

# AGENDA

- i> Multilayer Perceptron
- ii> Backpropagation intuition
- iii> ANN {forward propagation information flow}
- iv> Why we need activation function? (Proof)

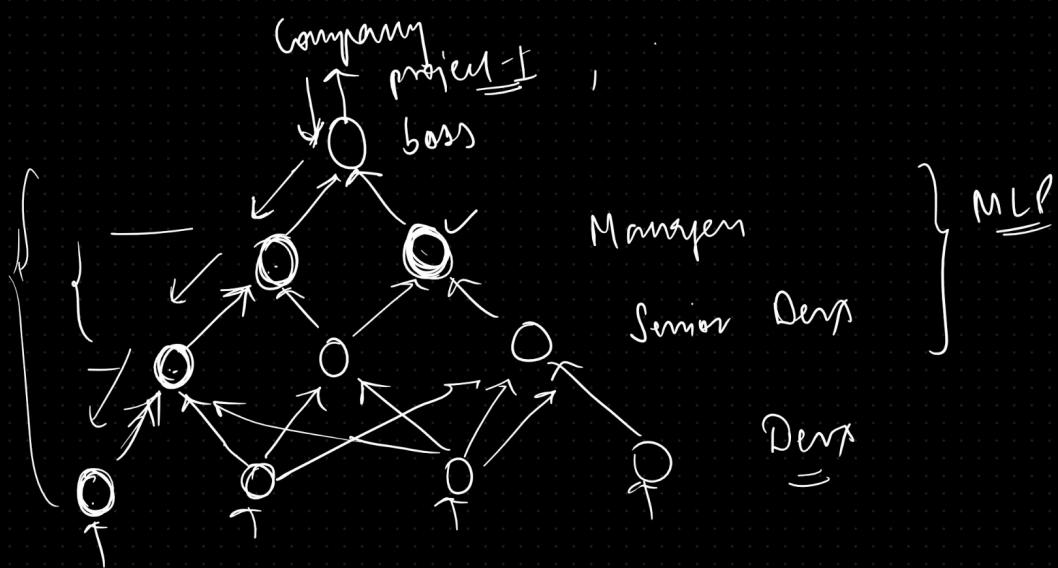
ANTS

One neuron

Unit

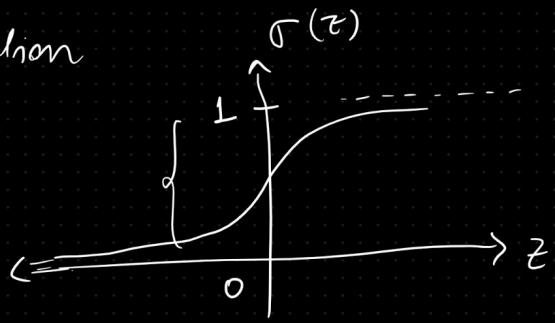
$$\text{error} = 0$$

Ant hill



Sigmoid activation function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



domain z values

$$(-\infty, \infty)$$

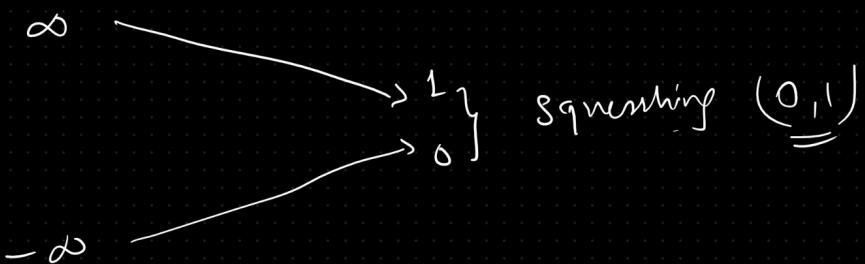
~~range~~

$$\begin{matrix} \uparrow & \downarrow \\ [1, 2] & (1, 2) \\ [1, 2] \end{matrix}$$

Range

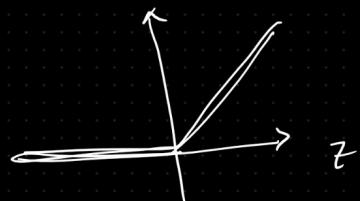
$$(0, 1)$$

$$\begin{aligned} z \rightarrow \infty &\Rightarrow 1 \\ z \rightarrow -\infty &\Rightarrow 0 \end{aligned}$$



ReLU  $\Rightarrow$  Rectified Linear Unit

$$\text{ReLU}(z) = \begin{cases} 0 & z < 0 \\ z & z \geq 0 \end{cases}$$

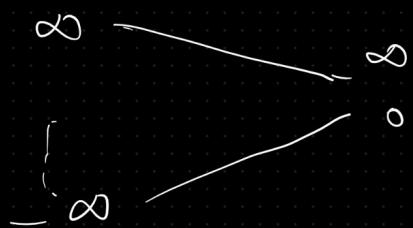


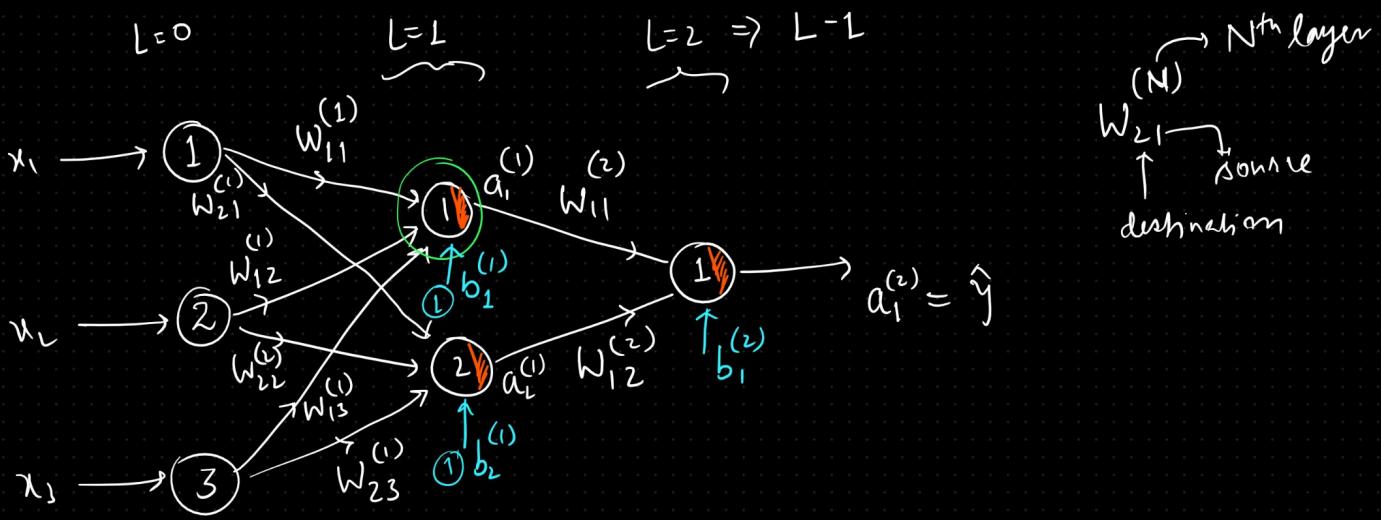
domain

$$(-\infty, \infty)$$

Range

$$[0, \infty)$$





$$\begin{aligned}
 & \text{at layer } 1. \\
 (1) \quad & \left\{ \begin{array}{l} z_1^{(1)} = w_{11}^{(1)} \cdot x_1 + w_{12}^{(1)} x_2 + w_{13}^{(1)} x_3 + b_1^{(1)} \\ a_1^{(1)} = \sigma(z_1^{(1)}) \end{array} \right. \\
 (2) \quad & \left\{ \begin{array}{l} z_2^{(2)} = w_{21}^{(1)} \cdot x_1 + w_{22}^{(1)} x_2 + w_{23}^{(1)} x_3 + b_2^{(1)} \\ a_2^{(1)} = \sigma(z_2^{(2)}) \end{array} \right.
 \end{aligned}$$

final layer  $\setminus$  o/p layer

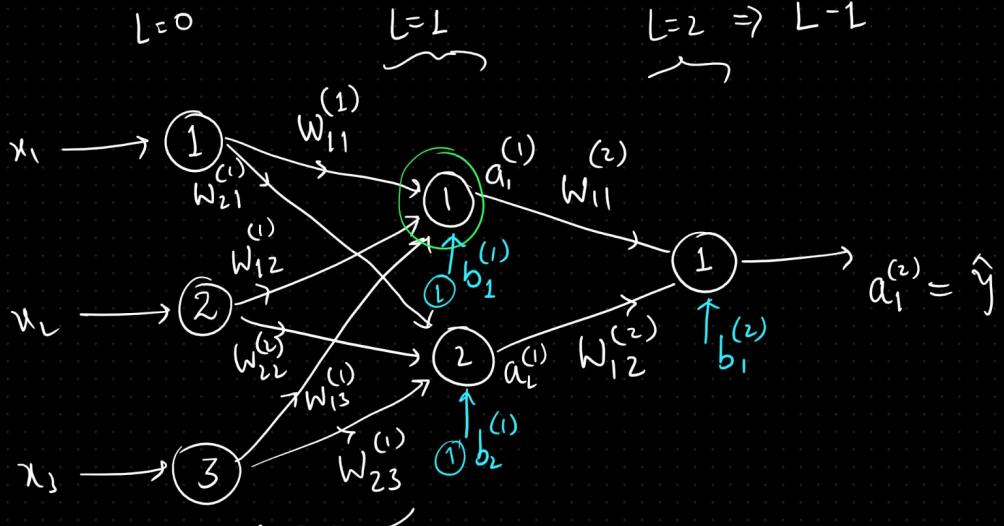
$$\begin{aligned}
 z_1^{(2)} &= w_{11}^{(2)} a_1^{(1)} + w_{12}^{(2)} a_2^{(1)} + b_1^{(2)} \\
 a_1^{(2)} &= \sigma(z_1^{(2)}) \longrightarrow \hat{y}
 \end{aligned}$$

Weight update rule (General)

$$\left\{
 \begin{array}{l}
 w = w + \Delta w \\
 \Delta w = -\eta \frac{\partial e}{\partial w}
 \end{array}
 \right\}
 \left\{
 \begin{array}{l}
 b = b + \Delta b \\
 \Delta b = -\eta \frac{\partial e}{\partial b}
 \end{array}
 \right\}$$

$$\frac{\partial y}{\partial x} \downarrow$$

Change in y  
w.r.t. small  
change in x



at layer  $L$

$$\begin{bmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \end{bmatrix}_{2 \times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3 \times 1} + \begin{bmatrix} b_1^{(1)} \\ b_2^{(1)} \end{bmatrix}_{2 \times 1} = \begin{bmatrix} z_1^{(1)} \\ z_2^{(1)} \end{bmatrix}_{2 \times 1} \xrightarrow{\text{act}} \begin{bmatrix} \sigma(z_1) \\ \sigma(z_2) \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(1)} \\ a_2^{(1)} \end{bmatrix}$$

at final layer

$$\begin{bmatrix} w_{11}^{(2)} & w_{12}^{(2)} \end{bmatrix}_{2 \times 2} \begin{bmatrix} a_1^{(1)} \\ a_2^{(1)} \end{bmatrix}_{2 \times 1} + \begin{bmatrix} b_1^{(2)} \end{bmatrix}_{1 \times 1} = \begin{bmatrix} z_1^{(2)} \end{bmatrix}_{1 \times 1} \xrightarrow{\text{act}} \begin{bmatrix} \sigma(z_1) \end{bmatrix} \downarrow \hat{y}$$

$$w = w - \eta \frac{\partial e}{\partial w} \quad \text{no weight update}$$

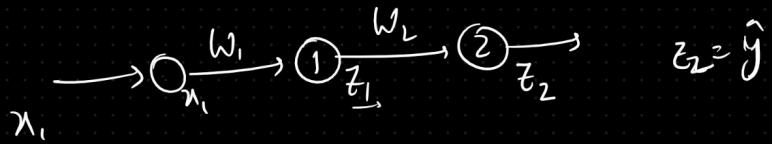
$$b = b - \eta \frac{\partial e}{\partial b}$$

$b = b \Rightarrow$  no bias update

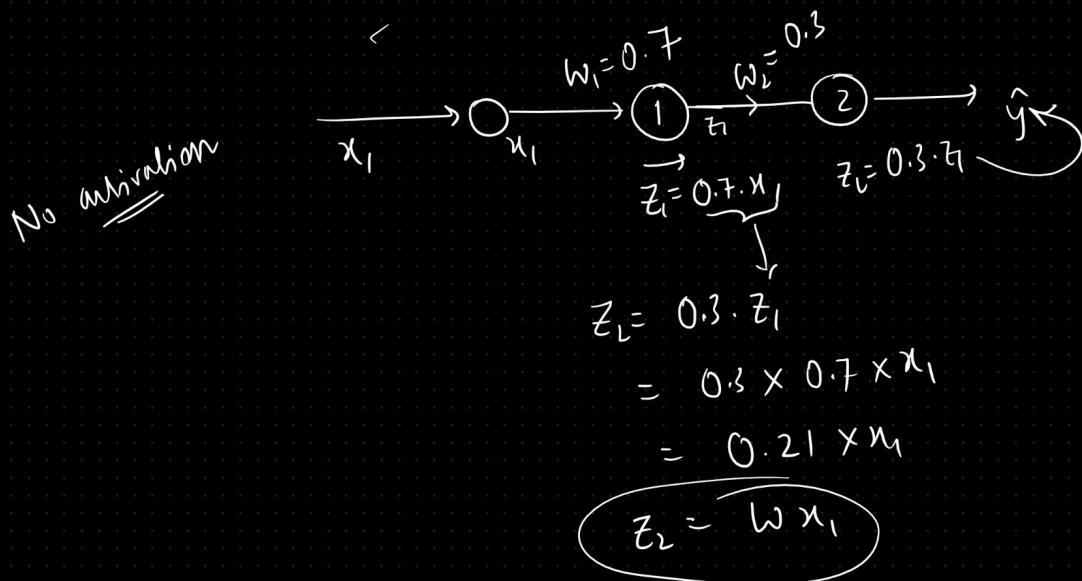
$\nabla \Rightarrow$

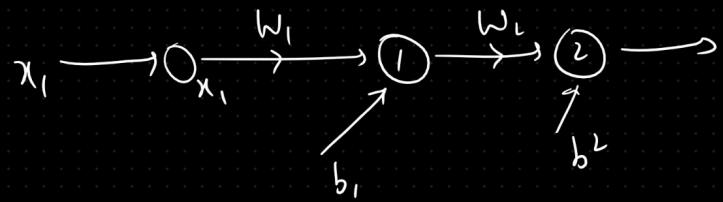
Why we need activation function?

$\sigma X$



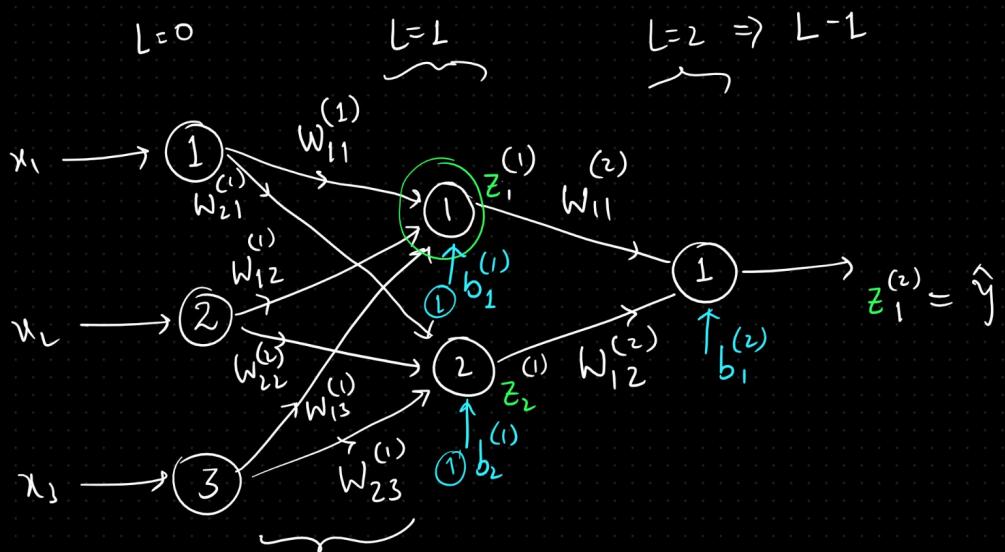
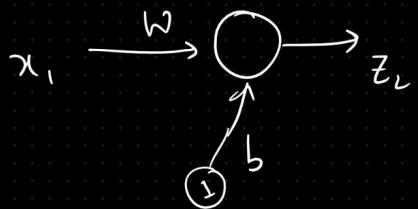
$$z_1 = w_1 x_1 - \textcircled{1}, \quad z_L = w_2 \underbrace{z_1}_{x} \\ z_2 = \underbrace{w_2 w_1}_{\text{circled}} x_1 \\ z_L = \underbrace{w x_1}_{W x_1}$$





$$z_1 = w_1 x_1 + b_1 \quad z_2 = w_2 z_1 + b_2$$

$$\begin{aligned} z_2 &= w_2 (w_1 x_1 + b_1) + b_2 \\ &= \underbrace{w_2 w_1}_{\downarrow} x_1 + \underbrace{w_2 b_1 + b_2}_{b} \end{aligned}$$



at layer 1

$$\textcircled{1} \quad \left\{ \begin{array}{l} z_1^{(1)} = w_{11}^{(1)}x_1 + w_{12}^{(1)}x_2 + w_{13}^{(1)}x_3 + b_1^{(1)} \end{array} \right.$$

$$\textcircled{2} \quad \left\{ \begin{array}{l} z_2^{(2)} = w_{21}^{(1)}x_1 + w_{22}^{(1)}x_2 + w_{23}^{(1)}x_3 + b_2^{(1)} \end{array} \right.$$

final layer { o/p layer }

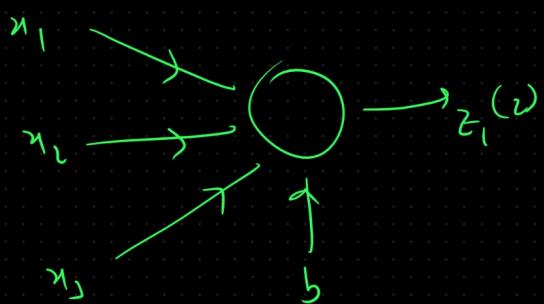
$$z_1^{(2)} = w_{11}^{(2)}z_1^{(1)} + w_{12}^{(2)}z_2^{(1)} + b_1^{(2)}$$

$$\begin{aligned} z_1^{(2)} &\longrightarrow \hat{y} \\ &= w_{11}^{(2)} \left\{ w_{11}^{(1)}x_1 + w_{12}^{(1)}x_2 + w_{13}^{(1)}x_3 + b_1^{(1)} \right\} \\ &+ w_{12}^{(2)} \left\{ w_{21}^{(1)}x_1 + w_{22}^{(1)}x_2 + w_{23}^{(1)}x_3 + b_2^{(1)} \right\} \\ &+ b_1^{(2)} \end{aligned}$$

$$\begin{aligned} &= Ax_1 + Bx_2 + Cx_3 + D_1 \\ &+ Px_1 + Qx_2 + Rx_3 + D_2 \\ &+ b_1^{(2)} \end{aligned}$$

$$= (\underbrace{A+P}_{\text{b}})x_1 + (\underbrace{B+Q}_{\text{b}})x_2 + (\underbrace{C+R}_{\text{b}})x_3 + \underbrace{D_1 + D_2}_{\text{b}} + b_1^{(2)}$$

$$z_1^{(2)} = \omega_1x_1 + \omega_2x_2 + \omega_3x_3 + b$$



① Beginner → Python Machine Learning (ML+DL)  
→ Sebastian Raschka

② Advance → Hands on Machine Learning (ML+DL)  
→ Alfonso Gomez



$$z_1 = \uparrow\uparrow \rightarrow \widehat{(0,1)} \quad z_1 = \uparrow\uparrow \rightarrow y \\ z_2 = \uparrow\uparrow$$

