Influence of speculative behaviour on the real estate economic bubbles

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I. Introduction

As the recent history shows, dramatic and unnatural growth of the real estate prices may cause serious damage to the economies and is one reason of the financial crisis in the last few years.

Figure 1: UK nominal house prices between 1975 and 2012. [Source: Nationwide Building Society, http://www.nationwide.co.uk]

High prices on the real estate market make the flats not affordable for many people. Thus, financial companies create new products dedicated to people with weak credit history. And because many of the risky or "subprime" loans were repackaged into mortgage-backed securities, the problem loans spread to the financial sector, prompting lenders to become more cautious. [1]

Basing on the bottom-up agent-based modeling we want to investigate if the occurence of the real estate bubble can be explained only by speculative behaviour of the market players. If agents make decisions on the basis of the recent events, the process of the real estate prices should show the autocorrelations.

We want to check if the low interest rates increase and accelerate the effect of the economic bubble. We want to investigate as well if there is certain level of the interest rate such that speculative bubble are not likely to occur.

We will also see that if the demand is bigger than supply the prices and economic bubble are more significant.

We will investigate if the distribution of wealth has an effect on occurence of economic bubbles and in the number of mortgage loans taken, as well as on the final distribution of houses. In the end we will check if the average house prices on the market have significant autocorrelations.

In the next section we will briefly state what economic literature say about investigated phenomena. Later, we describe model and implementation. Finally we will show the results obtained from the model and finish with the conclusions.

II. Literature

Economists prove that the expansion of real estate speculation and the excess lending are conceived as the main reasons that bring about a real estate bubble. Real estate speculation brings treat to the sustainable, stable and balanced development of real estate market. [2]

Speculator agent is designed as a model of individual speculator, which has internal states and behavioral rules. Each speculator agent acts simultaneously at a time step as follows: [3]

1. Predict the future price,
2. Choose the sell or buy order,
3. Decide the order price,
4. Decide the order amount,
5. Submit the order to the market

In our simulation we will use very similar algorithm to determine the behaviour of the agents.

The influence of the interest rates on housing bubbles was also investigated in the literature. Hoth and Jokipii [4] basing on the research of the sample of 14 OECD countries show that the impact of short-term interest rates on housing bubbles is especially strong when they are "too low" for "too long".

III. Model

We model a population of N agents, which may be perceived as individuals or households. Each agent has a salary which is a Pareto distributed random variable. It's distribution function equals:

$$F(x) = 1 - \left(\frac{x\_m}{x}\right)^{\alpha}$$

where x_m is a minimum salary and α is a Pareto index.

In our model we assume that agents, in order to buy a property, take a loan. For each agent we calculate it's solvency, assuming that they do not take loans exceeding a certain fraction of their salaries. The highest amount one can borrow from the bank is then:

$$K = \frac{R}{q^n} \cdot \frac{q^n - 1}{q - 1}$$

where R is a monthly installment, n is a number of installments and $q = 1 + \frac{r}{12}$ and r is an annual interest rate. We will investigate how the level of r influences real estate prices growth.

To avoid an unnatural rush on buying in the beginning of simulation we provide each agent a determination parameter, which indicates if it is likely that in a certain moment this agent wants to trade. In the beginning we set this parameter to 0.1, which means that on average in 1 of 10 simulation step the agent will participate in auctions. However every simulation step the agent does not buy a flat we will increase this parameter by 0.01. It means that after 90 simulation steps all agents will have this parameter equal to 1 – so in fact they want to buy a flat then.

We assume that new flats arrive on the market according to Poisson process. The rate of this process will be set to ensure that total number of flats delivered in the simulation will be close to the number of agents. This can be considered as equalizing the demand and supply. A flat will have two properties: quality and size.

In our model each agents has it's own preferences on quality and size of the flat. These preferences will be described by vector (p_1, p_2), such that $p\_1 + p\_2 = 1$, where p_1 is a preference for quality and p_2 is a preference of size. Such description shows if the agent pays more attention to quality or to size of the flat. We use these parameters to create the unique utility function for each agent, according to the formula:

$$u = \sqrt{p\_1q + p\_2s}$$

there q is the quality of the flat and s is it's size. It's a concave function with respect to both q and s.

Our real estate trading mechanism will be based on auctions and collective intelligence. In each simulation step, if an agent wants to buy a property, he evaluates all available flats according to his utility function. Then he will choose the most suitable flat for him and check in the trading history of a market (which we assume is available for everyone) what was the price for the flat with similar utility in the last few simulation steps. Next, an agent makes an offer for the flat choosing the almost the same price. We allow the agent to make the offer up to $\pm 10\%$ different from this price. This difference will be chosen randomly. Of course if higher price is chosen by the agent it is more likely that his offer will be accepted. The agent who offer the most for a flat wins an auction and buys the flat.

For the first few months, when there is no information available we assume that the agents offer amount equal to $u \cdot K$. It means that they offer such part of their solvency as the utility function indicates. We also bound this value to avoid unnaturally high prices in the beginning, that could affect the whole simulation.

If an agent see that he cannot afford the most suitable flat, he chooses next, according to the utility function. If he cannot afford any available flat for ten months, he may decide to take a subprime mortgage, which monthly installment in the beginning is significantly lower. However, this kind of loan is more risky. In our model maximum subprime mortgage loan is given by formula:

$$K = R\frac{12}{r}$$

We allow agents to take this kind of loans after some time, not earlier. We want to see if occurence of this kind of loans changes the properties of our market.

If even this kind of loan does not help, an agent remains without an own flat. We will investigate how many agents are forced by the market to take subprime mortgages in the simulation and how many of them remains 'homeless'.

If it happens on the market, that the interest rate of return on properties is higher than interest rates of credits, it is good for the rich agents to start investing their money in real estate. To model this behaviour we will calculate in each step an average price of the sold properties. When such event occurs, all agents that still have enough solvency to buy properties will try to do it. We will see if this kind of speculative behaviour results in the economic bubble in our system.

Of course when the prices stop to grow the speculative behaviour is turned off and the market has again it's normal properties.

IV. Implementation

Although the implementation of this model can be done in several ways, we decided to choose MATLAB as a platform that allows fast calculations and has a lot of build in functions and properties that can be quite useful for a problem like ours. The fact that MATLAB is completely matrix based was quite helpful in some situations but then maybe a little bit confusing in other We will try to explain the implementation in the next few pages, giving appropriate pseudocode of the main program, since the other two are pretty straightforward. Before going through the months, first we want to create our agents, the participants of the auctions and set their parameters.

empty matrix agents[ ][ ]
FOR $t = 1$ to number_of_agents
agents[t][1]<-t
agents[t][2]<-salary
agents[t][3]<-solvency
agents[t][4]<-determination

```
agents[t][5]<-size_preferences
agents[t][6]<-quality_preferences
END
```

The script is designed in a way that for every month of the simulation we generate new flats, The distribution of the number of flats generated each month is made in a way that after 120 months the number of flats is similar to the number of agents in model. Each month we have a function call to ASgenerateflats, that sets all the needed parameters of the flats. At the beginning of each flat we want to see if the model enters the speculation mode. Also there is a need to determine how many of the agents are participating in the auction, as well as how many of the flats are free to be sold.

```
final_flats<-new flats for month m
empty matrix participating_agents[ ][ ]
empty matrix available_flats[][]
empty matrix utility[][]
empty matrix value_utility[][]
empty matrix index_utility[][]
    speculative_mode<-0;

IF comparation_interest_rate>interest_rate
speculative_mode<-1
END
    FOR i=1 to number_of_final_flats
        IF the i-flat is not yet sold
        available_flats<-final_flats(i)
        END
    END

FOR a=1 to number_of_agents
IF (speculative behavior is ON and agent a solvency is bigger than 0) or (agent a determination is
bigger than a random number in range[0,1])
participating_agents<-agents(a)
    FOR f=1 to available_flats
    utility[a][f]=sqrt(available_flats[f][size]*participating_agents[a][size_preference]+
                available_flats[f][quality]*participating_agents[a][quality_preference]);
    END
    [ value_utility, index_utility ]=sort(utility,decreasing_order)
ELSE
agents[a][determination]=agents[a][determination]+0.01
END
END
```

We have different ways of setting the prices offered in the auction as mentioned above in the description of the model. For the first 6 months we fill the price_flat matrix in which as rows we have the participating agents in a month auction and as columns the prices these agents offer for the flats, sorted in descending order. So the first column corresponds to the max prices each participating agent offer for a flat, regardless the ID of the flat. This sorting of the prices is also present in the p_flat matrix which is the one used in the remaining flats as it will be described below.

```
IF month>=1 and month=<6
FOR p=1 to number_of_participating_agents
    price_flat[p][1 to number_of_available_flats]<-value_utility[p][1 to number_of_available_flats]*
          participating_agents[p][solvency]
END
```

The lines of code that follow should present the actual auction. Still because this part is the same for the both cases, first let us see how we determine the prices of the flats for the other months. Before actually setting the prices of the flats, we should see if an agent needs a more risky loan, one that will increase his solvency- the subprime mortgage. We allow it only after 60 months of the simulation are passed.

```
ELSE
IF month>60
    FOR b=1 to number_of_participating_agents
        count: the number an participating_agent failed to gain a flat in the last 60 months
        IF count>10 and agent has no flat till now
            agents[participating_agents[b][ID]][solvency]<-bigger_solvency
            participating_agents[b][solvency]<-bigger_solvency
            mortgage<-participating_agents[b][ID], month
        END
    END
END
```

```
create empty array utility_differences[]
create empty matrix final_prices[][]
create empty array price[]
FOR a=1 to number_participating_agents
    FOR f=1 to number_available_flats
        FOR r=1 to number_ownership_information
            IF ownership_information==month-3 OR ownership_information=
           =month-2 OR ownership_information==month-1
               utility_differences[r]<-calculated utility differences between all the r flats in the last
               three months and the utility of the f flat of agent a
               price[r]<-min(price of flat r,solvency of agent)
           END
           index<-min(utility_differences)
           price_offer<-price(index)± price(index) *10%
           final_prices<- all price_offer values from agent a
        END
    END
    p_flat[a][f]=final_prices(a)
END
```

As one can see from this pseudocode we take the information about successful trancations from the last 3 months and then calculate the differences of the utilities these sold flats have with respect to the current participating agent and the utility of the agent's chosen flats for this month. The most similar utility is the one with minimal difference, so we choose the price offered for that flat, information that we get from the trading history or our ownership_information matrix. In this way we form the p_flat matrix that again has number of participating agents as rows and number of available flats as columns.

Now we can see how the auction part is implemented. As mentioned above it is the same principle for both cases of first 6 months and all other months. This pseudocode is applicable for the first 6 months case. One should only replace the price_flat matrix with p_flat matrix to get the auction for the rest of the months. Note that the number of auction in a month corresponds to the number of available flats in the same month. Each auction lasts until each column of the price_flat (or p_flat) matrix is equal to 0, meaning that in that way all the bids in auction are made. The idea is that in each moment the maximum price in the auction wins. The first auction is "strongest" which is quite expected as the prices are sorted by in descending order.

```
FOR num_auction=1 to number_of_available_flats
    WHILE bids are made in the num_auction auction
        max_bid=max( price_flat[1 to number_of_participating_agents][num_auction] )
        IF more than one agent offer max_bid
            id_random<-choose random agent id from ones who offer same price
            help_id_flat<-index_utility[id_random][num_auction]
            utility_sold_flat<-value_utility[id_random][num_auction]
            FOR f=1 to number_available_flats
                IF utility_sold_flat == utility[id_random][f]
                    id_sold_flat=available_flats[f][flat_id]
                END
            END
            owner_id<- participating_agents[id_random][agentID];
            ownership_information<-owner_id, id_sold_flat, utility_sold_flat, max_bid, month
            agents[owner_id][determination]<-0
            agents[owner_id][solvency]<- agents[owner_id][solvency]/2
            put 0s in all positions where help_id_flat is found in index_utility matrix
            put 0s in all positions where help_id_flat is found in price_flat matrix
            put 0s in all positions in the id_random row on price_flat matrix
            final_flats[id_sold_flat][owner]<-1
        ELSE
            /* case when only one agent wants the flat*/
            help_id_flat<-index_utility[id_agent][num_auction]
            utility_sold_flat<-value_utility[id_agent][num_auction]
            FOR f=1 to number_available_flats
                IF utility_sold_flat == utility[id_agent][f]
                    id_sold_flat=available_flats[f][flat_id]
                END
            END
```

```
            owner_id<- participating_agents[id_agent][agentID];
            ownership_information<-owner_id, id_sold_flat, utility_sold_flat, max_bid, month
            agents[owner_id][determination]<-0
            agents[owner_id][solvency]<-agents[agent_id][solvency]/2
            put 0s in all positions where help_id_flat is found in index_utility matrix
            put 0s in all positions where help_id_flat is found in price_flat matrix
            put 0s in all positions in the id_agent row on price_flat matrix
            final_flats[id_sold_flat][owner]<-1
        END
    END
END
```

At the end of each month simulation we also want to calculate how many of the participating agents have not bought a flat, as this information is useful to determine which agent can take subprime mortgage.

In this part of the code we also calculate the value of the interest rate of return of properties, which at the beginning of each month is compared to the interest rate to see if the model enters speculative mode or not.

```
rejected_agents<-participating_agents, month
FOR o=1 to size_of_ownership_information
    IF ownership_information[o][month]==month
         delete the agent from rejected_agents matrix, to keep inside only ones who did not
        bought a flat
    END
END
average_p<-mean of prices of the flats sold in a month
IF month>12
    comparation_interest_rate=average_p(month)/average_p(month-12)-1;
ELSE
    comparation_interest_rate=0;
END
```

These are the main concepts we used to program our model. To see the actual implementation of the model, one can see our scripts ASgenerateflats.m, ASpareto.m and simulation.m. Since the first two are pretty straightforward, in the implementation part we described the last one, one that holds the whole model actually. All the three scripts are well commented to provide the reader a better experience while going through the lines of the code.

V. Results

For each experiment we simulate a 10-year period of time with simulation step equal to one month. In total it gives us T=120 simulation steps. We assume that the agents take 30-years loans, minimum salary is 1000.

We allow agents to take subprime loans after 60 months.

Parameters of our model are:

Table 1: Parameters of the model used for simulations.

Parameter:	Description
α	Pareto index, responsible for the distribution of wealth
λ	arrival rate of the supply of flats
n	number of agents
r	interest rate of the loans

V.1 Interest rates

To describe the of influence of interest rates on the occurence of economic bubbles we simulate a market consisting of 1000 agents, where wealth is distributed according to Pareto 80-20 rule. It means that in that case (on average) 20% of agents own 80% of money ($\alpha = 1.16$).

We perform simulations for interest rates: 2%, 5%, 7%, 10% and 20%.

We will investigate scatterplots of all transactions made during the simulation to see how the prices change (not only average prices, but also high prices for luxury houses and low prices for small low-quality flats).

Let us first see the most evident case of the market, where interest rate equals 2% (Figure 2.). Here, the lending costs are very small and it it easy to convince agents that buying properties is a good investment. One could expect the occurence of really big economic bubbles.

In this simulation the average salary was 5219. In first few simulation steps we can see that bounding the prices help to restrain the behaviour of the agents. However, once this bound is turned off, the prices start to rise extremely quickly, so in next 18 months the maximal prices grow more than 100%.

When the prices stop to grow anymore, we can see that this growth was of the speculative nature. We observe then dramatic cut-off of the prices. We can say that they return to some kind of normal, equilibrium value of the market. For some time the prices are now stable and the growth is very slow. This is due to the fact that lot of capital was spent in the first economic bubble. The market has no more potential to blow up again.

After 60 months we allow agents to take subprime loans. What in fact we do is supply new money to the market. In the simulation steps 60-80 we can see the effect of this operation. A new bubble emerges – prices again grow more than 100%.

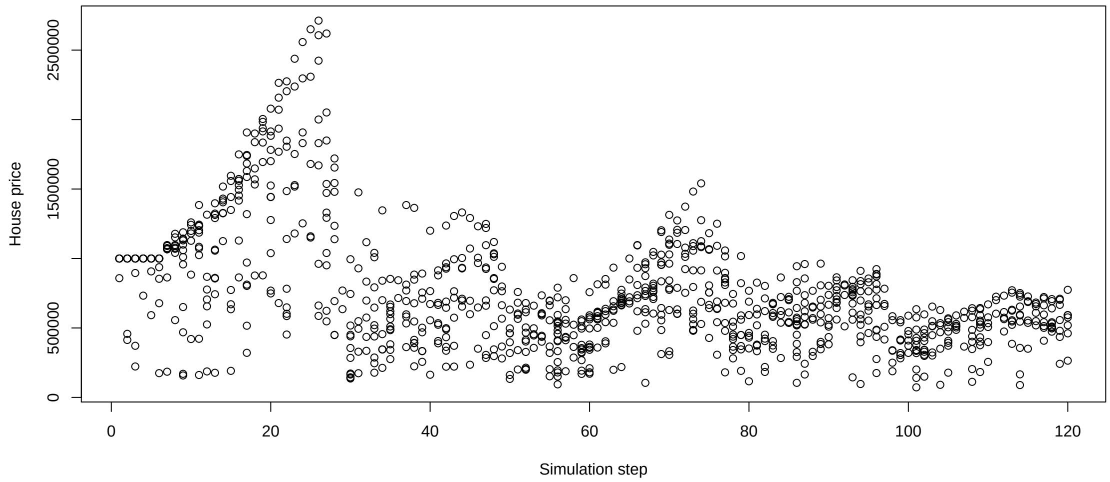

Figure 2: Trades made. Simulation of 120 steps, using 1000 agents, interest rate 2%

In the last simulation steps the market is rather stable. This is due to lack of the capital on the market, as we do not provide new agents during the simulation.

On the next page we presents results for interest rates 5%, 7%, 10% and 20% (Figure 3.). In the case of $r = 5$ we can observe again a large economic bubble in the first period. The prices again grow more than 100%. But after this bubble the market is much more stable than in the 2% case. What is more, the effect of subprime mortgages is less significant. One more thing that can be observed is an exponential trend of the prices. This is due to the decrease of capital on the market during the simulation.

In case of $r = 7\%$ we can see probably the most desirable behaviour of the market. Prices still vary, but it's not that significant as in the previous cases. The prices decrease gradually and the most smoothly.

When the interest rates are higher (10% or 20%), the market is totally suppressed. After first period of rather high prices, there is a dramatic cutoff and then prices remain very low and stable, as nobody can afford to pay more and it is very unlikely that the prices start to grow. Properties are not a good investment in that case, so speculative behaviour almost does not occur.

Finally, to compare the models we present a graph of average prices for all simulations run by now (Figure 4.). Here we can see the differences between economic bubbles obtained, especially in first simulation steps. This graph may be a convincing argument that interest rates have significant influence on the economic bubble phenomenon.

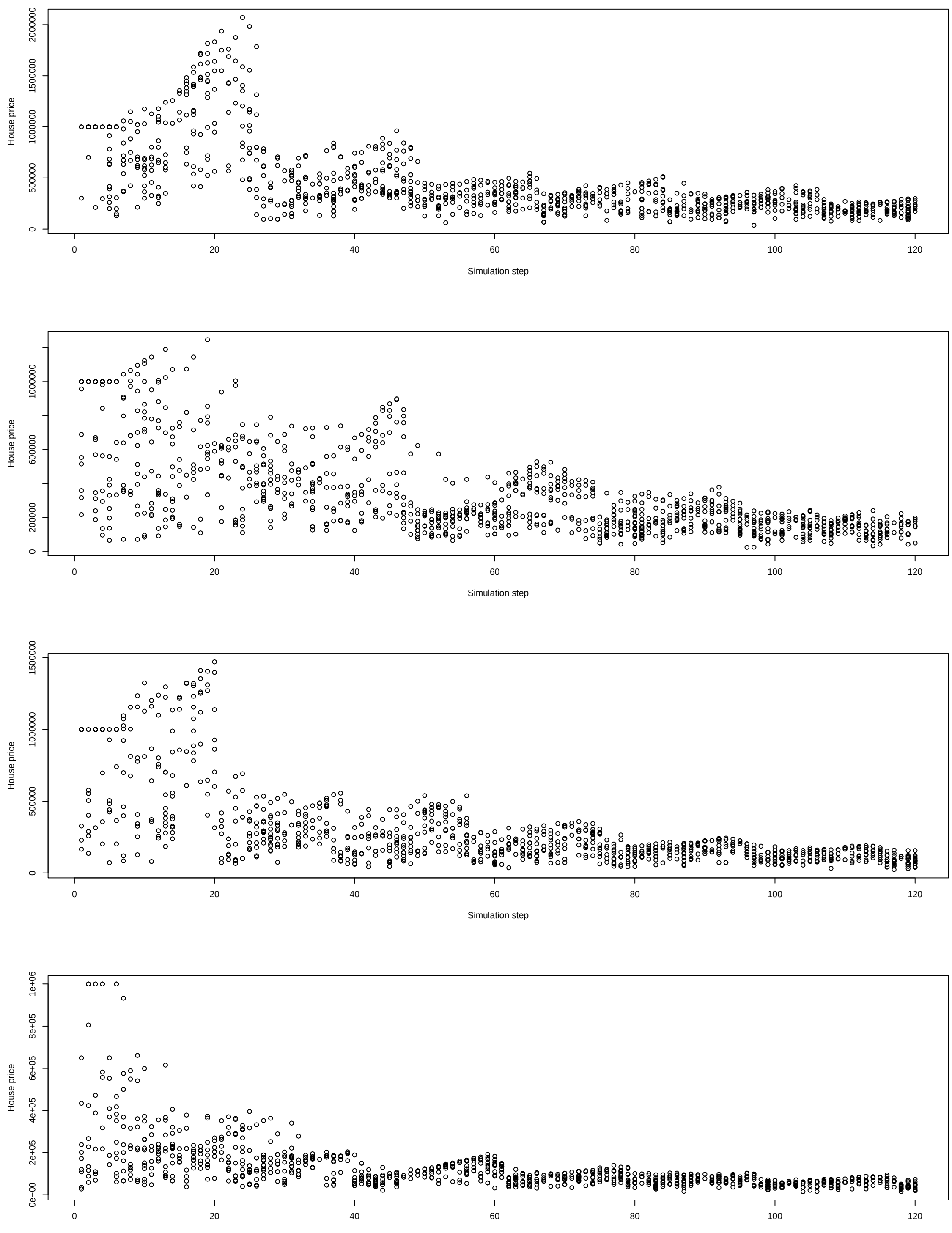

Figure 3: Trades made. Simulation of 120 steps, using 1000 agents, interest rate 5%, 7%, 10%, 20%, respectively.

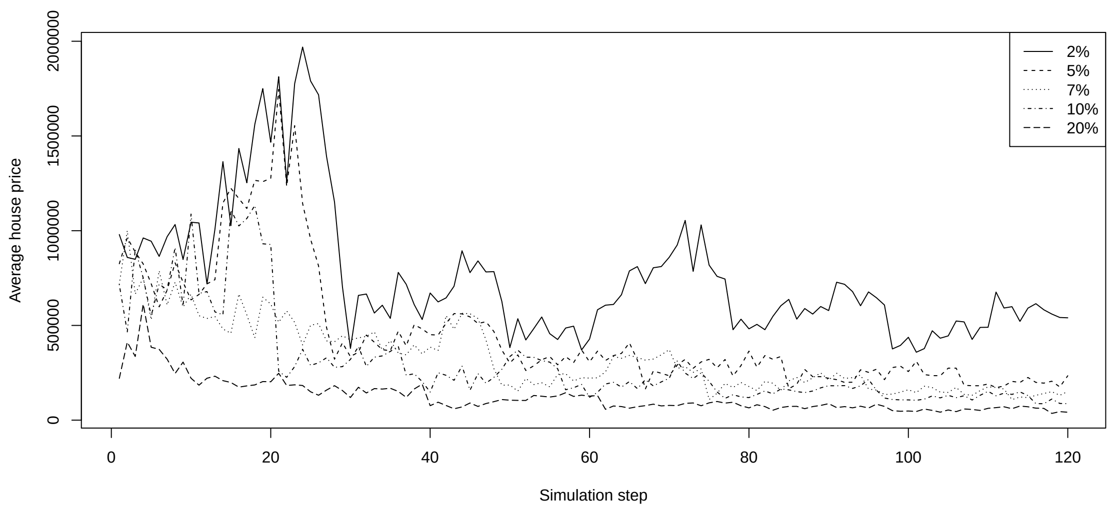

Figure 4: Average house prices for different interest rates. Simulation of 120 steps, using 1000 agents.

V.2 Demand and supply

Here we briefly show how the number of agents affects the prices on the market. We performed one more simulation, for $\alpha = 1.16$, $r = 5\%$, supply parameter $\lambda = 8.33$ remains the same, but this time there are 2000 agents on the market. Figure 5. presents the comparison of the average prices in these 2 simulations.

What is interesting, the first economic bubbles are very similar. Prices reach the same maximum level, then drastically decrease. Later we see that the behavior is quite similar as well. The only difference is, that on the market with 2000 agents prices are about twice as high as on the smaller market. This is maybe not a ground-breaking result, but it shows that if the demand is higher, the prices on a market are also higher. However we think that the effect on economic bubble is not really significant.

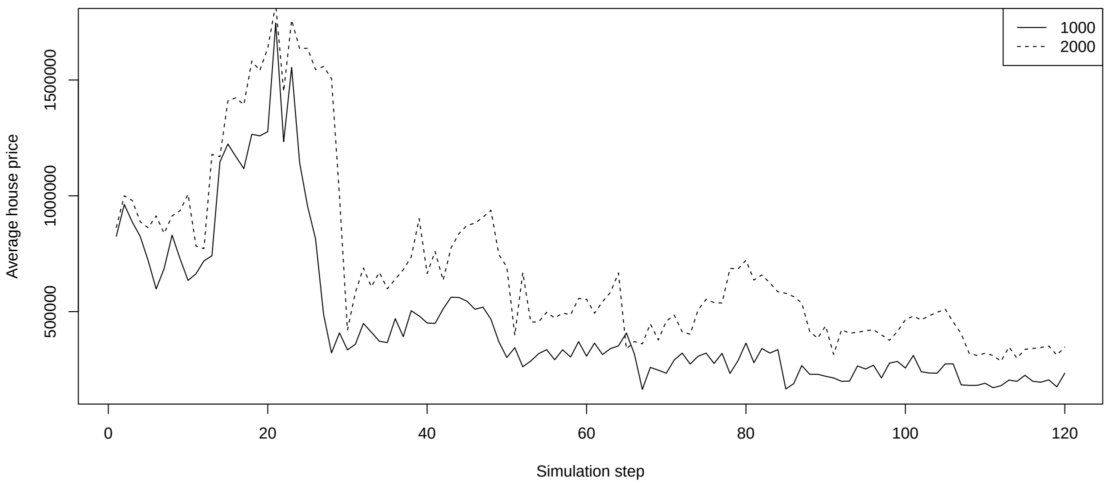

Figure 5: Average house prices for different number of agents. Simulation of 120 steps, interest rate 5%.

V.3 Distribution of wealth

In our final simulation we assumed that the distribution of wealth is more uniform. Instead of 80-20 rule we took the parameter $\alpha = 1.42$ according to the 70-30 rule. We use also $n = 1000$ agents and $r = 5\%$. It means that 30% of the population owns 70% of the money. Our society is now much more egalitarian.

First it is important to notice that the average salary is now equal to 2776, so it's almost 50% than in the first case. Therefore the prices will be actually smaller. Figure 6. presents all the transactions made. What is important, there is no expansion of the economic bubble in the beginning of the simulation anymore. Later, there is another growth of the prices. However, only high quality and big flats become more expensive. Standard flats' prices remain almost the same, which can be deduced from Figure 7. where we present average prices. There, the growth is less significant. After this, the market becomes very stable.

Figure 7. confirms that now the market is much more stable than in the first case. Therefore we may assume that uniform distribution of wealth reduces speculative behaviour on the market.

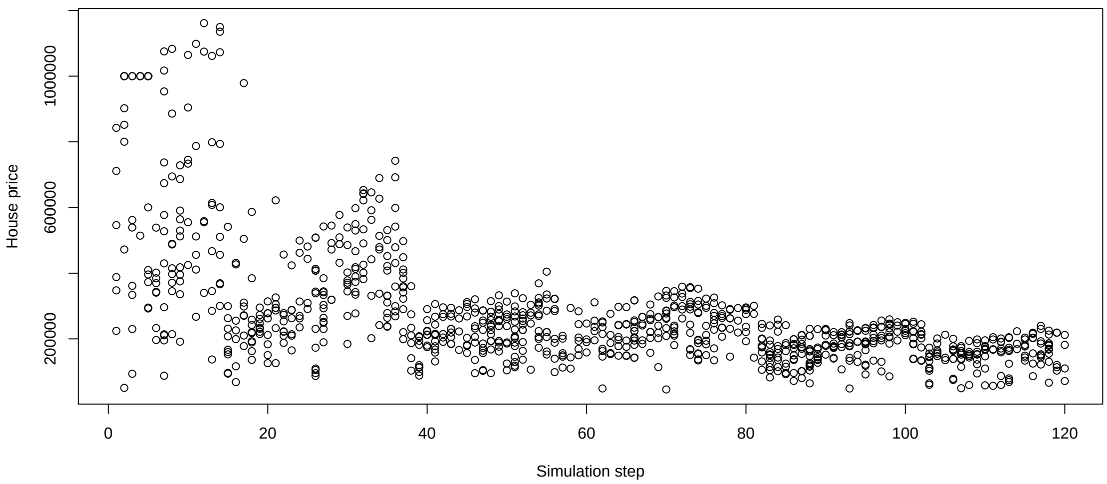

Figure 6: House prices for pareto index 1.42. Simulation of 120 steps, 1000 agents, interest rate 5%.

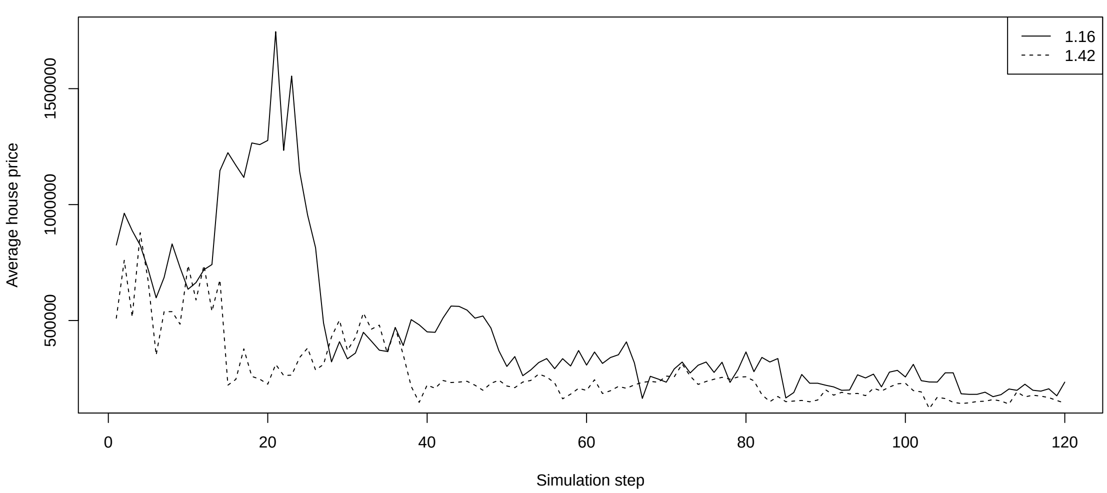

Figure 7: Average house prices for different Pareto indices. Simulation of 120 steps, 1000 agents, interest rate 5%.

V.4 Subprime loans and distribution of houses

In all our simulations we investigated also things: number of agents who could not buy a flat without taking subprime loans and the distribution of houses – it means, we checked how many agents after the simulation remained without their own property. Table 2. presents the results.

Table 2: Number of necessary subprime loans and homeless agents in the end of simulation for $\alpha = 1.16$, $n = 1000$, $\lambda = 8.33$, $r = 5\%$ *if not stated otherwise.*

Model parameter:	Subprime loans	Homeless agents
$r = 2\%$	68.9%	34,0%
$r = 5\%$	69,4%	42,9%
$r = 7\%$	69,2%	43,1%
$r = 10\%$	67,9%	46,6%
$r = 20\%$	69,0%	43,9%
$n = 2000$	82.9%	69.4%
$\alpha = 1.42$	66.9%	32.2%

What we can see is that the interest rate has almost no influence on the number of agents who cannot afford their own flat. However, we can see that low interest rates allow more agents to buy houses.

Next result we get is that when the demand is bigger than supply it causes serious problem to the agents to buy their own property. In that case government should take action to decrease the social consequences of the market properties.

Perhaps the most important conclusion of this part of the report is that uniform distribution of wealth cause also more uniform distribution of real estate. Potential speculative behaviour does not deprive other agents the possibility of finding affordable house.

V.5 Autocorrelations

Let S_t be the average house price at time t. We define then the logarithmic rate of return of the average house prices as [5]:

$$r\_t = \log\left(\frac{S_t}{S_{t-1}}\right) = \log(S_t) - \log(S_{t-1})$$

We will investigate if the process r_t resembles white noise or there are autocorrelations in the time series. This is an important property of our artificial market.

For the models with $n = 1000$ agents, $\lambda = 8.33$, $\alpha = 1.16$ and r =2%, 5%, 7%, 10%, 20% we test hypothesis H_0: process r_t resembles white noise against H_1: there are significant autocorrelation in the process.

We use Box-Pierce statistic:

$$Q_{BP} = n \sum_{k=1}^{h} \hat{\rho}_k^2$$

where $\hat{\rho}_l$ is an estimated value of autocorrelation function at lag k. This statistic has χ^2 distribution with h degrees of freedom. Table 3. presents values of the Box-Pierce test at significance level $\alpha = 0.05$.

Table 3: Box-Pierce test for correlations in logarithmic rates of return for the average house price for different interest rates, $h = 10$.

Interest rate:	Q_BP	p-value	null hypothesis
$r = 2\%$	14.9776	0.1329	not rejected
$r = 5\%$	15.3462	0.1199	not rejected
$r = 7\%$	25.9838	0.003762	rejected
$r = 10\%$	20.6408	0.02374	rejected
$r = 20\%$	18.7059	0.04416	rejected

For the interest rates 2$ and 5% there is not enough evidence to reject hypotheses which state that processes r_t resemble white noise.

It indicates that actually there are no significant correlations in the rates of returns on our markets. It means that it is hard to predict whether the trend will continue or will change. Even if, due to speculative behaviour, the prices grow, we cannot say when this process starts or when the bubble bursts.

The case of higher interest rates, where markets are more stable, have more significant correlations. We could use therefore autoregressive moving-average models as a description of these markets.

VI. Conclusions

In our researched we investigated the influence of speculative behaviour on the real estate bubble phenomenon. We were checking how interest rates, excess of demand over supply and distribution of wealth affects the behaviour of the agents, house prices, necessity of taking subprime loans and distribution of properties.

Our model used in simulation assumed that agents make decisions according to their individual preferences, history of the process and have speculative nature. This nature causes that agents were willing to invest their money in property when they notices that it brings profit.

In our experiments we have shown that:

1. speculative nature of the agents, who make decisions after analyzing the market in the past is enough to induce occurence of economic bubbles,
2. low interest rates heighten the effect of the economic bubble and reduce the stability of the market,

3. high interets rates cause that real estate is not a good subject to investment and the market is suppressed such that economic bubbles are not likely to occur and the prices are low and stable,
4. increase of the demand causes increase of the prices, but do not necessarily has influence on the size of the economic bubble,
5. distribution of wealth is an important factor affecting the occurence of economic bubble. What is more, uniform distribution of wealth contributes to more uniform distribution of houses,
6. interest rates and distribution of wealth does not affect the need of taking risky subprime loans – therefore markets will always have to deal with this risk, unless supply is significantly bigger than demand,
7. in case of low interest rates market is not stable and there is not enough evidence to state that return rates of the average house prices do not resemble white noise – therefore it is hard to predict the trend on the real estate market

VII. References

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