Deloitte.

Credit Risk - Economic Capital Model

ETH, March 2020

Agenda

1. Introduction

2. Economical Capital Model

3. Case Study

Introduction

What is a bank?

Basic overview of a bank

Assets
(e.g. loans)

Liabilities
(e.g. deposits)

How does a bank make money?

Interest Rates (Net Interest Income)

Fees and Commissions

Trading Revenues

What are the risks?

Overview of the key risks for a bank

Credit Risk	• Risk of a counterparty failing to meet its obligations (default) • Risk of not receiving promised payments
Market Risk	• Risk of loss in a financial position due to changes in the referenced underlyings (e.g. interest rates, price changes in stock, bond, commodity prices)
Liquidity Risk	• Lack of marketability of an investment that cannot be bought or sold quickly enough to prevent/minimize a loss • "Granny" wants money back
Operational Risk	• Risk of loss resulting from inadequate or failed internal processes, people and systems or from external events (such as cyber attacks) • Fines, Frauds, …

Quantifying risks

Overview of risk measures and metrics

Loss-based risk metrics	Description	Often risks can often be translated directly into "losses". Risk metrics can then be expressed in terms of **"extreme event" losses**.
	Example	• **Credit Risk**: Suppose a severe housing crisis occurs in Switzerland, how much money would the bank lose due to defaulting mortgages? (Stress Testing) • **Market Risk**: Assuming a certain distribution of the stock returns – what is the 99th percentile worst case return over a 1-year horizon? (Value at Risk)
Absorbing capacity	Description	• Sometimes, a risk is such that a particular threshold cannot be breached. Here one tests whether or not a breach would occur under a given stressed scenario.
	Example	• **Liquidity Risk:** Under a "bank run" scenario (e.g. Greece in 2011), will the bank have sufficient cash to pay-off its liabilities?

Risk and Regulation

Various regulations exist to prevent financial institutions from taking excessive risks

Q **Would you trust a bank to manage these risks without any supervision? (especially after 2009)**

➡ **Regulations** are introduced to ensure financial institutions "act responsibly"…

Consumer Protection	Anti Money Laundering	Capital Requirements	Market Infrastructure

Capital Requirements ensure that a bank has enough assets & liquidity set aside to withstand a new financial crisis.

Credit Risk	Market Risk	Liquidity Risk	Op. Risk

Regulatory Capital versus Economic Capital

Regulatory Capital Models

- Determines an institution's **minimum capital requirements**.
- Standardisation across institutions
- Used for regulatory reporting

Economic Capital Models

- **Company-specific** measure of risk
- Risk appetite setting
- Risk-adjusted return on capital: **economic profitability** of individual businesses, portfolios, regions and assets classes.

Economical Capital Model

Portfolio Credit Risk model
Example of portfolio credit losses

Context:

- A bank has a portfolio of many loans of various rating classes (credit quality) and different regions.
- Losses can occur in this portfolio due to loans defaulting.
  - Each loan has a particular **probability of default** (PD) depending on its rating class (highly rated loans have lower default probabilities)
  - In case of a loan defaulting, some proportion of the loss can still be recovered (e.g., in case of a mortgage, the bank can sell the underlying property to cover their losses). The loss of a defaulted loan is given by:

$$Loss_{loan} = Exposure_{loan} \cdot LGD_{loan},$$

Where the **loss given default** (LGD) is a number between 0% and 100%.

Aim:

- The bank wants to assess the **loss distribution** of the portfolio. From this they can identify:
  - **What are expected levels of loss?**
  - **How bad can the losses get in a severe scenario (quantile)?**

Portfolio Credit Risk model
Example of portfolio credit losses

- Tables below show portfolio composition and 1 year probability of default for each rating class.

Loan ID	Exposure (CHFm)	Rating	Region	LGD
Loan 1	10	AA	CH	60%
Loan 2	1	A	EU	70%
Loan 3	3	BB	CH	20%
…				

Rating	PD (1yr)
AAA	0.01%
AA	0.02%
A	0.06%
…	…
Default	100%

- The portfolio-level loss distribution is mathematically very complex:
  - Takes into account that certain loans are more likely to default than other (rating classes)
  - Takes into account correlations of loans defaulting (regional concentrations)
  - Takes into account the fact that very large loan exposures carry a higher risk (“single-name” concentrations)

➔Finding an analytical solution of the loss distribution is very tricky (and only works in very simple cases)

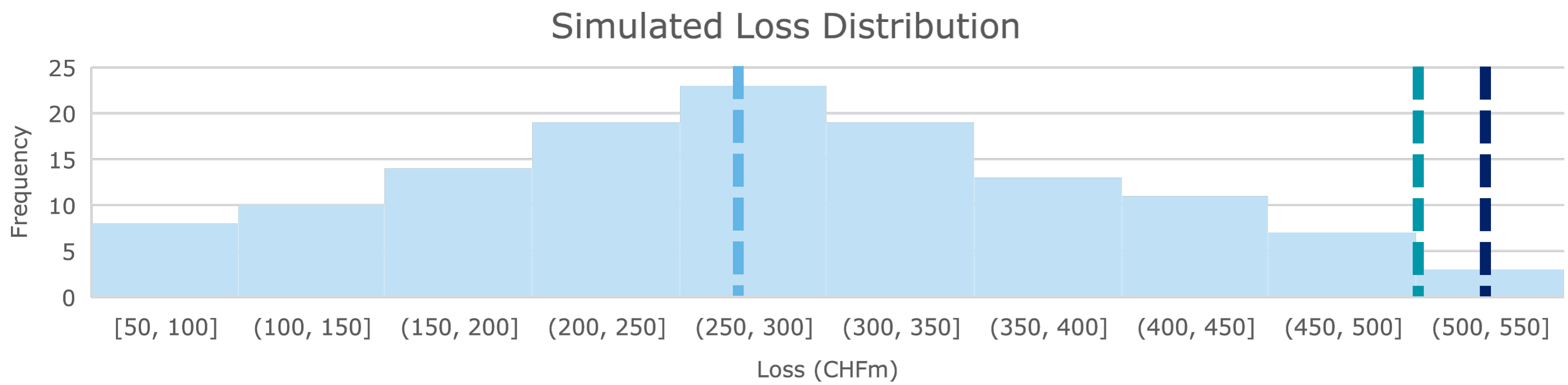
➔**Monte Carlo simulation** approach is used to estimate the loss distribution

	Scenario 1		Scenario 2	
Loan ID	**Status**	**Loss**	**Status**	**Loss**
Loan 1	Def.	6	No Def.	0
Loan 2	No Def.	0	No Def.	0
Loan 3	No Def.	0	Def.	0.6
…				
Total		**390**		**255**

Portfolio Credit Risk model

Loss Risk Measures

- Portfolio losses can be simulated numerous times, producing a simulated loss distribution



- There are different **risk measures** based on loss distributions:

Expected Loss

- Average portfolio simulated losses
- It can be calculated without performing a simulation
- EL is an **expected**-based risk measure

Value-at-Risk

- $VaR_\alpha(L)$ represents the α-th quantile of the cumulative distribution function of L (F_L), i.e. $VaR_\alpha(L) = \inf\{x \in \mathbb{R} | F_L(x) \geq \alpha\}$
- Example: $VaR_{0.95}(L)$ represents the portfolio losses that are expected to be exceeded only once every 20 years.
- VaR is a **frequency**-based risk measure

Expected Shortfall

- $ES_\alpha(L)$ represents the expected losses above $VaR_\alpha(L)$, i.e. the average losses that are higher than $VaR_\alpha(L)$. It is given by $ES_\alpha(L) = \mathbb{E}(L]L > VaR_\alpha(L))$
- Example: $ES_{0.95}(L)$ represents the average portfolio losses that are expected to occur once every 20 years.
- ES is a **severity**-based risk measure

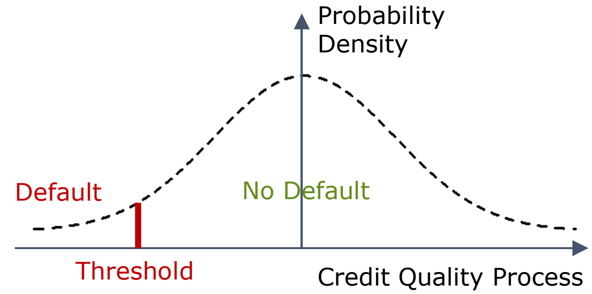
Portfolio Credit Risk model
Simulating default for a single loan

Define for each loan i a "**Credit Quality Process**" X_i so that:

- $X_i \sim N(0,1)$
- $Loan_i$ defaults within 1 year if and only if $X_i < threshold_i$, for some threshold

We can then simulate defaults of loan i as follows:

1. Generate an observation of X_i
2. If $X_i \geq threshold_i$ then no default occurs, else $Loan_i$ defaults



Determining the threshold $threshold_i$

- For each loan i we have a rating, which corresponds to a 1-year default probability (PD_i)
- We have that:

$$\begin{aligned} PD_i &= \mathbb{P}[X_i < threshold_i] \\ &= \Phi(threshold_i) \qquad \text{as } X_i \sim N(0,1) \end{aligned}$$

- Hence: $$threshold_i = \Phi^{-1}(PD_i)$$

➔ Given the rating class (and hence the PD) of each loan, we can determine the threshold

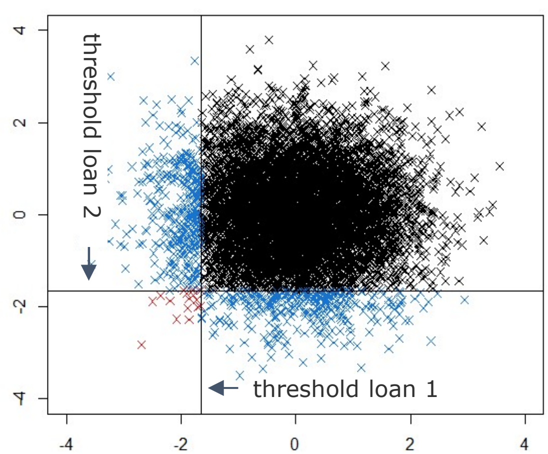
Portfolio Credit Risk model
Simulating joint defaults (1/3)

Consider two loans: $Loan_i$ and $Loan_j$

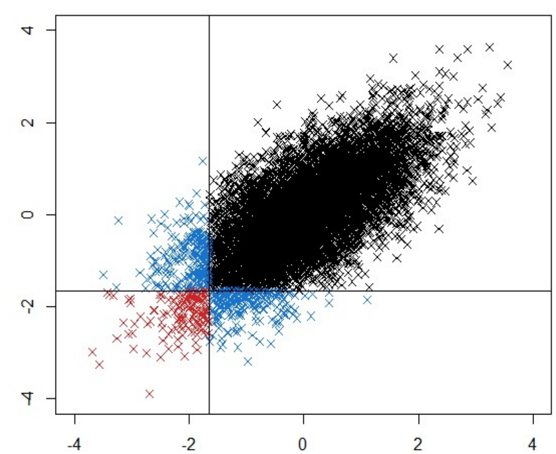
- If the default of $Loan_i$ and $Loan_j$ would be completely independent, then we could just simply simulate X_i and X_j independently and determine joint default observations
- However, suppose that $Loan_i$ and $Loan_j$ are both mortgages with underlying properties in Zürich. A significant drop in Zürich house prices could lead to both loans defaulting.

➔ **Loan defaults in similar regions or industries are typically correlated**

Returns & Defaults for two uncorrelated loans



Returns & Defaults for two correlated loans



Portfolio Credit Risk model
Simulating joint defaults (2/3)

Capturing default correlations: Factor Model Approach

Correlations are introduced by **systematic risk factors** corresponding to specific regions (or industries). For our example, we express:

$$X_i = \alpha_{CH,i} \cdot Z_{CH} + \alpha_{EU,i} \cdot Z_{EU} + \alpha_{US,i} \cdot Z_{US} + \gamma_i \cdot \epsilon_i$$

Where:

- Z_{CH}, Z_{EU}, Z_{US} are regional **systematic risk factors**. We assume that each risk factor are standard normally distributed ($\sim N(0,1)$) and correlated with covariance Σ (next slide)
- $\epsilon_i \sim N(0,1)$ is the loan-specific **idiosyncratic risk factor**
- $\alpha_{CH,i}$, $\alpha_{EU,i}$, $\alpha_{US,i}$, γ_i are loan-specific systematic and idiosyncratic factor loadings

Note the indices:

- The systematic factor Z_{region} is the same for each loan i
- The idiosyncratic factor is unique for each loan i
- The factor loadings ($\alpha_{region,i}$ and γ_i) are loan specific

Portfolio Credit Risk model
Simulating joint defaults (3/3)

Correlations of systematic factors:

The systematic risk factors follow a *multi-variate normal* distribution:

$$(Z_{CH}, Z_{EU}, Z_{US}) \sim N(\mathbf{0}, \Sigma)$$

The covariance matrix Σ is given.

	CH	EU	US
CH	1	$\rho_{CH,EU} = 0.6$	$\rho_{CH,US} = 0.3$
EU	$\rho_{CH,EU} = 0.6$	1	$\rho_{EU,US} = 0.4$
US	$\rho_{CH,US} = 0.3$	$\rho_{EU,US} = 0.4$	1

Constraint on factor loadings

Since we require $X_i \sim N(0,1)$, we need:

- $\mathbb{E}[X_i] = 0$. This is trivial (as Z_{region} and ϵ_i have zero mean, by construction)
- $Variance[X_i] = 1$

➔ $\alpha_{CH,i}^2 + \alpha_{EU,i}^2 + \alpha_{US,i}^2 + 2 \cdot \rho_{CH,EU} \cdot \alpha_{CH,i} \cdot \alpha_{EU,i} + 2 \cdot \rho_{CH,US} \cdot \alpha_{CH,i} \cdot \alpha_{US,i} + 2 \cdot \rho_{EU,US} \cdot \alpha_{EU,i} \cdot \alpha_{US,i} + \gamma_i^2 = 1$

Note, in our example, only one of the three regional factor loadings ($\alpha_{region,i}$) will be non-zero (i.e., a loan can only be in one of the three regions at one time).

Suppose e.g., the loan is in CH (so $\alpha_{CH\,i} \neq 0$ and $\alpha_{EU\,i} = 0$, $\alpha_{US\,i} = 0$), then the above constraint on the factor loadings yields:

$$\gamma_i = \sqrt{1 - \alpha_{CH,i}^2}$$

Portfolio Credit Risk model

Example of a loss simulation (1/4)

Starting point: We are given the following information:

Loan ID	Exposure (CHFm)	Rating	Region	LGD	α_{region}
Loan 1	10	AA	CH	60%	0.5
Loan 2	1	A	EU	70%	0.3
Loan 3	3	BB	CH	20%	0.5
Loan 4	12	CCC	US	80%	0.3
…					

Rating	PD (1yr)
AAA	0.01%
AA	0.02%
A	0.06%
BBB	0.64%
BB	1.48%
B	5.68%
CCC	7.58%
Default	100%

Step 1: Extend dataset to include relevant variables (γ_i, PD_i and $Threshold_i$)

Loan ID	Exposure (CHFm)	Rating	Region	LGD	α_{region}	γ_i	PD_i	$Threshold_i$
Loan 1	10	AA	CH	60%	0.5	**0.87**	**0.02%**	**-3.54**
Loan 2	1	A	EU	70%	0.3	**0.95**	**0.06%**	**-3.24**
Loan 3	3	BB	CH	20%	0.5	**0.87**	**1.48%**	**-2.18**
Loan 4	12	CCC	US	80%	0.3	**0.95**	**7.58%**	**-1.43**
…								

Portfolio Credit Risk model

Example of a loss simulation (2/4)

Step 2: Monte Carlo Simulation

Repeat the following four steps several times (i.e., we generate several different scenarios):

1. Generate an observation for each of the systematic risk factors: Z_{CH}, Z_{EU} and Z_{US}

   ➔ a random draw from a multivariate normal distribution: $(Z_{CH}, Z_{EU}, Z_{US}) \sim N(\mathbf{0}, \Sigma)$

$$\Sigma = \begin{pmatrix} 1 & 0.6 & 0.3 \\ 0.6 & 1 & 0.4 \\ 0.3 & 0.4 & 1 \end{pmatrix}$$

Risk Factor	Scenario Realisation
Z_{CH}	-1.94
Z_{EU}	-2.24
Z_{US}	-0.82

2. Generate an idiosyncratic risk factor realisation (ϵ_i) for each loan i

   ➔ Draw independent, standard normally distributed numbers for each loan

Loan ID	Scenario realisation of ϵ_i
Loan 1	0.17
Loan 2	0.70
Loan 3	-2.50
Loan 4	-1.93
…	

Portfolio Credit Risk model
Example of a loss simulation (3/4)

Step 2: Monte Carlo Simulation (continued)

3. Determine loan defaults and corresponding losses
   - Calculate the loan-level Credit Quality Processes: $X_i = \alpha_{region,i} \cdot Z_{region} + \gamma_i \cdot \epsilon_i$
   - Assess whether a default occurs ($X_i < Threshold_i$)
   - Calculate losses for defaulted loans ($Loss_i = LGD_i \cdot Exposure_i$)

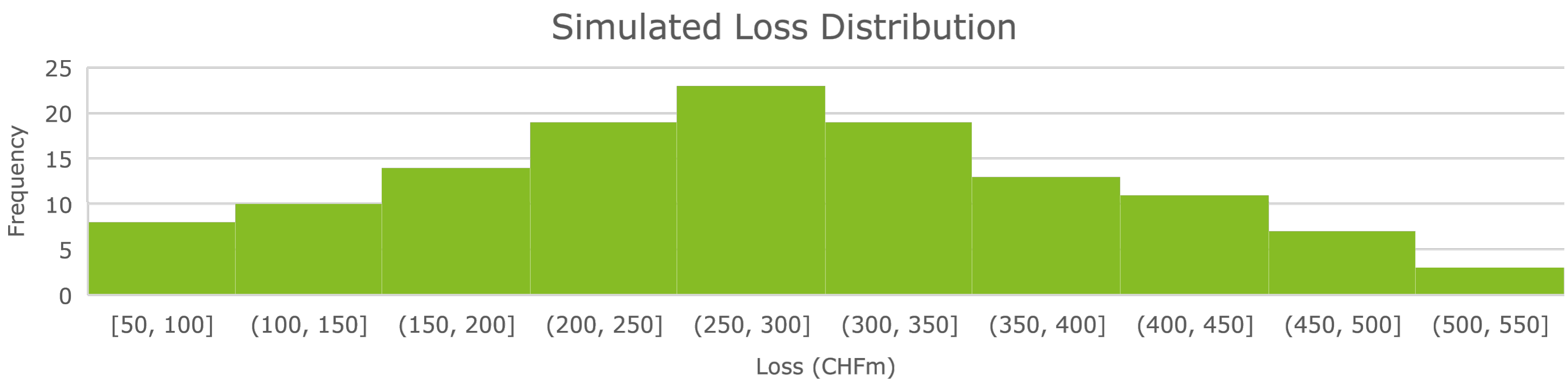
	Exposure (CHFm)	LGD	$Threshold_i$	X_i	Default	Loss (CHFm)
Loan 1	10	60%	-3.54	-0.82	No	0
Loan 2	1	70%	-3.24	0.00	No	0
Loan 3	3	20%	-2.18	-3.14	Yes	0.6
Loan 4	12	80%	-1.43	-2.19	Yes	9.6
…						

4. Calculate and store scenario aggregate loss
   - Sum over all losses within scenario

Portfolio Credit Risk model
Example of a loss simulation (4/4)

Step 3: Assessment of aggregated losses

- Plot histogram of (portfolio-aggregated) scenario losses



- Calculate risk metrics
  - Value at Risk (95^{th} and 99^{th} quantile)
  - Expected Shortfall (95^{th} and 99^{th} quantile)
  - Expected loss

Case Study

Case Study

Exercise (1/2)

- The aim of this case study is to simulate 1 year default losses for a portfolio containing loans from different regions, with different credit ratings and exposures and to calculate loss risk measures.
- The inputs available are:
  - **Portfolio:** list of loans containing the following information: region, rating, exposure and LGD. The portfolio is stored in a csv file
  - **Probability of Default:** table containing PDs for each rating class (csv file)
  - **Factor loadings:** file containing factor loadings for each loan (csv file)
  - **Systematic Risk Factors Correlation Matrix:** file containing correlations between systematic risk factors
- In order to complete the case study, the following steps need to be produced:

Step 1	**Data preparation**	• Upload inputs into your prototype • Correctly map PDs and factor loadings to each loan. Store combined information in a single table • Calculate default thresholds for each loan • Calculate idiosyncratic loading for each loan.
Step 2	**Monte Carlo Simulation**	• Generate 100’000 realisations of systematic and idiosyncratic risk factors (consider systematic risk factors correlation matrix) • Identify defaulted loans in scenario • Determine loan and portfolio level scenario losses
Step 3	**Risk Measures**	• Loss distribution histogram • Calculation of risk metrics (expected loss, VaR and ES at 95th and 99th quantile)

Case Study
Exercise (2/2)

Output:

As deliverable of the assignment, please provide the following:

- A histogram/plot of the loss distribution
- A table with the various risk metrics:

Metric	Result
Expected Loss	…
VaR (95%)	…
VaR (99%)	…
ES (95%)	…
ES (99%)	…

- The time it takes to run the Monte Carlo simulation
- Programming language used
- CPU details: number of cores & processing speed (e.g., 2.5 GHz)

Case Study

Optional Extra Teaser

If you have additional time available and you are intrigued by the model, have a think about the following:

- **Stability of risk metrics**: how would you assess the convergence of the risk measures, i.e. Monte Carlo error? Is 100’000 simulations acceptable?
- **Risk contributions**: a bank would want to assign the expected shortfall metric to the regional, rating class and loan level. How can this be achieved?
- **Model extensions:** suppose a bank has a more complex portfolio including different types of investment
    - Capturing industry concentrations
    - Credit insurances against specific counterparties
    - Various loans to various entities of the same company (e.g. IBM Spain and IBM Germany).

References

- Vasicek, O. (2002). The distribution of loan portfolio value. *Risk,* 15(12), 160-162.
- Pykhtin, M. (2004). Portfolio credit risk multi-factor adjustment. *Risk*, 17(3), 85-90.
- Schönbucher, P. J. (2000). Factor models for portofolio credit risk (No. 16/2001). *Bonn Econ Discussion Papers*.