1 Surrounding a Square

In the figure below, squares (rotated by $\pi/2$) are built iteratively from one to the next, beginning from the unit square, where each iteration builds a new layer around the inner area.

One posibble way to look at the process of creation is that the central column "bifurcates" and and a new taller one takes its place.

So for example, the central column $N = 3$ has height $2N - 1$ and bifurcates adding a clone. In its place, a column with $N + 1 = 4$ and height $2(N + 1) - 1$ is added. With this picture we can write that the area a is equal to,

$$a_N = \left[2\sum_{n=1}^{N}(2n-1)\right] - (2N-1)$$

As an example look at $N = 4$,

$$a_4 = 2\left[1+3+5+7\right] - 7 = 25$$

From the description of the creation process, at each new iteration, there are an additional $4N$ squares added. Use mathematical induction,

$$a_{N+1} = \left[2\sum_{n=1}^{N}(2n-1)\right] - (2N-1) + 4N$$

$$a_{N+1} = \left[2\sum_{n=1}^{N}(2n-1)\right] + 2(2N+1) - (2N+1)$$

$$a_{N+1} = \left[2\sum_{n=1}^{N}(2n-1)\right] + 2[2(N+1)-1] - [2(N+1)-1]$$

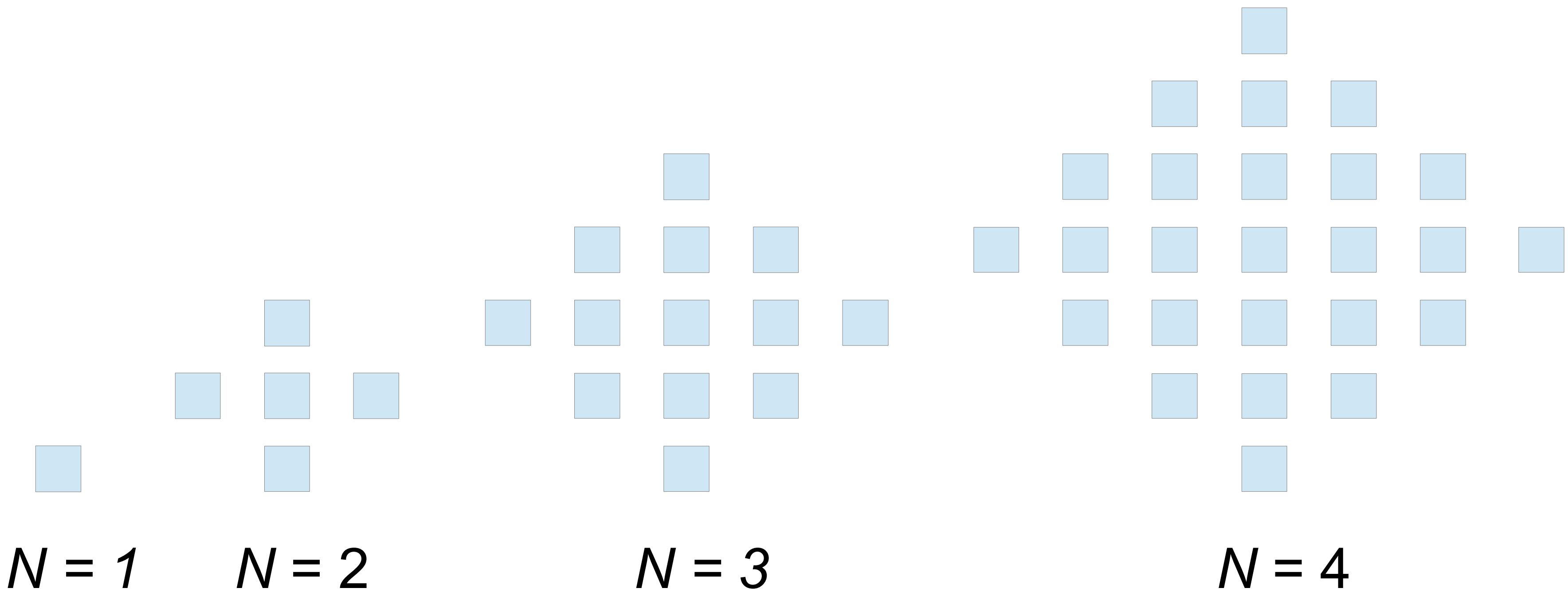
$$a_{N+1} = \left[2\sum_{n=1}^{N+1}(2n-1)\right] + 2[2(N+1)-1] - [2(N+1)-1]$$

This is our proof. We can make some further modifications by recognizing that the sum over odd integers is just N^2. Here is the refactored relation,

$$a_N = N^2 + (N-1)^2$$

From the second equation there is the recusion relation,

$$a_{N+1} = a_N + 4N$$



The iterative process of building bigger squares