PHƯƠNG PHÁP TÍNH – ĐA THỨC NỘI SUY

Đề bài: Cho hàm f(x) thỏa mãn:

x_i	0	1	2	3	4	5
$f(x_i)$	5	16	43	92	169	280

Tìm hàm nội suy của f(x), tính gần đúng: f(2.5), f(3.8)

Cách 1:

Ta có:

$$L_5(x) = 5\frac{(x-1)(x-2)(x-3)(x-4)(x-5)}{(-1)(-2)(-3)(-4)(-5)} + 16\frac{(x-0)(x-2)(x-3)(x-4)(x-5)}{1(-1)(-2)(-3)(-4)} + 43\frac{(x-0)(x-1)(x-3)(x-4)(x-5)}{2.1.(-1)(-2)(-3)} + 92\frac{(x-0)(x-1)(x-2)(x-4)(x-5)}{3.2.1.(-1)(-2)} + 169\frac{(x-0)(x-1)(x-2)(x-3)(x-5)}{4.3.2.1.(-1)} + 280\frac{(x-0)(x-1)(x-2)(x-3)(x-4)}{5.4.3.2.1}$$

$$= -\frac{1}{24}(x-1)(x-2)(x-3)(x-4)(x-5) + \frac{2}{3}x(x-2)(x-3)(x-4)(x-5) - \frac{43}{12}x(x-1)(x-3)(x-4)(x-5) + \frac{23}{3}x(x-1)(x-2)(x-4)(x-5) - \frac{169}{24}x(x-1)(x-2)(x-3)(x-5) + \frac{7}{3}x(x-1)(x-2)(x-3)(x-4)$$

Vậy:

f(2.5) = L_5(2.5) $= -\frac{1}{24}(2.5-1)(2.5-2)(2.5-3)(2.5-4)(2.5-5) + \frac{2}{3}2.5(2.5-2)(2.5-3)(2.5-4)(2.5-5) - \frac{43}{12}2.5(2.5-1)(2.5-3)(2.5-4)(2.5-5) + \frac{23}{3}2.5(2.5-1)(2.5-2)(2.5-4)(2.5-5) - \frac{169}{24}2.5(2.5-1)(2.5-2)(2.5-3)(2.5-5) + \frac{7}{3}2.5(2.5-1)(2.5-2)(2.5-3)(2.5-4) = 64.375$

f(3.8) = L_5(3.8) $= = \frac{1}{24}(3.8-1)(3.8-2)(3.8-3)(3.8-4)(3.8-5) + \frac{2}{3}3.8(3.8-2)(3.8-3)(3.8-4)(3.8-5) - \frac{43}{12}3.8(3.8-1)(3.8-3)(3.8-4)(3.8-5) + \frac{23}{3}3.8(3.8-1)(3.8-2)(3.8-4)(3.8-5) - \frac{169}{24}3.8(3.8-1)(3.8-2)(3.8-3)(3.8-5) + \frac{7}{3}3.8(3.8-1)(3.8-2)(3.8-3)(3.8-4) = 151.072$

Mã nguồn:

```
#include<stdio.h>
```

```
double Lagrange(int x[], int y[], double value) {
    double result = 0;
    for (int i = 0; i < 5; i++) {
        double temp = y[i];
        for (int j = 0; j < 5; j++) {
            if (i != j) {
                temp *= (value - x[j]) / (x[i] - x[j]);
            }
        }
        result += temp;
    }
    return result;
}

int main() {

    int x[6] = {0, 1, 2, 3, 4, 5};
    int y[6] = {5, 16, 43, 92, 169, 280};

    double result1 = Lagrange(x, y, 2.5);
    double result2 = Lagrange(x, y, 3.8);

    printf("f(2.5) = %lf\nf(3.8) = %lf\n",result1, result2);

    return 0;
}
```

Kết quả chương trình:

Cách 2:

Ta có:

W(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)

W(0) = (0-1)(0-2)(0-3)(0-4)(0-5) = -120

W(1) = 1(1-2)(1-3)(1-4)(1-5) = 24

W(2) = 2(2-1)(2-3)(2-4)(2-5) = -12

W(3) = 3(3-1)(3-2)(3-4)(3-5) = 12

W(4) = 4(4-1)(4-2)(4-3)(4-5) = -24

W(5) = 5(5-1)(5-2)(5-3)(5-4) = 120

$L_5(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)(-\frac{1}{24x} + \frac{2}{(x-1).3} - \frac{43}{(x-2).12} + \frac{23}{(x-3).3} + \frac{169}{(x-4).(-24)} + \frac{7}{(x-5).3})$

Vậy:

f(2.5) $= 2.5(2.5-1)(2.5-2)(2.5-3)(2.5-4)(2.5-5)(-\frac{1}{24.2.5} + \frac{2}{(2.5-1).3} - \frac{43}{(2.5-2).12} + \frac{23}{(2.5-3).3} - \frac{169}{(2.5-4).24} + \frac{7}{(2.5-5).3}) = 64.375$

f(3.8) $= 3.8(3.8-1)(3.8-2)(3.8-3)(3.8-4)(3.8-5)(-\frac{1}{243.8} + \frac{2}{(3.8-1).3} - \frac{43}{(3.8-2).12} + \frac{23}{(3.8-3).3} + \frac{169}{(3.8-4).(-24)} + \frac{7}{(3.8-5).3}) = 151.072$

Mã nguồn:

```
#include<stdio.h>

double Wp(int x[], int val) {
    double result = 1;
    for (int i = 0; i < 5; i++) {
        if (val != x[i]) {
            result *= (val - x[i]);
        }
    }
    return result;
}

double Lagrange(int x[], int y[], double value) {
```

```
    double result = 0, w =1;

    for (int i = 0; i < 5; i++) {
        w = w*(value - x[i]);
    }

    for (int i = 0; i < 5; i++) {
        double temp = y[i] / ((value - x[i]) * Wp(x, x[i]));
        result += temp;
    }
    return w * result;
}

int main() {

    int x[6] = {0, 1, 2, 3, 4, 5};
    int y[6] = {5, 16, 43, 92, 169, 280};

    double result1 = Lagrange(x, y, 2.5);
    double result2 = Lagrange(x, y, 3.8);

    printf("Cach 2: \n");
    printf("f(2.5) = %lf\nf(3.8) = %lf\n",result1, result2);

    return 0;
}
```

Kết quả chương trình:

```
Cach 2:
f(2.5) = 64.375000
f(3.8) = 151.072000
Press any key to continue . . .
```

Đề bài: Cho hàm f(x) thỏa mãn:

x_i	0	1	2	3	4	5
$f(x_i)$	20	16.2	7.2	0.2	7.2	45

Tìm hàm nội suy của f(x), tính gần đúng: f(1.5), f(3.1)

Cách 1:

Ta có:

$$L_5(x) = 20\frac{(x-1)(x-2)(x-3)(x-4)(x-5)}{(-1)(-2)(-3)(-4)(-5)} + 16.2\frac{(x-0)(x-2)(x-3)(x-4)(x-5)}{1(-1)(-2)(-3)(-4)} + 7.2\frac{(x-0)(x-1)(x-3)(x-4)(x-5)}{2.1.(-1)(-2)(-3)} + 0.2\frac{(x-0)(x-1)(x-2)(x-4)(x-5)}{3.2.1.(-1)(-2)} + 7.2\frac{(x-0)(x-1)(x-2)(x-3)(x-5)}{4.3.2.1.(-1)} + 45\frac{(x-0)(x-1)(x-2)(x-3)(x-4)}{5.4.3.2.1}$$

$$= -\frac{1}{6}(x-1)(x-2)(x-3)(x-4)(x-5) + \frac{27}{40}x(x-2)(x-3)(x-4)(x-5) - \frac{3}{5}x(x-1)(x-3)(x-4)(x-5) + \frac{1}{60}x(x-1)(x-2)(x-4)(x-5) - \frac{3}{10}x(x-1)(x-2)(x-3)(x-5) + \frac{3}{8}x(x-1)(x-2)(x-3)(x-4)$$

Vậy:

f(1.5) $= -\frac{1}{6}(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5) + \frac{27}{40}1.5(1.5-2)(1.5-3)(1.5-4)(1.5-5) - \frac{3}{5}1.5(1.5-1)(1.5-3)(1.5-4)(1.5-5) + \frac{1}{60}1.5(1.5-1)(1.5-2)(1.5-4)(1.5-5) - \frac{3}{10}1.5(1.5-1)(1.5-2)(1.5-3)(1.5-5) + \frac{3}{8}1.5(1.5-1)(1.5-2)(1.5-3)(1.5-4) = 12.0125$

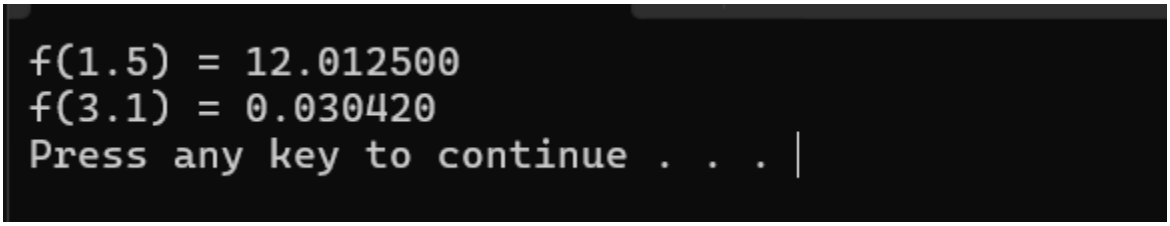
f(3.1) $= = -\frac{1}{6}(3.1-1)(3.1-2)(3.1-3)(3.1-4)(3.1-5) + \frac{27}{40}3.1(3.1-2)(3.1-3)(3.1-4)(3.1-5) - \frac{3}{5}3.1(3.1-1)(3.1-3)(3.1-4)(3.1-5) + \frac{1}{60}3.1(3.1-1)(3.1-2)(3.1-4)(3.1-5) - \frac{3}{10}3.1(3.1-1)(3.1-2)(3.1-3)(3.1-5) + \frac{3}{8}3.1(3.1-1)(3.1-2)(3.1-3)(3.1-4) = 0.030420$

Mã nguồn:

```
#include<stdio.h>

double Lagrange(int x[], double y[], double value) {
    double result = 0;
    for (int i = 0; i < 5; i++) {
        double temp = y[i];
```

```
        for (int j = 0; j < 5; j++) {
            if (i != j) {
                temp *= (value - x[j]) / (x[i] - x[j]);
            }
        }
        result += temp;
    }
    return result;
}

int main() {

    int x[6] = {0, 1, 2, 3, 4, 5};
    double y[6] = {20,  16.2, 7.2, 0.2, 7.2, 45};

    double result1 = Lagrange(x, y, 1.5);
    double result2 = Lagrange(x, y, 3.1);

    printf("f(1.5) = %lf\nf(6) = %lf\n",result1, result2);

    return 0;
}
```

Kết quả chương trình:



Cách 2:

Ta có:

W(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)

W(0) = (0-1)(0-2)(0-3)(0-4)(0-5) = -120

W(1) = 1(1-2)(1-3)(1-4)(1-5) = 24

W(2) = 2(2-1)(2-3)(2-4)(2-5) = -12

W(3) = 3(3-1)(3-2)(3-4)(3-5) = 12

W(4) = 4(4-1)(4-2)(4-3)(4-5) = -24

W(5) = 5(5-1)(5-2)(5-3)(5-4) = 120

$L_5(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)(\frac{1}{-6x} + \frac{27}{(x-1).40} + \frac{-3}{(x-2).5} + \frac{1}{(x-3).60} + \frac{3}{(x-4).10} + \frac{3}{(x-5).8})$

Vậy:

f(1.5) $= 1.5(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)(\frac{1}{-6.1,5} + \frac{27}{(1,5-1).40} + \frac{-3}{(1,5-2).5} + \frac{1}{(1,5-3).60} + \frac{3}{(1,5-4).10} + \frac{3}{(1,5-5).8}) = 12.0125$

f(3.1) $= 3.1(3.1-1)(3.1-2)(3.1-3)(3.1-4)(3.1-5)(\frac{1}{-6.3,1} + \frac{27}{(3,1-1).40} + \frac{-3}{(3,1-2).5} + \frac{1}{(3,1-3).60} + \frac{3}{(3,1-4).10} + \frac{3}{(3,1-5).8}) = 0.030420$

Mã nguồn:

```
#include<stdio.h>

double Wp(int x[], int val) {
    double result = 1;
    for (int i = 0; i < 5; i++) {
        if (val != x[i]) {
            result *= (val - x[i]);
        }
    }
    return result;
}

double Lagrange(int x[], double y[], double value) {
    double result = 0, w =1;

    for (int i = 0; i < 5; i++) {
        w = w*(value - x[i]);
    }

    for (int i = 0; i < 5; i++) {
```

```
        double temp = y[i] / ((value - x[i]) * Wp(x, x[i]));
        result += temp;
    }
    return w * result;
}

int main() {

    int x[6] = {0, 1, 2, 3, 4, 5};
    double y[6] = {20,  16.2, 7.2, 0.2, 7.2, 45};

    double result1 = Lagrange(x, y, 1.5);
    double result2 = Lagrange(x, y, 3.1);

    printf("Cach 2: \n");
    printf("f(2.5) = %lf\nf(3.1) = %lf\n",result1, result2);

    return 0;
}
```

Kết quả chương trình:

```
Cach 2:
f(2.5) = 12.012500
f(3.1) = 0.030420
Press any key to continue . . .
```

----------Hết----------