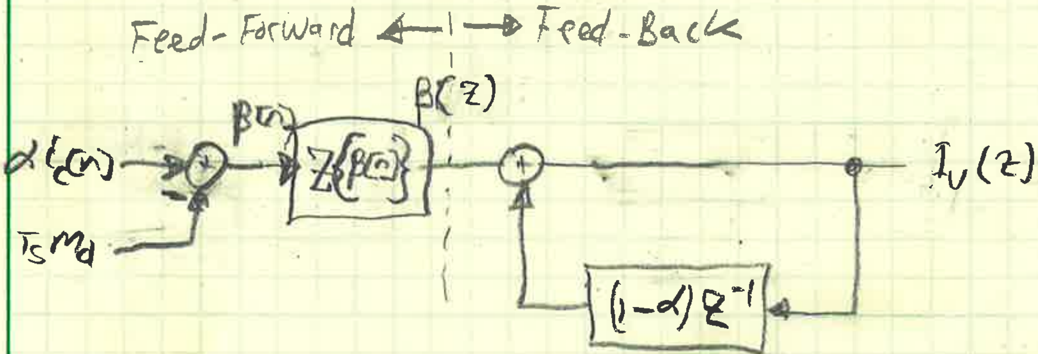
Z-transform of Peak current control

$$i_v[n] = (1-\alpha)\, i_v[n-1] + \underbrace{\alpha\, i_c[n] - T_s\, m_d}_{\beta[n]}$$

$$\alpha = \frac{m_c + m_d}{m_c + m_{cmp}}$$



For stability anaysis we only need to consider feedback,

$$H(z) = \frac{I_v(z)}{\beta(z)}$$

See Appendix sheet for approach eliminating the constant from transfer function, i.e., assert $I_c(z) = \beta(z)$ $(T_s(z) = m_d(z) = 0)$

If $I_c(z) = \beta(z)$, then

$$I_v(z) = (1-\alpha) I_v(z) z^{-1} + \alpha I_c(z)$$

$$I_v(z)\left(1 - (1-\alpha) z^{-1}\right) = \alpha I_c(z)$$

$$H(z) = \frac{I_v(z)}{I_c(z)} = \frac{\alpha}{1-(1-\alpha) z^{-1}}$$

Evaluate @ $\frac{\omega_s}{2}$

$$z^{-1} = e^{-j\omega \cdot T_s}\Big|_{\omega = \frac{\omega_s}{2} = \frac{2\pi}{2T_s} = \frac{\pi}{T_s}}$$

$$= e^{-j\pi} = \cos(\pi) - j\sin\pi$$

$$= -1 - 0$$

$$H(z)\Big|_{\frac{\omega_s}{2}} = \frac{\alpha}{1-(1-\alpha)\cdot(-1)}$$

$$= \frac{\alpha}{1-(\alpha-1)} = \frac{\alpha}{2-\alpha}$$

$$\boxed{\text{Unstable @ } \alpha \geq 2}$$

Recall

$$H(z)\Big|_{\frac{\omega_s}{2}} = H(-1) = \frac{\alpha}{2-\alpha}$$

Then perfect cancellation of peaking occurs at point

$$\frac{H(-1)}{H(0)} = 1 \text{; or } \underbrace{H(-1)} = H(0)$$

$$\frac{\alpha}{2-\alpha} = \frac{\alpha}{1-(1-\alpha)\underbrace{e^{-j2\pi\cdot 0\,T_s}}_{e^0=1}}$$

$$\frac{\alpha}{2-\alpha} = \frac{\alpha}{1-1+\alpha} = 1$$

$$\frac{\alpha}{2-\alpha} = 1, \quad \alpha = 2-\alpha$$

$$2\alpha = 2$$

$$\alpha = 1$$

Recall $\alpha = \frac{m_c + m_d}{m_c + m_{cmp}}$

(A) MIN STABILITY @

$$\alpha = 2, \quad \frac{m_c + m_d}{m_c + m_{cmp}} \geq 2, \quad m_c + m_d = 2(m_c + m_{cmp})$$

$$\frac{1}{2}m_c - m_c + \frac{1}{2}m_d = m_{cmp}$$

$$\boxed{m_{cmp} \geq \frac{m_d - m_c}{2}}$$

(B) No Peaking

$$\alpha = 1, \quad m_c + m_d = m_c + m_{cmp}$$

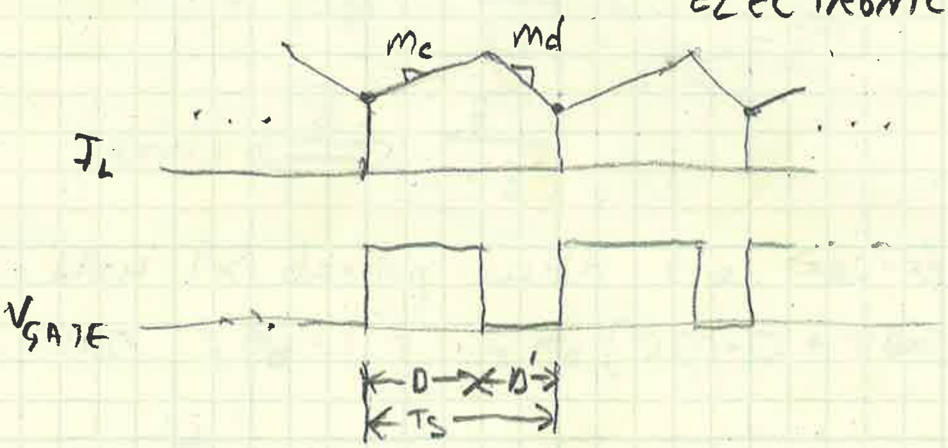
$$\boxed{m_{cmp} = m_d}$$

MIDDLEBROOK & TAN present peak current control stability in terms of duty cycle in following terms.

- Condition for stability

$$D'_{min} = \frac{0.5}{1 + \frac{M_c}{M_1}} \qquad \left\{ \begin{matrix} M_c = m_{cmp} \\ M_1 = m_c \end{matrix} \right\} \text{ Translating Notation} \qquad [1]$$

pg. 400, Eq. (11) IEEE TRANSACTIONS ON POWER ELECTRONICS, VOL 10, NO. 4 JULY 1995



When system is stable and reaches steady state the following is true

$$DT_s m_c - D' T_s m_d = 0$$

$$Dm_c = (1-D)\, m_d \,, \qquad Dm_c = m_d - D\, m_d$$

$$D(m_c + m_d) = m_d$$

$$D = \frac{m_d}{m_c + m_d}\,, \qquad D' = 1 - \frac{m_d}{m_c + m_d} = \frac{m_c}{m_c + m_d}$$

FROM [1],

$$\frac{m_c}{m_c + m_d} = \frac{0.5}{1 + \frac{m_{cmp}}{m_c}} = \frac{0.5\, m_c}{m_c + m_{cmp}}\,, \qquad \frac{2 m_c}{m_c + m_d} = \frac{m_c}{m_c + m_{cmp}}$$

$$m_c + m_{cmp} = \frac{m_c + m_d}{2}\,, \qquad 2m_c + 2m_{cmp} = m_c + m_d$$

$$2m_{cmp} = m_c - 2m_c + m_d = m_d - m_c$$

$$\boxed{\Rightarrow \quad m_{cmp} = \frac{m_d - m_c}{2}\,,} \text{ MATCHES (A), SHEET (2)}$$

Appendix: Support Assertion $\beta(z) = I_c(z)$

Function $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

$$u[n-1] + u[-n] = 1$$

$U(z) = \dfrac{1}{1-z^{-1}}$. By Time Reversal property,

$u[-n] = U(z^{-1})$

$$U(z^{-1}) = \frac{1}{1-z^{1}} = \frac{z^{-1}}{z^{-1}-1} = \frac{-z^{-1}}{1-z^{-1}}$$

$$u[n-1] \overset{Z}{\Longleftrightarrow} \frac{z^{-1}}{1-z^{-1}}$$

Now for dealing with the Constant,

$$c[n] = T_s m_d \cdot 1 = T_s m_d \left(u[n-1] + u[-n] \right)$$

$$c[n] \overset{Z}{\Longleftrightarrow} T_s m_d \left(U(z) z^{-1} + U(-z) \right)$$

$$= T_s m_d \left(\frac{z^{-1}}{1-z^{-1}} + \frac{-z^{-1}}{1-z^{-1}} \right) = T_s m_d \cdot (0)$$

$$= 0$$