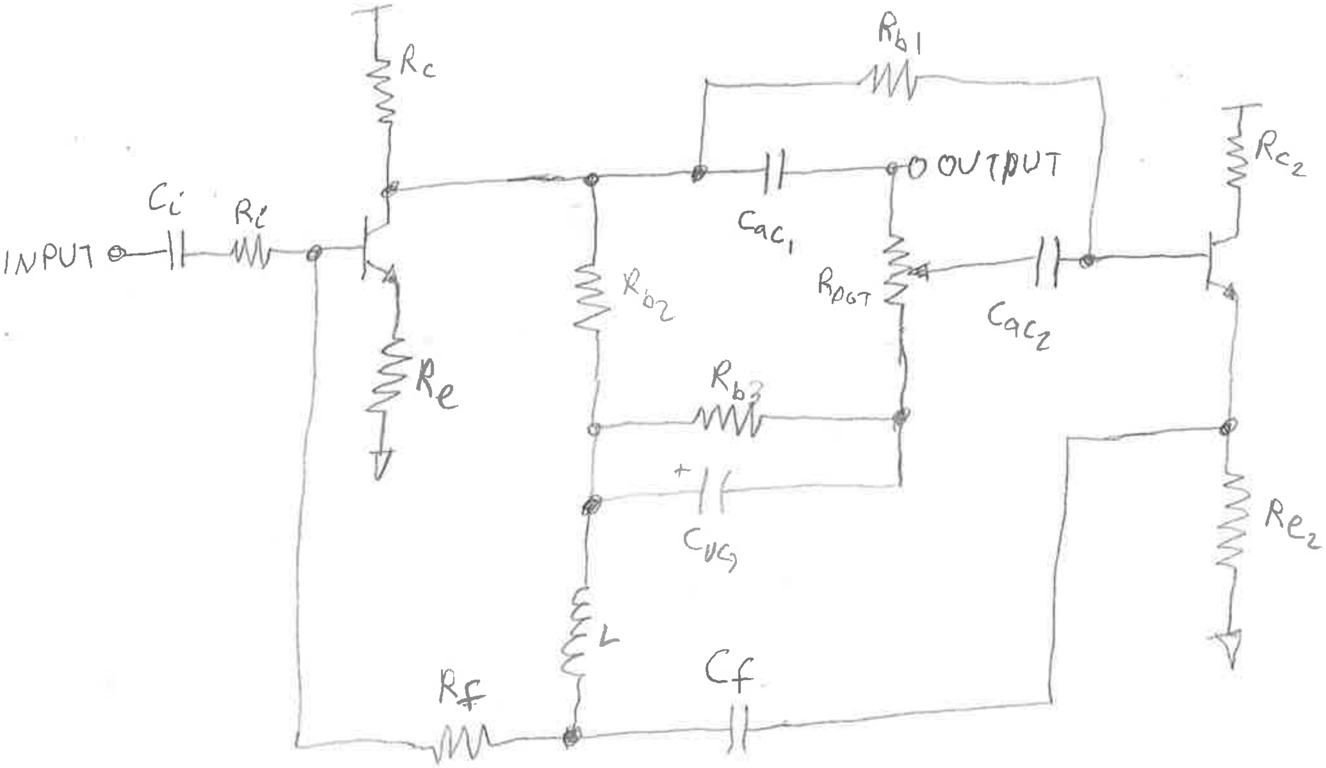
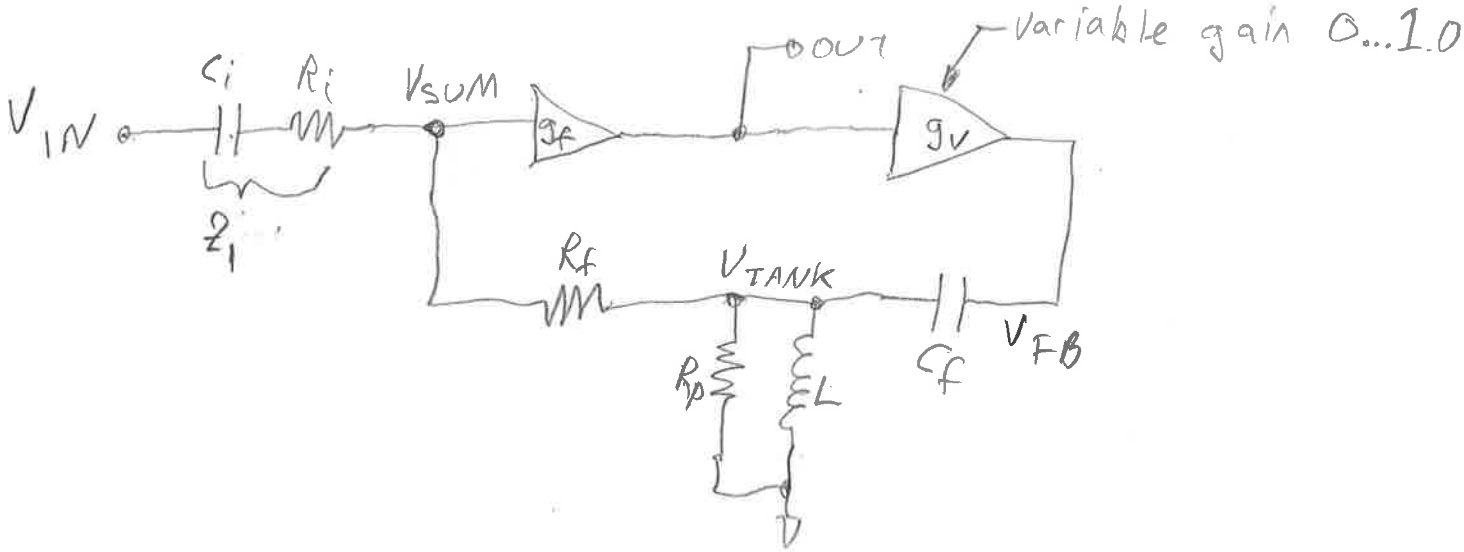
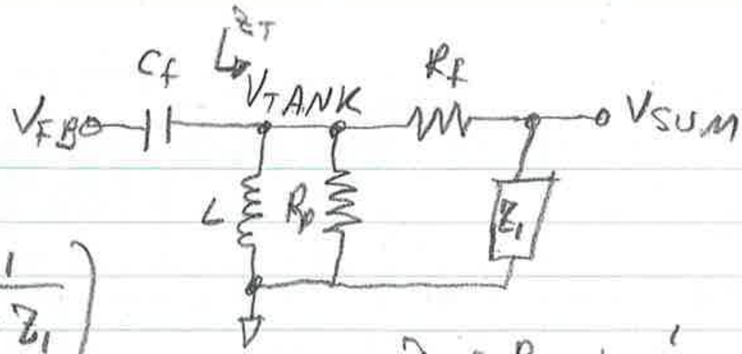


Synthesize to Linear amplifier model



SOLVE $\dfrac{V_{SUM}}{V_{FB}}$



$$\frac{V_{SUM}}{V_{FB}} = \left(\frac{V_{TANK}}{V_{FB}}\right)\left(\frac{Z_1}{R_f + Z_1}\right)$$

$$Z_i = R_i + \frac{1}{sC_i}$$

$$Z_T = \frac{1}{\frac{1}{sL} + \frac{1}{R_p} + \frac{1}{R_f + Z_1}} = \frac{sR_pL}{s\left(L + \frac{R_pL}{R_f + Z_1}\right) + R_p}$$

$$\frac{V_{TANK}}{V_{FB}} = \frac{Z_T}{Z_T + X_{C_f}} = \frac{Z_T}{Z_T + \frac{1}{sC_f}}$$

$$= \frac{\dfrac{sR_pL}{s\left(L + \frac{R_pL}{R_f+Z_1}\right) + R_p}}{\dfrac{sR_pL}{s\left(L+\frac{R_pL}{R_f+Z_1}\right) + R_p} + \dfrac{1}{sC_f}} = \frac{sR_pL}{sR_pL + \left(\frac{1}{sC_f}\right)\left(s\left(L + \frac{R_pL}{R_f+Z_1}\right) + R_p\right)}$$

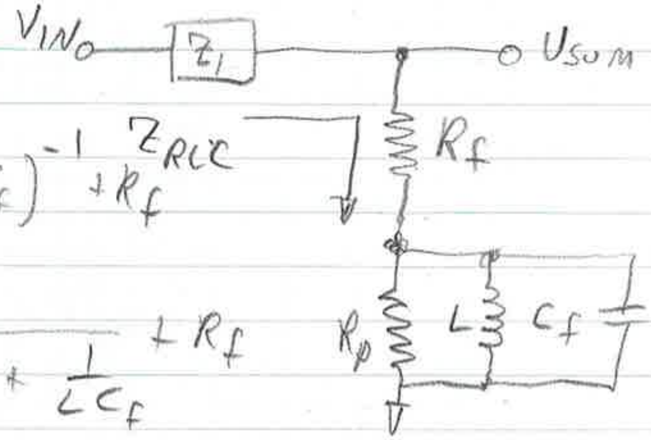
$$= \frac{s^2R_pC_fL}{s^2R_pC_fL + sL\left(1 + \frac{R_p}{R_f+Z_1}\right) + R_p}$$

$$= \frac{s^2}{s^2 + s\left(\frac{1}{R_pC_f}\right)\left(1 + \frac{R_p}{R_f + Z_1}\right) + \frac{1}{LC_f}}$$

$$\boxed{\frac{V_{SUM}}{V_{FB}} = \underbrace{\left(\frac{Z_1}{R_f + Z_1}\right)}_{\alpha_{HPF}}\left(\frac{\overbrace{\left\{s^2\right\}}^{N_{HPF}}}{\underbrace{\left\{s^2 + s\left(\frac{1}{R_pC_f}\right)\left(1 + \frac{R_p}{R_f+Z_1}\right) + \frac{1}{LC_f}\right\}}_{D}}\right)}$$

$$Z_1 = R_i + \frac{1}{sC_i}$$

SOLVE $\frac{V_{SUM}}{V_{IN}}$



$$Z_{RLC} = \left(\frac{1}{R_p} + \frac{1}{sL} + sC_f\right)^{-1} + R_f$$

$$= \frac{s\left(\frac{1}{C_f}\right)}{s^2 + s\left(\frac{1}{R_pC_f}\right) + \frac{1}{LC_f}} + R_f$$

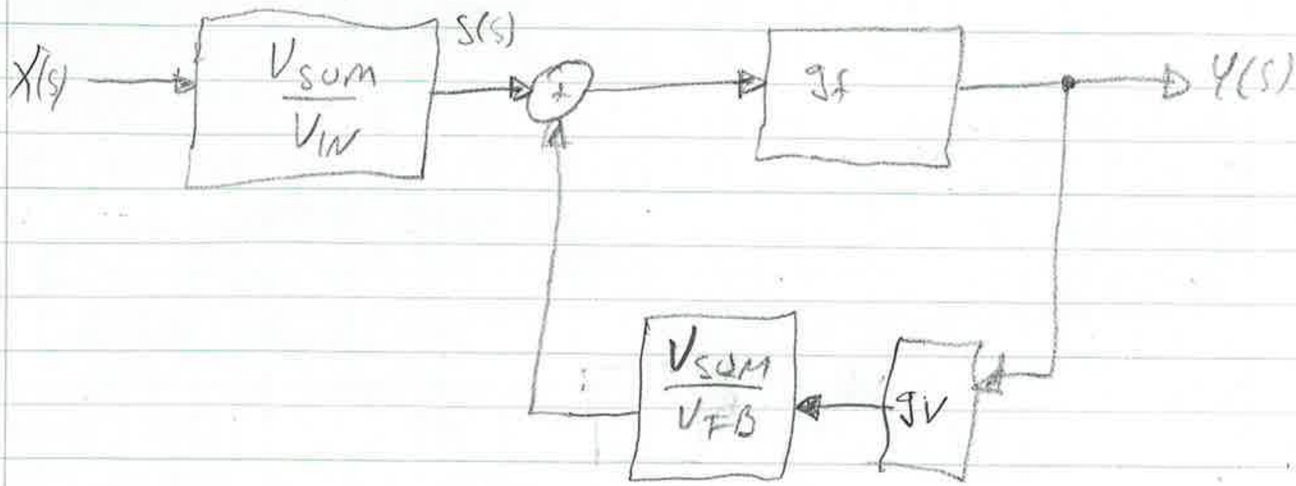
$$\frac{V_{SUM}}{V_{IN}} = \frac{Z_{RLC}}{Z_{RLC} + Z_1} = \frac{\left(\frac{s\left(\frac{1}{C_f}\right)}{s^2 + s\left(\frac{1}{R_pC_f}\right) + \frac{1}{LC_f}} + R_f\right)}{\frac{s\left(\frac{1}{C}\right)}{s^2 + s\left(\frac{1}{R_pC_f}\right) + \frac{1}{LC_f}} + (R_f + Z_1)}$$

$$= \frac{s\left(\frac{1}{C_f}\right) + s^2R_f + sR_f\left(\frac{1}{R_pC_f}\right) + \frac{R_f}{LC_f}}{s\left(\frac{1}{C_f}\right) + s^2(R_f + Z_1) + s\left(\frac{R_f + Z_1}{R_pC_f}\right) + \frac{R_f + Z_1}{LC_f}}$$

$$= \frac{\left(\frac{1}{R_f + Z_1}\right)\left(s^2R_f + s\left(\frac{1}{C_f} + \frac{R_f}{R_pC_f}\right) + \frac{R_f}{LC_f}\right)}{s^2 + s\left(\frac{1}{(R_f + Z_1)C_f} + \frac{1}{R_pC_f}\right) + \frac{1}{LC_f}}$$

$$\boxed{\frac{V_{SUM}}{V_{IN}} = \underbrace{\left(\frac{R_f}{R_f + Z_1}\right)}_{\alpha_{BPF}} \left[\frac{\overbrace{\left\{s^2 + s\left(\frac{1}{R_fC_f} + \frac{1}{R_pC_f}\right) + \frac{1}{LC_f}\right\}}^{N_{BPF}}}{\underbrace{\left\{s^2 + s\left(\frac{1}{R_pC_f}\right)\left(\frac{R_p}{R_f + Z_1} + 1\right) + \frac{1}{LC_f}\right\}}_{D}}\right]}$$

Synthesize Feedback Network Block Diagram



$$\frac{Y(s)}{S(s)} = \frac{g_f}{1 - g_f g_v \left(\frac{V_{sum}}{V_{FB}}\right)}$$

$$\frac{Y(s)}{X(s)} = \frac{\left(\frac{V_{sum}}{V_{IN}}\right) g_f}{1 - g_f g_v \left(\frac{V_{sum}}{V_{FB}}\right)} = \frac{\left(\frac{N_{BPF}}{D}\right) g_f a_{BPF}}{1 - g_f g_v a_{HPF} \frac{N_{HPF}}{D}}$$

EXPAND TERMS AND SIMPLIFY

$$= \frac{g_f a_{BPF} \cdot N_{BPF}}{D - g_f g_v \alpha_{HPF} N_{HPF}}$$

$$= \frac{g_f \alpha_{BPF} \cdot N_{BPF}}{s^2 + s\left(\frac{1}{R_p C_f}\right)\left(1 + \frac{R_p}{Z_1 + R_f}\right) + \frac{1}{LC_f} - s^2 g_f g_v \alpha_{HPF}}$$

$$= \frac{g_f \alpha_{BPF} N_{BPF}}{s^2(1 - g_f g_v \alpha_{HPF}) + s\left(\frac{1}{R_p C_f}\right)\left(1 + \frac{R_p}{Z_1 + R_f}\right) + \frac{1}{LC_f}}$$

$$\frac{Y(s)}{X(s)} = \underbrace{\left(\frac{\alpha_{BPF}\, g_f}{1-g_f g_v \alpha_{HPF}}\right)}\left(\frac{s^2 + s\left(\frac{1}{R_p C_f} + \frac{1}{R_f C_f}\right) + \frac{1}{LC_f}}{s^2 + s\left(\frac{1}{R_p C_f(1-g_f g_v \alpha_{HPF})}\right)\left(1+\frac{R_p}{Z_i + R_f}\right) + \frac{1}{LC_f\underbrace{(1-g_f g_v \alpha_{HPF})}_{1-g_f g_v \alpha_{HPF}}}}\right)$$

Expand α_{BPF} and α_{HPF}, collect terms
(See green sheet)

$$= \frac{s\left(\frac{g_f R_f}{R_f + R_i(1-g_f g_v)}\right)}{s + \frac{1}{C_i\left(\frac{R_f}{1-g_f g_v} + R_i\right)}}$$

← Variable gain term approaches 1.0 as $g_f \to -\infty$

This pole changes a subtle amount when $-\infty < g_f g_v < 0$

It approaches $\frac{1}{R_i C_i}$ as $g_f g_v \to -\infty$

RECALL:
$g_f < 0$, first stage inverting amplifier

SIMPLIFICATIONS
1) Assume $Z_i = R_i$ in biquad term
2) Assume 1PHP fixed pole @ $g_f g_v >> 1$ (heel pos)

$$\frac{Y(s)}{X(s)} \sim \left[\frac{s\left(\frac{g_f R_f}{R_f + R_i(1-g_f g_v)}\right)}{s + \frac{1}{R_i C_i}}\right]\left[\frac{s^2 + s\left(\frac{1}{R_p C_f} + \frac{1}{R_f C_f}\right) + \frac{1}{LC_f}}{s^2 + s\left[\frac{1}{R_p C_f G(g_v)}\right]\left[1 + \frac{R_p}{R_i + R_f}\right] + \frac{1}{LC_f\, G(g_v)}}\right]$$

$$G(g_v) = 1 - g_f g_v\left(\frac{R_i}{R_i + R_f}\right)$$

Expand 1-Pole HPF TERM

$$\alpha_{BPF} = \frac{R_f}{R_f + z_1} \qquad \alpha_{HPF} = \frac{z_1}{R_f + z_1}$$

$$\frac{g_f \dfrac{\alpha_{BPF}}{\alpha_{HPF}}}{\dfrac{1}{\alpha_{HPF}} - g_f g_v} = \frac{g_f \dfrac{R_f}{z_1}}{\dfrac{R_f + z_1}{z_1} - g_f g_v}$$

$$\frac{g_f R_f}{R_f + z_1 - g_f g_v z_1} = \boxed{\frac{g_f R_f}{R_f + z_1(1 - g_f g_v)}} \quad \text{Ⓐ}$$

$$\frac{g_f R_f}{= R_f + \left(R_i + \frac{1}{sC_i}\right)\underbrace{(1 - g_f g_v)}_{G}} = \frac{s g_f R_f C_i}{sC_i R_f + sC_i R_i G + G}$$

$$\frac{s g_f R_f C_i}{sC_i(R_f + R_i G) + G} = \frac{s \dfrac{g_f R_f C_i}{C_i(R_f + R_i G)}}{s + \dfrac{G}{C_i(R_f + R_i G)}}$$

$$\boxed{= \frac{s \dfrac{g_f R_f}{R_f + R_i(1 - g_f g_v)}}{s + \dfrac{1}{C_i\left(\dfrac{R_f}{(1 - g_f g_v)} + R_i\right)}}} \quad \text{Ⓑ}$$

Ⓐ Guitar output impedance could be considered by including this with z_1 before expanding

Ⓑ Response when source is low impedance.