# Sum-Product Networks

## Sum-Product Networks (SPNs)

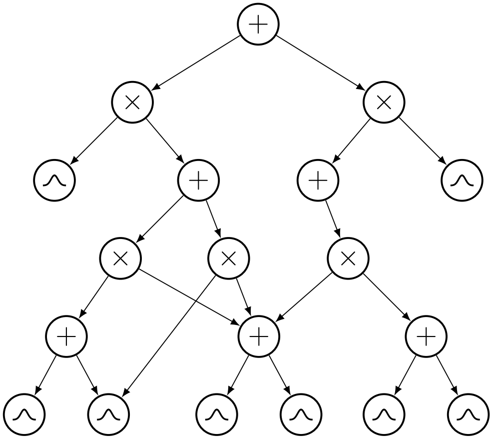
A sum-product network [Poon & Domingos 2011] over random variables $\boldsymbol{X}$ is a tuple $\mathcal{S} = (\mathcal{G}, \psi, \boldsymbol{w}, \theta)$, consisting of a *computational graph* $\mathcal{G}$, a *scope-function* $\psi$ [Trapp et al. 2019], network weights $\boldsymbol{w}$ and leaf node parameters $\theta$.

Why use SPNs? *Exact and efficient* computation of many probabilistic inference tasks, e.g. marginalisation.

SPNs are deep structured mixture models, generalising shallow mixtures, using an architecture that is a special type of a sparse *linear* neural network with non-linear inputs.

Every SPN is a multi-linear function in its weights.

# Computational Graph: $\mathcal{G}$



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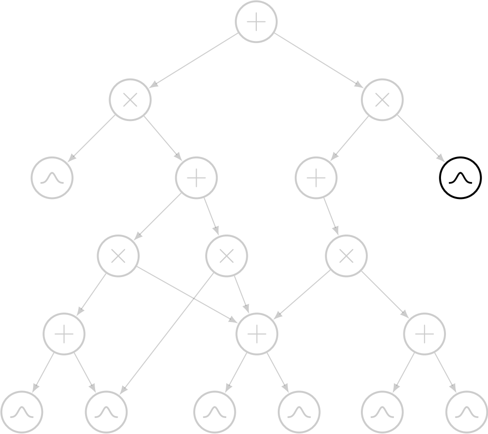
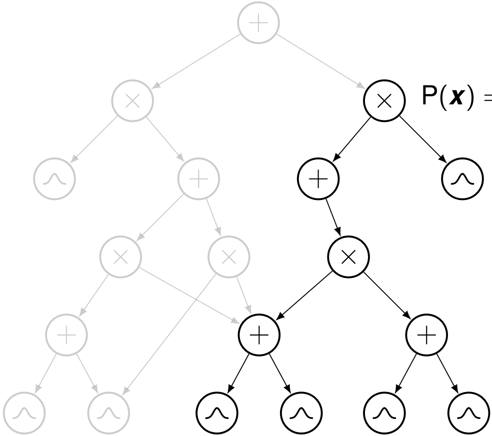


$\mathsf{L}(\boldsymbol{x}) = p(\boldsymbol{x}_\mathsf{L} \mid \theta_\mathsf{L})$

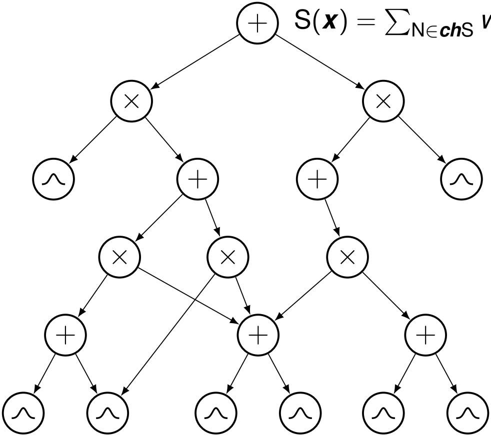
## Computational Graph: $\mathcal{G}$



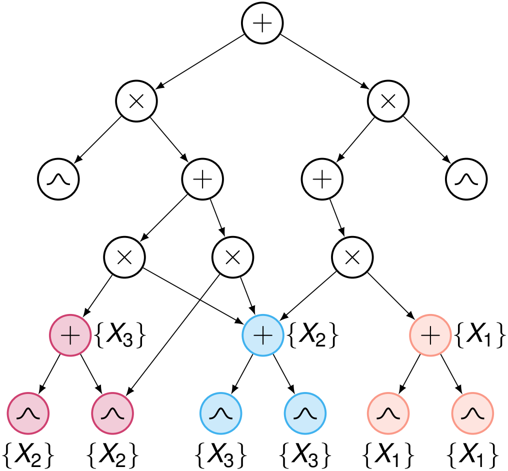
$\mathsf{P}(\boldsymbol{x}) = \prod_{\mathsf{N} \in \boldsymbol{ch}(\mathsf{P})} \mathsf{N}(\boldsymbol{x})$

# Computational Graph: $\mathcal{G}$

$$\mathsf{S}(\boldsymbol{x}) = \sum_{\mathsf{N} \in \boldsymbol{ch}\mathsf{S}} w_{\mathsf{S},\mathsf{N}} \mathsf{N}(\boldsymbol{x}) \quad (w_{\mathsf{S},\mathsf{N}} \geq 0)$$



# Scope Function: $\psi : \mathbf{N} \mapsto \mathscr{P}(\boldsymbol{X})$



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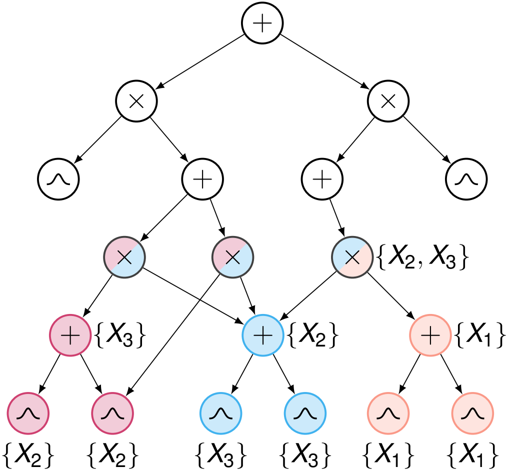

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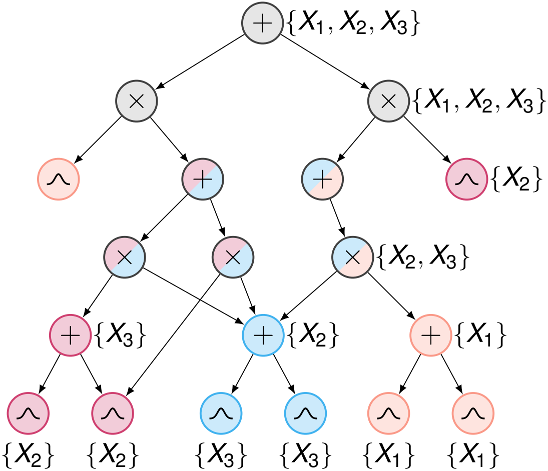
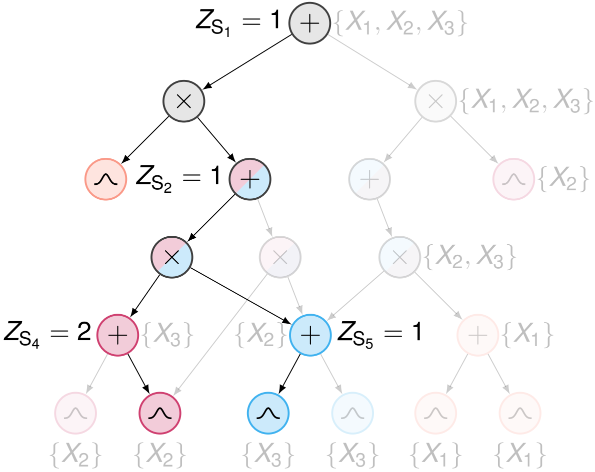


# SPNs as a Mixture of Induced Trees




$$\mathcal{S}(\boldsymbol{x}) = \sum_{i=1}^{K} \prod_{(\mathrm{S},\mathrm{N}) \in \mathcal{T}_{i,E}} w_{\mathrm{S},\mathrm{N}} \prod_{\mathrm{L} \in \mathcal{T}_{i,V}} p(\boldsymbol{x}_{\mathrm{L}} \mid \theta_{\mathrm{L}}) \tag{1}$$

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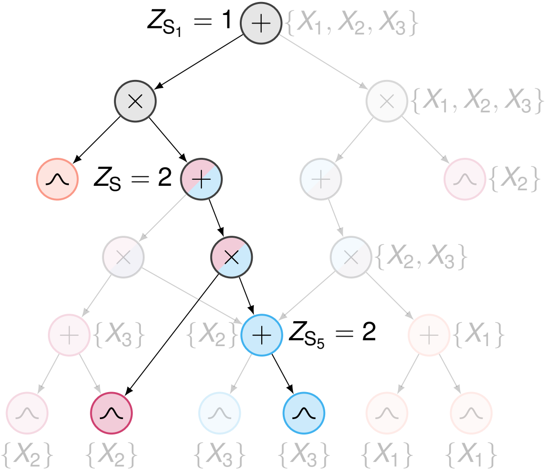


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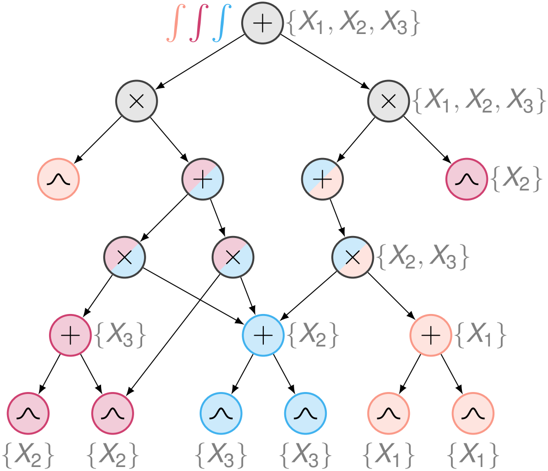
# Exact Marginalisation in SPNs



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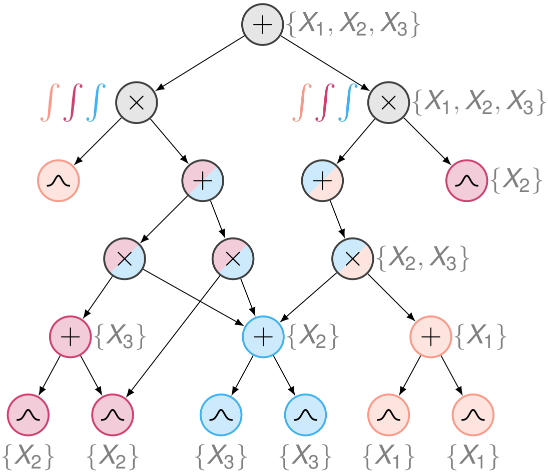

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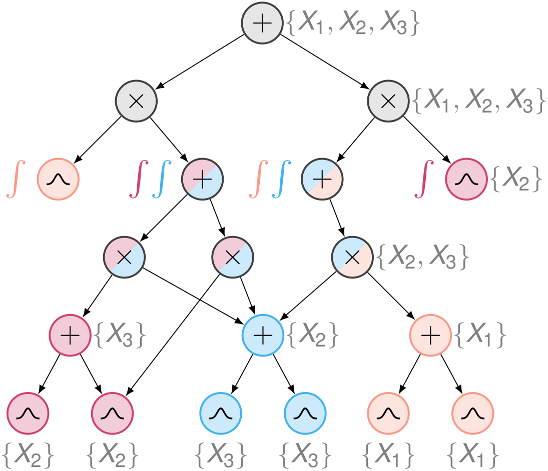

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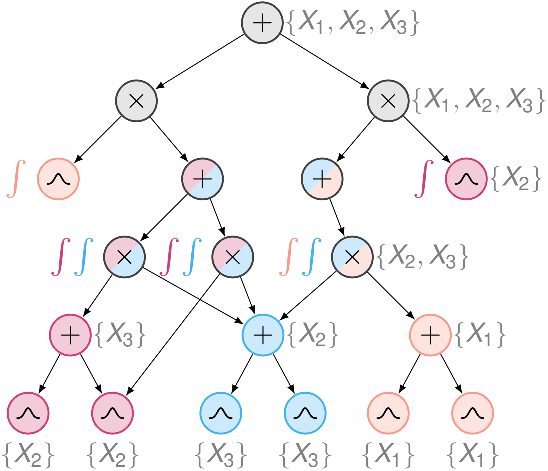

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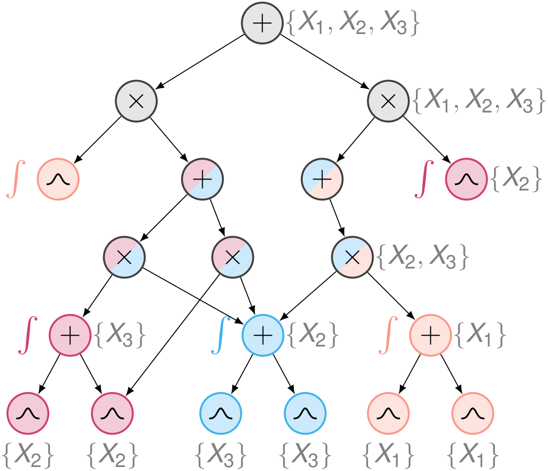

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