Performance comparison between Sliding Mode Controller SMC and Proportional-Integral-Derivative PID controller for a highly nonlinear two-wheeled balancing robot

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Performance Comparison between Sliding Mode Controller SMC and Proportional-Integral-Derivative PID Controller for a Highly Nonlinear Two-wheeled Balancing Robot

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Abstract – **The research on two-wheels balancing robot has gained momentum due to their functionality and reliability when completing certain tasks. This paper presents investigations into the performance comparison of Sliding Mode Controller (SMC) and Proportional-Integral-Derivative (PID) controller for a highly nonlinear 2-wheels balancing robot. The mathematical model of 2-wheels balancing robot that is highly nonlinear is derived. The final model is then represented in state-space form and the system suffers from mismatched condition. Two system responses namely the robot position and robot angular position are obtained. The performances of the SMC and PID controllers are examined in terms of input tracking and disturbances rejection capability. Simulation results of the responses of the nonlinear 2–wheels balancing robot are presented in time domain. A comparative assessment of both control schemes to the system performance is analyzed and discussed.**

Keywords - SMC, PID, balancing robot.

1 INTRODUCTION

The research on two-wheeled balancing robot has gained momentum over the last decade due to the nonlinear and unstable dynamics system. Various control strategies had been proposed by numerous researchers to control the two-wheeled balancing robot such that the robot able to balance itself. Two wheels balancing robot is a good platform for researchers to investigate the efficiency of various controllers in control system. The research on two wheels balancing robot is based on inverted pendulum model. Thus, a two wheels balancing robot needs a good controller to control itself in upright position without the needs from outside. Motion of two wheels balancing robot is governed by under-actuated configuration, i.e., the number of control inputs is less than the number of degrees of freedom to be stabilized [1], which makes it difficult to apply the conventional robotics approach for controlling the systems. Due to these reasons, increasing effort has been made towards control designs that guarantee stability and robustness for mobile wheeled inverted pendulums. Although two wheels balancing robot are intrinsically nonlinear and their dynamics will be described by nonlinear differential equations, it is often possible to obtain a linearized model of the system. If the system operates around an operating point and the signals involved are small signals, a linear model that approximates the nonlinear system in the region of operation can be obtained. Several techniques for the design of controllers and analysis techniques for linear systems were applied. In [2], motion control was proposed using linear state-space model. In [3], dynamics was derived using a Newtonian approach and the control was design by the equations linearized around an operating point. In [4], the dynamic equations were studied, with the balancing robot pitch and the rotation angles of the two wheels as the variables of interest, and a linear controller was designed for stabilization under the consider of its robustness in [5]. In [6], a linear stabilizing controller was derived by a planar model without considering vehicle yaw. The above control laws are designed on the linearized dynamics which only exhibits desirable behavior around the operating point, and do not have global applicability. In [7], the exact dynamics of two wheels inverted pendulum was investigated, and linear feedback control was developed on the dynamic model. In [8], a two-level velocity controller via partial feedback linearized and a stabilizing position controller were derived; however, the controller design is not robust with respect to parameter uncertainties. In [9], a controller using sliding mode approach was proposed to ensure robustness versus parameter uncertainties for controlling both the position and the orientation of the balancing robot. The mathematical model is established through a modelling process where the system is identified based on the conservation laws and property laws. This process is crucial since a controller is design solely based on this mathematical model. Thus, an accurate equation must be derived in order for the controller to response accordingly. This paper presents investigations of performance comparison between conventional control PID and modern control SMC schemes for a two wheels balancing robot. The mathematical model of the two wheels balancing robot system is presented in differential equation form. The dynamic model of the system with the permanent magnet DC motors dynamic included is derived based on [3] and [9]. Performances of both control strategy with respect to balancing robot outputs angular position θ and linear position x are examined. Comparative assessment of both control schemes to the two balancing robot system performance is analyzed and discussed.

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2 PROBLEM FORMULATION

Modeling is the process of identifying the principal physical dynamic effects to be considered in analyzing a system, writing the differential and algebraic equations from the conservative laws and property laws of the relevant discipline, and reducing the equations to a convenient differential equation model. This section provides a description on the modeling of the two wheels balancing robot, as a basis of a simulation environment for development and assessment of both control schemes. The robot with its three degrees of freedom is able to linearly move which is characterized by position x, able to rotate around the y-axis (yaw) with associated angle δ and able to rotate around z-axis (pitch) where the movement is described by angle θ. List of parameters for the two wheels balancing robot are shown in Table 1. These parameters are based on the project conducted by Ooi (2003) as stated by [9].

Tab. 1: List of Parameters of two-wheels balancing robot based on [9]

Symbols	Parameters	Values
D	distance between contact patches of the wheels	0.2m
g	gravitational force	9.81 m.s^{-2}
Jp	chassis's inertia	0.0041 kg.m^2
Jp	chassis's inertia during rotation	0.00018 kg.m^2
Jw	wheel's inertia	0.000039 kg.m^2
ke	back emf constant	0.006087 Vs/rad
km	motor torque constant	0.006123 Nm/A
l	distance between center of the wheels and the robot's CG	0.07 m
Mp	body's mass	1.13 kg
Mw	wheel's mass	0.03 kg
R	nominal terminal resistance	3 •
r	wheel's radius	0.051 m

The inputs of the system are the voltages V_{aR} and V_{aL} which both are applied to the two motors which located on right side and left side of the robot as shown in Figure 1. In order to obtain the dynamic model of the balancing robot some assumptions and limitations are introduced:

- Motor inductance and friction on the motor armature is neglected
- The wheels of the robot will always stay in contact with the ground
- There is no slip at the wheels
- Cornering forces are also negligible

Figure 2 shows a free body diagram of the balancing robot which contributed to the nonlinear dynamic equations of the system. Equation (1) represents linear acceleration $\ddot{x}$ in x direction, equation (2) represents angular acceleration $\ddot{\theta}$ about y-axis and equation (3) represents angular acceleration $\ddot{\delta}$ about z-axis. It can be seen from (1), (2), and (3), all nonlinear terms, sine and cosine present in the equations. All these nonlinear equations are used to design the proposed controllers which will be described in section 3.

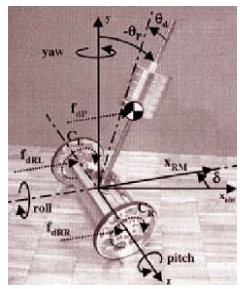

Fig. 1: A mobile balancing robot (Grasser et al., 2002)

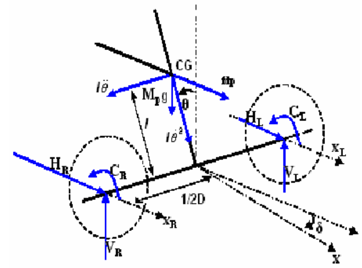

Fig. 2: Free body diagram of balancing robot

$$\ddot{x} = -\frac{2k_e k_m}{\alpha r R\beta}\left[\frac{1}{r}+\frac{M_p l\cos\theta}{\gamma}\right]\dot{x} + \frac{M_p^2 g l^2 \sin\theta\cos\theta}{\alpha\gamma\beta\theta}\theta + \frac{k_m}{\alpha R\beta}\left[\frac{1}{r}+\frac{M_p l\cos\theta}{\gamma}\right]V_{aR} + \frac{k_m}{\alpha R\beta}\left[\frac{1}{r}+\frac{M_p l\cos\theta}{\gamma}\right]V_{aL} + \frac{1}{\alpha\beta}f_{drR} + \frac{1}{\alpha\beta}f_{drL} + \frac{1}{\alpha\beta}\left[1+\frac{M_p l^2\cos^2\theta}{\gamma}\right]f_{dp} + \frac{M_p l\dot{\theta}^2\sin\theta}{\alpha\beta}. \quad (1)$$

$$\ddot{\theta} = \frac{2k_e k_m}{\gamma R\beta}\left[1+\frac{M_p l\cos\theta}{\alpha r}\right]\dot{x} + \frac{M_p g l\ \sin\theta}{\gamma\beta\theta}\theta - \frac{k_m}{\gamma R\beta}\left[1+\frac{M_p l\cos\theta}{\alpha r}\right]V_{aR} - \frac{k_m}{\gamma R\beta}\left[1+\frac{M_p l\cos\theta}{\alpha r}\right]V_{aL} - \frac{M_p l\cos\theta}{\alpha\gamma\beta}f_{drR} + \frac{M_p l\cos\theta}{\alpha\gamma\beta}f_{drL} + \frac{l\cos\theta}{\gamma\beta}\left[1-\frac{M_p}{\alpha}\right]f_{dp} - \frac{M_p^2 l^2\dot{\theta}^2\sin\theta\cos\theta}{\alpha\gamma\beta}. \quad (2)$$

$$\ddot{\delta} = -\frac{k_m D}{2J_{p\delta} rR} V_{aR} + \frac{k_m D}{2J_{p\delta} rR} V_{aL} - \frac{D}{2J_{p\delta}} f_{drR} + \frac{D}{2J_{p\delta}} f_{drL}. \quad (3)$$

The symbols of α, β, and γ in equations (1), (2), and (3) are defined as in equation (4), (5), and (6):

$$\alpha = 2M_w + \frac{2J_w}{r^2} + M_p \quad (4)$$

$$\beta = \frac{\alpha\gamma - M_p^2 l^2 \cos^2\theta}{\alpha\gamma} \quad (5)$$

$$\gamma = J_p + M_p l^2 \quad (6)$$

In order to make the system unstable, the angular position θ of the two-wheels balancing robot is set initially 0.6 radians. Hence, controller must be design to ensure the angular position, θ and linear position, x of the balancing robot to be at zero.

3 PROBLEM SOLUTION

In this section, two control schemes (SMC and PID) are proposed and described in detail. Furthermore, the following design requirements have been made to examine the performance of both control strategies.

- The system overshoot (*%OS*) of robot position, *x* is to be at most 30%.
- The Rise time (T_r) of robot position, *x* less than 5 s.
- The settling time (T_s) of robot position, *x* and robot angle θ is to be less than 5 seconds.
- Steady-state error is within 2% of the final value.

3.1 SMC Controller

SMC is a method in modern control theory that uses state-space approach to analyze such a system. Using state-space methods it is relatively simple to work with a multi-output system [8].The typical structure of a sliding mode controller (SMC) is composed of a nominal part and additional terms to deal with model uncertainty. The way SMC deals with uncertainty is to drive the plants state trajectory onto a sliding surface and maintain the error trajectory on this surface for all subsequent times. The advantage of SMC is that the controlled system becomes insensitive to system disturbances. The sliding surface is defined such that the state tracking error converges to zero with input reference. With the perspective to achieve zero steady state error, Cao and Xu (2001) and Sam et al. (2002) have proposed the proportional integral sliding mode control (PISMC) in their studies [3]. The proportional factor in this controller gives more freedom in selecting some parameters matrices such as parameters shown in Table 2 that will make the output response faster and the stability condition to be more easily satisfied. The proportional integral sliding surface equation can be represented as (7).

Tab. 2: Sliding mode controller parameters

Symbols	Values
K	[-331.662 -190.18 -400.42 -2.083]
S	[-331.662 -200.18 -350.42 -7.083]
δ_c	0.01
ρ	0.001

$$\sigma(t) = Sx(t) - \int_0^t [SA + SBK] x(\tau) d\tau \quad (7)$$

where $x \in \Re^n$ is a vector of measureable states, $A \in \Re^{nxn}$, $B \in \Re^m$ and $S \in \Re^{mxn}$ are constant matrices. $K \in \Re^{mxn}$ is a constant matrix such that the matrix (A+BK) is asymptotically stable and has the stable eigenvalues. The system can be stabilized using full state feedback. The schematic of this type of control system is shown in Figure 4.

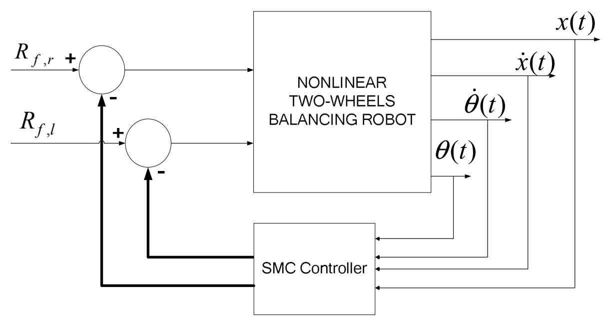

Fig. 3: Block diagram of SMC controller with balancing robot

The control law should be designed such that the reaching condition $\sigma(t)\dot{\sigma}(t) < 0$ is satisfied [10]. This criterion should be fulfilled to ensure that the state will move toward and reach the chosen sliding surface. The equivalent control method will be used for finding equations of ideal sliding mode. In this technique a time derivative of the sliding surface $\sigma(t)$ along the system trajectory is set equal to zero, and the resulting algebraic system is solved for the equivalent control. The control law that will be implemented into this system is presented as (8).

$$u(t) = u_{eq}(t) + \Delta u(t) \quad (8)$$

where $u_{eq}(t)$ is the equivalent sliding mode control and continuous function $\Delta u(t)$ is added to satisfy reaching condition. $u_{eq}(t)$ and $\Delta u(t)$ can be represented as (9) and (10) respectively.

$$u_{eq}(t) = Kx(t) \quad (9)$$

$$\Delta u(t) = -(SB)^{-1}\frac{\rho\sigma(t)}{|\sigma(t)| + \delta_c} \tag{10}$$

where δ_c is the boundary layer thickness and ρ is a design parameter. For this study, the values for controller parameters are tabulated in Table 2. All these parameters must be substituted into (9) and (10).

3.2 PID Controller

PID stands for Proportional-Integral-Derivative. This is a type of feedback controller whose output, a control variable (CV), is generally based on the error (e) between defined set point (SP) and some measured process variable (PV). Each element of the PID controller refers to a particular action taken on the error. In order to demonstrate the performance of the PID controller in locating the balancing robot to its desired position and angle, the collocated sensor signal of the position of the robot about roll axis, x*(s)* and angular position of the robot about yaw axis $\theta(s)$ are fed back and compared to the reference position, $x_f(s)$ and angle $\theta_f(s)$ respectively. Initially, the angular position of the robot which is position about pitch axis is set 50 degrees or 0.8727 radians. In this study, two PID controllers are required to control the position on the roll axis and the angular position about the yaw axis. The position and angular position errors are regulated through the proportional, integral and derivative gain for each PID. Block diagram of the PID controller is shown in Figure 3, where $u_1(s)$ and $u_2(s)$ represent the applied voltage at the right motor and left motor respectively. Both of the inputs of the balancing robot are limited to 20volts to -20volts. The control signal $u_1(s)$ and $u_2(s)$ in Figure 3 can be represented as in equations (11) and (12) respectively:

$$u_{PID}(s)_{pos} = -[K_{P1} + \frac{K_{I1}}{s} + K_{D1}s][r(s) - r_f(s)] \tag{11}$$

$$u_{PID}(s)_{ang} = -[K_{P2} + \frac{K_{I2}}{s} + K_{D2}s][\theta(s) - \theta_f(s)] \tag{12}$$

where s is the Laplace variable. Hence the closed-loop transfer function is obtained as in equation (13) and (14). In this paper, the Ziegler-Nichols approach is utilized to design both PID controllers.

$$\frac{r(s)}{r_f(s)} = \frac{\left(K_{P1} + K_{D1}s + \frac{K_{I1}}{s}\right)G(s)}{1 + \left(K_{P1} + K_{D1}s + \frac{K_{I1}}{s}\right)G(s)} \tag{13}$$

$$\frac{\theta(s)}{\theta_f(s)} = \frac{\left(K_{P2} + K_{D2}s + \frac{K_{I2}}{s}\right)G(s)}{1 + \left(K_{P2} + K_{D2}s + \frac{K_{I2}}{s}\right)G(s)} \tag{14}$$

Analyses the tuning process of the proportional, integral and derivative gains using Ziegler-Nichols technique shows that the optimum response of PID controller for controlling linear position is achieved by setting $K_{P1} = -8$, $K_{I1} = -0.921$ and $K_{D1} = -6$, while for controlling angular position, $K_{P2} = -63$, $K_{I2} = -60$ and $K_{D2} = -11$. All the PID1 and PID2 controller parameters must be tuned simultaneously to achieve the best responses as desired.

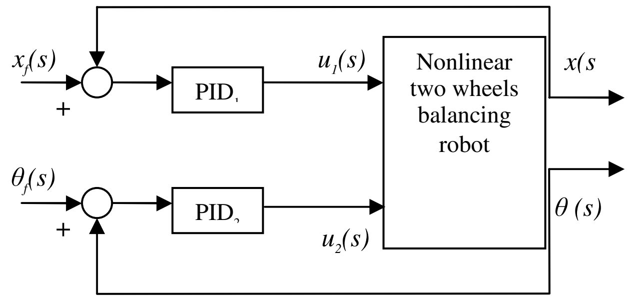

Fig. 3 Block diagram of PID controller

4 RESULTS AND ANALYSIS

In this section, the simulation results of the proposed controller, which is performed on the model of a two wheels balancing robot are presented. Comparative assessment of both control strategies to the system performance are also discussed in details in this section. Two wheels balancing robot systems with SMC and PID controller block diagram produced two responses, angular position θ and linear position x. As stated earlier, the initial value of the angular position θ of the balancing robot was set to 0.5 radians. It means that the initial condition of the robot is very unstable. Figure 5 shows the comparison of the balancing robot linear position response between SMC and PID controller graphically. In this figure, the response for the linear position of the robot with PID controller is represented by straight line or blue color line and the response for the linear position of the robot with SMC controller is represented by dotted line or red color line. Figure 5 shows that both of the controllers are capable to control the linear position of the nonlinear two wheels balancing robot with different performances.

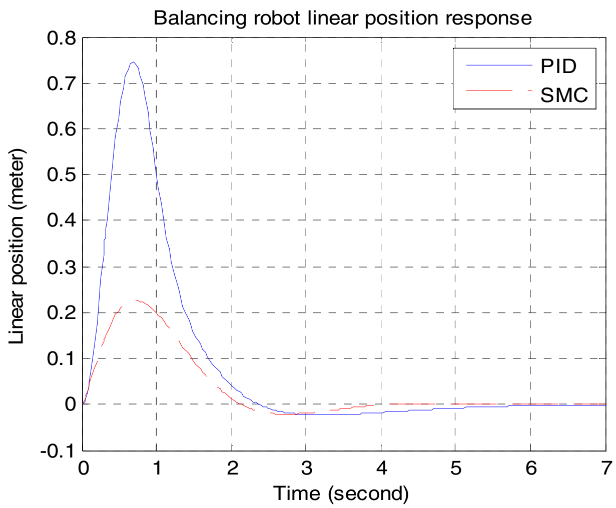

Fig. 5: Two-wheels balancing robot linear position response

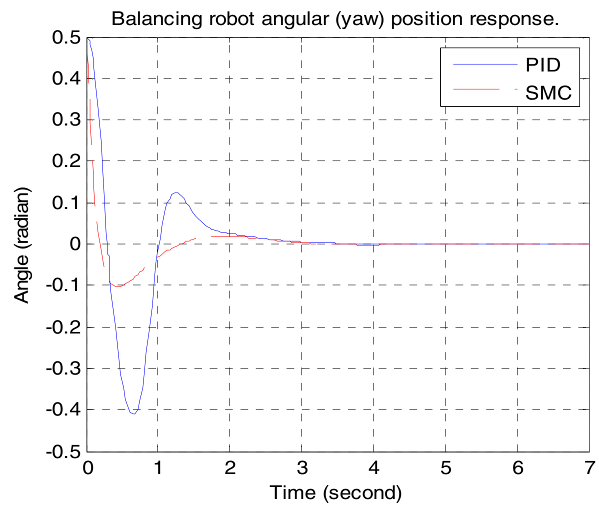

Fig. 6 Two-Wheels Balancing Robot Angular Position Response

Table 3 shows the summary of the performance characteristics of the balancing robot linear position between SMC and PID controller quantitatively. Based on the data tabulated in Table 3, SMC has the fastest settling time of 2.95 seconds while PID has the slowest settling time of 3.81 seconds. An extra of 0.86 seconds is required for the PID controller balancing robot to balance itself. Similarly, for the maximum overshoot, SMC controller has the best overshoot which is the lowest overshoot between two controllers. The maximum displacement of the balancing robot when SMC control signal applied to the system is 0.23 meters while

Tab. 3: Summary of performance characteristics of the balancing robot linear position, x between SMC and PID

Time response specification	SMC	PID
Rise Time	0.43 sec	0.41sec
Steady state error	0.00	0.00
Settling Time	3.20 sec	3.81 sec
Maximum overshoot	0.29m	0.74 m

maximum displacement of the balancing robot when PID control signal applied to the system is 0.74 meters. A distance of minimum 0.51 meters is required for the PID controller balancing robot to balance itself. Despite the large initial values for the displacement, the proposed PID controller is able to bring itself to the vertical position. In term of the rise time, balancing robot with PID controller has the fastest rise time 0.41 seconds while balancing robot with SMC controller needs an extra time of 0.02 seconds to rise from 10% to the 90% of the final value. In term of steady state error, both of the controllers had shown very excellence performance by giving zero error at time 6 seconds and more. The responses of the balancing robot linear position have acceptable overshoot and undershoot. Figure 6 shows the balancing robot with SMC and PID controller angular position responses. It shows that the result has got similar pattern and not much different. The initial value of the balancing robot angular position is 0.5 radians. The robot needs to balance itself by eliminating the angular position so that the body of the robot remains vertically straight in upright position. Figure 6 shows that both of the SMC and PID controllers are capable of controlling the nonlinear unstable balancing robot. Table 4 shows the summary of the performance characteristics of the balancing robot angular position between SMC and PID controller quantitatively.

Tab. 4: Summary of performance characteristics of the balancing robot angular position between SMC and PID

Time response specification	SMC	PID
Rise Time	0.20 sec	0.36sec
Steady state error	0.00	0.00
Settling Time	2.28 sec	2.21 sec
Maximum undershoot	-0.13 rad	0.41 rad

Based on the data tabulated in Table 4, SMC has the fastest settling time of 1.10 seconds while PID has the slowest settling time of 2.21 seconds. An extra time of 1.11 seconds is required for the PID controller balancing robot to balance itself. In contrast, for the maximum undershoot, SMC controller has the best undershoot which is the lowest undershoot between two controllers. The maximum angular displacement of the balancing robot when SMC control signal applied to the system is -0.10 radians while maximum angular displacement of the balancing robot when PID control signal applied to the system is 0.41 radians. An extra angle of minimum 0.31 meters is required for the PID controller balancing robot to balance itself. Despite the large initial values for the displacement, the proposed PID controller is able to bring itself to the vertical position. In term of the rise time, balancing robot with SMC controller has the fastest rise time 0.21 seconds while balancing robot with PID controller needs an extra time of 0.15 seconds to rise from 10% to the 90% of the final value. In term of steady state error, both of the controllers had shown very outstanding performance by giving zero error at time 4 seconds and more.

5 CONCLUSION

In this paper, two controllers such as SMC and PID are successfully designed. Based on the results and the analysis, a conclusion has been made that both of the control method, modern controller SMC and conventional controller PID are capable of controlling the nonlinear two wheels balancing robot angular and linear position. The responses of each controller were plotted in one window and are summarized in Table 3 and Table 4. Simulation results show that SMC controller has better performance compared to PID controller in controlling the nonlinear balancing robot system. PID controller should be further improved so that the maximum overshoot and undershoot for the linear and angular positions do not have very high range. The SMC controller has got the best performance characteristics. SMC has very high capability to track the input of the system and reject the unwanted disturbances that might affect the system performance.

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