z^{-1}\right)$$

IN APPENDIX A IT IS SHOWN HOW THIS CAN BE USED TO OBTAIN THE 3dB CUT-OFF FREQUENCY

TO SEE FREQUENCY COMPONENT SEPARATELY FROM DC GAIN, H(z) CAN BE EXPRESSED AS FOLLOWING

$$H(z) = I_\infty H_N(z)$$

APPENDIX A: FIND 3-dB CUT-OFF FREQUENCY FOR RHPZ

$$H_N(z) = \frac{A_1}{A_0+A_1}\left(\frac{A_0}{A_1} + z^{-1}\right)$$

$$= \frac{1}{\frac{A_0}{A_1}+1}\left(\frac{A_0}{A_1} + z^{-1}\right)$$

z-domain root does not coincide w/ 3dB cut-off

$$= \frac{1}{\frac{1}{\omega_z}+1}\left(\frac{1}{\omega_z} + z^{-1}\right), \qquad \omega_z = \frac{A_1}{A_0}$$

FIND ω in $z^{-1} = e^{-j\omega T}$ such that,

$|H_N(z)| \cdot 2^{\pm\frac{1}{2}} = |H_c|$, (either $\sqrt{2}$ or $\frac{1}{\sqrt{2}}$)

$$H_c\left(\frac{1}{\omega_z}+1\right) = \left|\frac{1}{\omega_z} + \cos(\omega_0) - j\sin(\omega_0)\right|$$

$$\sqrt{\left(\frac{1}{\omega_z} + \cos(\omega_0)\right)^2 - j^2\sin^2(\omega_0)}$$

$$\left[H_c\left(\frac{1}{\omega_z}+1\right)\right]^2 = \left(\frac{1}{\omega_z} + \cos(\omega_0)\right)^2 + \sin^2(\omega_0)$$

$$\left(\frac{1}{\omega_z}\right)^2 + \frac{2\cos(\omega_0)}{\omega_t} + \underbrace{\cos^2(\omega_0) + \sin^2(\omega_0)}_{1}$$

$$\cos^{-1}\left[\frac{\omega_z\left(\left[H_c\left(\frac{1}{\omega_z}+1\right)\right]^2 - 1 - \frac{1}{\omega_z^2}\right)}{2}\right] = \omega_0 = \omega_c' T$$

$$\omega_c = \frac{1}{T}\cos^{-1}\left[\frac{\omega_z}{2}\left(\left(H_c\left(\frac{1}{\omega_z}+1\right)\right)^2 - 1 - \frac{1}{\omega_z^2}\right)\right]$$