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An Exploration of Voting Methods & Utility Maximization

I Introduction

The goal of this paper is to explore various voting methods and their weaknesses and strengths. Specifically the effects of varying sizes of voters, number of candidates, and ratios of strategic voters will be studied. We will examine various case studies and theorems related to voting. However the main purpose of this paper is to present the results of simulations of different voting systems. The relative effectiveness of each voting system will be judged according to how well the voting system maximizes utility across voters. For the purpose of this paper we will look at three voting systems, which are outlined below.

II Voting Systems

i Plurality: *Single plurality voting* is the most simple form of voting. Each voter casts a single vote for a single candidate and the candidate with the most votes wins.

Another variation of Plurality Voting is the *plurality runoff*. Once again each voter casts a single vote for a single candidate, but now the two candidates with the most votes move on to a second round of voting. The second round of voting is simply a plurality vote between the two remaining candidates.

A third form of plurality voting is the *sequential runoff*. Each voter casts a single vote for a single candidate and the candidate with the fewest votes is eliminated. This process is repeated until only one candidate remains. Interestingly enough the *sequential runoff* is similar to how the American Presidential Primaries work (albeit unofficially).

Advantages:

· Plurality voting is the simplest voting method, in what it requires of voters. The voter must simply must cast one vote for their candidate of choice (in the case of the single plurality vote). Plurality requires voters have very little information about candidates and that they need to vote only once.

Disadvantages:

- Often favors extremist candidates in comparison to other voting systems
- Parties who run multiple candidates are punished because their candidates ”split” the vote.
- Lacks expression, voters can only express an opinion about one candidate.

ii Borda Count: Rather than express a preference for a single candidate, the *Borda Count* requires the voter express preferences about each candidate he or she votes for. Consider an election with five candidates, the voter awards four points to their favorite candidate, three points to their second favorite, two points to their third, one to their fourth, and none to their fifth. The points over all voters are added and the candidate with the most points wins.

Advantages:

- Borda Count is an expressive voting system, voters may express an opinion (positive or negative) about every candidate.
- When voters are honest, Borda Count can be effective as a voting method (in terms of utility maximization).

Disadvantages:

- Requires a large amount of information about each candidate.
- Extremely susceptible to strategic voting, more so than almost any other voting system.
- Incentivizes political parties to run as many candidates as possible, with enough ”clones”, it is almost guaranteed that one of them will win.

iii Score Voting: Like the *Borda Count*, *Score Voting* requires that voters express a preference for each candidate. Each vote consists of a numerical score in a given range,for example (0-10 or 0-100). Voters may also choose to vote ”no opinion”, in which case their vote for that candidate is irrelevant. The candidate with the average highest score wins.

Additionally, the candidates may agree before the voting to give each candidate T fake votes with a fixed score S. In this case, the highest average based on both the fake and real votes win. An explanation for why this is done will follow below.

Advantages:

- Score Voting is the most expressive voting system considered so far. Voters assign numerical values to each candidate, thus expressing quantitatively by how much one candidate is favored
over another.
- Range voting is extremely resistant to strategic voting, in it's worst case range voting is identical to approval voting (when the voter assigns scores of 0 to every candidate except their preferred one).
- As opposed to plurality and borda count voting methods, range voting is not susceptible to "candidate cloning."
- Range is the best known voting system in terms of utility maximization for voters(as measured by Bayesian Regret).

Disadvantages:

- Requires large amount of information from voters (though voters may elect to not vote or give a 0 to a voter they are unfamiliar with).
- Does not always result in the election of the candidate supported by the majority. Consider two candidates, Candidate A and Candidate B. Candidate A is the top-scoring (favorite) candidate of 55% of voters, whereas Candidate B is the top-scoring candidate of 45% of voters. Among those who scored Candidate B highest, most gave Candidate A a score of 0. However among those who voted for Candidate A, most gave Candidate B a score of 50. The result is that Candidate B has a higher average score than Candidate A. Thus despite the fact that Candidate B is the favorite candidate of the majority of voters, Candidate A will win the election.
- "Unknown Lunatic" - an unknown lunatic runs whose supporters give him the maximum score, while others simply vote no opinion for the unknown candidate, electing him into office with the highest score. This is the reason for the T fake votes mentioned earlier, which will have a much greater effect on candidates with lower vote counts. They will pull candidates with lower vote counts towards the set score S.

III Simulation Methodology

Inputs: n, the number of voters and m the number of candidates.

For each of the n voters m values are drawn from unif(0,100). That is m (not necessarily integer) random values from 0 to 100 are generated. Each one of these values corresponds to the amount of utility the voter gains from the election of that candidate. See *Figure 1* below for R code. How we proceed from here depends on the system of voting, we will outline the next steps for all three voting methods below.

Plurality - Assuming voters are honest, for each voter the candidate with the highest utility value get's the voter's vote. The candidate with the most

```r
### Function: preforder
### Inputs: X, some integer
### Output: a vector of X values between 0 and 100
preforder <- function(x) {
  runif(n = x, min = 0, max = 100) #return random utilities for candidates
}


### Function: Trial
### Inputs: N, the number of candidates that can be voted for
###         X, the number of voters
### Output: Random utility values for X voters, voting for N candidates
trial <- function(n, x) {
  base <- c(1:n) #Create a vector with values from 1 to n
  voters <- matrix(0, nrow = x, ncol = n, byrow = TRUE)
  for (row in 1:nrow(voters)) {
    voters[row,] = preforder(base) # Give each voter random utilities for each candidate
  }
  voters
}
```

Figure 1: Create Randomized Voter Preferences

votes wins and for the purpose of simplicity, the winner is randomly chosen in the case of a tie. See *Figure 2* below for reference.

```r
### Function: findPluralityCount
### Inputs: LISTY, a list of voters and their preferences
### Output: The Plurality counts
findPluralityCount <- function(matrixy) {
  len = nrow(matrixy)
  f = ncol(matrixy)
  winnerVec <- integer(f) #create a vector keeping track of how many votes each candidate has
  for (i in 1:len) {
    maxIndex = which.max(matrixy[i,])
    winnerVec[maxIndex] = winnerVec[maxIndex] + 1 #add 1 to the voter's top candidate
  }
  winnerVec
}

findPluralityWinner <- function(matrixy) {
  which.max(findPluralityCount(matrixy))
}
```

Figure 2: Find Plurality Winner

If we want to simulate strategic voting, we assign each voter either s or h. s if the voter is strategic h is they are honest. If we want to simulate 100% strategic voting, every voter would be assigned s. If a voter is assigned h, they simply vote as they did previously, for the candidate that provides them with the highest utility. Simulating strategic voting takes slightly more work.

First we simulate the election, acting as if all voters are honest. Now we count the number of votes for each candidate, now with those vote counts we create a probability distribution P. For a candidate c, $P(c) = \frac{\text{number of votes for } c}{\text{total number of votes}}$. Put simply, $P(c)$ is the probability that an arbitrarily chosen voter, will vote for c. P represents information voters have about other voters preferences before the election, for example the results of pre-election polls. Voters, however, do not have perfect information about other voters preferences. Our goal is to create a new probability distribution that simulates the uncertainty and imperfect knowledge that exists in real elections.

Thus we create a new probability distribution from P, let's call it F. For a candidate c, $F(c) \sim N(P(c), 0.1)$. That is each $F(c)$ is a normally distributed random variable with mean $P(c)$ and variance 0.1. Next we check to make sure each $F(c_i) \in [0, 1]$. Though we rarely see values outside of this interval, if we do we will round them up to 0 (if $val < 0$) or down to 1 (if $val > 1$). Lastly we divide each $F(c_i)$ by the sum of all the $F(c_i)$'s. This ensures that the sum of the $F(c_i)$'s is equal to 1 and thus that we have a valid probability distribution. Since each $F(c_i) \in [0, 1]$ and $\sum F(c_i) = 1$, F is a probability distribution.

We will now use F to determine the voting preferences of strategic voters. Let $pref(v_k)$ denote the vector of utilities for strategic voter k. For voter v_k, $pref(v_k) = pref(v_k) * F$, where $*$ is element-wise vector multiplication. After this transformation, the strategic voter v_k votes for the candidate with the maximum adjusted utility value. We perform this utility transformation for each strategic voter. Combining the results with those from honest voters, we get our plurality winner. Note that F is common for all voters, we assume for the sake of simplicity that each voter has relatively equal information about the election.

<u>Borda Count</u>: For each honest voter, we look at their vector of utilities and we rank each candidate according to their utility. If there are m candidates, the candidate with highest utility receives $m - 1$ points down to the candidate with the lowest utility who receives 0 points. The candidate with the most points over all voters wins.

For dishonest voters the process is the same as plurality voting except now for candidate c, $P(c) = \frac{\text{number of points for } c}{\text{total number of points}}$. Otherwise the process is identical.

```r
### Function: findBordaCount
### Inputs: LISTY, a list of voters and their preferences
### Output: The Borda Counts for each candidate
findBordaCount <- function(matrixy) {
  len = ncol(matrixy)
  voteCount <- integer(len) #make a vector to keep track of vote totals for each candidate
  numVoters <- nrow(matrixy)
  for (i in 1:numVoters) {
    matrixy[i,] = rank(matrixy[i,])
    for (j in 1:len) {
      voteCount[j] = voteCount[j] + matrixy[i,j] #add the current voter's points
    }
  }
 voteCount #return the vector containing each candidates point count
}

### Function: bordaCountWinner
### returns the Borda Count Winner
bordaCountWinner <- function(matrixy) {
  which.max(findBordaCount(matrixy)) #returns a randomly chosen winner if there is a tie
}
```

Figure 3: Find Borda Count Winner

Score Voting: For the purpose of our simulation, we will use a scoring system from 0-100. Honest Voters simply give each candidate a score equal to the voter's utility associated with that candidate. The candidate with the highest average a score wins. Simulating strategic voters is quite simple, each voter has some randomly generated cutoff from 65-100. For all candidates with a utility value below the cutoff, the voter will give the candidate a score of 0. For all candidates with a utility value of at least the cutoff, the voter will give the candidate a score of 100.

IV Results

In the following tables, the numerical values represent the proportion of the the time (over 1000 trials) that the winning candidate is the candidate with the greatest total utility over all voters. Let's first consider when all voters vote honestly.

List of Tables

Number of Voters

Number of Candidates	1	10	100	1000
1	1.00	1.00	1.00	1.00
2	1.00	0.78	0.79	0.82
3	1.00	0.74	0.76	0.75
4	1.00	0.74	0.77	0.75
5	1.00	0.73	0.76	0.75
6	1.00	0.74	0.73	0.74
7	1.00	0.75	0.74	0.73
8	1.00	0.73	0.73	0.76
9	1.00	0.75	0.74	0.74
10	1.00	0.77	0.76	0.77

Table 1: Utility Maximization of Borda Count with Honest Voters

Number of Voters

Number of Candidates	1	10	100	1000
1	1.00	1.00	1.00	1.00
2	1.00	0.80	0.82	0.79
3	1.00	0.70	0.66	0.68
4	1.00	0.59	0.56	0.58
5	1.00	0.51	0.50	0.49
6	1.00	0.47	0.45	0.47
7	1.00	0.41	0.38	0.36
8	1.00	0.39	0.35	0.31
9	1.00	0.37	0.34	0.34
10	1.00	0.33	0.30	0.30

Table 2: Utility Maximization of Plurality Voting with Honest Voters

Number of Voters

# of Candidates	1	10	100	1000
1	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	1.00
3	1.00	1.00	1.00	1.00
4	1.00	1.00	1.00	1.00
5	1.00	1.00	1.00	1.00

Table 3: Utility Maximization of Score Voting with Honest Voters

With honest voters, score voting performs the best in terms of utility maximization, as is expected. By definition when voters are honest and have perfect information, Score Voting elects the candidate of maximum utility. Plurality voting suffers greatly as the number of candidates increases. Whereas the performance of the Borda Count was relatively consistent across different numbers of candidates and voters.

Let's now consider when all voters vote strategically, that is 100 % of voters vote strategically. The results are below.

# of Candidates	Number of Voters: 1	10	100	1000
1	1.00	1.00	1.00	1.00
2	0.99	0.74	0.60	0.51
3	0.90	0.63	0.46	0.35
4	0.78	0.53	0.38	0.31
5	0.75	0.52	0.35	0.24
6	0.69	0.52	0.32	0.21
7	0.67	0.48	0.27	0.19
8	0.61	0.43	0.28	0.16
9	0.55	0.42	0.24	0.15
10	0.55	0.42	0.25	0.14

Table 4: Utility Maximization of Borda Count with Strategic Voters

# of Candidates	Number of Voters: 1	10	100	1000
1	1.00	1.00	1.00	1.00
2	0.99	0.72	0.60	0.55
3	0.98	0.66	0.48	0.37
4	0.98	0.55	0.39	0.32
5	0.98	0.53	0.36	0.26
6	0.98	0.47	0.32	0.23
7	0.98	0.39	0.28	0.20
8	0.98	0.37	0.25	0.16
9	0.99	0.36	0.28	0.16
10	0.98	0.36	0.23	0.17

Table 5: Utility Maximization of Plurality Voting with Strategic Voters

		Number of Voters			
		1	10	100	1000
# of Candidates	1	1.00	1.00	1.00	1.00
	2	0.68	0.76	0.76	0.77
	3	0.63	0.66	0.68	0.68
	4	0.61	0.62	0.60	0.59
	5	0.57	0.59	0.58	0.58
	6	0.56	0.55	0.55	0.52
	7	0.56	0.50	0.51	0.51
	8	0.50	0.51	0.50	0.48
	9	0.49	0.49	0.52	0.48
	10	0.45	0.49	0.46	0.47

Table 6: Utility Maximization of Score Voting with Strategic Voters

The borda count and plurality vote both suffer greatly as the number of voters and candidates increase. Score voting also suffers but only as the number of candidates increases. Score Voting appears to be unaffected by increases in the number of voters. Additionally for higher numbers of candidates, score voting performs much better than the plurality vote and borda count (as judged by our metric).

V Conclusion

The results of simulation imply that in terms of utility maximization, score voting is a superior method to plurality voting and the borda count. However the simulation has questionable application to the real world. The randomly generated voting preferences and the simulated strategic voters and not necessarily completely representative of how strategic voting actually operates. With more time I would hope to run the simulation with varying combinations of strategic and honest voters, and fine-tune the strategic voting algorithm. Additionally I would consider more types of voting systems and variations of already considered voting systems. One addition that would make the simulation more realistic, would be to incorporate more uncertainty into honest voters decisions. In the current model, honest voters have perfect information about every candidate.

VI References

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