22.5 Prove that the series $\sum_{n=1}^{\infty} n/(n+1)!$ converges and find its sum.

Proof The following equation

$$\frac{n}{(n+1)!} = \frac{1}{n!} - \frac{1}{(n+1)!}$$

justifies

$$s_n = \sum_{i=1}^{n} \frac{i}{(i+1)!} = \sum_{i=1}^{n} \left(\frac{1}{i!} - \frac{1}{(i+1)!} \right) = 1 - \frac{1}{(n+1)!}$$

and

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1.$$

22.6 Suppose that the series $\sum_{n=1}^{\infty} a_n$ converges. Prove that the series $\sum_{n=p}^{\infty} a_n$ converges for every positive integer p and $\lim\limits_{p\to\infty} \sum_{n=p}^{\infty} a_n = 0$.

Solution Let $s_n = \sum_{i=1}^{n} a_i$, $s = \sum_{n=1}^{\infty} a_n$ and $s'_n = \sum_{i=p}^{n} a_i$ $(n \geq p)$.
First, the following equation

$$s'_n = s_n - s_{p-1}$$

holds where s_{p-1} is constant. so $\lim_{n\to\infty} s'_n = \sum_{n=p}^{\infty} a_n$ converges.
Second, in the same manner

$$\sum_{n=p}^{\infty} a_n = s - s_{p-1}$$

holds. No further explains is needed.