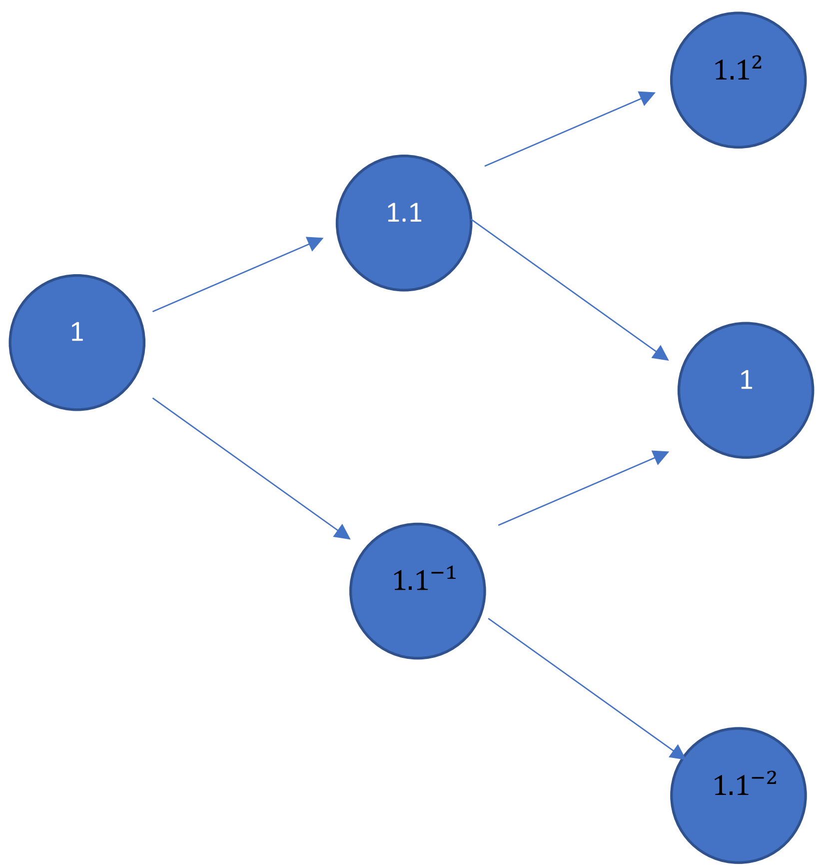
APM466 Assignment 2

Objective:

- calculate the price of 4-upswing option, 4-downswing option
- show optimal exercise nodes in the option trees

Diagram of the underlying prices for the first 3 periods:



Risk-neutral probabilities

Let's first find the risk-neutral probabilities of all possible movements. We will assume zero interest rates.

At any node with price p_i, the next up state is $1.1p_i$, the down state is $1.1^{-1}p_i$. Thus, we get the following equation:

$$p_i = P(u) \cdot 1.1^{-1}p_i + P(d) \cdot 1.1^{-1}p_i \Rightarrow$$

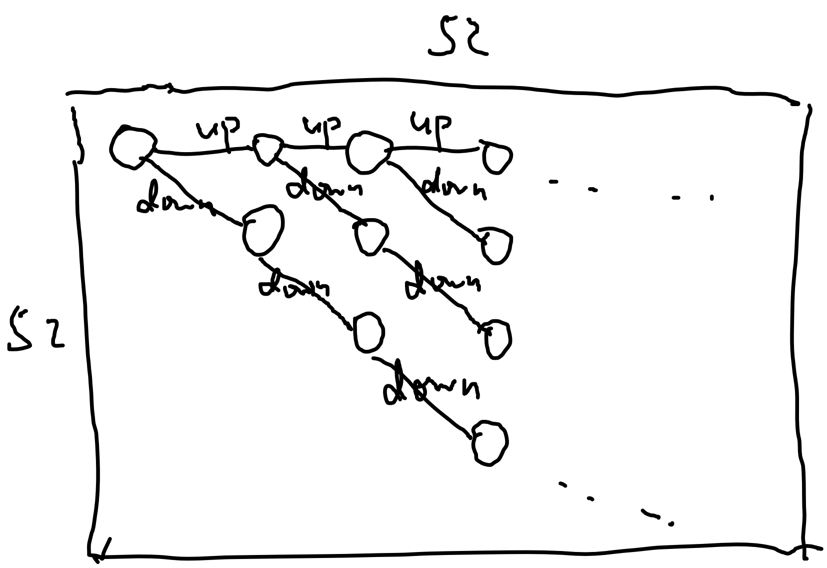
$$1 = P(u) \cdot 1.1 + P(d) \cdot 1.1^{-1}$$

$$P(u) = 10/21, P(d) = 11/21$$

For each problem of pricing option, I will create 6 trees. For example, for 4-upswing option, I will have one tree for the underlying prices, the 4 others for 1-upswing, 2-upswing, 3-upswing and 4-upswing option values. And 1 more tree to indicate whether a node is an optimal exercise node.

Data structure

I will use 52x52 numpy array to represent a tree



Each entry in the matrix represents the value of underlying/option.

Note that entries under the diagonals are irrelevant to our problem. They are filled with 0's.

Algorithm

1. create the tree for the underlying prices

2. create the tree for the n-swing-option

2.1 Starting from the last week and calculate option values backwards

2.2 Calculate $value\_if\_exercise$, which is equal to cash value from exercising one option plus the option value of n-1-swing option at the same tree position. Note that option value of n-1-swing option here has an extra constraint that it cannot be exercised in the same period, i.e., it has to be calculated the same way as 2.3

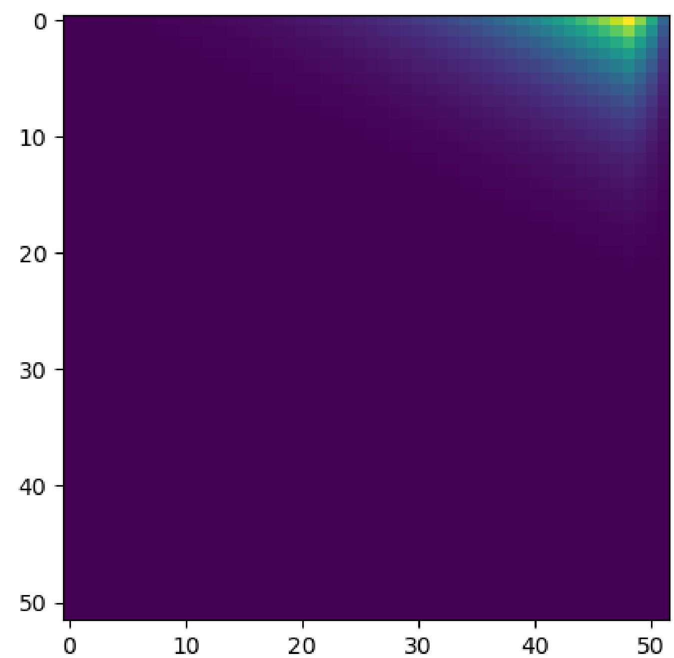
2.3 Calculate $value\_not\_exercise$ as the expected option value one period later under risk neutral probabilities.

2.4 At each node, the option value is the higher of $value\_if\_exercise$ and $value\_not\_exercise$.

Result

The price of 4-upswing option is $52.504.

I print out the option value tree as image below. Light color represents high value and dark color represents low value. Note that area under the diagonal is irrelevant to our problem.



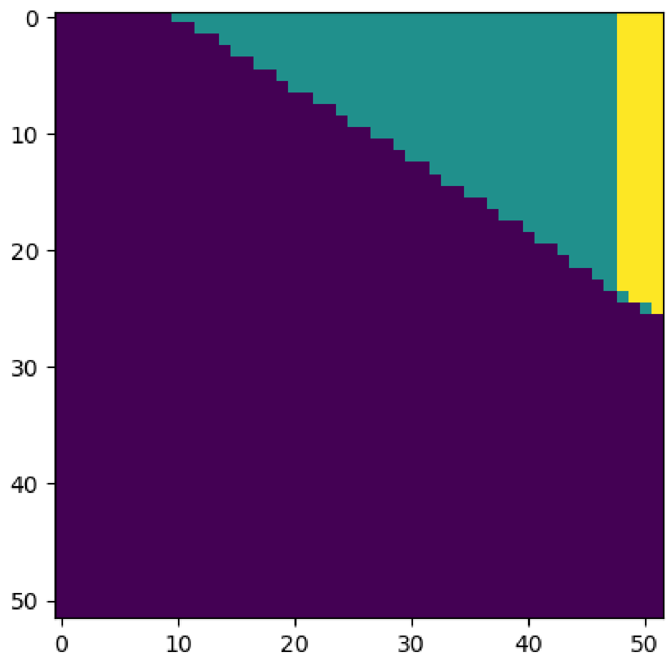
The optimal exercise nodes are indicated in the array called $exercised$ with:

1 represents $should\ exercise$,

0.5 represents $indifferent$,

0 represents $should\ not\ exercise$

I print out the result as image below:



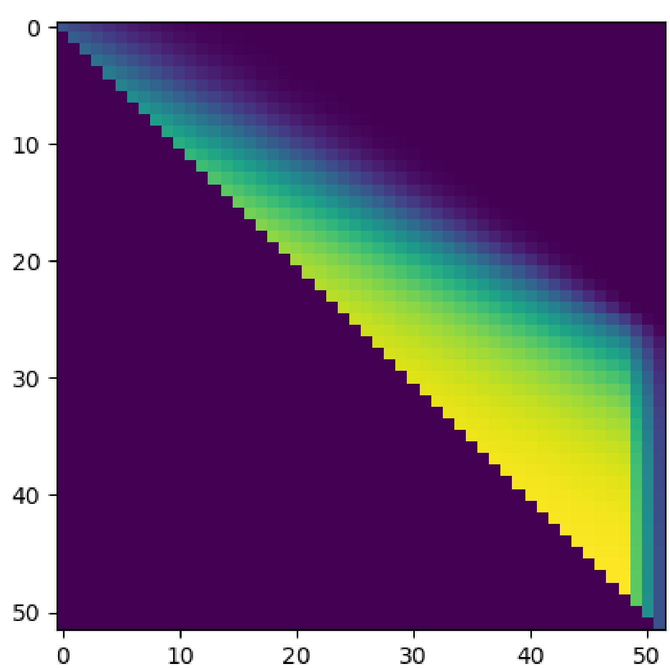
yellow represents $should\ exercise$,

green represents $indifferent$,

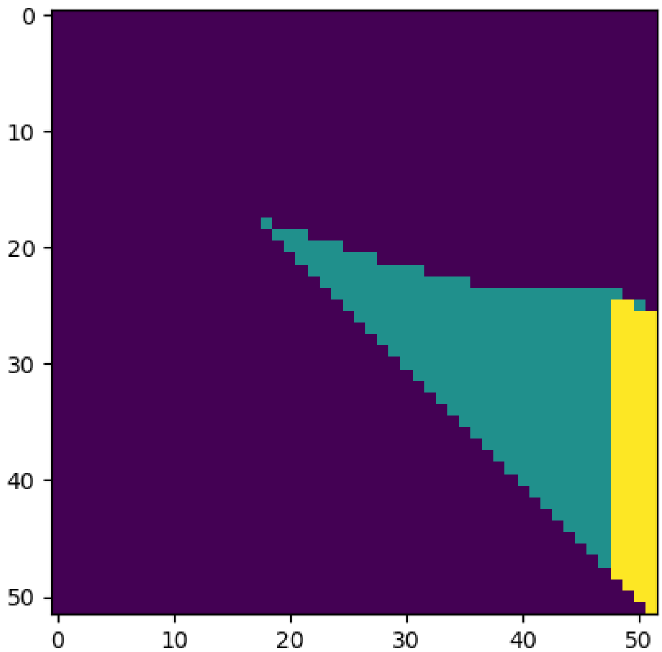
purple represents $should\ not\ exercise$

The price of 4-downswing option is $52504.

The option value tree is represented below:



The optimal exercise nodes:



Reference

Used only the lecture slides

No collaborators

GitHub Link to Code
https://github.com/travelwithwind/APM466/blob/master/a1/a2.py