A Robust Control Method of Two-Wheeled Self-Balancing Robot

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***Abstract*—The research on two-wheeled self-balancing robot has gained momentum over the last decade at research, industrial and hobby level around the world. This paper deals with the modeling of two-wheeled self-balancing robot and the design of Sliding Mode Control (SMC) for the system. The mathematical model of two-wheeled self-balancing robot system which is highly nonlinear is derived. The final model is then represented in state-space after decoupling ignorance of the rotation. A robust controller based on Sliding Mode Control is proposed to perform the robust stabilization and disturbance rejection of the system. A computer simulation study is carried out to access the performance of the proposed control law.**

Keywords-sliding mode control;two-wheeled self-balancing robot; control theory

I. INTRODUCTION

Recently many investigations have been devoted to problems of controlling two-wheeled self-balancing robot, which are widely taken into applications in the field of autonomous robotics and intelligent vehicles. The two-wheeled self-balancing robot models are not only of theoretical interest but also of practical interest. Many practical systems have been implemented based on two-wheeled self-balancing robot models[1]. Among these applications, the Segway PT has been a popular personal transporter since invented in 2001[2]. Such systems are characterized by the ability to balance on its two wheels and spin on the spot. This additional maneuverability allows easy navigation on various terrains, turn sharp corners and traverse small steps or curbs. In addition, people can drive such vehicles to travel in a small area instead of using cars which are more pollutive[3].

From the point of view of theories, the two-wheeled self-balancing robot models have attracted much attention in the field of control theory and engineering because they are nonlinear with inherent unstable dynamics. Many control techniques have been studied in the past decades for the control of benchmark underactuated systems such as the inverted pendulum, the acrobot and the rotating pendulum.

This paper makes a study of the GBOT1001 two-wheeled self-balancing robot produced by Googol Technology Limited, establishing the mathematical model of this system, using sliding mode control theory to control the position and speed of the robot.

A prominent advantage of sliding mode control is that it can make the perturbation and the system completely irrelevant, so the system is robust to the outside disturbance and the parameter perturbation.

II. SYSTEM MODELING

In order to develop the control system, we need a dynamic model of the system that will link the system's behavior (described by the state space variables) to its inputs (defined in Introduction).this model is characterized by the system's parameters (i.e. size, mass and moment of inertia of the vehicle)[4].

A mechanical 3 DOF system can be model using six state space variables. For GBOT1001, the following variables have been chosen:

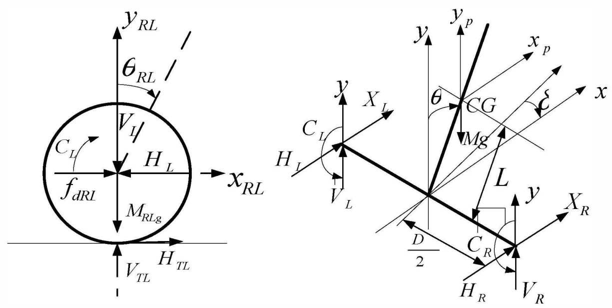

Figure 1. Free body diagram of the vehicle.

x_r ——Straight line position [m]

$\dot{x}_r$ ——Straight line speed [m/s]

θ_p ——Pitch angle [rad]

δ ——Yaw angle [rad]

The following variables have been chosen to describe the vehicle

J_{RL}, J_{RR} ——moment of inertia of the rotating masses with respect to the z axis.

M_r ——mass of rotating masses connected to the left and right wheel ,since the left hand wheel and the ones for the right-hand wheel are completely analogous,

$M_{RL} = M_{RR} = M_r$.

J_p——moment of inertia of the chassis with respect to z axis.

J_δ——moment of inertia of the chassis with respect to the y axis.

M_p——mass of the chassis.

R——radius of the wheel.

L——distance between the z axis and the CG of the chassis.

D——lateral distance between the contact patches of the wheels.

y_r—— shift position of the wheel with respect to the y axis.

x_p——shift position of the chassis with respect to the x axis.

According to the definitions in Figure 1, the following equations of motion can be defined:

For left hand wheel:

$$\ddot{x}_{RL}M_r = H_{TL} - H_L + (f_{dRL} + f_{dRR})$$

$$\ddot{y}_{RL}M_r = V_{TL} - V_L - M_r g$$

$$\ddot{\theta}_{RL}J_{RL} = C_L - H_{TL}R$$

$$\dot{x}_{RL} = R\dot{\theta}_{RL}, \dot{y}_p = -\dot{\theta}_p L\sin\theta_p$$

$$\dot{x}_p = \dot{\theta}_p L\cos\theta_p + \frac{\dot{x}_{RL} + \dot{x}_{RR}}{2}$$

$$\dot{\delta} = \frac{\dot{x}_{RL} + \dot{x}_{RR}}{2f}$$

For the chassis：

$$\ddot{x}_p M_p = (H_R + H_L) + f_{dp}$$

$$\ddot{y}_p M_p = V_R + V_L - M_p g + F_{C\theta}$$

$$\ddot{\delta}J_\delta = (H_L - H_R)\frac{D}{2}$$

Where $H_{TL,}H_{TR}, H_L, H_R, V_{TL}, V_{TR}, V_L, V_R$ and represent reaction forces between the different free bodies. Modifying the equations above and then linearizing the result around the operating point($\theta_p = 0, x_r = 0, \delta = 0$, then $\sin\theta_p \approx \theta_p$, $\cos\theta_p = 1$)the state space equations of the system can be written in matrix form as:

$$\begin{bmatrix} \dot{x}_r \\ \ddot{x}_r \\ \dot{\theta}_p \\ \ddot{\theta}_p \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ \dot{x}_r \\ \theta_p \\ \dot{\theta}_p \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} \\ 0 & 0 & 0 & 0 & 0 \\ B_{41} & B_{42} & B_{43} & B_{44} & B_{45} \\ 0 & 0 & 0 & 0 & 0 \\ B_{61} & B_{62} & B_{63} & B_{64} & B_{65} \end{bmatrix} \begin{bmatrix} C_L \\ C_R \\ f_{dRL} \\ f_{dRR} \\ f_{dP} \end{bmatrix} \quad (1)$$

Ignoring roll friction and disturbance forces, the state-space equations can be simplified as following:

$$\begin{bmatrix} \dot{x}_r \\ \ddot{x}_r \\ \dot{\theta}_p \\ \ddot{\theta}_p \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ \dot{x}_r \\ \theta_p \\ \dot{\theta}_p \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{21} & B_{22} \\ 0 & 0 \\ B_{41} & B_{42} \\ 0 & 0 \\ B_{61} & B_{62} \end{bmatrix} \begin{bmatrix} C_L \\ C_R \end{bmatrix} \quad (2)$$

The state-space equations for the vehicle can now be written as two different systems: "pendulum" and "rotation", one system describing the rotation about the z axis and another system describing the rotation about the y axis.

Ignoring the rotation of （2）, we have:

$$\begin{bmatrix} \dot{x}_r \\ \ddot{x}_r \\ \dot{\theta}_p \\ \ddot{\theta}_p \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & A_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & A_{43} & 0 \end{bmatrix} \begin{bmatrix} x_r \\ \dot{x}_r \\ \theta_p \\ \dot{\theta}_p \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \\ 0 \\ B_4 \end{bmatrix} [C_\theta] \quad (3)$$

$$A_{23} = \frac{M_p^2 L^2 (g + \ddot{y}_r)}{M_p^2 L^2 - (M_p L^2 + J_p)(M_p + M_r + J_r/R^2)}$$

$$A_{43} = \frac{-M_p g L (M_p + M_r + J_r/R^2)}{M_p^2 L^2 - (M_p L^2 + J_p)(M_p + M_r + J_r/R^2)}$$

$$B_2 = \frac{-(M_p L^2 + J_p) - M_p L R}{R\left[M_p^2 L^2 - (M_p L^2 + J_p)(M_p + M_r + J_r/R^2)\right]}$$

$$B_4 = \frac{M_p L + (M_p + M_r + J_r/R^2)}{R\left[M_p^2 L^2 - (M_p L^2 + J_p)(M_p + M_r + J_r/R^2)\right]}$$

III. DESIGN OF THE CONTROLLER

A. *Sliding-Mode Controller Design*

Generally speaking, using the SMC technique to control a two-wheeled self-balancing robot system involves two major steps. The first step is to select an appropriate switching surface which can guarantee the stability of the equivalent dynamics in the sliding mode such that the dynamics can converge to zero. The second step is to determine a SMC to guarantee the hitting of the switching surfaces. As mentioned above, we first need to design a proper switching surface to ensure the stability of the system in the sliding mode. To reach this goal, a switching sliding surface is defined as

$$s = Cx, \tag{4}$$

$$C = e^T P(A) \tag{5}$$

Here we use Ackermann Formula to calculate the value of c .Ackermann formula can be described as

$$e^T = [0,\cdots,0,1][b, Ab,\cdots, A^{n-1}b]^{-1} \tag{6}$$

$$P(\lambda) = (\lambda-\lambda_1)(\lambda-\lambda_2)\cdots(\lambda-\lambda_{n-1})(\lambda-\lambda_n)$$
$$= P_1 + P_2\lambda + \cdots + P_{n-1}\lambda^{n-2} + \lambda^{n-1} \tag{7}$$

Where matrix c is relate to the converge rate of system states. It is obvious that the sliding plane is stable. An equivalent control u_{eq} can be obtained by considering

$$\dot{s} = c\dot{x} = 0, \tag{8}$$

$$\Rightarrow c[Ax + B\cdot u_{eq}] = 0,$$

$$\Rightarrow u_{eq} = -(cB)^{-1} cAx. \tag{9}$$

The final control is realized as $u = u_{eq} + u_{sw}$, where

$$\dot{V} = s\dot{s} = s(cAx + cBu), \tag{10}$$

k is a positive constant, and

$$sgn(s) = \begin{cases} 1, & if \quad s > 0 \\ 0, & if \quad s = 0 \\ -1, & if \quad s < 0 \end{cases} \tag{11}$$

Defining a lyapunov function

$$V = \frac{1}{2}s^2 \tag{12}$$

$$\dot{V} = s\dot{s}$$
$$= s\left[cAx + cB\left(-(cB)^{-1}(cAx + k\cdot sgn(s))\right)\right]$$
$$= -s\cdot k\cdot sgn(s)$$
$$= -k\cdot|s|$$
$$\le 0 \tag{13}$$

It confirming the presence of reaching condition $(s\cdot\dot{s} < 0)$, that is, the sliding surface $s = 0$ is an attracting surface. It guarantees the robust stability of the SMC[5]. We take exponential velocity trending law in this paper:

$$\dot{s} = -ks - \varepsilon\,\mathrm{sgn}(s) \tag{14}$$

Take the state-space equation $\dot{x} = Ax + Bu$ of system into the expression above, $\dot{s}$ can be obtained as:

$$\dot{s} = C\dot{x} = CAx + CBu \tag{15}$$

When the matrix CB is nonsingular, we can get the control law u:

$$u = -(CB)^{-1}(CAx + ks + \varepsilon sgn(s)) \tag{16}$$

Defining a lyapunov function

$V = \frac{1}{2}s^2$ take the derivation of the formula above:

$$\dot{V} = s\dot{s} = s(cAx + cBu) = s(-ks - \varepsilon\,\mathrm{sgn}(s))$$
$$\le -ks^2 - \varepsilon|s| \tag{17}$$

As both k and ε are positive, we can get:

$$\dot{V} < 0,$$

So the sliding-model of pendulum system exists and can be hit.

In this way, sliding mode controller can be determined from (4) and (16) .Among them, the selection of the parameter k in (16) is mainly based on the dynamic characteristics of the system that approach the sliding surface. The selection of ε is to control errors and other factors caused by approximate linear system which can make the system has good robustness. In the control system, in order to ensure fast approaching and weaken the system chattering at the same time, the value of parameter ε should be reduced while increasing the value of parameter k [6].

B. The simulation and analysis

The simulation using exponential velocity trending law is made without interference to the system.Parameters for the two-wheeled self-balancing robot are as follows:

$g = 9.8m/s^2$ $M_p = 21$, $M_r = 0.420$, $L = 0.3$, $L = 0.3$,

So we can get

$A_{23} = -23.7097$, $A_{43} = 83.7742$, $B2 = 3.6663$ $B4 = -9.9595$.

The system performance Expect:

Overshoots $\sigma_p \le 25\%$;

Adjusting time $t_s \le 3.5s$;

Three poles were selected by calculating the overshoots and adjust time. $s_{1,2} = -1.5 \pm \frac{3\sqrt{3}}{2}i$, $s_3 = -8$

Make the drawing at the sampling time $T = 0.02s$, simulation time of $6s$. The initial condition is chosen as

$$x(0) = 0, \dot{x}(0) = 0, \theta(0) = 0.3, \dot{\theta}(0) = 0$$

Where, radian is the swing angle unit. Take parameters $\varepsilon = 0.8$, $k = 25$.

In actual system, the uncertainty of parameters is a widespread problem for modeling errors, time-varying parameter changes with the time and the approximate linearization of dynamic equation in the system. For a general linear controller, perturbation of the internal parameters has a direct impact both on the dynamic characteristics and static characteristics of the system. When the parameters change greatly, system is unstable as a positive real part of pole may be appeared. SMC also has a fully adaptability for the uncertain of parameters in the system. When there are parameter perturbations, the differential equations of normal movement will also be changed in the system while differential equations of motion sliding mode won't be change. Sliding mode has a complete adaptability on perturbation. But for general linear systems, the establishment of invariance is conditional and it needs to meet the matching conditions of the sliding mode. As for the existence of perturbation in the system, ΔA is perturbation of A.

1) System with interference

$$\dot{x} = Ax + Bu + Df \quad (18)$$

Where Df presents the outside disturbance to the system, the sufficient and necessary condition that sliding mode suffering from interference of Df is

$$rank[B, D] = rankB \quad (19)$$

The system can be translated into

$$\dot{x} = Ax + B\left(u + \tilde{D}f\right) \quad (20)$$

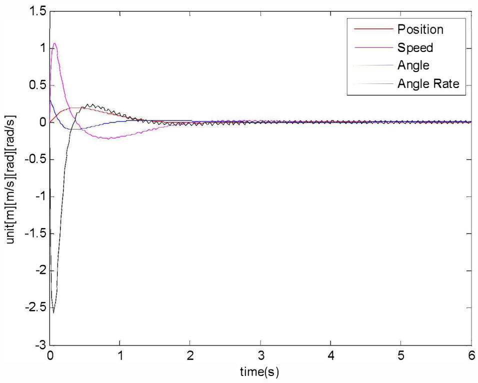

Figure 2. System trajectories by using the SMC controller with disturbance.

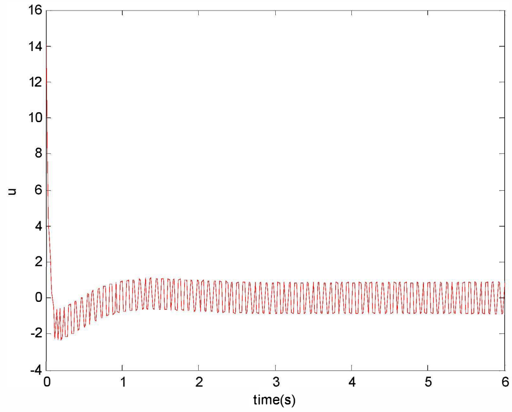

Figure 3. The trajectory of input u

If (19) is fulfilled, where $\tilde{D} = B^{-1}D$, Then the complete compensation to interference can be realized through the control law design of u. Equation (19) is called completely matching conditions of the interference to system. Take the disturbance to system as:

$$f(t) = 0.5\sin(3t).$$

2) Chattering reductionon

From the simulation results, we can see that despite the output chattering of the controller is large and the control effect of exponential reaching law is not the best, however, the stability control of Inverted pendulum car and swing angle can be achieved when the error is acceptable.

The sliding model variable structure controller can make the system stable on the sliding surface movement, but the output chattering of controller is large, by the same time the control state will be back on the sliding surface, it not only affect control precision of the system, increase energy consumption and reduce the machine service life, and also may inspire forced vibration to the un-modeled system, even can cause system instability. Although the simulation meets the requirements, but in the actual process, the entire system will increase the jitter, therefore, it is necessary to weak chattering.

In this paper we reduce the high frequency chattering of the control signal through replacing the ideal sliding mode of symbol function by saturated function.

$$sat(s) = \begin{cases} 1, & if \quad s > \Delta \\ ks, & if \quad |s| \le \Delta \\ -1, & if \quad s < -\Delta \end{cases}, \quad k = \frac{1}{\Delta}, \quad \Delta = 0.05 \quad (21)$$

3) System with both interference and parameter perturbation

$$\dot{x} = Ax + \Delta Ax + Bu + Df \quad (22)$$

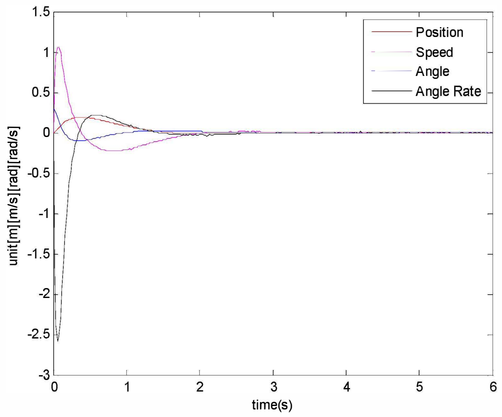

Figure 4. System trajectories by using the SMC controller

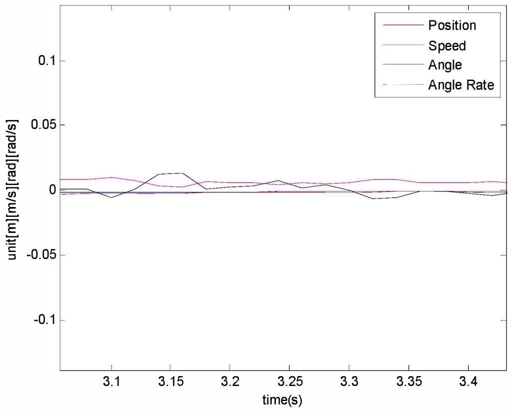

Figure 5. The amplified curves to fig.4

If (19) and (20) are fulfilled, the system can be described as

$$\dot{x} = Ax + B\left(u + \Delta\tilde{A}x + \tilde{D}f\right) \tag{23}$$

Choose parameter perturbation

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \end{bmatrix}$$

The transient process of the system is longer when a parameter perturbation added to the system, but it will be stable in the end, compared fig.6 with fig.4, the system will not be influenced by parameter perturbation after system is stable, it proved that the uncertainty of system parameters did influence the approaching equation, but it is irrelevant to the sliding hyperplanes, not affecting steady-state process of the system.

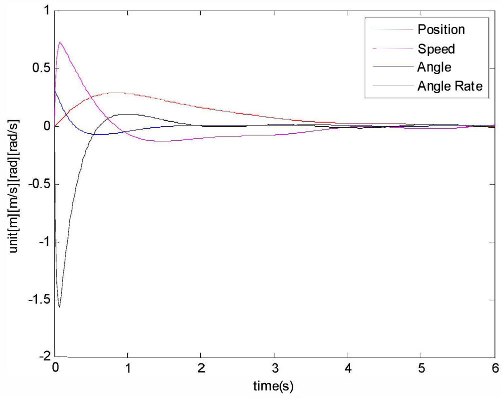

Figure 6. System trajectories by using the SMC controller with both disturbance and parameter perturbation.

IV. CONCLUSION

Comparing the three simulation results using the designed SMC controller, both angle and the position of the robot can be stable at a high speed rate which could reach the expected requirement. From the results, we can reduce the system chattering to improve the quality of the control system through replacing the ideal sliding mode of symbol function by saturated function. Sliding mode control is robust to the outside interference and the parameter perturbation. The work of this paper is still basic, for future work we can combine sliding mode control, fuzzy control and neural control together using adaptive parameter to control the complex nonlinear system to obtain high control quality.

REFERENCES

[1] Ching-Chih Tsai, Shang-Yu Ju, Shih-Min Hsieh,"Trajectory Tracking of a Self-Balancing Two-Wheeled Robot Using Back stepping Sliding-Mode Control and Fuzzy Basis Function Networks," The 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems, October 18-22, 2010, Taipei,Taiwan. pp.3943-3948.

[2] Shui-Chun Lin, Ching-Chih Tsai, Hsu-Chih Huang,"Nonlinear Adaptive Sliding-Mode Control Design for Two-Wheeled Human Transportation Vehicle," Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics San Antonio, TX, USA - October 2009, pp.1965-1970.

[3] S. C. Lin, C. C. Tsai, and W. L. Luo, "Adaptive neural network control of a self-balancing two-wheeled scooter," In proc. 33rd Annu. IEEE IECON, Nov. 5-8, 2007, pp.868-873.

[4] Googol Technology LTD.Googol Technology Self-Balancing Robot GBOT1001 User Manual V1.0.2007, pp.10-15.

[5] Metin U. SALAMCI, G. Serdar TOMBUL, "Sliding Mode Control Design with Time Varying Sliding Surfaces for a Class of Nonlinear Systems," Proceedings of the 2006 IEEE International Conference on Control Applications Munich, Germany, October 4-6, 2006, pp.996-1001.

[6] M. L. Corradini, T. Leo, G. Orlando, "Robust stabilization of a class of nonlinear systems via multiple model sliding mode control," Proceeding of the American Control Conference, Chicago, IL, 631-635, June 2000.