Current Balance

Abstract

The Current Balance experiment attempts to measure magnetic constant μo through the curve fitting the formula $\frac{F}{l} = \frac{\mu_o I^2}{2\pi d}$. The experiment result yields estimated $\mu_0 = 9.80 * 10^{-8} \pm 2.25 * 10^{-16}$Tm/A, which is far from the theoretical value. The result indicates the deficiency in data collection and cast doubt in the validity of approximations made in calculation.

Introduction

This fundamental experiment defines the unit of electric current: the Ampere. The French physicist Andre Marie Ampere (1777-1863) showed that two parallel currents attract each other if the currents are in the same direction and repel each other if the currents are in opposite directions. The magnitude of force per unit length between two current-carrying wires is given by:

$$\frac{F}{l} = \frac{\mu_o I^2}{2\pi d}$$

where I is the current through each wire, d is the distance between the wires and μo is the fundamental physical constant to be determined in this experiment

Equipment

- Step-down transformer
- Movable conductor with mirror on the axis
- Stand with Ruler and Scope
- Ammeter
- Tape measure
- Weights(20-500mg)

Units:
Current: A
Length: m
weight: kg

Method and Procedure

In this experiment the deflection of the movable conductor is measured by an optical system. A small mirror is attached to the base of the frame. This mirror is tilted at small angles when the movable frame is deflected due to the magnetic force between two current-carrying wires. The deflection is measured indirectly by the displacement of the ruler's scale reflected by the mirror.

Now I will describe the method used to calculate the distance between the two conductors from the displacement of the image of the ruler observed in the eyepiece.

Quantities that we need to measure:

- Distance from the scope to mirror. (L) Note, this is not horizontal distance, i.e., not the distance from the base of the scope stand to mirror, which would lead to a bigger approximation error.

- Distance from the rotating axis of the conductor wire to the conductor wire. (m)

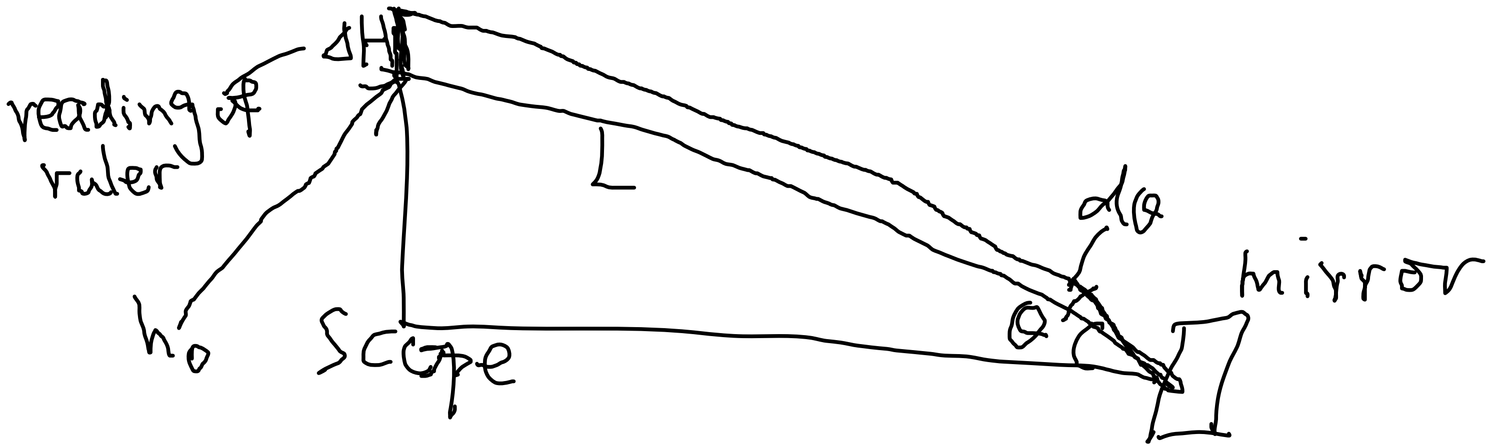
- Length of conductor wire(l)

- Reading of the ruler through the scope(H)

- Reading of current (I)

I will collect 4 sets of data with different weights placed on the conductor pan. Each set of data will include the current readings and ruler readings.

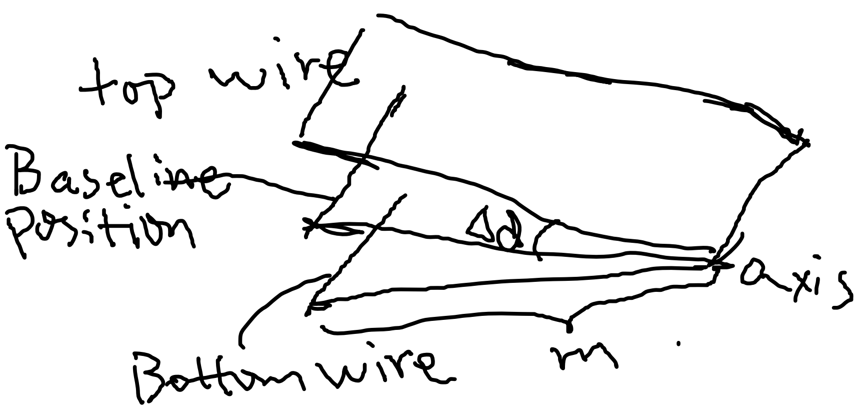
The weights chosen are 0, 20mg, 40mg, and 60mg. For each set, 10 observations will be made with the power transformer setting ranging from 10 to 30. When the power transformer is set to 30, it will provide approximately 10A of current, which we do not want to go over to avoid burning the fuse. The reason to start with transformer value of 10 is that the force needs to large enough for distances to accurately measured.



Let’s denote the initial reading of ruler without using any weight and current as h_0. This will be baseline for us to calculate the change in reading of ruler (ΔH). The corresponding baseline distance between wires is d_o. Note that the d_o is not zero. There’s a small separation. And d will be calculated by adding to d_o.

Assume the top narrow triangle is right triangle, then $\Delta\theta = \tan^{-1}(\frac{\Delta H}{L})$
Thus, we can relate the change of height ΔH to change of angle $\Delta\theta$.

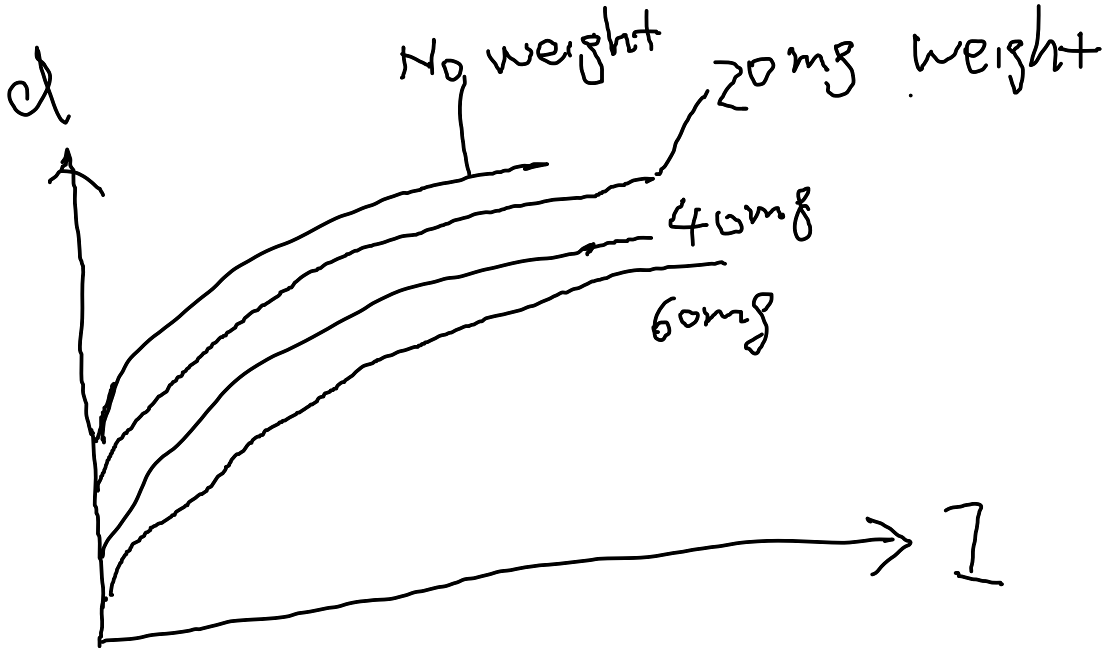
Let’s denote the rotation of the axis as $\Delta\alpha$. $\Delta\theta$=2 $\Delta\alpha$



$$d = m * \tan(\,\Delta\alpha) + d_0$$

Putting them together, we get $d = m * \tan\left(\frac{1}{2}\tan^{-1}(\frac{\Delta H}{L})\right) + d_0$

Plot distance d against current I



At a fixed distance d, we will expect heavy weight corresponds with stronger current. And I will take a set of four points from these four curves at a fixed distance d, and use them to curve fit the function $\frac{F}{l} = \frac{\mu_o I^2}{2\pi d}$ and estimate μ_o

Let's denote f_0 as the weight of the conductor wire itself and w as the weight placed on the conductor pan. Then, $F = f_0 + w$. Thus, the curve fitting function becomes

$$w = \frac{\mu_o I^2 l}{2\pi d} - f_0$$

Results

Here are the initial measurements before turning on the current:

$h_0 = 0.149 \pm 0.0005\, m$ # the initial reading of ruler without using any weight and current
$d_0 = 0.007 \pm 0.0005\, m$ # the initial distance between wires without using any weight and current
$L = 1.9 \pm 0.02 m$ # distance between scope and mirror
$l = 0.29 \pm 0.001\, m$ # length of conductor wire
$m = 0.215 \pm 0.005 m$ # distance between conductor wire and mirror

Here's the first set of data collected with current on and weight=0:

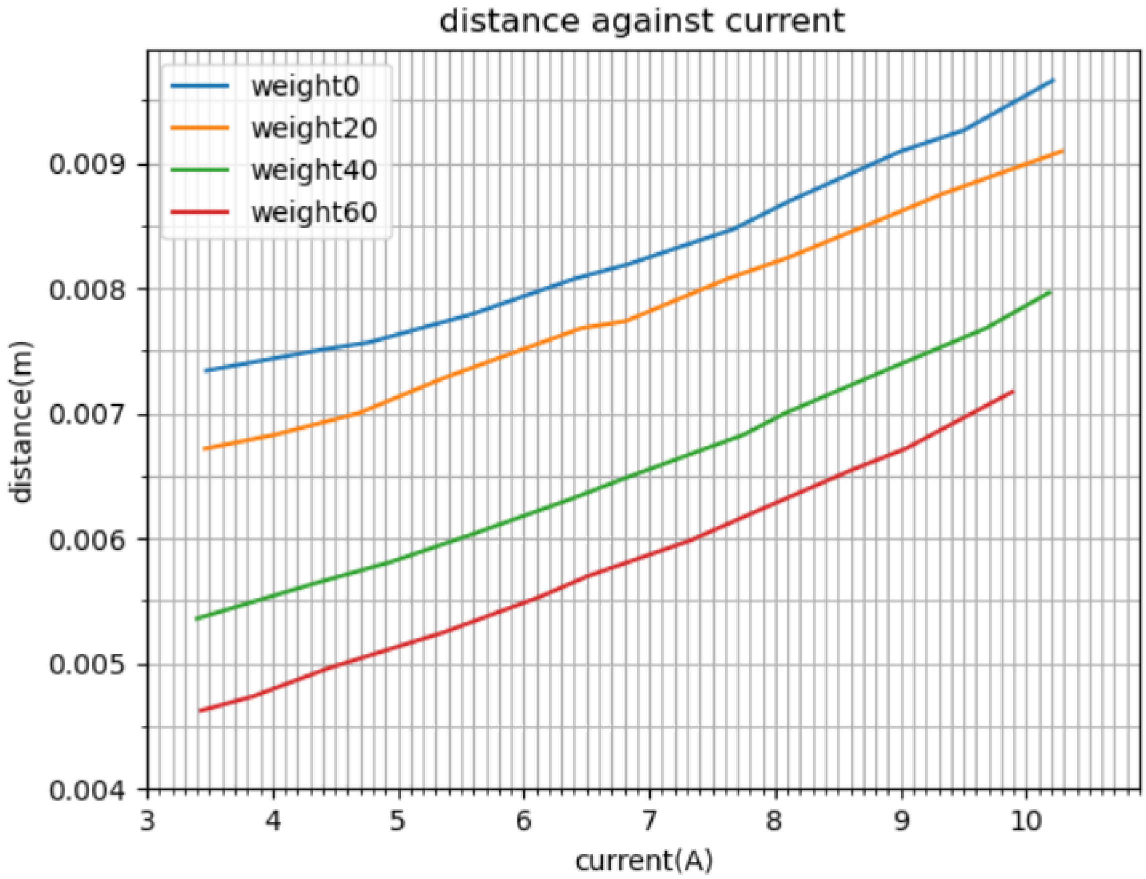
transformer	current(A)		ruler(cm)
	lower	upper	
0	0	0	14.9
10	3.47	3.475	15.5
12	4.401	4.405	15.8
14	4.765	4.773	15.9
16	5.587	5.592	16.3
18	6.4	6.409	16.8
20	6.822	6.826	17
22	7.664	7.672	17.5
24	8.115	8.12	17.9
26	8.996	9.002	18.6
28	9.5	9.512	18.9
30	10.207	10.212	19.6

During the measurement, the readings of the ammeter keep jumping. I recorded the lower and upper bound of the readings and will use the average as expected value for calculation. I will use half of the jump at each point as the uncertainties.

The ruler has precision error of 0.0005m, which will be used as the uncertainties.

I have collected another 3 sets of data with weight 20mg, 40mg, and 60 mg. They have the same way of calculating uncertainties. Complete data and codes are available in my GitHub page for reference. https://github.com/travelwithwind/PHY224/tree/master/current balance

With the 4 sets of data, we can draw the plot of distance between conductor wires against the current, as below.



We will fix distance to be 0.0075m and get the four points from the four curves.

	weight	current
1	0.00000	4.3
2	0.00002	6.0
3	0.00004	9.2
4	0.00006	10.3

Note that the last data point comes from extrapolation of the curve.
In order to curve fit, we would need to know the uncertainties in d. Here's the formula of d.

$$d = m * \tan\left(\frac{1}{2}\tan^{-1}(\frac{\Delta H}{L})\right) + d_0$$

To estimate the uncertainties of d, let's use small angle approximation on tangent function.

$$\tan(x) \approx x \; for \; small \; x$$

$$d \approx \frac{1}{2} m \frac{\Delta H}{L} + d_o$$

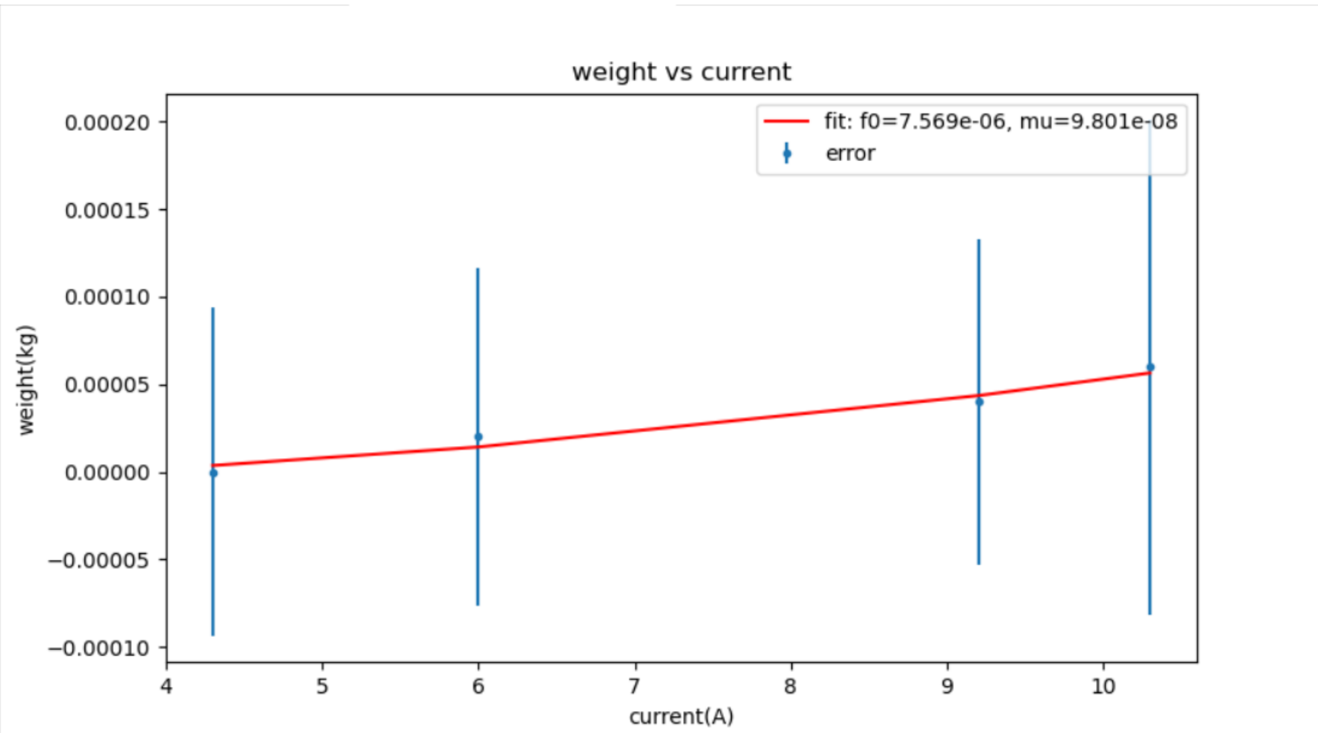
$$\sigma_d = d \sqrt{\left(\frac{\sigma_{\Delta H}}{\Delta H}\right)^2 + \left(\frac{\sigma_{\mathrm{L}}}{\mathrm{L}}\right)^2}$$

Where $\sigma_{\Delta H}$=0.0005 and $\sigma_{\mathrm{L}} = 0.02$

Because the exact point of with fixed distance fix of 0.0075m were not measured, we will approximate by the average of neighboring points. For example, the first point with current 4.3A on the first curve falls between the first two observations of the curve, which have ΔH values 0.006 and 0.009 and d values 0.007339 and 0.007509. Plug numbers in the formula above, and we get error in d for first point to be 9.40e-5.

Now that we have obtained the uncertainties in d, let's apply the curve fitting algorithm to the following function

$$w = \frac{\mu_o I^2 l}{2\pi d} - f_0$$



The estimated $\mu_0 = 9.80 * 10^{-8} \pm 2.25 * 10^{-16}$Tm/A. The theoretical $\mu_0 = 1.26 * 10^{-6}$Tm/A. Our estimation is quite bad.

However, the chi-squared goodness of fit test has test statistics 0.0036, which indicates the fit is good.

Discussion

The curve fitting result has good fit, yet the estimate is quiet far from the theoretical values. There are a few possible reasons. First, I could have overstated the uncertainties. In calculating σ_d, I used small angle approximation, the error from this approximation might be too big. I claimed that $\sigma_L = 0.02m$, but this value is just based on my gut feeling when I measured the distance from the scope to mirror.

Secondly, the method I used to calculate the distance between the two conductors involves approximating the two triangles as right triangles so that I could invoke the Pythagorean theorem. It is possible that this approximation is not valid.

Thirdly, I have collected data to draw four curves of wire distance versus current for various loads. Thus, in the last step to estimate μ_0, only four points are available for curve fitting. Perhaps four points is too prone to error and we should have collected more data.

It is also interesting to discuss why we use AC current instead of DC in this experiment. The earth's magnetic field can affect the wires with current. With DC, the ammeter readings are associated two sources of magnetic forces, the earth's magnetic field, and the wire's magnetic field. And it's hard to isolate the effect from only wire's magnetic field. With AC, the earth's effect on the wire changes directions as the current changes directions. And because the AC frequency is 60, which is fast, the oscillating effect on the reading is invisible to the naked eyes. Thus, we can ignore the earth's magnetic field with AC.

Conclusion

The estimated $\mu_0 = 9.80 * 10^{-8} \pm 2.25 * 10^{-16} \mathrm{Tm/A}$. Compared with the theoretical value$1.26 * 10^{-6}$ Tm/A, our estimation is quite bad.

There are a few things to work on to get better results. We need to improve on the accuracy of measurements and collect more data. When approximation is used, we need to make sure the approximation is valid.