# A PD Sliding Mode Controller for Two-Wheeled Self-Balancing Robot

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# A PD SLIDING MODE CONTROLLER FOR TWO-WHEELED SELF-BALANCING ROBOT
# BỘ ĐIỀU KHIỂN PD TRƯỢT CHO ROBOT HAI BÁNH TỰ CÂN BẰNG

***Nguyen Gia Minh Thao*, *Duong Hoai Nghia*, *Phan Quang An***
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**ABSTRACT**

*This paper presents a method to design and control a two-wheeled self-balancing robot. It focuses on hardware description and PD sliding mode controller design. The signals from angle sensors are filtered by a discrete Kalman filter before being fed to the PD sliding mode controller. The objectives of the proposed controller are twofold: regulation of the pitch angle and tracking the desired position. The proposed controller has two loops. The first loop is a sliding mode pitch angle regulator. The second loop is a PD position tracking controller. Simulations and experimental results show that the proposed controller has good performance and robust against disturbances.*
Keywords: Two wheeled self-balancing robot, Discrete Kalman filter, PID control, Sliding mode control, Accelerator, Gyroscope.

**TÓM TẮT**

*Bài báo này trình bày một phương pháp để thiết kế và điều khiển robot hai bánh tự cân bằng. Các nội dung chính bao gồm mô tả phần cứng robot và thiết kế bộ điều khiển PD trượt. Các tín hiệu từ các cảm biến góc được lọc bởi bộ lọc Kalman trước khi đưa vào bộ điều khiển PD trượt. Mục tiêu của bộ điều khiển được đề xuất là ổn định hóa góc nghiêng thân robot và robot bám theo vị trí mong muốn. Bộ điều khiển được đề xuất gồm hai vòng. Vòng thứ nhất là một bộ điều khiển trượt để ổn định góc nghiêng thân robot. Vòng thứ hai là một bộ điều khiển PD để điều khiển vị trí của robot. Các kết quả mô phỏng và thực nghiệm cho thấy bộ điều khiển được đề xuất có chất lượng tốt và bền vững với nhiễu.*

## I. INTRODUCTION

Two-wheeled self-balancing robot is a multi-variable and uncertain nonlinear system [1], [2], [7], [8]. Thus, the performance of the robot depends heavily on the signal processing technique and the control method in use. In recent years, the number of researches on sliding mode (SM) control has increased [4], [5],[6],[9]. The SM approach provides a powerful design tool for nonlinear system. So it is of great interest and feasible to use SM approach to design a compatible controller for two-wheeled self-balancing robot when the mathematical model of the robot is identified.

The designed two-wheeled self-balancing robot is given in *Fig.1.* Signals from angle sensors are filtered by a discrete Kalman filter before being fed to the PD sliding mode controller. The purposes of the controller are to stabilize the robot and keep the motion of the robot to track a reference signal. The proposed PD SM controller has two loops (*Fig.7*). The first loop is a nonlinear controller based on SM approach to maintain the tracking error of the pitch angle at zero. The second loop uses a PD controller to control the position of the robot.

## II. HARDWARE DESCRIPTION

### A. The hardware of the robot

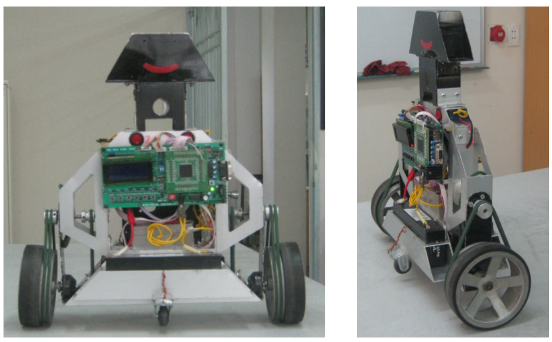

*Figure 1. The prototype two-wheeled self-balancing robot*

The prototype design of the robot is shown in *Fig.1*, *Fig.2*. It has an aluminum chassis, two 24V-30W DC-servo motors for actuation, an acceleration sensor and a gyroscope sensor for measuring the pitch angle and the angular velocity of the body, two incremental encoders for measuring the position of the wheels. The signal processing and control algorithm are embedded in a 16-bit two-core micro-controller MC9S12XDP512. The operations of the robot (upright and balance, moving forward, moving backward, etc.) can be set from buttons on the central control module or from a RF remote controller.

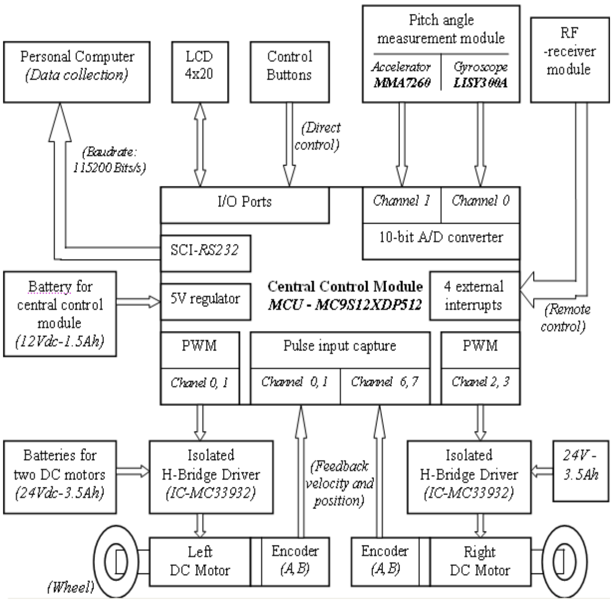


*Figure 2. Hardware description*

### B. Sensor Signal Processing

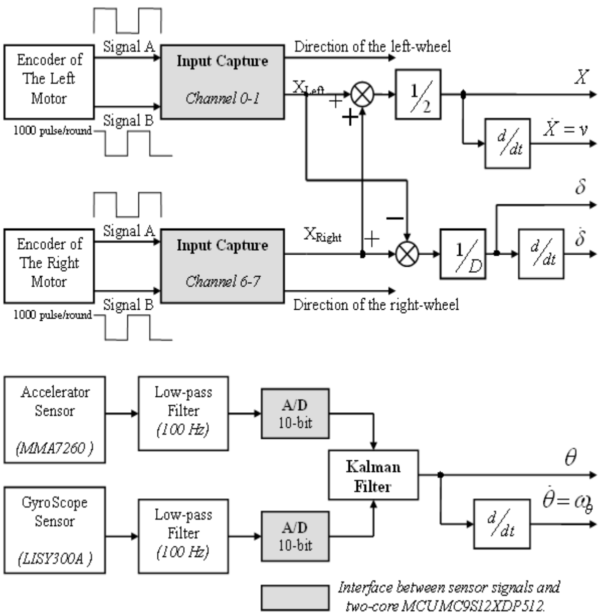


*Figure 3. Sensor signal processing*

An acceleration sensor, a gyroscope sensor and two incremental encoders provide full state of the robot (*Fig.3*). The signals from the acceleration sensor and the gyroscope sensor are filtered by a discrete Kalman filter (presented in Section III) before being fed to the PD sliding mode controller.

## III. DISCRETE KALMAN FILTER

The Kalman filter estimates a process by using a form of feedback control [3]. The filter estimates the process state at some times and then obtains feedback in the form of measurements. The equations of the discrete Kalman filter can be decomposed into two groups: *Time Update* equations and *Measurement Update* equations, as shown in *Fig.4*.

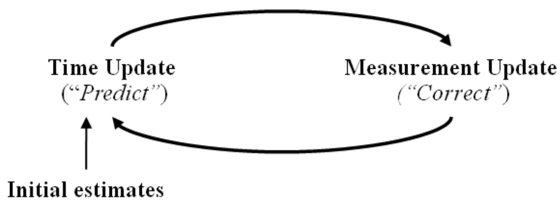


*Figure 4. The discrete Kalman filter algorithm*

- *Time Update* equations

  Project the state ahead

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k \tag{1}$$

  Project the error covariance ahead

$$P_k^- = AP_{k-1}A^T + Q \tag{2}$$

- *Measurement Update* equations

  Compute the Kalman gain

$$K_k = P_k^- H^T \left(HP_k^- H^T + R\right)^{-1} \tag{3}$$

  Update estimate with measurement $z_k$

$$\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - H\hat{x}_k^-\right) \tag{4}$$

  Update the error covariance

$$P_k = \left(I - K_k H\right)P_k^- \tag{5}$$

The parameters of discrete Kalman filter are as follows

$$A = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 0 \end{bmatrix};\ X = \begin{bmatrix} angle \\ gyro \end{bmatrix};\ R = 0.17$$

$$P_{initial} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ Q = \begin{bmatrix} 0.002 & 0 \\ 0 & 0.005 \end{bmatrix};\ H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Where: R and Q are determined by a trial and error method.

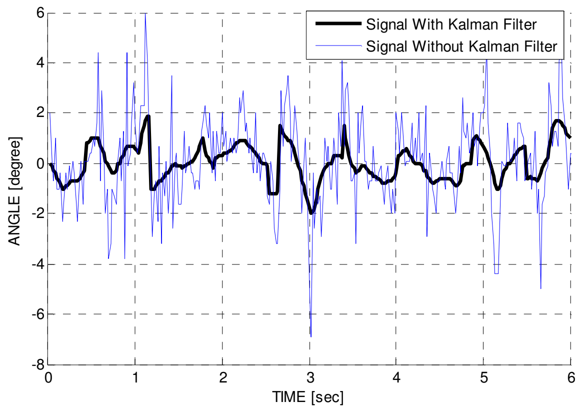


*Figure 5. Filtered and unfiltered angle measurement*

## IV. MATHEMATICAL MODEL

The coordinate system of the robot is shown in *Fig.6*. The mathematical model of the robot is derived based on Newton's 2nd law of motion [1], [2], [7], [8].

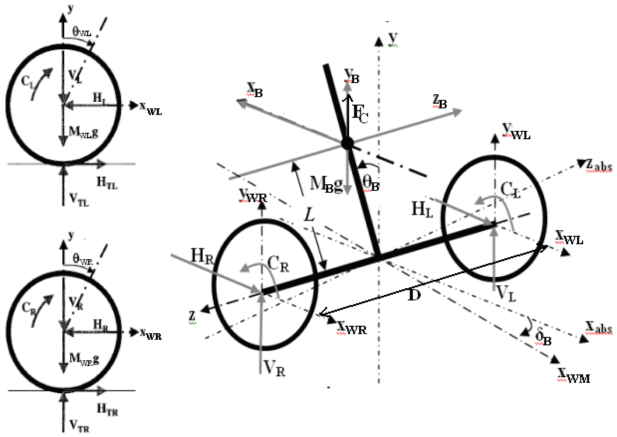

*Figure 6. The coordinate system*

Where:

$\theta$: Pitch angle [rad] ; $\delta$: Yaw angle [rad].

$M_W$: Mass of wheel, 0.5[kg].

$M_B$: Mass of body, 7[kg].

$R$ : Radius of wheel, 0.07[m].

$L$: Distance between the centre of the wheels and the robot's centre of gravity, 0.3[m].

$D$: Distance between the contact patches of the wheels, 0.41[m].

$g$ : Gravity constant, 9.8 [ms$^{-2}$].

$C_L, C_R$ : Input torque for left and right wheels [Nm].

$V_{TL}, V_{TR}, H_{TL}, H_{TR}, H_L, H_R, V_L, V_R$: Reaction forces [N].

- For the left wheel (same as the right wheel).

$$M_W \ddot{x}_{WL} = H_{TL} - H_L \tag{6}$$

$$M_W \ddot{y}_{WL} = V_{TL} - V_L - M_W g \tag{7}$$

$$J_{WL} \ddot{\theta}_{WL} = C_L - H_{TL} R \tag{8}$$

$$x_{WL} = \theta_{WL} R \tag{9}$$

$$J_{WL} = \frac{1}{2} M_{WL} R^2 \tag{10}$$

- For the body of the robot.

$$x_B = L \sin\theta_B + \left(\frac{x_{WL} + x_{WR}}{2}\right) \tag{11}$$

$$y_B = -L(1 - \cos\theta_B) \tag{12}$$

$$M_B \ddot{x}_B = H_L + H_R \tag{13}$$

$$\begin{aligned} M_B \ddot{y}_B &= V_L + V_R - M_B g + F_C \\ &= V_L + V_R - M_B g + \frac{(C_L + C_R)}{L} \sin\theta_B \end{aligned} \tag{14}$$

$$\begin{aligned} J_B \ddot{\theta}_B &= (V_L + V_R) L \sin\theta_B - \\ &- (H_L + H_R) L\cos\theta_B - (C_L + C_R) \end{aligned} \tag{15}$$

$$J_B = \frac{1}{3} M_B L^2 \tag{16}$$

$$\theta = \theta_B \approx \theta_W \tag{17}$$

Let $x_1 = \theta,\ x_2 = \dot{\theta},\ x_3 = x,\ x_4 = \dot{x}$ . The state equations follow from (6)-(17) are:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x_1) + f_2(x_1, x_2) + g_1(x_1)\mathrm{C} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_3(x_1) + f_4(x_1, x_2) + g_2(x_1)\mathrm{C} \end{cases} \tag{18}$$

Where:

$\mathrm{C} = C_L + C_R$

$$f_1(x_1) = \frac{\left(\dfrac{-0.75 g(\sin x_1)}{L}\right)}{\left(\dfrac{0.75(M_w R + M_B L(\cos x_1))(\cos x_1)}{(2M_W + M_B)L} - 1\right)}$$

$$f_2(x_1, x_2) = \frac{\left(\dfrac{0.75 M_B L(\sin x_1)(\cos x_1)}{(2M_W + M_B)L}(x_2)^2\right)}{\left(\dfrac{0.75(M_w R + M_B L(\cos x_1))(\cos x_1)}{(2M_W + M_B)L} - 1\right)}$$

$$g_1(x_1)=\frac{\left(\dfrac{0.75\left(M_wR+M_BL(\cos x_1)\right)\left(1+(\sin x_1)^2\right)}{M_BL^2}+\dfrac{1}{R}\right)}{\left(2M_W+M_B-\dfrac{0.75\left(M_wR+M_BL(\cos x_1)\right)(\cos x_1)}{L}\right)}$$

$$f_3(x_1)=\frac{\left(\dfrac{-0.75g\left(M_wR+M_BL(\cos x_1)\right)(\sin x_1)}{L}\right)}{\left(2M_W+M_B-\dfrac{0.75\left(M_wR+M_BL(\cos x_1)\right)(\cos x_1)}{L}\right)}$$

$$f_4(x_1,x_2)=\frac{\left(M_BL(\sin x_1)(x_2)^2\right)}{\left(2M_W+M_B-\dfrac{0.75\left(M_wR+M_BL(\cos x_1)\right)(\cos x_1)}{L}\right)}$$

$$g_2(x_1)=\frac{\left(\dfrac{0.75\left(M_wR+M_BL(\cos x_1)\right)\left(1+(\sin x_1)^2\right)}{M_BL^2}+\dfrac{1}{R}\right)}{\left(2M_W+M_B-\dfrac{0.75\left(M_wR+M_BL(\cos x_1)\right)(\cos x_1)}{L}\right)}$$

## V. CONTROLLER DESIGN

The proposed control system (described in *Fig.7*) has two loops. The inner loop is a PD position tracking controller with slow dynamic (*Fig.8*). The output of this controller is clamped between $\pm 25\%$ of the rated torque. The outer loop is a SM pitch angle regulator.

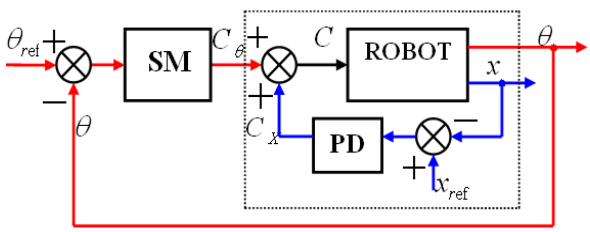


*Figure 7. The proposed controller*

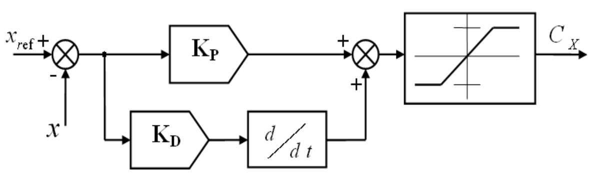


*Figure 8. The PD position tracking controller*

Define the tracking error as

$$e = x_{1ref} - x_1 \tag{19}$$

Where $x_{1ref} = \theta_{ref}$ is the reference value of the pitch angle ($\theta$). We have

$$\dot{e} = \dot{x}_{1ref} - \dot{x}_1 = \dot{x}_{1ref} - x_2 \tag{20}$$

$$\ddot{e} = \ddot{x}_{1ref} - \ddot{x}_1 = \ddot{x}_{1ref} - \dot{x}_2 \tag{21}$$

The sliding function is defined as

$$S = \tau\dot{e} + e \tag{22}$$

Where $\tau > 0$ is the desired time constant of the sliding phase. We have

$$\dot{S} = \tau\ddot{e} + \dot{e} \tag{23}$$

Substituting (20),(21) into (23), we obtain

$$\begin{aligned}\dot{S} &= \tau\left(\ddot{x}_{1ref} - \dot{x}_2\right) + \left(\dot{x}_{1ref} - x_2\right)\\ &= \tau\left(\ddot{x}_{1ref} - f_1(x_1) - f_2(x_1,x_2) - g_1(x_1)C\right) + \dot{x}_{1ref} - x_2\end{aligned} \tag{24}$$

Where $C = C_X + C_\theta$ (see *Fig.7*)

The control signal $C$ is determined such that

$$\dot{S} = -K\left[sat(S)\right] \tag{25}$$

Where

- $sat(.)$ is the saturation function

$$sat(S) = \begin{cases} -0.4\ , & \text{if } S < -0.4 \\ S\ , & \text{if } -0.4 \le S \le 0.4 \\ 0.4\ , & \text{if } S > 0.4 \end{cases} \tag{26}$$

The saturation value of *sat(.)* is chosen by a trial and error method.

- $K$ is positive constant. A large value of $K$ results in a robust controller but it also causes chattering phenomenon.

It follows from (24), (25) that

$$C_\theta = \frac{\ddot{x}_{1ref} - f_1(x_1) - f_2(x_1,x_2) + \frac{K}{\tau}[sat(S)] - \frac{x_2}{\tau} + \frac{\dot{x}_{1ref}}{\tau}}{g_1(x_1)} - C_X \tag{27}$$

## VI. SIMULATIONS, EXPERIMENTAL RESULTS

### A. Simulations

Responses of the proposed PD SM controller are compared with a PID multi-loop controller (*Fig.9*).

The parameters of the two controllers (*Table I*) are chosen from simulations and experiments.

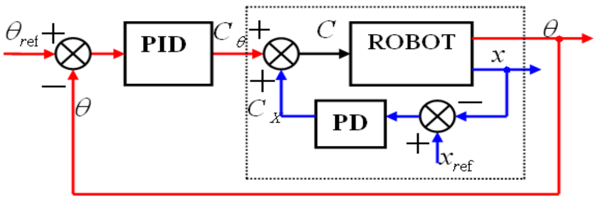


*Figure 9. PID multi-loop controller*

*Table I. Parameters of the controllers*

| | | |
|---|---|---|
| **PD SM controller** | **SM** | $\tau = 0.4$; K=1.35 |
| | **PD** | $K_P$=0.21; $K_D$=0.39 |
| **PID multi-loop** | **PID** | $K_P$=4; $K_I$=0.4; $K_D$=0.7 |
| | **PD** | $K_P$ = 0.21 ; $K_D$ = 0.39 |

- Pitch angle regulation

  $\theta_{ref} = 0^0$ , $x_{ref} = 0[m]$. (*Fig.10*)

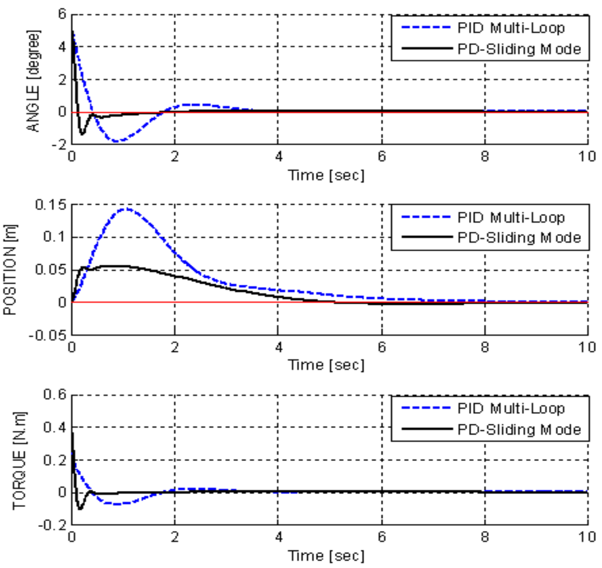


*Figure 10. Pitch angle regulation*

- Position tracking

  $\theta_{ref} = 0^0$, $x_{ref} = 1[m]$. (*Fig.11*)

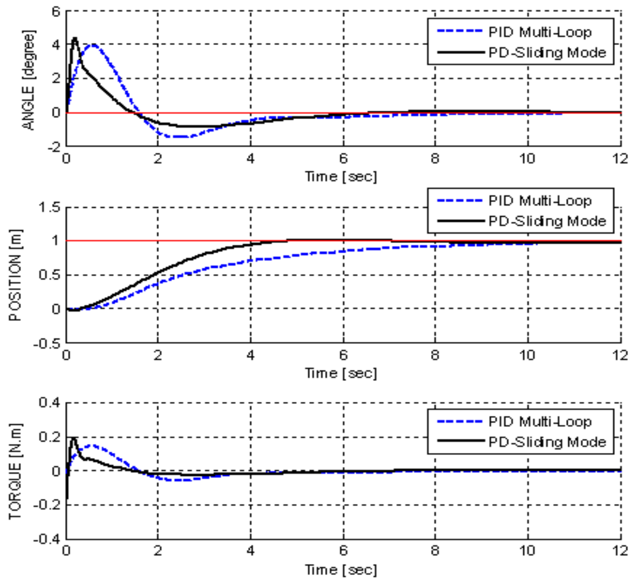


*Figure 11. Position tracking*

We observe that the proposed controller has better performance in terms of setting time and overshoot.

### B. Experimental Results

- Pitch angle regulation.

  ($\theta_{ref} = 0^0$, $x_{ref} = 0$ [m]) (*Fig.12*)

The pitch angle of the robot ( $\theta$ ) is regulated under $1.1^0$, and the position of the robot ( $x$ ) is regulated under 0.04 [m].

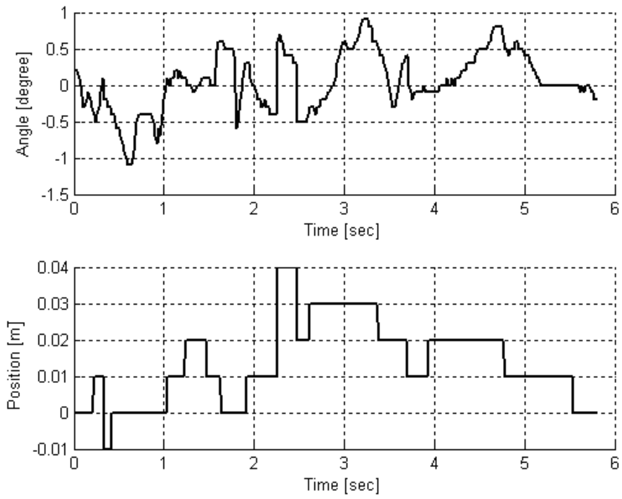


*Figure 12. Pitch angle regulation*

- Response to disturbance.

  ($\theta_{ref} = 0^0$ )

In this experiment, the robot is affected by an external disturbance (the robot is pushed at t=3[sec]). The result is given in *Fig.13*.

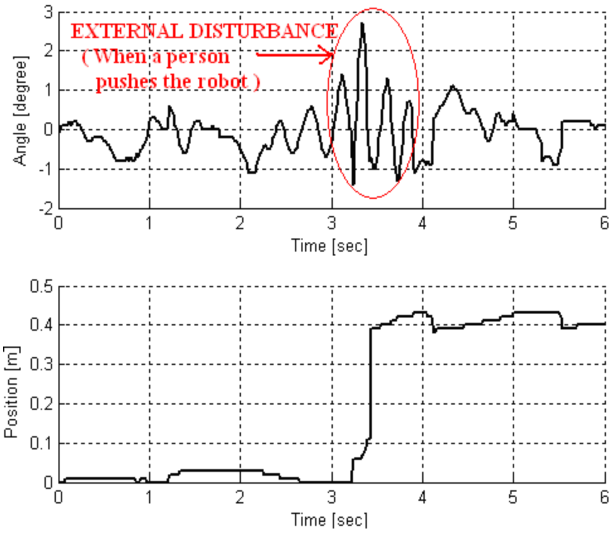


*Figure 13. Response to disturbance*

- Position tracking

$$(\theta_{ref} = 0^0,\ x_{ref} = 1\,[\mathrm{m}])$$

The result is given in *Fig.14*. The robot starts to move at t=2[sec]. We see that the setting time is 6[sec].

## VII. CONCLUSION

This paper presented a method to design and control a two-wheeled self-balancing robot. Simulations and experimental results show that the nonlinear controller based on SM approach has good performance (in terms of setting time, overshoot and robust against disturbance). The operations of the robot can be observed in [10], [11], [12]. We can conclude that the proposed PD SM controller is compatible with a nonlinear system as the two-wheeled self-balancing robot. The future development of this project is implementation of an adaptive SM controller for the two-wheeled self-balancing robot.

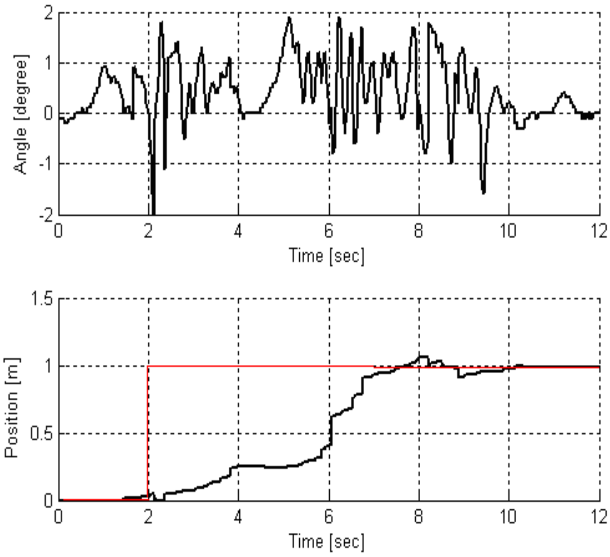


*Figure 14. Position tracking.*

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