Inhomogen DGL

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Buch S. 169

6. $y' = -2x \cdot y + 2x$

$y_h' = -2x \cdot y$
$y^{-1} dy = -2x\, dx \quad \mid \int$
$\ln|y| = -x^2 + c$
$y = e^{-x^2} \cdot c$

$y = c(x) \cdot e^{-x^2}$
$y' = c'(x) \cdot e^{-x^2} + c(x) \cdot e^{-x^2} \cdot (-2x)$
$= c'(x) \cdot e^{-x^2} - 2x \cdot c(x) \cdot e^{-x^2}$

$c'(x) \cdot e^{-x^2} \cancel{- 2x \cdot c(x) \cdot e^{-x^2}} = \cancel{-2x\, c(x) \cdot e^{-x^2}} + 2x$

$c'(x) \cdot e^{-x^2} = 2x$

$c'(x) = 2x e^{x^2}$

$C(x) = e^{x^2}$

$2xe^{x^2}$

Durch Substitution-Methode

$\int 2x e^u \, dx$ $\qquad u = x^2$
$\qquad u' = 2x = \frac{du}{dx}$

$\int \cancel{2x} e^u \cdot \frac{1}{\cancel{2x}} \cdot du$

$\frac{du}{dx} = 2x$ nach dx auflösen

$\int e^u \, du$

$du = 2x \cdot dx$
$dx = \frac{1}{2x} \cdot du$

$[e^u] = u = x^2$

$\Rightarrow e^{x^2}$

$2xe^{x^2}$ $\qquad u = x^2$
$\qquad \frac{du}{dx} = u' = 2x$

$\int \cancel{2x} e^u \cdot \frac{1}{\cancel{2x}} \cdot du$

$du = 2x\,dx$ nach dx umstellen
$\frac{1}{2x} \cdot du = dx$

$\int e^u$

$e^u \Rightarrow e^{2x}$

$\Rightarrow y_1 = e^{x^2} \cdot e^{-x^2} = \frac{e^{x^2}}{e^{x^2}} = 1 \Rightarrow$ spezielle Lös

Allgemeine: $y_{INH} = y_1 + y_{Hom} \Rightarrow 1 + e^{-x^2} \cdot D$

d) $y = 1 + e^{-x^2} \cdot D$

$y(0) = 0 \Rightarrow D = -1$

$y' = -2x \cdot e^{-x^2}$

$y'' = -2 \cdot e^{-x^2} + -2x \cdot e^{-x^2} \cdot (-2x)$
$= -2 \cdot e^{-x^2} + 4x^2 \cdot e^{-x^2}$
$= e^{-x^2}(4x^2 - 2)$

$y'' \overset{!}{=} 0 \Rightarrow 4x^2 - 2 = 0$
$x_{1,2} = \pm\sqrt{\frac{1}{2}}$

$y = 1 + e^{-\frac{1}{2}} \cdot D$
$= \frac{D}{\sqrt{e}} + 1$

$W_{1,2}\left(\pm\sqrt{\frac{1}{2}} \;\middle|\; \frac{D}{\sqrt{e}} + 1\right)$

7. $y' = \frac{1}{x} y + x^3$

$y' = \frac{1}{x} y$

$dy = \frac{1}{x} \cdot y\, dx$

$y^{-1} dy = x^{-1} dx \quad \mid \int$
$\ln|y| = \ln|x| + c$
$y = e^{\ln|x| + c} = x \cdot D$

$y_{HOM} = x \cdot D$

$y_1(x) = c(x) \cdot x$

$y_1'(x) = c'(x) \cdot x + c'(x)$

$\Rightarrow c'(x) \cdot x + \cancel{c'(x)} = \cancel{\frac{1}{x}} \cdot \cancel{c(x) \cdot x} + x^3$

$c'(x) = x^2$

$c(x) = \frac{1}{3} x^3$

$y_{INH} = \frac{1}{3} x^3 \cdot x + x \cdot D$
$= \frac{1}{3} x^4 + x \cdot D$

10. a) $y' = x^{-2} \cdot y + 3 \cdot x^2 \cdot e^{-\frac{1}{x}}$

$y^{-1} dy = x^{-2} dx$

$\ln|y| = -x^{-1} + c$

$y_{Hom} = e^{-\frac{1}{x} + c} = e^{-\frac{1}{x}} \cdot c$

$y_1(x) = e^{-\frac{1}{x}} \cdot c(x)$

$y_1'(x) = c'(x) \cdot e^{-\frac{1}{x}} + c(x) \cdot e^{-\frac{1}{x}} \cdot x^{-2}$

$c'(x) \cdot e^{-\frac{1}{x}} + \cancel{c(x) \cdot e^{-\frac{1}{x}} \cdot x^{-2}} = \cancel{x^{-2} \cdot e^{-\frac{1}{x}} \cdot c(x)} + 3x^2 e^{-\frac{1}{x}}$

$c'(x) = \frac{3x^2 \cdot e^{-\frac{1}{x}}}{e^{-\frac{1}{x}}}$

$c'(x) = 3x^2$

$c(x) = x^3 \Rightarrow y_1 = e^{-\frac{1}{x}} \cdot x^3$

Allgem: $e^{-\frac{1}{x}} \cdot x^3 + e^{-\frac{1}{x}} \cdot C$

$y_{INH} = e^{-\frac{1}{x}}(x^3 + c)$

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b) $y' = 3y + x$

$\frac{dy}{dx} = 3y$

$y^{-1} \cdot dy = 3 \cdot dx \quad \mid \int$

$\ln|y| = 3x + c$

$y_{Hom} = e^{3x} \cdot c$

$y_1(x) = e^{3x} \cdot c(x)$

$y_1'(x) = c'(x) \cdot e^{3x} + c(x) \cdot e^{3x} \cdot 3$

$c'(x) \cdot e^{3x} + \cancel{3 \cdot e^{3x} \cdot c(x)} = \cancel{3 \cdot e^{3x} \cdot c(x)} + x$

$c'(x) = \frac{x}{e^{3x}} = x \cdot e^{-3x}$

$c(x) = \int \underset{u}{x} \cdot \underset{v'}{e^{-3x}}\, dx \qquad v = -\frac{1}{3} \cdot e^{-3x} \quad v' = e^{-3x}$

$= -\frac{1}{3} \cdot e^{-3x} \cdot x - \int -\frac{1}{3} \cdot e^{-3x}$

$= -\frac{1}{3} \cdot e^{-3x} \cdot x - \frac{1}{9} \cdot e^{-3x}$

$c(x) = e^{-3x}\left(-\frac{1}{9} - \frac{x}{3}\right)$

$y_1(x) = e^{3x} \cdot e^{-3x}\left(-\frac{1}{9} - \frac{x}{3}\right)$

$= -\frac{1}{9} - \frac{x}{3}$

$y_{INH} = e^{3x} + C - \frac{1}{9} - \frac{x}{3}$

c) $y' = \cos(x) - y$

$y' = -y \qquad \mid : y$

$y^{-1} dy = -1\, dx$
$\ln|y| = -x + c$
$y_{Hom} = e^{-x} \cdot c$

$y_1(x) = e^{-x} \cdot c(x)$
$y_1'(x) = e^{-x} \cdot c'(x) - e^{-x} \cdot c(x)$

$e^{-x} \cdot c'(x) \cancel{- e^{-x} \cdot c(x)} = \cos x \cancel{- e^{-x} \cdot c(x)}$

$c'(x) = \overset{u}{\cos x} \cdot \overset{v'}{e^x} \quad \mid \int$
$c(x) = \cos x \cdot e^x - \int \overset{v}{e^x} \cdot (-\sin x)$
$= \cos x \cdot e^x + \int e^x \cdot \overset{u}{\sin x}$
$= \cos x \cdot e^x + \sin x \cdot e^x - \int \cos x \cdot e^x\, dx = \int \cos x \cdot e^x \cdot dx = c(x)$
$\cos x \cdot e^x + \sin x \cdot e^x = 2 \cdot \int \cos x \cdot e^x\, dx \quad \mid : 2$

$c(x) = \frac{\cos x \cdot e^x + \sin x \cdot e^x}{2}$

$c(x) = \frac{e^x}{2} \cdot (\cos x + \sin x)$

$y_1(x) = \frac{1}{2} \cdot (\cos x + \sin x)$

$y_{INH} = y_1(x) + y_{Hom} = \frac{1}{2} \cdot (\cos x + \sin x) + \frac{C}{e^x}$