

# Package ‘GpOutput2D’

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**Type** Package

**Title** What the Package Does (Title Case)

**Version** 0.1.0

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**Description** Package for two-dimensional functional data analysis (for instance : data set of images or maps). Functions for performing Functional Principal Component Analysis (FPCA) on such data have been developped. Furthermore, the package contains methods for Computer Experiments by using kriging methods : metamodeling of models with two-dimensional functional output.

**Imports** lhs,DiceDesign,stats,graphics,doParallel,fields,pracma,foreach

**Depends** waveslim, orthogonalsplinebasis,DiceKriging,kergp

**License** GPL-2

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 7.1.1

**NeedsCompilation** no

## R topics documented:

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GpOutput2D-package	<i>Memodelling for model with two-dimensional functional output by using Functional Principal Component Analysis and kriging methods</i>
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**Description**

Package for two-dimensional functional data analysis (for instance : data set of images or maps). Functions for performing Functional Principal Component Analysis (FPCA) on such data have been developped. Furthermore, the package contains methods for Computer Experiments by using kriging methods : metamodeling of models with two-dimensional functional output.

**Details**

Package :	GpOutput2D
Type :	Package
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License :	GPL (=2.0)
LazyLoad :	yes

**Note**

Important functions or methods :

- Fpca2d : Functional Principal Component Analysis (FPCA) for two-dimensional data (images, maps,...).
- km\_Fpca2d : Gaussian Process Model on principal components of Fpca2d, by using DiceKriging package.
- gp\_Fpca2d : Gaussian Process Model on principal components of Fpca2d, by using kergp package.
- predict : Prediction of the objective function at new points using km\_Fpca2d or gp\_Fpca2d model (Simple and Universal Kriging).

**Author(s)**

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**References**

test

Campbell2D

*Example of function called Campbell2D***Description**

Example of function called Campbell2D

**Usage**

```
Campbell2D(X, z1, z2)
```

**Arguments**

**X** a matrix with eight columns, which correspond to the function input.  
**z1, z2** vectors with the spatial coordinates.

**Value**

a three-dimensional array with the output maps. The first two dimensions correspond to the maps dimension. The third dimension corresponds to the number of maps (or simulations).

**Examples**

```
# inputs of the Campbell2D function
x1<-rep(-1,8); x2<-rep(5,8); x3<-c(5,3,1,-1,5,3,1,-1)
X <- rbind(x1,x2,x3)

# spatial domain of the Campbell2D output
nz<-64 # root of the size of the spatial domain
z<-seq(-90,90,length=nz)

# Campbell2D function
Y = Campbell2D(X,z,z)
```

coef.OrthoNormalBsplines2D

*Get control points of two-dimensional B-splines basis***Description**

Get control points of two-dimensional B-splines basis

**Usage**

```
## S3 method for class 'OrthoNormalBsplines2D'
coef(object, y, ...)
```

**Arguments**

object	an object of class <a href="#">OrthoNormalBsplines2D</a> with functions of the B-splines basis.
y	two-dimensional data to approximate on B-splines basis.
...	other arguments.

**Value**

a matrix with control points on two-dimensional B-splines basis.

**See Also**

[OrthoNormalBsplines2D Inverse2D](#). [OrthoNormalBsplines2D Inverse2D](#)

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error.predict	<i>Prediction accuracy</i>
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**Description**

Prediction accuracy is measured by the spatial RMSE and Q2. `error.predict` also computes RMSE and Q2 of the FPCA scores.

**Usage**

```
error.predict(
  y,
  py,
  fpca,
  sd.epsilon = 0,
  scores.epsilon = 0,
  rtx.scores = FALSE
)
```

**Arguments**

y	an array with the real two dimensional functional outputs. The two first dimensions correspond to data dimensions, which are denoted M and N. The third one is the size of the data set.
py	predictions which are given by <a href="#">predict.km_Fpca2d</a> or <a href="#">predict.gp_Fpca2d</a> .
fpca	an object of class <a href="#">Fpca2d</a> .
sd.epsilon	a value with y standard deviation threshold. Q2 are NA, where spatial standard deviations are less than or equal to sd.epsilon. The default is 0. This is to avoid infinite values of Q2.
scores.epsilon	a value with y score standard deviation threshold. Q2 of scores are NA, where score standard deviations are less than or equal to scores.epsilon. scores.epsilon are NA. The default is 0. This is to avoid infinite values of Q2.
rtx.scores	a logical value. If TRUE, rmse and Q2 of the predicted scores are returned. The default is FALSE

**Value**

- if `rtx.scores=FALSE`, a list with the following items is returned :
  - `rmse` : the spatial rmse of y prediction.
  - `Q2` : the spatial Q2 of y prediction.
- if `rtx.scores=TRUE`, a list with the following items is returned :
  - `scores` :
    - \* `rmse` : rmse of y score prediction.
    - \* `Q2` : Q2 of y score prediction.
  - `y` : the same list as for the case `rtx.scores=FALSE`

Fpca2d

*Two-dimensional Functional Principal Component Analysis (FPCA).***Description**

A wrapper function to perform FPCA on two-dimensional data (images, maps, etc.), given a projection method. For instance, the valid projections are two methods based on projection method (here, "Wavelets" and "Bsplines").

**Usage**

```
Fpca2d(method = c("Bsplines", "Wavelets"), ...)
```

**Arguments**

<code>method</code>	a character string which specifies the method used to implement FPCA.
<code>...</code>	further parameters for the projection method: see <a href="#">Fpca2d.Wavelets</a> , for using wavelets, and <code>Fpca2d.Bsplines</code> , for using orthonormal B-splines.

**Value**

an object of class "Fpca2d" which is a list with the following items :

- `sdev` : the standard deviations of the principal components (i.e., the square roots of the eigenvalues of the covariance operator).
- `EigFct` : a three dimensional array with the eigenfunctions. The two first dimensions correspond to data (images, maps, etc.) dimensions. The third one is the number of modeled principal component.
- `x` : if `retx` is TRUE (see [prcomp](#)). Data scores (coordinates in the eigen basis) are returned.
- `center, scale` : a logical value which indicates if data coefficients are respectively centered or scaled.

**Author(s)**

Tran Vi-vi Elodie PERRIN

**See Also**

[Fpca2d.Wavelets](#)

**Examples**

```
#####
### two-dimensional data set ###
#####
n<-200 # size of the learning sample
nz<-64; z <- seq(-90,90,length=nz) # spatial domain

### inputs of Campbell2D ###
library(lhs)
library(DiceDesign)

x <- maximinLHS(n=n,k=8)
X <-maximinSA_LHS(x)$design
X<-X*6 -1

# Campbell2D
Y <- Campbell2D(X,z,z)

#####
### FPCA ###
#####

### using wavelet basis ###
fpca_w<- Fpca2d(Y,method="Wavelets",
                wf="d4", J=1, # wavelet parameters
                ncoeff=1200, rank.=5) # FPCA configuration

plot(fpca_w,type="eigenfunctions")
plot(fpca_w)

## Not run:
### using B-splines basis ###

# knots for B-splines basis
K<-35
z.knots <- seq(-90,90,length=K)

fpca_Bs<- Fpca2d(Y,method="Bsplines",
                 z1=z,z2=z,z1.knots=z.knots,z2.knots=z.knots, # wavelet parameters
                 ncoeff=1225, rank.=2) # FPCA configuration

plot(fpca_Bs,type="eigenfunctions")
plot(fpca_Bs)

## End(Not run)
```

Fpca2d.Bsplines

---

*Two dimensional functional principal component analysis (FPCA) by  
using B-splines basis*

---

**Description**

Two dimensional functional principal component analysis (FPCA) by using B-splines basis

**Usage**

```
Fpca2d.Bsplines(
  x,
  z1,
  z2,
  z1.knots,
  z2.knots,
  norder = 2,
  ortho = "GS",
  expand_knots = FALSE,
  p = 1,
  ncoeff = NULL,
  center = TRUE,
  scale. = FALSE,
  ...
)
```

**Arguments**

x	a three dimensional array, which contains two-dimensional data (images, maps, etc.). The two first dimensions correspond to data dimensions, which are denoted M and N. The third one is the size of the data set.
z1, z2	two vectors of argument values at which the B-spline basis functions are to be evaluated.
z1.knots, z2.knots	The full set of knots used to define the basis functions.
norder	Order of the spline fit (degree= order-1). The default is 2.
ortho	a character string which specify the orthogonalization method. If it is "Redd", the <a href="#">orthogonalsplinebasis</a> package is used. If it is "GS", the Gram-Schmidt is used. The default is "GS".
expand_knots	a boolean. If it is TRUE, knots are expanded for appropriate number of knots in bsplines (see <a href="#">expand.knots</a> ). The default is FALSE.
p	a value which fixes the total mean proportion of energy (or mean spatial variance). The number of coefficients (ncoeff) used for PCA is calibrated according to its value. The default is 1. If a value is given in ncoeff, p is not used.
ncoeff	a value which fixes the number of coefficients used for PCA. The default is NULL. If it is NULL, the number of coefficients is calibrated by using the parameter p.
center	a logical value indicating whether the coefficients should be shifted to be zero centered. The default is TRUE.
scale.	a logical value indicating whether the coefficients should be scaled to have unit variance before the analysis takes place. The default is FALSE.
...	arguments of <a href="#">prcomp</a> , which can fix characteristics of PCA.

**Value**

an object of class "Fpca2d" which is a list with the following items :

- sdev : the standard deviations of the principal components (i.e., the square roots of the eigenvalues of the covariance operator).

- `EigFct` : a three dimensional array with the eigenfunctions. The two first dimensions correspond to data (images, maps, etc.) dimensions. The third one is the maximal number of principal component used.
- `x` : if `retx` is TRUE (see [prcomp](#)). Scores (coordinates in the eigen basis) of the data are returned.
- `center, scale` : a logical value which indicates if data coefficients are respectively centered or scaled.

## See Also

[Fpca2d](#)

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Fpca2d.Wavelets	<i>Two dimensional functional principal component analysis (FPCA) by using wavelet basis</i>
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## Description

performs FPCA on two-dimensional functional data (data set of images, maps, ...). The implementation of FPCA is based on wavelet basis.

## Usage

```
Fpca2d.Wavelets(
  x,
  wf,
  J,
  boundary = "periodic",
  p = 1,
  ncoeff = NULL,
  center = TRUE,
  scale. = FALSE,
  ...
)
```

## Arguments

<code>x</code>	a three dimensional array, which contains two-dimensional data (images, maps, etc.). The two first dimensions correspond to data dimensions, which are denoted M and N. The third one is the size of the data set.
<code>wf</code>	a character string which specifies the wavelet filter (see <a href="#">dwt.2d</a> ).
<code>J</code>	depth of the wavelet decomposition, must be a number less than or equal to $\log(\min(M,N),2)$ .
<code>boundary</code>	a character string which specifies how boundaries are treated. Only "periodic" is currently implemented (see <a href="#">dwt.2d</a> ).
<code>p</code>	a value which fixes the total mean proportion of energy (or mean spatial variance). The number of coefficients ( <code>ncoeff</code> ) used for PCA is calibrated according to its value. The default is 1. If a value is given in <code>ncoeff</code> , <code>p</code> is not used.
<code>ncoeff</code>	a value which fixes the number of coefficients used for PCA. The default is NULL. If it is NULL, the number of coefficients is calibrated by using the parameter <code>p</code> .



center	a logical value indicating whether the coefficients should be shifted to be zero centered. The default is TRUE.
scale.	a logical value indicating whether the coefficients should be scaled to have unit variance before the analysis takes place. The default is FALSE.
...	arguments of <a href="#">prcomp</a> , which can fix characteristics of PCA.

### Value

an object of class "Fpca2d" which is a list with the following items :

- sdev : the standard deviations of the principal components (i.e., the square roots of the eigenvalues of the covariance operator).
- EigFct : a three dimensional array with the eigenfunctions. The two first dimensions correspond to data (images, maps, etc.) dimensions. The third one is the maximal number of principal component used.
- x : if retx is TRUE (see [prcomp](#)). Scores (coordinates in the eigen basis) of the data are returned.
- center, scale : a logical value which indicates if data coefficients are respectively centered or scaled.

### See Also

[Fpca2d](#)

---

gp_Fpca2d	<i>Gaussian Process Model on principal components of Fpca2d, by using kergp package</i>
-----------	---

---

### Description

the function gp of the [kergp](#) package is use to fit kriging models on each principal component modeled on [Fpca2d](#) (for more details, see [gp](#) ).

### Usage

```
gp_Fpca2d(formula = ~1, design, response, cov, estim = TRUE, ...)
```

### Arguments

formula	an object of class "formula" (or a list of "formula" which the length is equal to the number of modeled principal component) specifying the linear trend of the kriging model (see <a href="#">lm</a> ) on each principal component. This formula should concern only the input variables (design), and not the output (response). The default is ~1, which defines a constant trend on each principal component.
design	a data frame representing the design of experiments. The ith row contains the values of the d input variables corresponding to the ith evaluation.
response	n object of class Fpca2d which contains eigen decomposition of the model/function output.
cov	a covariance kernel object or call (or a list of covariance kernel objects or call )

`estim` Logical. If TRUE, the model parameters are estimated by Maximum Likelihood. The initial values can then be specified using the `parCovIni` and `varNoiseIni` arguments of `mle,covAll`-method passed through dots. If FALSE, a simple Generalized Least Squares estimation will be used, see `gls,covAll`-method. Then the value of `varNoise` must be given and passed through dots in case noise is TRUE.

`...` other inputs of `gp`.

### Value

a list of object of class `gp` for each modeled principal component.

### See Also

`gp kergp`

### Examples

```
#####
### two-dimensional data set ###
#####
n<-200 # size of the learning sample
nz<-64; z <- seq(-90,90,length=nz) # spatial domain

### inputs of Campbell2D ###
library(lhs)
library(DiceDesign)

x <- maximinLHS(n=n,k=8)
X <-maximinSA_LHS(x)$design
X<-X*6 -1

# Campbell2D
Y <- Campbell2D(X,z,z)

# change X on data.frame
colnames(X)<-paste("x",1:8,sep="")

#####
### FPCA ###
#####

### by using wavelet basis ###
fpca_w<- Fpca2d(Y,method="Wavelets",
                wf="d4", J=1, # wavelet parameters
                ncoeff=1200, rank.=2) # FPCA configuration

#####
###          ###
### Kriging model ###
###          ###
#####

#-----#
#-----#
#   Example by using wavelet basis   #
#-----#
```

```

#-----#

#-----#
# Same kernel for all principal components #
#-----#

## kernel
myCov <- covTS(inputs = colnames(X),
kernel = "k1Matern5_2",
dep = c(range = "input"),
value = c(range = 0.4))

myGp<- gp_Fpca2d(design=X, response=fpca_w, cov=myCov,estim=FALSE)

#-----#
# Different kernel and formula for each principal component #
#-----#

## Not run:
## kernel of first principal component
myCov1<-myCov

## kernel of second principal component
myCov2 <- covTS(inputs = colnames(X),
kernel = "k1Matern3_2",
dep = c(range = "input"),
value = c(range = 0.4))

## List of both kernels
myCovList <- list(myCov1,myCov2)

## Gp model
myGp2<- gp_Fpca2d(formula=list(~1,~x1+x2+x3+x4+x5+x6+x7+x8),
design=X, response=fpca_w, cov=myCovList,estim=FALSE)

## End(Not run)
#####
### Prediction ###
#####

NewX<-matrix(runif(5*8,min=-1,max=5),ncol=8) # newdata
RealY <- Campbell12D(NewX,z,z)# real maps

# change NewX on data.frame
colnames(NewX)<-colnames(X)

#-----#
#-----#
# Example by using wavelet basis #
#-----#
#-----#
pw.UK <- predict(myGp,NewX,"UK")

#####
### Prediction RMSE and Q2 ###
#####

```

```

#-----#
#-----#
#   Example by using wavelet basis   #
#-----#
#-----#
err.pw.UK <-error.predict(RealY,pw.UK,fpca_w,rtx.scores=TRUE)

### scores ###
print(err.pw.UK$scores$rmse)
print(err.pw.UK$scores$Q2)

### images/maps ###
library(fields)
image.plot(err.pw.UK$y$rmse, main="RMSE")
image.plot(err.pw.UK$y$Q2, main="Q2")

```

---

Inverse2D

---

*Inverse transform of wavelet or orthonormal B-splines basis.*


---

## Description

embeds the coefficients of "Wavelet2D" or "OrthonormalBsplines2D" basis onto two-dimensional domain.

## Usage

```
Inverse2D(object, ...)
```

## Arguments

object	an object of class <a href="#">Wavelet2D</a> or <a href="#">OrthonormalBsplines2D</a> .
...	if object is an object of class <a href="#">OrthonormalBsplines2D</a> , the matrix of control points must be given (see <a href="#">Inverse2D.OrthonormalBsplines2D</a> ).

## Value

a three dimensional array. The first two dimensions correspond to maps dimensions. The third dimension corresponds to the size of the data set.

## Author(s)

Tran Vi-vi Elodie PERRIN

## See Also

[Inverse2D.OrthonormalBsplines2D](#) [Inverse2D.Wavelet2D](#)

---

Inverse2D.OrthoNormalBsplines2D

*Inverse transform of control points of two-dimensional B-splines basis*


---

**Description**

Inverse transform of control points of two-dimensional B-splines basis

**Usage**

```
## S3 method for class 'OrthoNormalBsplines2D'
Inverse2D(object, coeff, ...)
```

**Arguments**

object	an object of class <a href="#">OrthoNormalBsplines2D</a> with functions of the B-splines basis.
coeff	matrix with control points on two-dimensional B-splines basis.
...	other arguments.

**Value**

an array with two-dimensional approximated data.

**See Also**

[coef.OrthoNormalBsplines2D](#) [OrthoNormalBsplines2D](#) [Inverse2D](#)

---

Inverse2D.Wavelet2D

*Multiple two-dimensional inverse wavelet transform*


---

**Description**

performs inverse wavelet transform of each set of wavelet coefficients from a data set.

**Usage**

```
## S3 method for class 'Wavelet2D'
Inverse2D(object, ...)
```

**Arguments**

object	an object of class <a href="#">Wavelet2D</a> .
...	other arguments

**Value**

a three dimensional array. The first two dimensions correspond to maps dimensions. The third dimension corresponds to the size of the data set

**Author(s)**

Tran Vi-vi Elodie PERRIN

**See Also**[Wavelet2D](#)

km\_Fpca2d

*Gaussian Process Model on principal components of Fpca2d, by using DiceKriging package.***Description**

the function km of the [DiceKriging](#) package is use to fit kriging models on each principal component modeled on [Fpca2d](#) (for more details, see [km](#) ).

**Usage**

```
km_Fpca2d(
  formula = ~1,
  design,
  response,
  parallel = FALSE,
  covtype = "matern5_2",
  coef.trend = NULL,
  coef.cov = NULL,
  coef.var = NULL,
  nugget = NULL,
  noise.var = NULL,
  lower = NULL,
  upper = NULL,
  parinit = NULL,
  multistart = 1,
  kernel = NULL,
  ...
)
```

**Arguments**

formula	an object of class "formula" (or a list of "formula" which the length is equal to the number of modeled principal components) specifying the linear trend of the kriging model (see <a href="#">lm</a> ) on each principal component. This formula should concern only the input variables (design), and not the output (response). The default is ~1, which defines a constant trend on each principal component.
design	a data frame representing the design of experiments. The ith row contains the values of the d input variables corresponding to the ith evaluation.
response	an object of class Fpca2d which contains eigen decomposition of the model/function output.
parallel	a logical value specifying if parallelization is done on principal components. The default is FALSE.

covtype	optional character string or vector of character strings specifying the covariance structure to be used on each modeled principal component (see <a href="#">km</a> for possible inputs of covtype). If a vector, the length should be equal to the number of modeled principal components.
coef.trend, coef.cov, coef.var	optional vectors or matrices containing the values for the trend, covariance and variance parameters. If matrices, the number of rows should be equal to the number of modeled principal components. For details, see <a href="#">km</a> ).
nugget	an optional variance value or vector standing for the homogeneous nugget effect. If vector, the length should be equal to the number of modeled principal components.
noise.var	an optional vector or matrix containing the noise variance at each observation on each modeled principal component.
lower, upper	optional vectors or matrices containing the bounds of the correlation parameters of each principal component for optimization. For details, see <a href="#">km</a> ). If matrices, the number of rows should be equal to the number of modeled principal components.
parinit	an optional vector or matrix containing the initial values for the variables to be optimized over. For details, see <a href="#">km</a> ).
multistart	an optional integer indicating the number of initial points from which running the BFGS optimizer. (see <a href="#">km</a> ).
kernel	an optional function or list of functions containing a new covariance structure for each principal component. At this stage, the parameters must be provided as well, and are not estimated.
...	other parameters of <a href="#">km</a> function from DiceKriging. (see <a href="#">DiceKriging</a> )

**Value**

a list of object of class [km](#) for each modeled principal component.

**Author(s)**

Tran Vi-vi Elodie PERRIN

**See Also**

[km DiceKriging](#)

**Examples**

```
#####
### Learning Sample      ###
#####
n<-200 # size of the learning sample
nz<-64; z <- seq(-90,90,length=nz) # spatial domain

### inputs of Campbell2D ###
library(lhs)
library(DiceDesign)

x <- maximinLHS(n=n,k=8)
X <-maximinSA_LHS(x)$design
```

```

X<-X*6 -1

# Campbell12D
Y <- Campbell12D(X,z,z)

# change X on data.frame
colnames(X)<-paste("x",1:8,sep="")

#####
###   Test Sample       ###
#####

NewX<-matrix(runif(5*8,min=-1,max=5),ncol=8) # newdata
RealY <- Campbell12D(NewX,z,z)# real maps

# change NewX on data.frame
colnames(NewX)<-colnames(X)

#####
#
#   Example by using wavelets
#
#####

#####
### FPCA ###
#####

fpca_w<- Fpca2d(Y,method="Wavelets",
               wf="d4", J=1, # wavelet parameters
               ncoeff=1200, rank.=2) # FPCA configuration

#####
### Kriging model ###
#####

mw <- km_Fpca2d(design=X,response=fpca_w,control=list(trace=FALSE))

#-----
# (same example) To fix different kernel and formula
#               for each principal component
#-----
## Not run:
mw <- km_Fpca2d(formula=list(~.,~1)),
               design=X,response=fpca_w,
               covtype = c("matern5_2","matern3_2"),
               control=list(trace=FALSE))

## End(Not run)

#-----
# (same example) how to use the multistart argument of km
#-----
## Not run:
nCores <- 2
require(doParallel)

```



```

cl <- makeCluster(nCores)
registerDoParallel(cl)
mw <- km_Fpca2d(design=X,response=fpca_w,multistart=4,control=list(trace=FALSE))
stopCluster(cl)

## End(Not run)

#####
### Prediction ###
#####

pw.UK <- predict(mw,NewX,"UK")

#####
### RMSE and Q2 ###
#####

err.pw.UK <-error.predict(RealY,pw.UK,fpca_w,rtx.scores=TRUE)

### scores ###
print(err.pw.UK$scores$rmse)
print(err.pw.UK$scores$Q2)

### images/maps ###
library(fields)
image.plot(err.pw.UK$y$rmse, main="RMSE")
image.plot(err.pw.UK$y$Q2, main="Q2")

#####
#
# Example by using B-splines
#
#####

## Not run:
#####
### FPCA ###
#####

### using B-splines basis ###

# knots for B-splines basis
K<-35
z.knots <- seq(-90,90,length=K)

fpca_Bs<- Fpca2d(Y,method="Bsplines",
                 z1=z,z2=z,z1.knots=z.knots,z2.knots=z.knots, ortho="GS",
                 expand_knots=TRUE,# B-splines parameters
                 ncoeff=1225, rank.=5) # FPCA configuration

#####
### Kriging model ###
#####

mB <- km_Fpca2d(design=X,response=fpca_Bs,control=list(trace=FALSE))

```

```
#####
### Prediction ###
#####

pB.UK <- predict(mB,NewX,"UK")

#####
### RMSE and Q2 ###
#####

err.pB.UK <-error.predict(RealY,pB.UK,fpca_Bs,rtx.scores=TRUE)

### scores ###
print(err.pB.UK$scores$rmse)
print(err.pB.UK$scores$Q2)

### images/maps ###
library(fields)
image.plot(err.pB.UK$y$rmse, main="RMSE")
image.plot(err.pB.UK$y$Q2, main="Q2")

## End(Not run)
```

---

OrthoNormalBsplines2D *Creating a two-dimensional ortonormal B-splines basis*

---

## Description

Creating a two-dimensional ortonormal B-splines basis

## Usage

```
OrthoNormalBsplines2D(
  x,
  y,
  x.knots,
  y.knots,
  order = 2,
  ortho = "GS",
  expand_knots = TRUE,
  ...
)
```

## Arguments

<code>x, y</code>	two vectors of argument values at which the B-spline basis functions are to be evaluated.
<code>x.knots, y.knots</code>	The full set of knots used to define the basis functions.
<code>order</code>	Order of the spline fit (degree= order-1). The default is 2.

ortho	a character string which specify the orthogonalization method. If it is "Redd", the <a href="#">orthogonalsplinebasis</a> package is used. If it is "GS", the Gram-Schmidt is used. The default is "Redd".
expand_knots	a boolean. If it is TRUE, knots are expanded for appropriate number of knots in bsplines (see <a href="#">expand.knots</a> ). The default is TRUE.
...	other arguments (see <a href="#">SplineBasis</a> ).

**Value**

an array with two dimensional orthonormal B-splines. The two first dimensions correspond to the data dimensions. The last two dimensions correspond to the basis dimensions.

**See Also**

[coef.OrthoNormalBsplines2D](#) [Inverse2D.OrthoNormalBsplines2D](#) [Inverse2D](#)

**Examples**

```
#####
#
# Create two-dimensional data set
#
#####

# inputs of the Campbell2D function
x1<-rep(-1,8); x2<-rep(5,8); x3<-c(5,3,1,-1,5,3,1,-1)
X <- rbind(x1,x2,x3)

# spatial domain of the Campbell2D output
nz<-64 # root of the size of the spatial domain
z<-seq(-90,90,length=nz)

# Campbell2D function
Y = Campbell2D(X,z,z)

#####
#
# Create two-dimensional B-splines basis
#
#####

# knots for B-splines basis
K<-35
z.knots <- seq(-90,90,length=K)

# Generating a two-dimensional orthonormal B-splines basis
OPhi <-OrthoNormalBsplines2D(z,z,z.knots,z.knots,ortho="GS")

#####
#
# Get control points
#
#####

coeff_Y<-coef(OPhi,Y)
```

```
#####
#
# Approximation of Y
#
#####

hatY <-Inverse2D(OPhi,coeff_Y)
```

---

plot.Fpca2d

*Plot for Fpca2d object*


---

### Description

plots characteristic of Fpca2d decomposition : eigenfunctions, coefficients total mean proportion of energy, coefficients truncation etc.

### Usage

```
## S3 method for class 'Fpca2d'
plot(
  x,
  type = c("inertia", "energy", "MeanPoe"),
  PC = 1:2,
  p = seq(0.5, 1, by = 0.05),
  z1 = NULL,
  z2 = NULL,
  ...
)
```

### Arguments

x	an object of class "Fpca2d".
type	a character vector, which the characteristics of x are plotted. The default is c("inertia","energy","MeanPoe","SelectedCoeff"). It means that the barplot of proportion of inertia of each principal component (type="inertia"), the one of the number of coefficients estimated with PCA according to the total mean proportion of energy (type="energy"), the image of the coefficients mean proportion of energy (type="MeanPoe"), and the image which indicates which coefficients are estimated by PCA or by empirical mean (type="SelectedCoeff"). These graphics can be separately plotted. The eigenfunctions can be plotted by specifying (type="eigenfunctions").
PC	a vector with the numbers of which principal components are plotted. The default is c(1,2).
p	a vector with the proportion of energy which are plotted. The default is seq(0.5,1,by=0.05).
z1, z2	spatial coordinates.
...	plot parameters (see <a href="#">plot</a> , <a href="#">image</a> or <a href="#">image.plot</a> ).

---

predict.gp_Fpca2d	<i>Predict values and confidence intervals at newdata for a km_Fpca2d object</i>
-------------------	--

---

## Description

Predict values and confidence intervals at newdata for a km\_Fpca2d object

## Usage

```
## S3 method for class 'gp_Fpca2d'  
predict(object, newdata, type, compute = TRUE, ...)
```

## Arguments

object	an object of class <a href="#">gp_Fpca2d</a> .
newdata	a vector, matrix or data frame containing the points where to perform predictions.
type	a character string corresponding to the kriging family, to be chosen between simple kriging ("SK"), or universal kriging ("UK").
compute	an optional boolean. If FALSE, only the kriging mean is computed. If TRUE, the kriging variance (the corresponding standard deviation) and confidence intervals are computed too.
...	see <a href="#">predict.km</a> .

## Value

a list with the following items :

- `scores_predict` : prediction of scores from two-dimensional FPCA (see [Fpca2d](#) and [predict.km](#)).
- `mean` : an array with three dimensions which contains image (or map) kriging means. The two first dimensions corresponds to the output data dimensions. The third correspond to the number of predictions.
- `sd` : it is return only if `compute = TRUE`. If TRUE, an array which contains image (or map) prediction standard deviation.
- `lower95, upper95` : bounds of the 95 Not computed if `compute=FALSE`.

## See Also

[predict.km](#)

---

predict.km_Fpca2d	<i>Predict values and confidence intervals at newdata for a km_Fpca2d object</i>
-------------------	--

---

## Description

Predict values and confidence intervals at newdata for a km\_Fpca2d object

## Usage

```
## S3 method for class 'km_Fpca2d'
predict(object, newdata, type, compute = TRUE, ...)
```

## Arguments

object	an object of class <a href="#">km_Fpca2d</a> .
newdata	a vector, matrix or data frame containing the points where to perform predictions.
type	a character string corresponding to the kriging family, to be chosen between simple kriging ("SK"), or universal kriging ("UK").
compute	an optional boolean. If FALSE, only the kriging mean is computed. If TRUE, the kriging variance (the corresponding standard deviation) and confidence intervals are computed too.
...	see <a href="#">predict.km</a> .

## Value

a list with the following items :

- `scores_predict` : prediction of scores from two-dimensional FPCA (see [Fpca2d](#) and [predict.km](#)).
- `mean` : an array with three dimensions which contains image (or map) kriging means. The two first dimensions corresponds to the output data dimensions. The third correspond to the number of predictions.
- `sd` : it is return only if `compute = TRUE`. If TRUE, an array which contains image (or map) prediction standard deviation.
- `lower95, upper95` : bounds of the 95 Not computed if `compute=FALSE`.

## See Also

[predict.km](#)

Wavelet2D

*Multiple two dimensional wavelet transform***Description**

performs two dimensional wavelet transforms on all images/maps of a data set.

**Usage**

```
Wavelet2D(x, wf, J = 1, boundary = "periodic")
```

**Arguments**

x	a matrix (for one map) or a three dimensional array (for several maps). Two first dimensions correspond to maps dimensions. The third one is the number of maps.
wf	name of the wavelet filter to use in the decomposition.
J	depth of the decomposition, must be a number less than or equal to $\log(\min(M,N),2)$ , with M and N are respectively the number of rows and columns of each map.
boundary	a character string which specified the method used for side effect. Only "periodic" is currently implemented.

**Value**

an object of class "Wavelet2D", which corresponds to a matrix (for one map) or an array (for several maps) of same dimension as x, with the wavelet coefficients.

**Author(s)**

Tran Vi-vi Elodie PERRIN

**See Also**

[Inverse2D.Wavelet2D Inverse2D](#)

**Examples**

```
# inputs of the Campbell2D function
x1<-rep(-1,8); x2<-rep(5,8); x3<-c(5,3,1,-1,5,3,1,-1)
X <- rbind(x1,x2,x3)

# spatial domain of the Campbell2D output
nz<-64
z<-seq(-90,90,length=nz)

# Campbell2D function
Y = Campbell2D(X,z,z)

# Wavelet transform
w <-Wavelet2D(Y,wf="d4",J=2)

# Inverse wavelet transform
```

```
hatY <- Inverse2D(w)
```



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