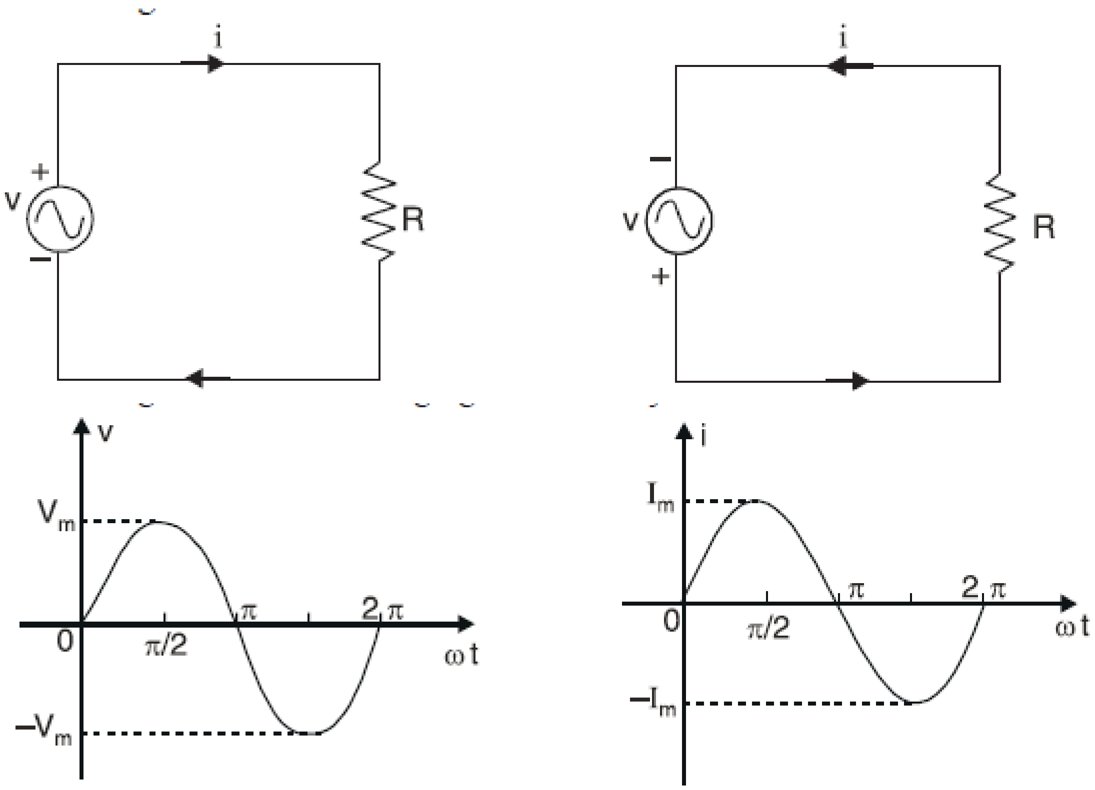
MODULE II

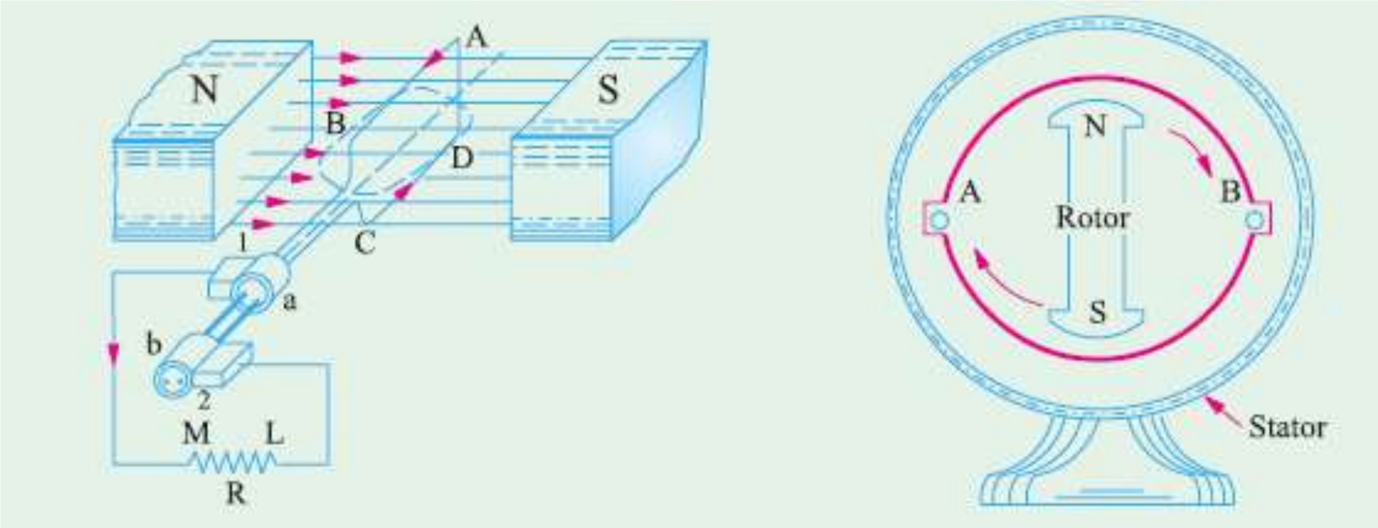
A.C. FUNDAMENTALS



Advantages

- Alternating voltages can be stepped up or stepped down efficiently by means of a transformer.
- A.c. motors (induction motors) are cheaper and simpler in construction than d.c. motors.
- The switchgear (*e.g*. switches, circuit breakers etc.) for a.c. system is simpler than the d.c. system

Generation of Alternating Voltages and Currents

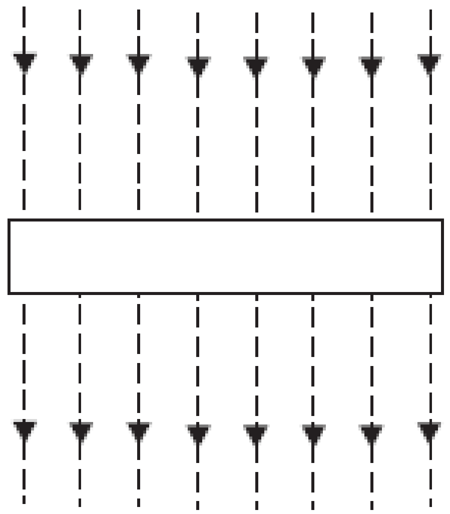


An alternating voltage may be generated :
(i) by rotating a coil at constant angular velocity in a uniform magnetic field
or
(ii) by rotating a magnetic field at a constant angular velocity within a stationary coil

The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. The first method is used for small a.c. generators while the second method is employed for large a.c. generators

Equation of Alternating Voltage and Current

Consider a rectangular coil of *n* turns rotating in anticlockwise direction with an angular velocity of w rad/sec in a uniform magnetic field as shown in Fig



ϕ_{max}

Let the time be measured from the instant the plane of the coil coincides with *OX*-axis. In this position of the coil, the flux linking with the coil has its maximum value f_{max}. Let the coil turn through an angle q (= w*t*) in anticlockwise direction in *t* seconds. In this position, the maximum flux f_{max} acting vertically downward can be resolved into two perpendicular components *viz*.

Flux linkages of the coil

= No. of turns × Flux linking = n Φmax coswt

According to Faraday's laws of electromagnetic induction, the e.m.f. induced in a coil is equal to the rate of change of flux linkages of the coil

$$v = -\frac{d}{dt}(n\,\phi_{max}\cos\omega t) = -n\,\phi_{max}\,\omega\,(-\sin\omega t)$$

$$v = n\,\phi_{max}\,\omega\sin\omega t$$

The value of v will be maximum (call it V_m) when sin wt = 1 *i.e.*, when the coil has turned through 90º in anticlockwise direction from the reference axis.

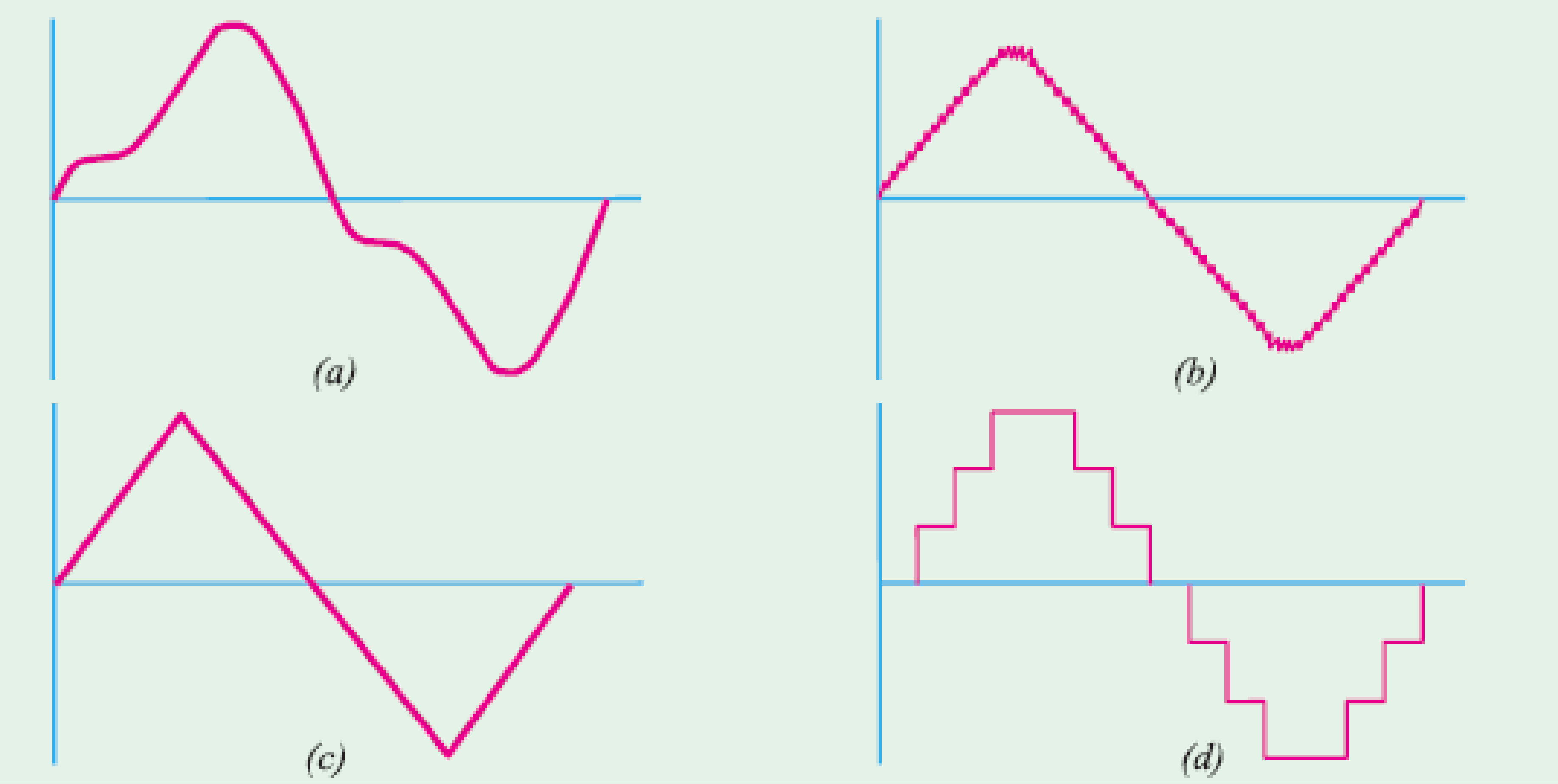
$$V_m = n\, \phi_{max}\, \omega$$
$$v = V_m \sin \omega t \quad \text{where} \quad V_m = n\, \phi_{max}\, \omega$$
$$v = V_m \sin \theta$$

Thus a coil rotating with a constant angular velocity in a uniform magnetic field produces a sinusoidal alternating e.m.f. If this alternating voltage ($v = V_m$ sin wt) is applied across a load, alternating current flows through the circuit which would also vary sinusoidally *i.e.*, following a sine law. The equation of the alternating current is given by ;

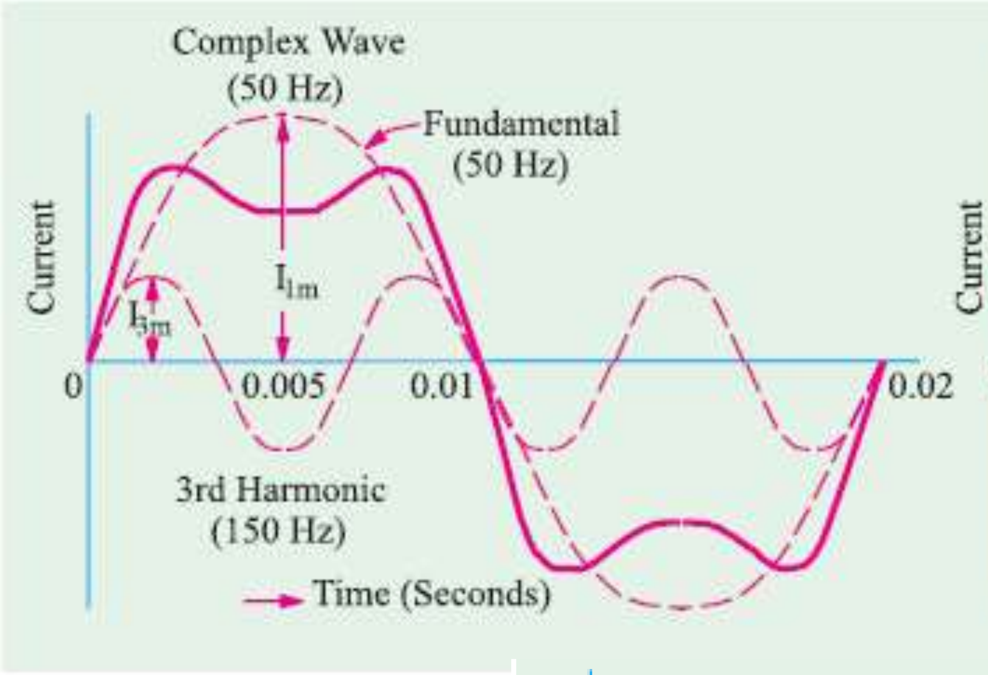
$$i = I_m \sin \omega t$$

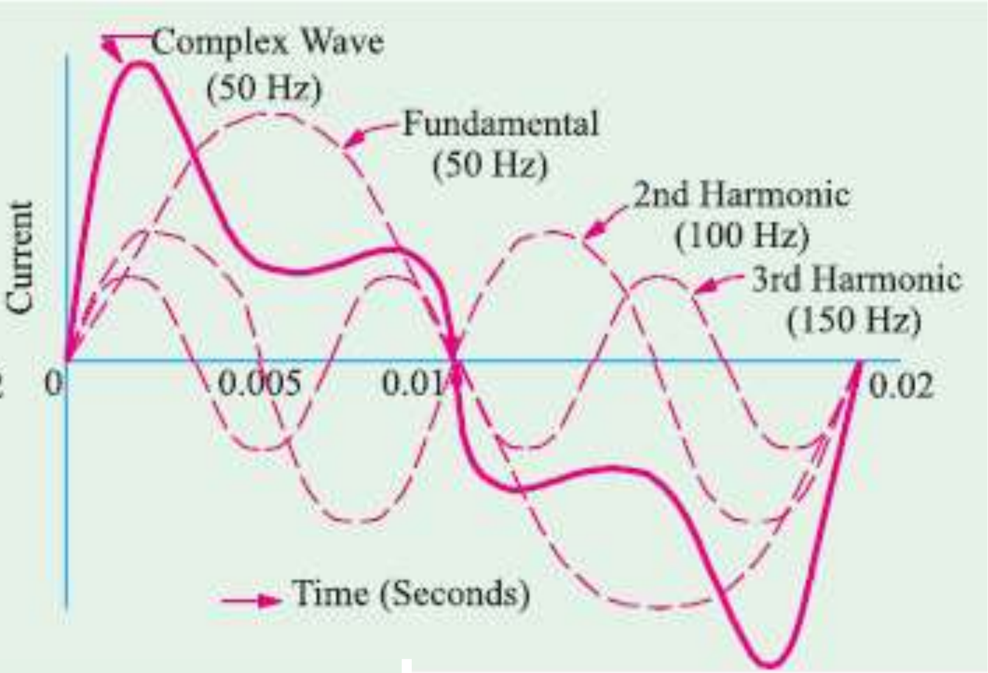
$$E_m = 2\, \pi\, f\, N\, B\, A \text{ volts}$$

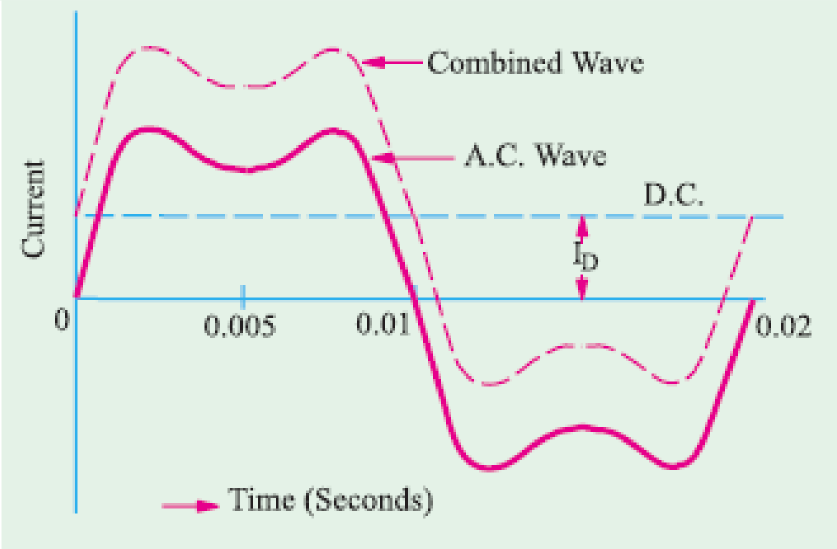
Simple Waveforms



Complex Waveforms







Important A.C. Terminology

An alternating voltage or current changes continuously in magnitude and alternates in direction at regular intervals of time. It rises from zero to maximum positive value, falls to zero, increases to a maximum in the reverse direction and falls back to zero again

Waveform. The shape of the curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abcissa is called its *waveform* or *waveshape.*

Instantaneous value. The value of an alternating quantity at any instant is called instantaneous value.

Cycle. One complete set of positive and negative values of an alternating quantity is known as a cycle. A cycle can also be defined in terms of angular measure. One cycle corresponds to 360º electrical or 2π radians.

Alternation. One-half cycle of an alternating quantity is called an alternation. An alternation spans 180º electrical. The positive or negative half of alternating voltage is the alternation.

Time period. The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.

Frequency. The number of cycles that occur in one second is called the frequency (f) of the alternating quantity. It is measured in cycles/sec (C/s) or Hertz (Hz). One Hertz is equal to 1C/s.

Amplitude. The maximum value (positive or negative) attained by an alternating quantity is called its amplitude or peak value. The amplitude of an alternating voltage or current is designated by V_m (or E_m) or I_m.

Time period and frequency.

Time taken to complete *f* cycles = 1 second
Time taken to complete 1 cycle = 1/*f* second

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

Angular velocity and frequency

the coil is rotating with an angular velocity of ω rad/sec in a uniform magnetic field. In one revolution of the coil, the angle turned is 2π radians and the voltage wave completes 1 cycle. The time taken to complete one cycle is the time period *T* of the alternating voltage.

$$\text{Angular velocity, } \omega = \frac{\text{Angle turned}}{\text{Time taken}} = \frac{2\pi}{T}$$

$$\omega = 2\pi f$$

Frequency and speed

A coil rotating at a speed of *N* r.p.m. in the field of *P* poles. As the coil moves past successive north and south poles, one complete cycle is generated. Obviously, in one revolution of the coil, *P*/2 cycles will be generated

$$\begin{aligned} \text{Frequency, } f &= \text{No. of cycles/sec} \\ &= (\text{No. of cycles/revolution}) \times (\text{No. of revolutions/sec}) \\ &= \left(\frac{P}{2}\right) \times \left(\frac{N}{60}\right) = \frac{PN}{120} \end{aligned}$$

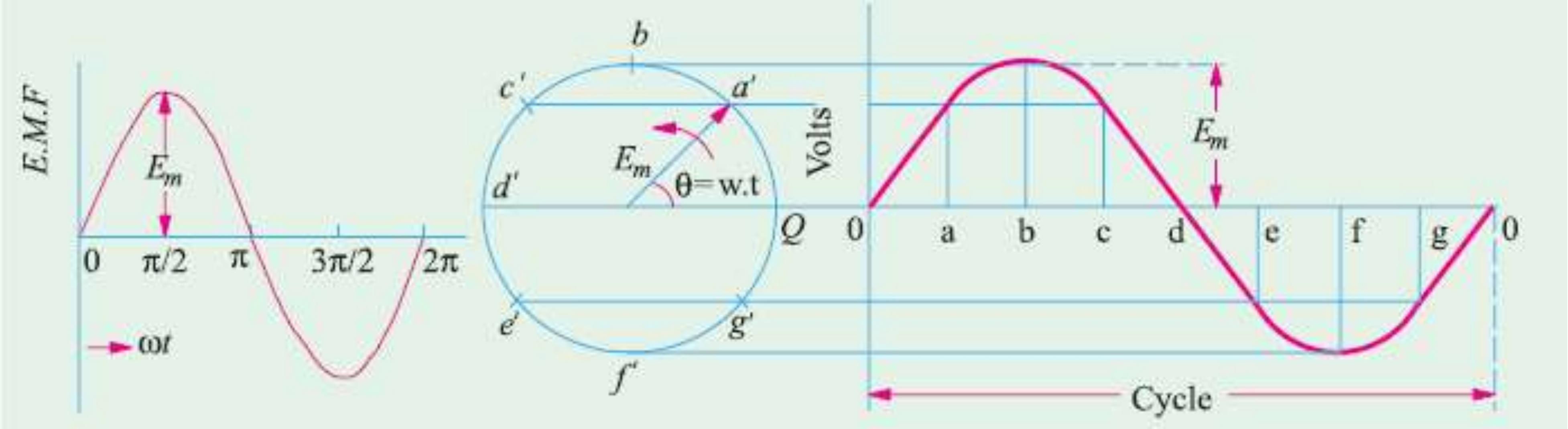
$$f = \frac{PN}{120}$$

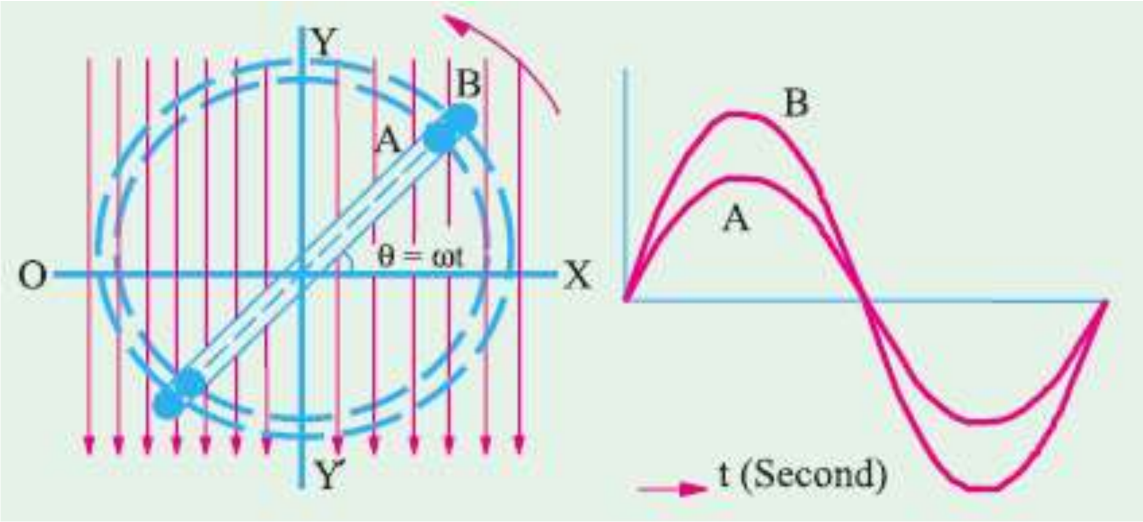
Different Forms of Alternating Voltage

$$v = V_m \sin\theta$$
$$= V_m \sin\omega t$$
$$= V_m \sin 2\pi f t \qquad (\because \omega = 2\pi f)$$
$$= V_m \sin \frac{2\pi}{T} t \qquad (\because f = 1/T)$$

Phase

Phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

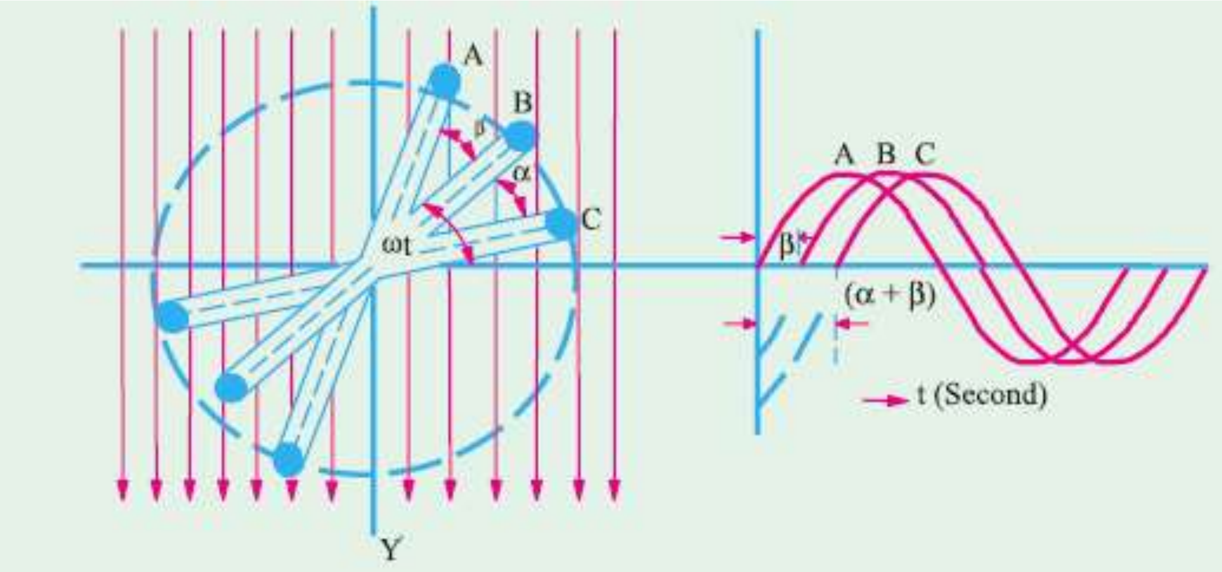




$e_1 = E_{m1} \sin \omega t$ and $e_2 = E_{m2} \sin \omega t$

Phase Difference

when two alternating quantities of the same frequency have different zero points, they are said to have a **phase difference.**

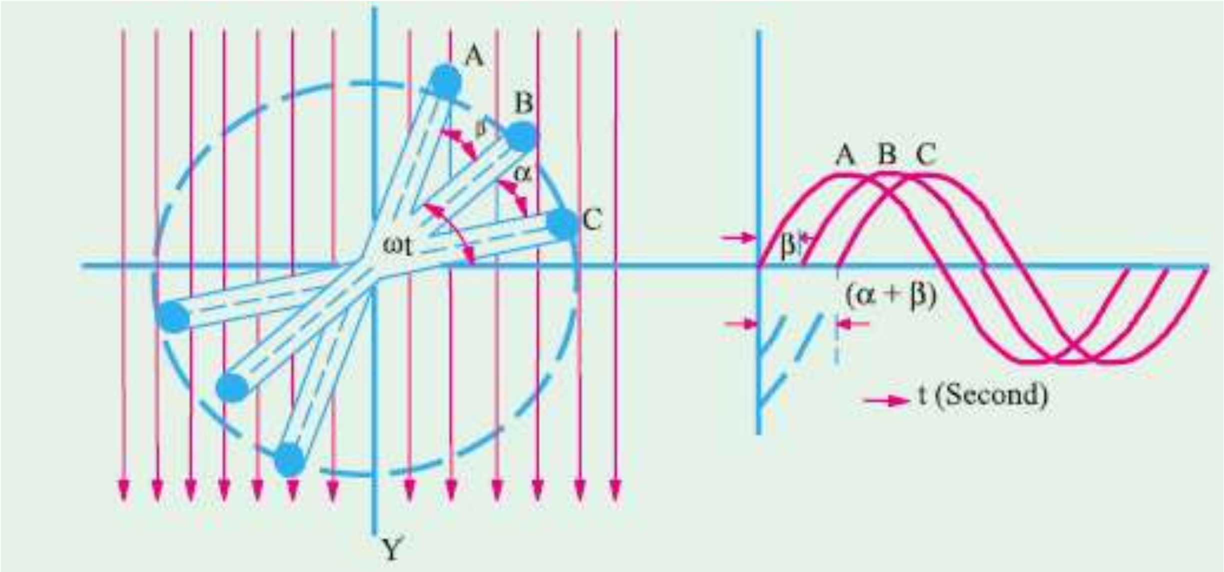


B lags behind *A* by α and *C* lags behind *A* by (α + β) because they reach their maximum values later

‘lag’ or **‘lead’**

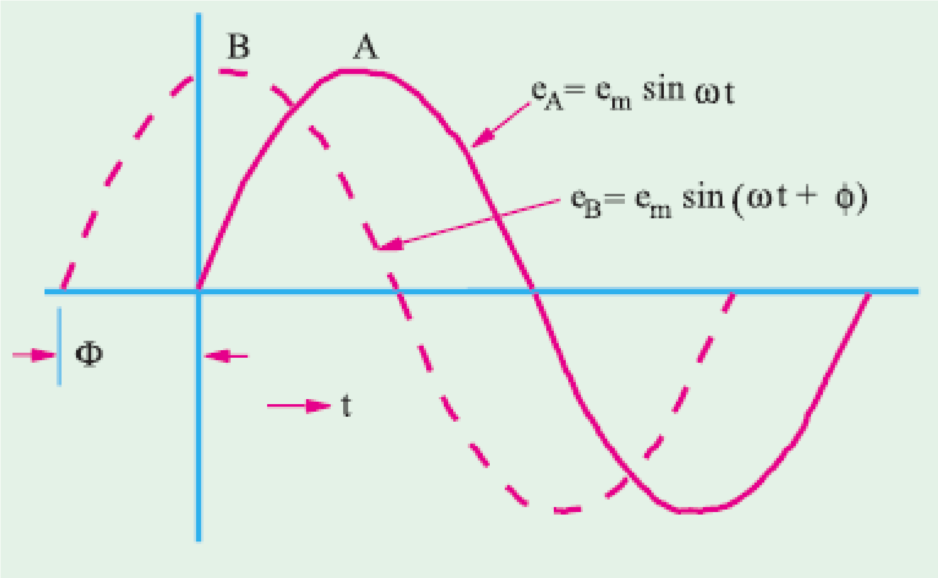
A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared to the other quantity.

Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity



B lags behind A by α and C lags behind A by (α + β) because they reach their maximum values later

$$
\begin{aligned}
e_A &= E_m \sin \omega t \text{ ...reference quantity} \\
e_B &= E_m \sin (\omega t - \beta) \\
e_C &= E_m \sin [\omega t - (\alpha + \beta)]
\end{aligned}
$$

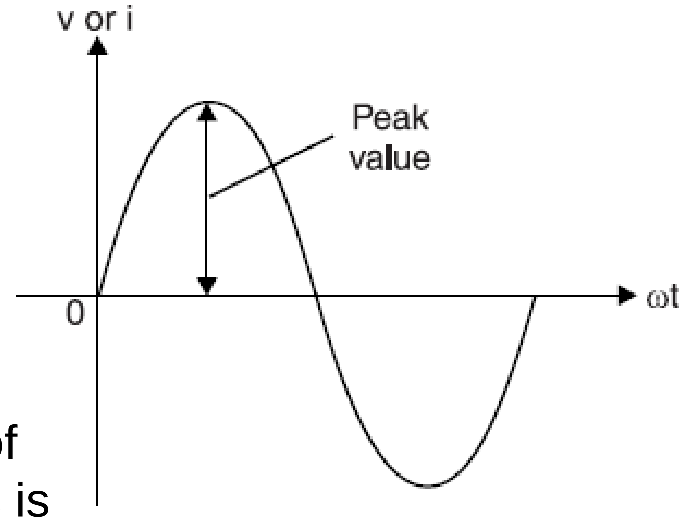


quantity B leads A by an angle Φ

$$e_A = E_m \sin \omega t \text{ ...reference quantity}$$
$$e_B = E_m \sin(\omega t - \phi)$$

Peak Value

It is the maximum value attained by an alternating quantity. The peak or maximum value of an alternating voltage or current is represented by V_m or I_m.



Average Value

The average value of a waveform is the average of all its values over a period of time. That steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

$$\text{Average value} = \frac{\text{Total (net) area under curve for time } T}{\text{Time } T}$$

In case of *symmetrical waves (*e.g.* sinusoidal voltage or current), the average value over one cycle is zero. It is because positive half is exactly equal to the negative half so that net area is zero. However, the average value of positive or negative half is not zero. Hence in case of symmetrical waves, average value means the average value of half-cycle or one alternation

$$\text{Average value of a symmetrical wave} = \frac{\text{Area of one alternation}}{\text{Base length of one alternation}}$$

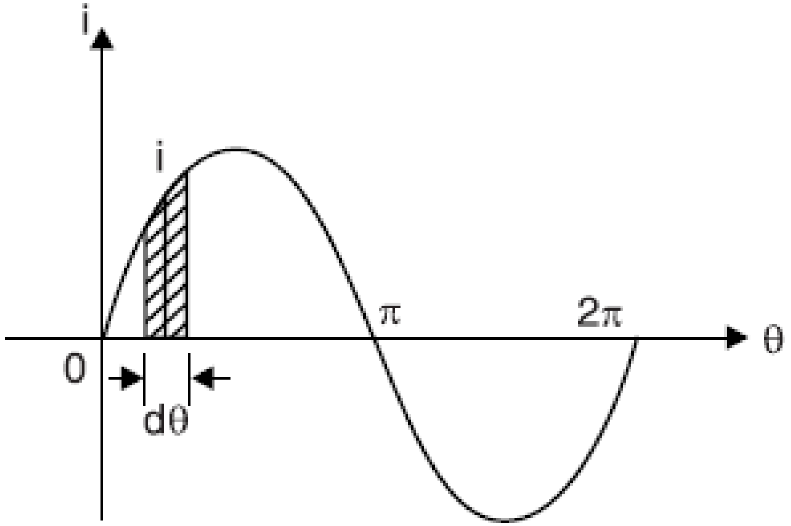
In case of unsymmetrical waves (e.g. half-wave rectified voltage etc.), the average value is taken over the full cycle.

$$\text{Average value of an unsymmetrical wave} = \frac{\text{Area over one cycle}}{\text{Base length of 1 cycle}}$$

$$= \frac{\text{Sum of *mid-ordinates over one alternation}}{\text{No. of mid-ordinates}}$$

$$= \frac{i_1 + i_2 + i_3 + \ldots + i_n}{n}$$

Average Value of Sinusoidal Current



$$i = I_m \sin\theta$$

$$\text{Area of strip} = i\,d\theta$$

$$\text{Area of half-cycle} = \int_0^{\pi} i\,d\theta$$

$$= \int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= I_m[-\cos\theta]_0^{\pi} = 2I_m$$

$$\therefore \quad \text{Average value, } I_{av} = \frac{\text{Area of half-cycle}}{\text{Base length of half-cycle}} = \frac{2I_m}{\pi}$$

$$\text{or} \quad I_{av} = 0{\cdot}637\, I_m$$

Hence, the half-cycle average value of a.c. is 0·637 times the peak value of a.c.
For positive half-cycle, $I_{av} = +\ 0{\cdot}637\ I_m$
For negative half-cycle, $I_{av} = -\ 0{\cdot}637\ I_m$
Clearly, average value of a.c. over a complete cycle is zero. Similarly, it can be proved that for alternating voltage varying sinusoidally, $\mathbf{V_{av} = 0{\cdot}637\ V_m}$

R.M.S.(Root Mean Square Value) or Effective Value

The **effective or r.m.s. value** of an alternating current is that steady current (d.c.) which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistance for the same time.

$$i = I_m \sin\theta$$

$$\text{Area of strip} = i^2 d\theta$$

Area of half-cycle of the squared wave

$$= \int_0^{\pi} i^2 d\theta$$

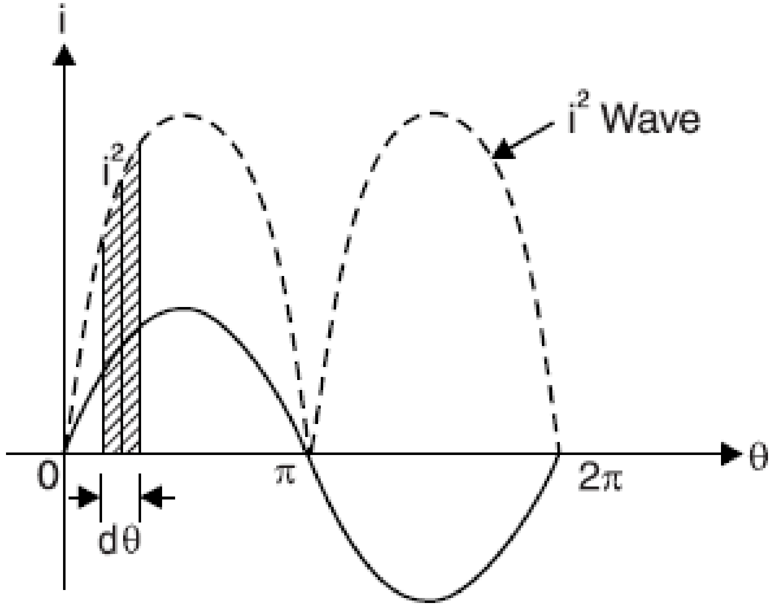
$$= \int_0^{\pi} I_m^2 \sin^2\theta d\theta$$

$$= I_m^2 \int_0^{\pi} \sin^2\theta d\theta = \frac{{}^*\pi I_m^2}{2}$$

$$\therefore \quad I_{r.m.s.} = \sqrt{\frac{\text{Area of half-cycle squared wave}}{\text{Half-cycle base}}}$$

$$= \sqrt{\frac{\pi I_m^2/2}{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$$\therefore \quad I_{r.m.s.} = 0{\cdot}707\, I_m$$



$$I = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots + i_n^2}{n}}$$

Form Factor

The ratio of r.m.s. value to the average value of an alternating quantity is known as **form factor** i.e.

$$\text{Form factor} = \frac{\text{R.M.S. value}}{\text{Average value}}$$

For a sinusoidal voltage or current,

$$\text{Form factor} = \frac{0.707 \times \text{Max. value}}{0.637 \times \text{Max. value}} = 1.11$$

The form factor gives a measure of the “peakiness” of the waveform. The peakier the wave, the greater is its form factor and vice-versa. For instance, a sine wave is peakier than a square wave. Similarly, a triangular wave is more peaky than a sine wave and has a form factor of 1·15.

Peak factor

The ratio of maximum value to the r.m.s. value of an alternating quantity is known as **peak factor**

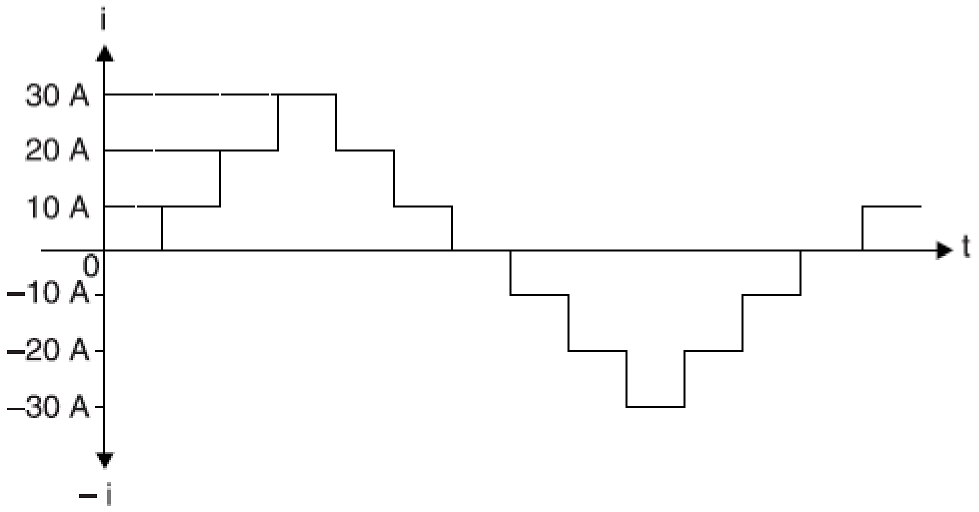
$$\text{Peak factor} = \frac{\text{Max. value}}{\text{R.M.S. value}}$$

For a sinusoidal voltage or current,

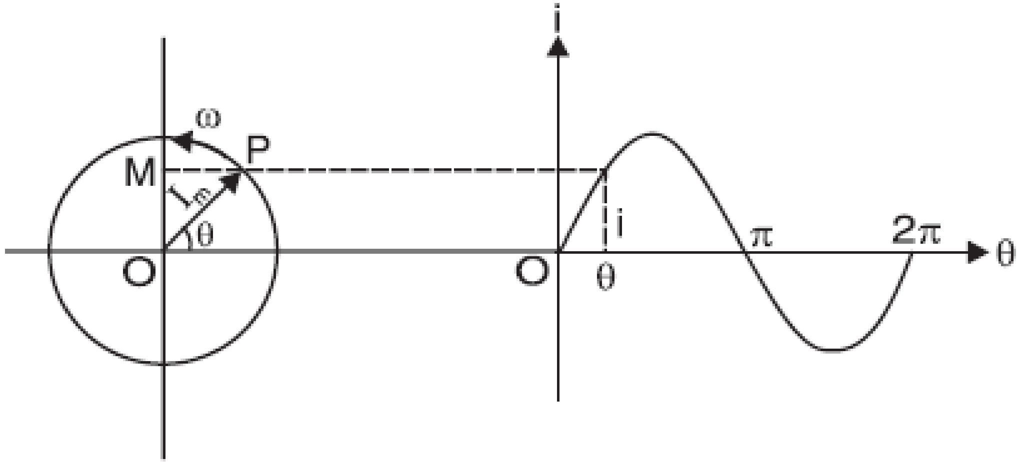
$$\text{Peak factor} = \frac{\text{Max. value}}{0.707 \times \text{Max. value}} = 1.414$$

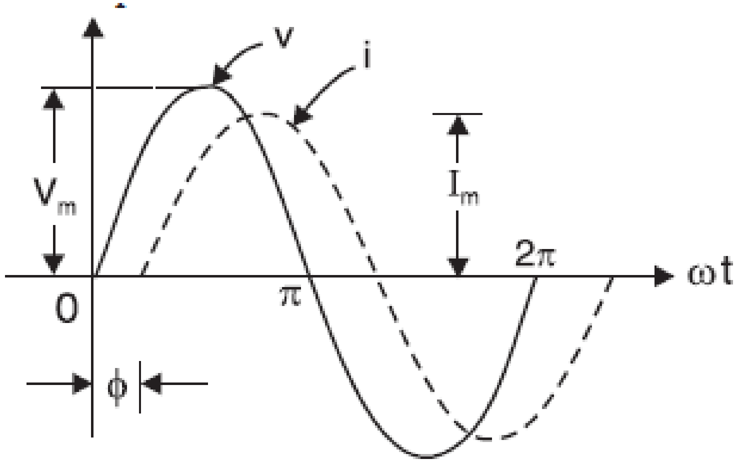
Q. An alternating voltage $v = 200 \sin 314t$ is applied to a device which offers an ohmic resistance of 20 Ω to the flow of current in one direction while entirely preventing the flow of current in the opposite direction. Calculate the current r.m.s. value, average value and form factor.

Q. A current has the following steady values in amperes for equal intervals of time changing instantaneously from one value to the next .0, 10, 20, 30, 20, 10, 0, −10, −20, −30, −20, −10, 0, etc. Calculate (i) average value (ii) r.m.s. value (iii) form factor and (iv) peak factor.

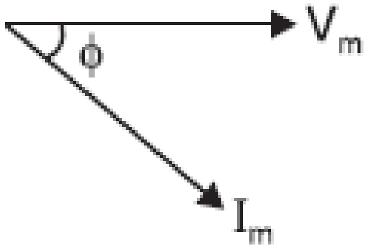


Phasor Representation of Sinusoidal Quantities

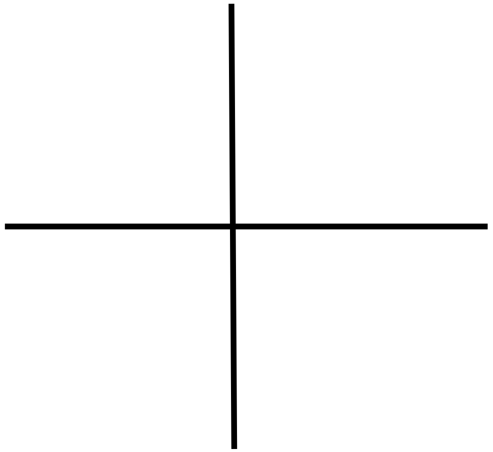


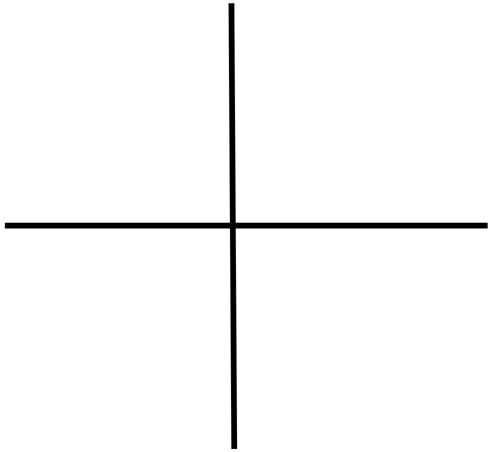


$$v = V_m \sin \omega t$$
$$i = I_m \sin (\omega t - \phi)$$



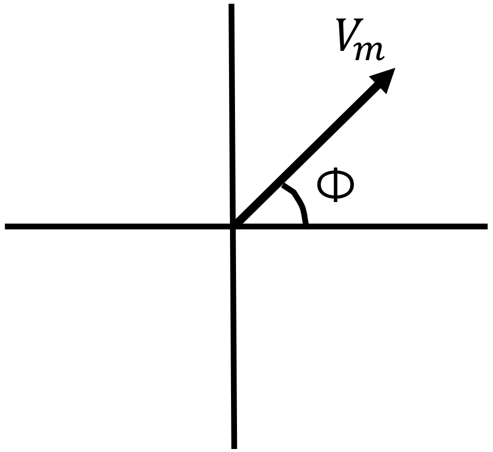
Representing Phasors





Representation

Polar Form



Rectangular Form A + j B

Exponential Form Vm $e^{-j\emptyset}$

Three resistors in parallel take the following currents :

$i_1 = 20 \sin 314\, t$;

$i_2 = 30 \sin (314\, t - \pi/4)$;

$i_3 = 40 \text{ Sin } (314\, t + 2\pi/3)$

Find the expression for the resultant current.

its r.m.s. value and frequency

A circuit consists of three loads in series ; the voltage across these loads are given by the following relations measured in volts :

$v_1 = 50 \sin \omega t$;

$v_2 = 25 \sin (\omega t + 60^\circ)$

$v_3 = 30 \sin (\omega t - 45^\circ)$

Calculate the supply voltage giving the relation in similar form.

A.C. Circuit Containing Resistance Only

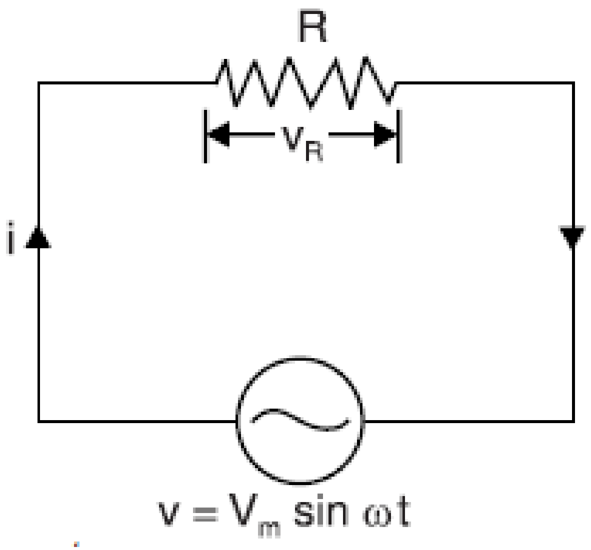
Let the alternating voltage be given by the equation

$$v = V_m \sin \omega t$$

$$v = i R$$

$$i = \frac{v}{R}$$

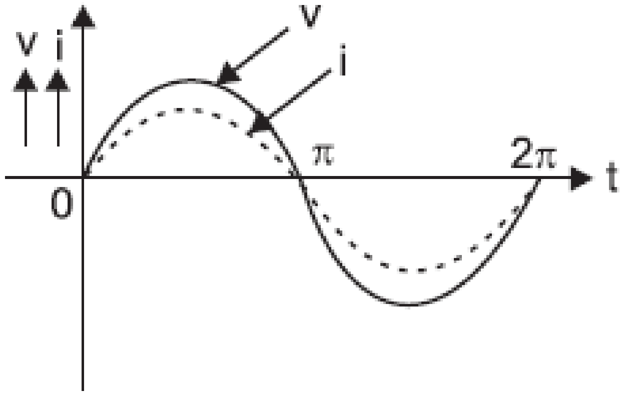
$$i = \frac{V_m}{R} \sin \omega t$$

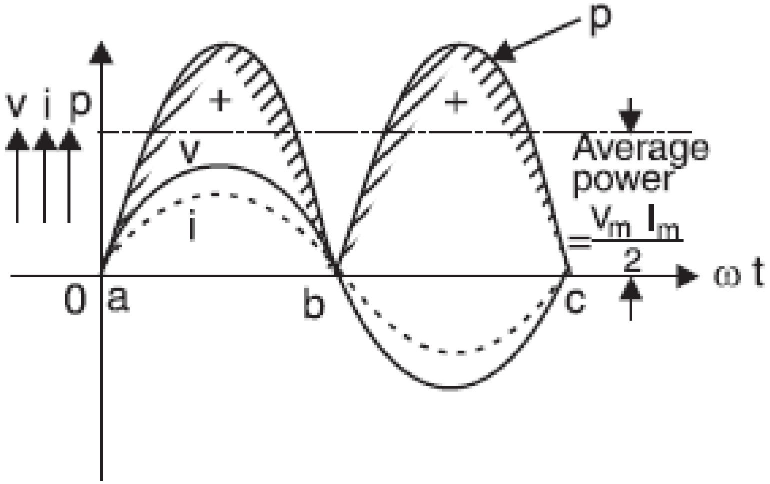


The value of *i* will be maximum (*i.e.* I_m) when $\sin \omega t = 1$.

$$I_m = V_m / R$$

$$i = I_m \sin \omega t$$





Power

Instantaneous power, $p = v\,i = (V_m \sin \omega t)(I_m \sin \omega t) = V_m I_m \sin^2 \omega t$

$$= V_m I_m \frac{(1-\cos 2\omega t)}{2} = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Thus power consists of two parts *viz*. a constant part ($V_m I_m/2$) and a fluctuating part ($V_m I_m/2$) $\cos 2\omega t$. Since power is a scalar quantity, average power over a complete cycle is to be considered.

Power consumed, $P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} d(\omega t) + \frac{1}{2\pi} * \int_0^{2\pi} \frac{V_m I_m}{2} \cos 2\omega t \, d(\omega t)$

$$= \frac{V_m I_m}{2} + 0 = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$P = V_R I = V I$$

where $V = V_R =$ r.m.s. value of the applied voltage

$I =$ r.m.s. value of the circuit current

A.C. Circuit Containing Pure Inductance Only

Let the equation of the applied alternating voltage be :

$$v = V_m \sin \omega t$$

Clearly, $$V_m \sin \omega t = L \frac{di}{dt}$$

or $$di = \frac{V_m}{L} \sin \omega t \, dt$$

Integrating both sides, we get, $i = \frac{V_m}{L} \int \sin \omega t \, dt = \frac{V_m}{\omega L}(-\cos \omega t)$

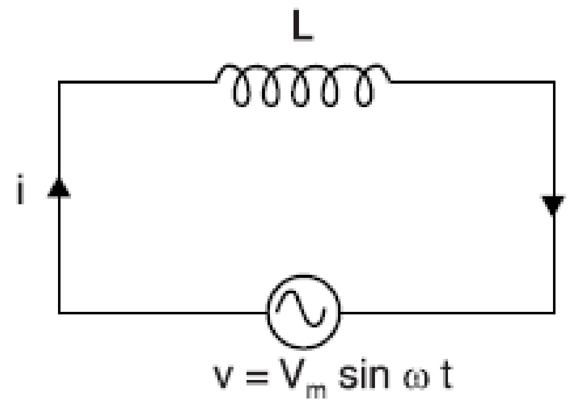
$$\therefore \quad i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$

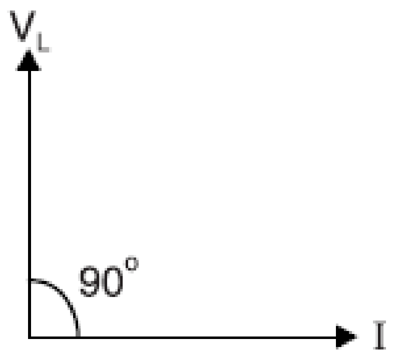
The value of i will be maximum (*i.e.* I_m) when $\sin(\omega t - \pi/2)$ is unity.

$$\therefore \quad I_m = V_m / \omega L$$

Substituting the value of $V_m/\omega L = I_m$ in eq. , we get,

$$i = I_m \sin(\omega t - \pi/2)$$



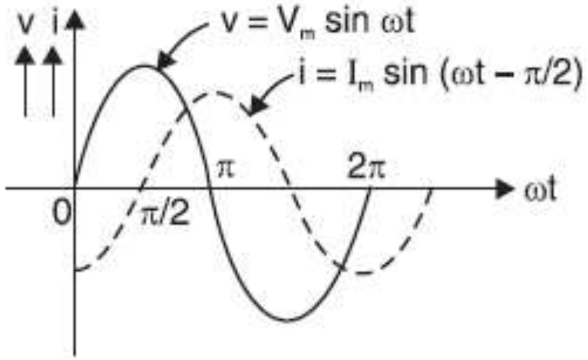


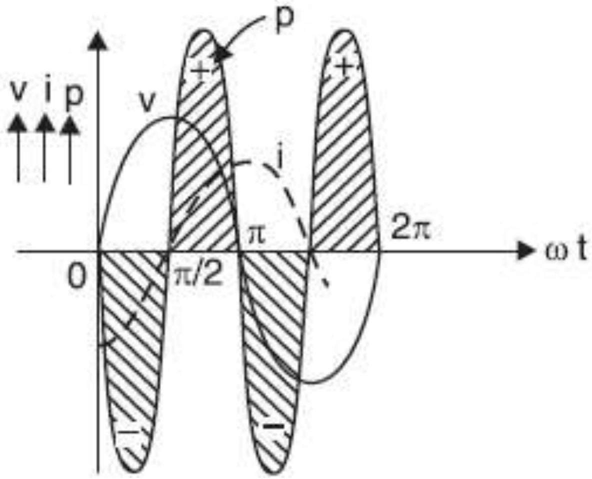
Inductive reactance

$$I_m = V_m/\omega L$$

$$\frac{V_m}{I_m} = \omega L$$

$$X_L = \omega L = 2\pi f L$$





Power

$$\text{Instantaneous power, } p = v\,i = V_m \sin \omega t \times I_m \sin(\omega t - \pi/2)$$

$$= -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t$$

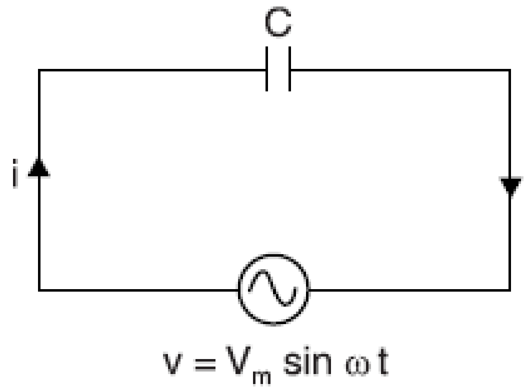
$$\therefore \quad \text{Average power, } P = \text{Average of } p \text{ over one cycle}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

Hence power absorbed in pure inductance is zero.

In a resistance, current and p.d. are in phase and power is absorbed. In an inductance, current and p.d. are 90° out of phase and there is no net power consumed

A.C. Circuit Containing Capacitance Only



$$v = V_m \sin \omega t$$

Let at any instant *i* be the current and *q* be the charge on the plates.

$$\text{Charge on capacitor, } q = C\,v = C\,V_m \sin \omega t$$

$$\therefore \quad \text{Circuit current, } i = \frac{d}{dt}(q) = \frac{d}{dt}(C\,V_m \sin \omega t) = \omega\,C\,V_m \cos \omega\,t$$

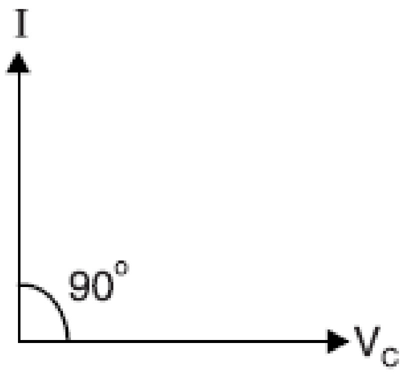
$$\therefore \quad i = \omega\,C\,V_m \sin(\omega\,t + \pi/2)$$

The value of i will be maximum (*i..e.* I_m) when $\sin(\omega\,t + \pi/2)$ is unity.

$$\therefore \quad I_m = \omega\,C\,V_m$$

Substituting the value $\omega C\,V_m = I_m$

$$i = I_m \sin(\omega\,t + \pi/2)$$

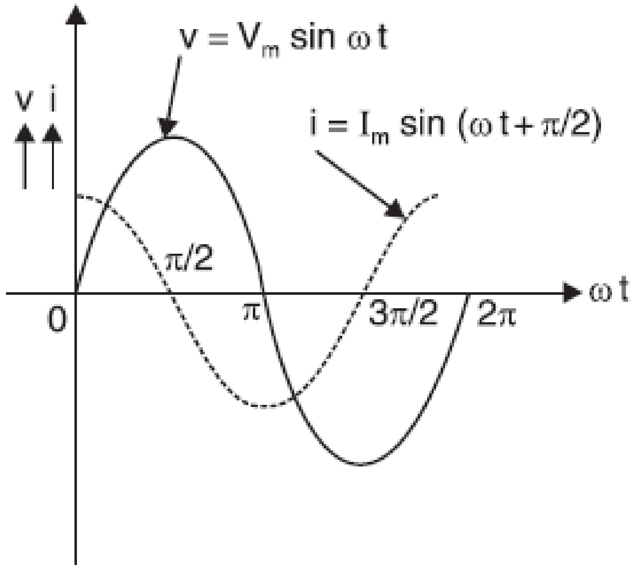


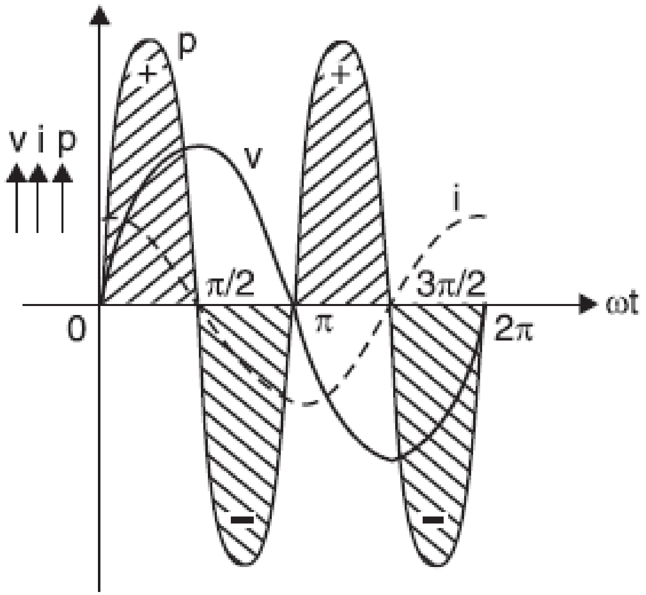
Capacitive reactance

$$I_m = \omega C V_m$$

$$\frac{V_m}{I_m} = \frac{1}{\omega C}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$





Power

Instantaneous power, $p = v\,i = V_m \sin \omega t \times I_m \sin(\omega t + \pi/2) = V_m I_m \sin \omega t \cos \omega t$

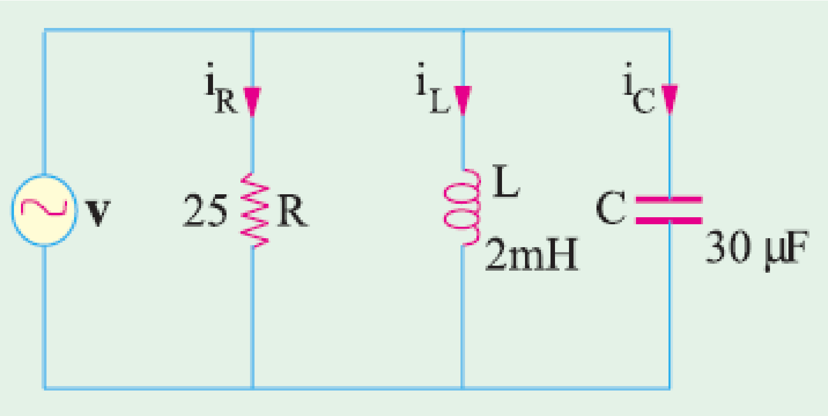
$\therefore$ $$p = \frac{V_m I_m}{2} \sin 2\omega t$$

$\therefore$ Average power, P = Average of p over one cycle

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

Hence power absorbed in a pure capacitance is zero.

The voltage applied across 3-branched circuit of Fig. is given by v = 100 sin (5000t + π/4). Calculate the branch currents and total current.



The current in a circuit is given by (4.5 + j12) A when the applied voltage is (100 + j150) V. Determine (i) the magnitude of impedance and (ii) phase angle.

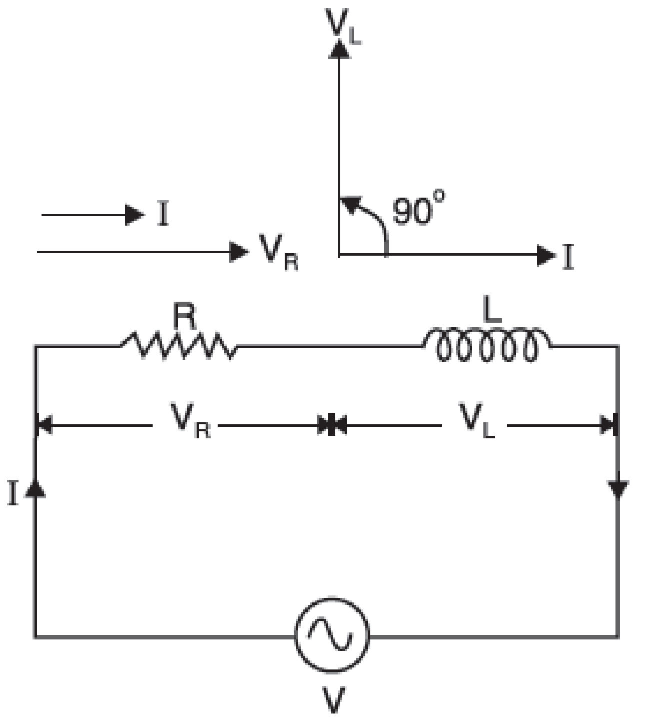
R- L Series A.C. Circuit

V = r.m.s. value of the applied voltage

I = r.m.s. value of the circuit current

$V_R = I\,R$ .... where V_R is in phase with I

$V_L = I\,X_L$ .... where V_L leads I by 90°



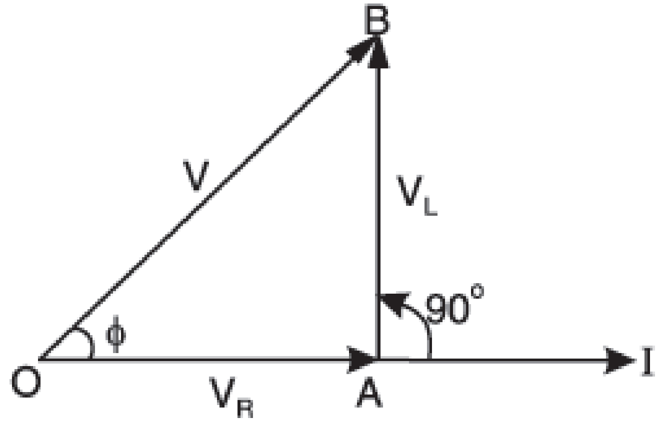
The applied voltage V is the phasor sum of

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

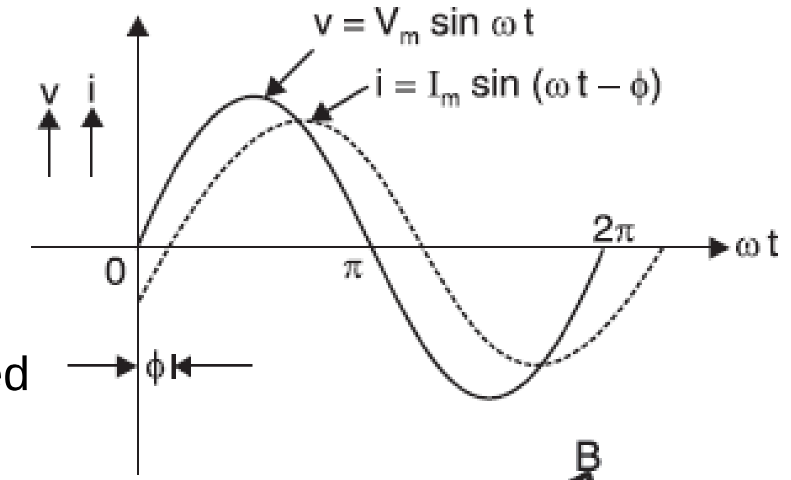
The quantity $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called **impedance** of the circuit. It is represented by Z and is measured in ohms (Ω).

$$\therefore \quad I = \frac{V}{Z} \quad \text{where } Z = \sqrt{R^2 + X_L^2}$$

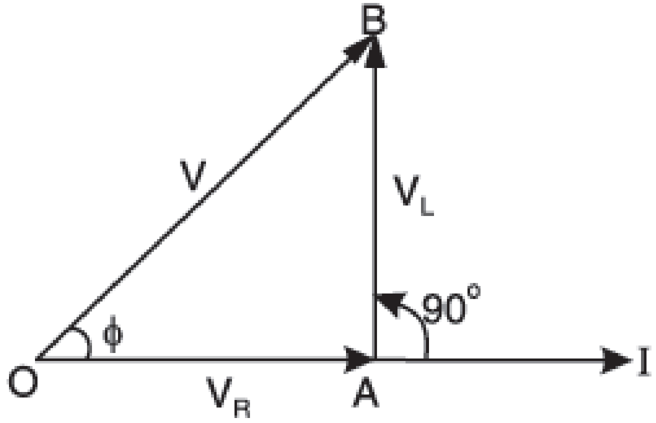


The value of phase angle Φ is

$$\tan\phi = \frac{V_L}{V_R} = \frac{I\,X_L}{I\,R} = \frac{X_L}{R}$$



In an inductive circuit, current lags behind the applied voltage. The angle of lag (i.e. Φ) is greater than 0° but less than 90°. It is determined by the ratio of inductive reactance to resistance ($\tan \Phi = X_L/R$) in the circuit. The greater the value of this ratio, the greater will be the phase angle Φ and vice-versa.



$$\text{Impedance, } Z = \sqrt{R^2 + X_L^2} \quad \text{where } X_L = 2\pi f L$$

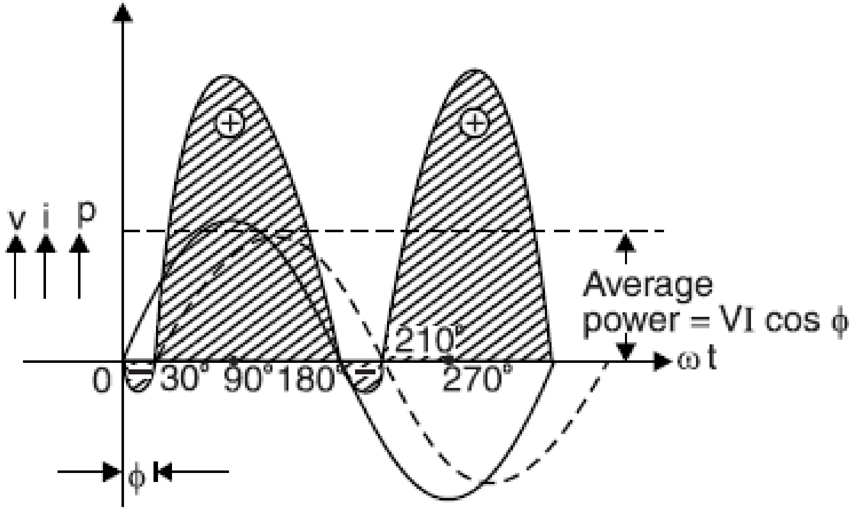
Power

$$\text{Instantaneous power, } p = v\,i = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$= \frac{1}{2} V_m\, I_m\, [2 \sin \omega t \sin(\omega t - \phi)]$$

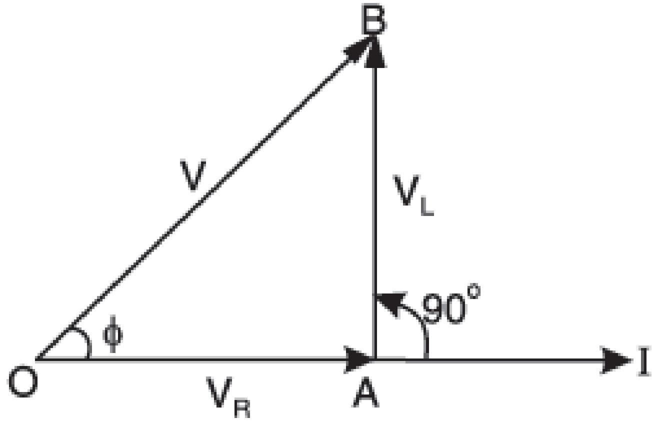
$$= \frac{1}{2} V_m I_m [\cos \phi - \cos(2\omega t - \phi)]$$

$$= \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$



$$\text{Average power, } P = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos \phi$$

$$P = V I \cos \phi$$



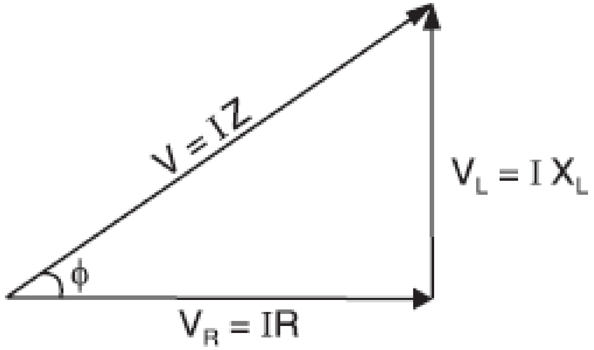
The term cos Φ is called **power factor** of the circuit and its value is given by

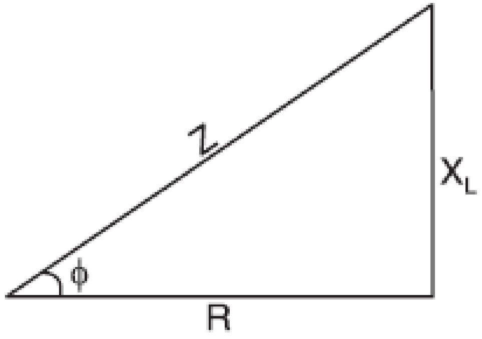
$$\text{Power factor, } \cos \phi = \frac{IR}{IZ} = \frac{R}{Z}$$

$$P = VI \cos \phi = (IZ)\, I\, (R/Z) = I^2 R \qquad [\because\ V = IZ \text{ and } \cos \phi = R/Z]$$

This is expected because power is consumed in resistance only; inductance does not consume any power.

Impedance Triangle





Impedance triangle is a useful concept in a.c. circuits as it enables us to calculate :

(*i*) the impedance of the circuit *i.e.*, $Z = \sqrt{R^2 + X_L^2}$

(*ii*) power factor of the circuit *i.e.*, $\cos\phi = R/Z$

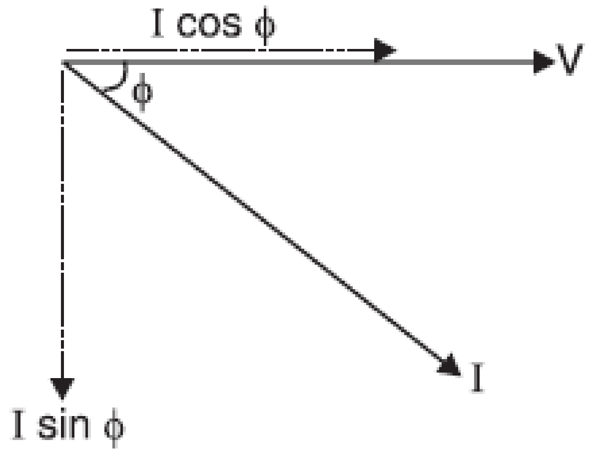
(*iii*) phase angle ϕ *i.e.*, *$\tan\phi = X_L/R$

(*iv*) whether current leads or lags the voltage.

Therefore, it is always profitable to draw the impedance triangle while analysing an a.c. circuit.

(i) $I \cos \phi$ in phase with V.

(ii) $I \sin \phi$; 90° out of phase with V.



Apparent power (S). The total power that appears to be transferred between the source and load is called **apparent power.** It is equal to the product of applied voltage (V) and circuit current (I) i.e.

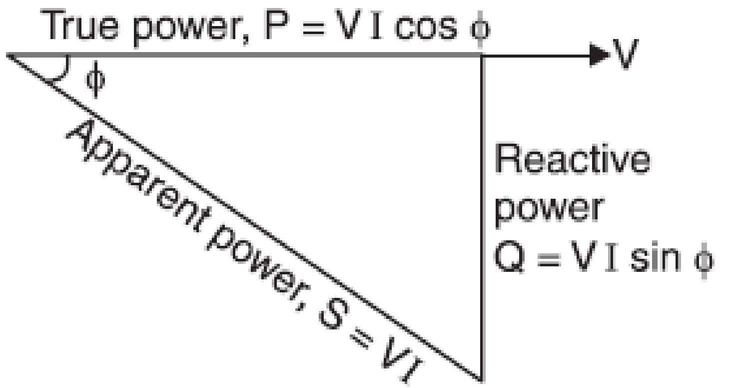
$$\text{Apparent power, } S = V \times I = VI = (P+jQ)$$

It is measured in volt-ampers (VA).

Apparent power has two components viz true power and reactive power.

$$S = \text{Phasor voltage} \times \text{Conjugate of phasor current}$$

$$P + jQ = \mathbf{V} \times \mathbf{I}^*$$



True power (P). The power which is actually consumed in the circuit is called **true power** or **active power.** The product of voltage (V) and component of total current in phase with voltage (I cos Φ) is equal to **true power** i.e.

$$\begin{aligned} \text{True power, } P &= \text{Voltage} \times \text{Component of total current in phase with voltage} \\ &= V \times I \cos \phi \\ P &= VI \cos \phi \end{aligned}$$

It is measured in watts (W). The component *I* cos Φ is called in-phase component or **wattful component** because it is this component of total current which contributes to true power (*i.e. VI* cos Φ). It may be noted that it is the true power which is used for producing torque in motors and supply heat, light *etc*. It is used up in the circuit and cannot be recovered.

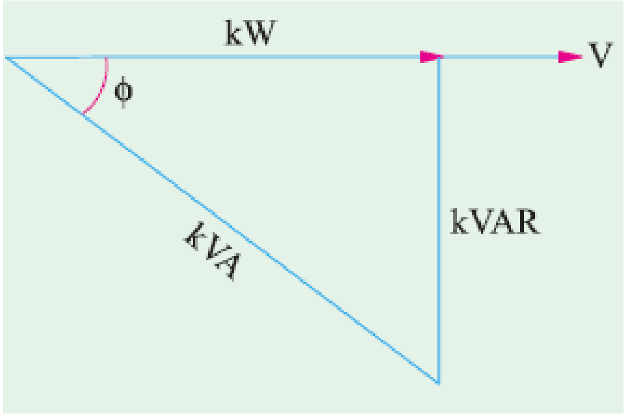
Reactive power (Q). The component of apparent power which is neither consumed nor does any useful work in the circuit is called **reactive power.** The power consumed (or true power) in L and C is zero because all the power received from the source in one quarter-cycle is returned to the source in the next quarter-cycle. This circulating power is called reactive power

The product of voltage (V) and component of total current 90° out of phase with voltage (I sin Φ) is equal to **reactive power** i.e.

$$\text{Reactive power, } Q = \text{Voltage} \times \text{Component of total current } 90^\circ \text{ out of phase with voltage}$$
$$= V \times I \sin\phi$$
$$\therefore \quad Q = VI \sin\phi$$

It is measured in volt-amperes reactive (VAR). The component *I* sin Φ is called the **reactive component** (or **wattless component**) and contributes to reactive power (*i.e. VI* sin Φ). It does no useful work in the circuit and merely flows back and forth in both directions in the circuit. A wattmeter does not measure the reactive power.

Power triangle



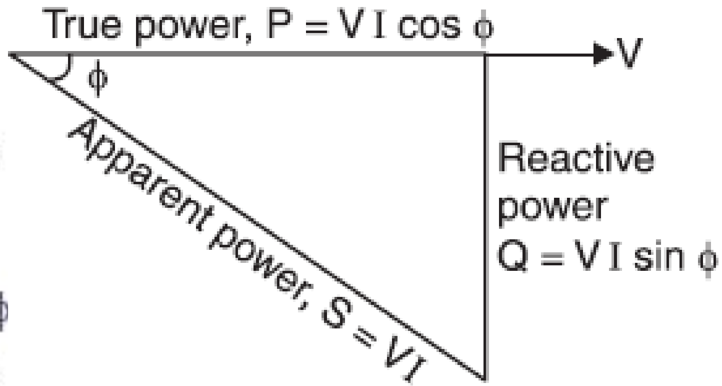
(*i*) $\text{Power factor, } \cos\phi = \dfrac{\text{True power}}{\text{Apparent power}} = \dfrac{VI\cos\phi}{VI}$

(*ii*) $(\text{Apparent power})^2 = (\text{True power})^2 + (\text{Reactive power})^2$

or $S^2 = P^2 + Q^2$

(*iii*) $\text{True power, } P = \text{Apparent power} \times \cos\phi = VI\cos\phi$

$\text{Reactive power, } Q = \text{Apparent power} \times \sin\phi = VI\sin\phi$



Power Factor

The power factor (*i.e*. cos Φ) of a circuit can be defined in one of the following ways :

(*i*) Power factor = $\cos\phi$ = cosine of angle between V and I

(*ii*) Power factor = $\dfrac{R}{Z} = \dfrac{\text{Resistance}}{\text{Impedance}}$

(*iii*) Power factor = $\dfrac{VI\cos\phi}{VI} = \dfrac{\text{True power}}{\text{Apparent power}}$

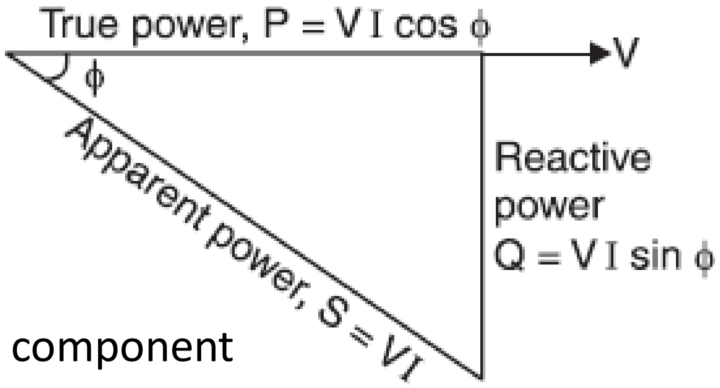
In a resistor, the current and voltage are in phase i.e. Φ = 0°. Therefore, power factor of a pure resistive circuit is cos 0° = 1. Similarly, phase difference between voltage and current in a pure inductance or capacitance is 90°. Hence power factor of pure L or C is zero. This is the reason that power consumed by pure L or C is zero. For a circuit having R, L and C in varying proportions, the value of power factor will lie between 0 and 1.

power factor can never have a value greater than 1.

Power factor expressed as:

- 'lagging' or 'leading'
- Percentage-0.8 lagging power factor may be expressed as 80% lagging

Apparent Power → True power
→ reactive power



True power should be large → does useful work in the circuit
So reactive power should be small
Smaller the phase angle Φ → greater the power factor → smaller the reactive power component

Thus power factor of a circuit is a measure of its effectiveness in utilising the apparent power drawn by it. The greater the power factor of a circuit, the greater is its ability to utilise the apparent power

Q-factor of a Coil

The ratio of the inductive reactance (X_L) of a coil to its resistance (R) at a given frequency is known as Q-factor of the coil at that frequency *i.e.*,

$$Q\text{-factor} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$Q\text{-factor} = 2\pi \times \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

The Q-factor is used to describe the quality or effectiveness of a coil. A coil is usually designed to have high value of L compared to its resistance R. The greater the value of Q-factor of a coil, the greater is its inductance (L) as compared to its resistance (R).

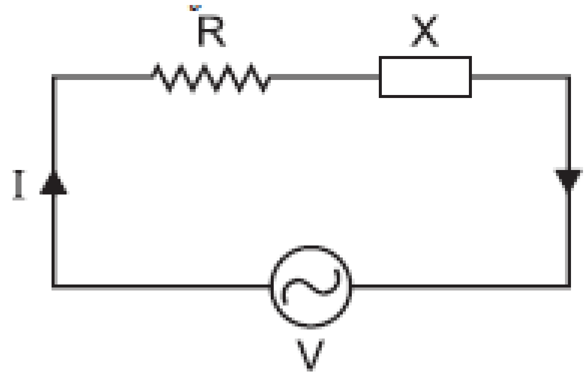
A current of 10 A at a voltage of 200 V and its power factor is 0.8 lagging. Find Apparent power, true power and reactive power.

The reactive power does no useful work, it merely flows into and out of the circuit periodically. In fact, reactive power is a liability on the source because the source has to supply the additional current (*i.e.* $I \sin \Phi$) to provide for this power

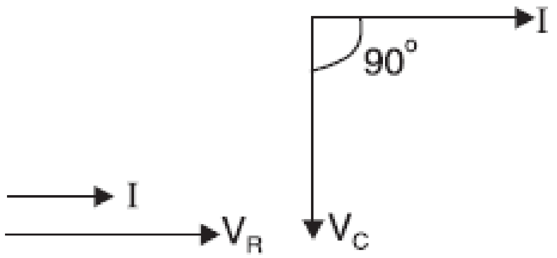
In an R-L series circuit, $R = 10\ \Omega$ and $X_L = 8.66\ \Omega$. If current in the circuit is $(5 - j\,10)$A, find (i) the applied voltage (ii) power factor and (iii) active power and reactive power

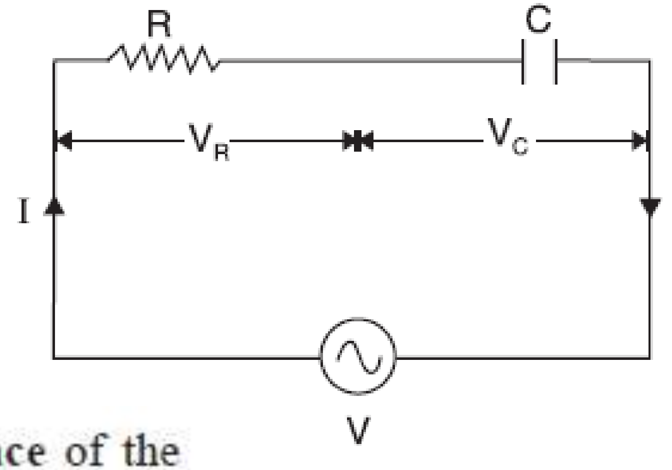
In a given R–L series circuit, $R = 35\ \Omega$ and $L = 0.1$ H. Find (i) current through the circuit (ii) power factor if a 50 Hz frequency, voltage $V = 220 \angle 30°$V is applied across the circuit.

In the circuit, applied voltage V is given by (0 + j 10) V and the current is (0.8 + j 0.6) A. Determine the values of R and X also indicate if X is inductive or capacitive.



R-C Series A.C. Circuit





V = r.m.s. value of applied voltage

I = r.m.s. value of the circuit current

$V_R = IR$ .... where V_R is in phase with I

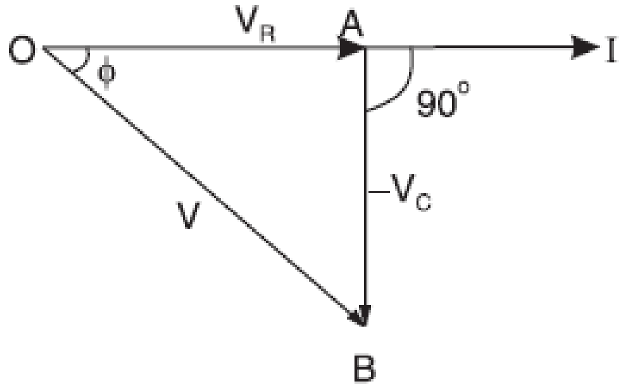
$V_C = IX_C$ .... where V_C lags I by 90°

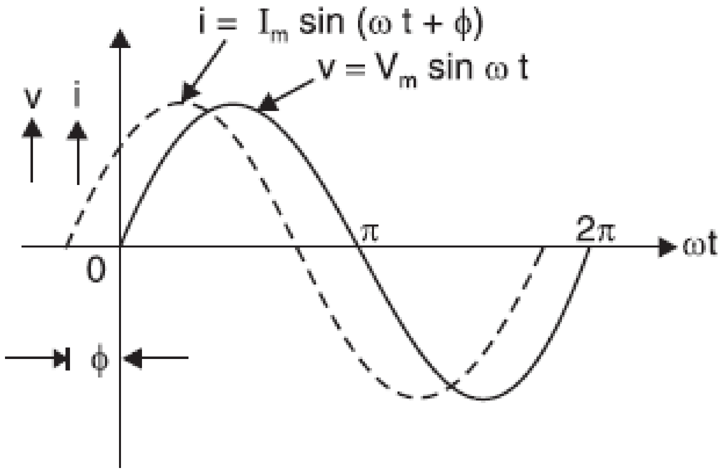
$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

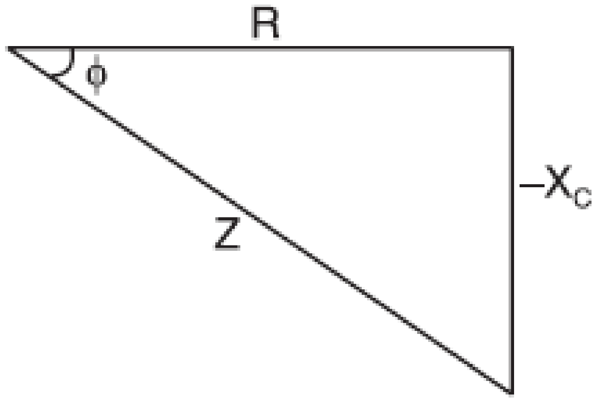
$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{R^2 + X_C^2}$ offers opposition to current flow and is called **impedance** of the circuit.

$\therefore$ $$I = V/Z \text{ where } Z = \sqrt{R^2 + X_C^2}$$







The value of phase angle Φ is

$$\tan\phi = -\frac{V_C}{V_R} = -\frac{IX_C}{IR} = -\frac{X_C}{R}$$

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t + \phi)$$

Power

$\therefore$ Average power, P = Average of vi

$= VI\cos\phi$

Alternatively. P = Power in R + Power in C

$$= I^2 R + 0 = IR \times I = IR \times \frac{V}{Z} = VI \times \frac{R}{Z} = VI\cos\phi$$

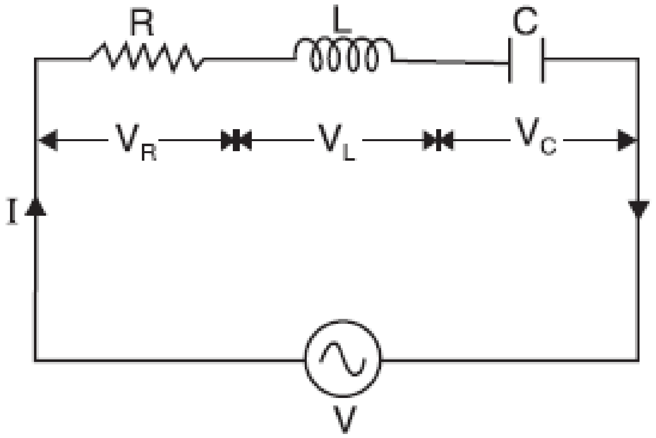
A capacitor of capacitance 79·5 μ F is connected in series with a non-inductive resistance of 30 Ω across 100 V, 50 Hz supply. Find (i) impedance (ii) current (iii) phase angle and (iv) equation for the instantaneous value of current.

R-L-C Series A.C. Circuit

Voltage across R, $V_R = I R$ ... V_R is in phase with I

Voltage across L, $V_L = I X_L$ ... where V_L leads I by 90°

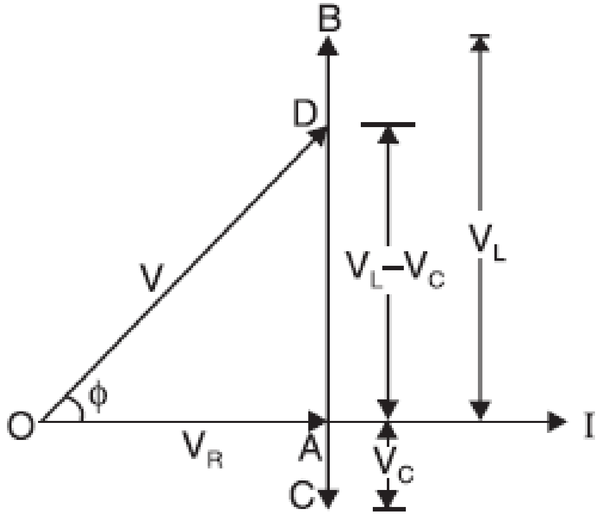
Voltage across C, $V_C = I X_C$ ... where V_C lags I by 90°



The circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(I R)^2 + (I X_L - I X_C)^2}$$

The quantity $\sqrt{R^2 + (X_L - X_C)^2}$ offers opposition to current flow and is called **impedance** of the circuit.



$$\text{Circuit power factor, } \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \quad ...(i)$$

Also,

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad ...(ii)$$

Since X_L, X_C and R are known, phase angle ϕ of the circuit can be determined.

$$\text{Power consumed, } P = V I \cos\phi = {}^*I^2 R$$

Three cases of *R-L-C* series circuit. We have seen that the impedance of a *R-L-C* series circuit is given by ;

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

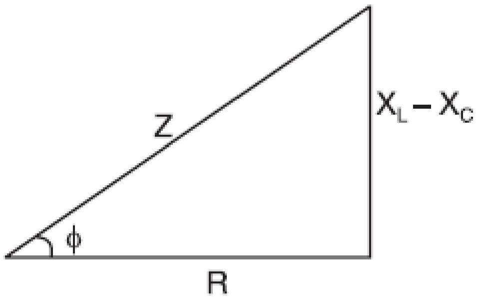
(*i*) **When $X_L - X_C$ is positive** (*i.e.* $X_L > X_C$), phase angle ϕ is positive and the circuit will be inductive. In other words, in such a case, the circuit current I will lag behind the applied voltage V by ϕ

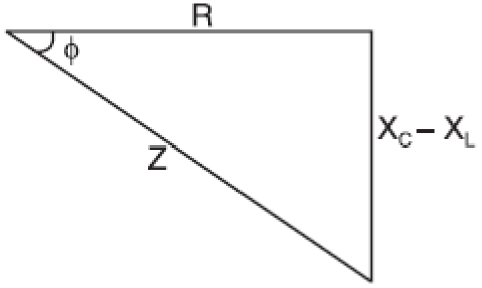
(*ii*) **When $X_L - X_C$ is negative** (*i.e.* $X_C > X_L$), phase angle ϕ is negative and the circuit is capacitive. That is to say the circuit current I leads the applied voltage V by ϕ

(*iii*) **When $X_L - X_C$ is zero** (*i.e.* $X_L = X_C$), the circuit is purely resistive. In other words, circuit current I and applied voltage V will be in phase *i.e.* $\phi = 0°$. The circuit will then have unity power factor.

$$v = V_m \sin \omega t,$$

$$i = I_m \sin(\omega t \pm \phi) \quad \text{where } I_m = V_m / Z$$

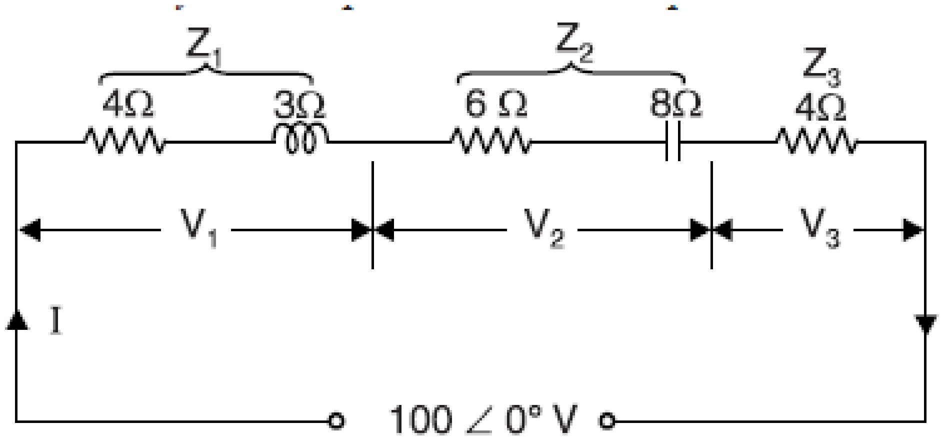




A coil of resistance 12 Ω and inductive reactance of 25 Ω is connected in series with a capacitive reactance of 41 Ω. The combination is connected to a supply of 230 V, 50Hz. Using phasor algebra, find (i) circuit impedance (ii) current and (iii) power consumed.

Ans: $Z = (12-16j)\Omega$; $I = (6.92+9.18j)A$; $P = 1591.8W$

In the circuit shown in Fig. 13.20, calculate (i) current (ii) voltage drops V1, V2 and V3



Ans: I = (6.33+2.26j)A; V1 = (18.54+28.06j)V; V2 = (56.22-36.97j)V; V3 = (25.34+ 9.06j)V

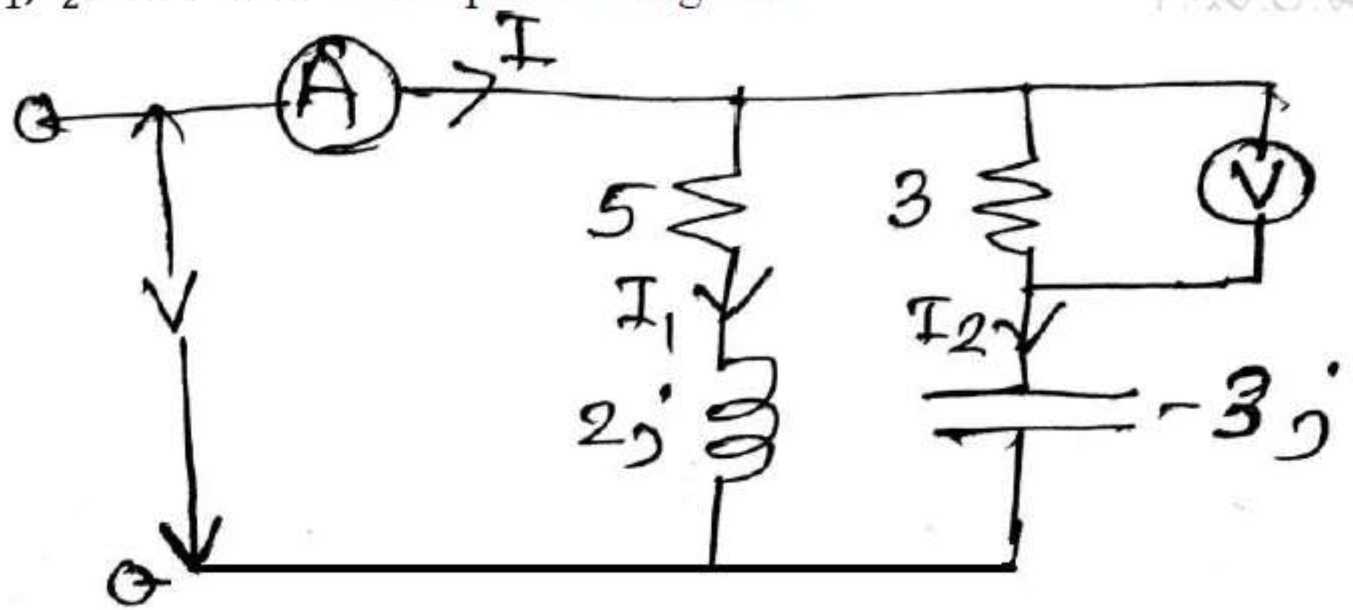
A 230 V, 50 Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5Ω resistance connected in series with a 6·8 μF capacitor. Calculate (i) impedance (ii) current (iii) phase angle between current and voltage (iv) power factor and (v) power consumed.

Ans: $Z = (2.5-449.26j)\Omega$; $I = (2.85+0.511j)A$; $P = 0.6830W$

A 230 V, 50 Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5Ω resistance connected in series with a 6·8 μF capacitor. Calculate (i) impedance (ii) current (iii) phase angle between current and voltage (iv) power factor and (v) power consumed.

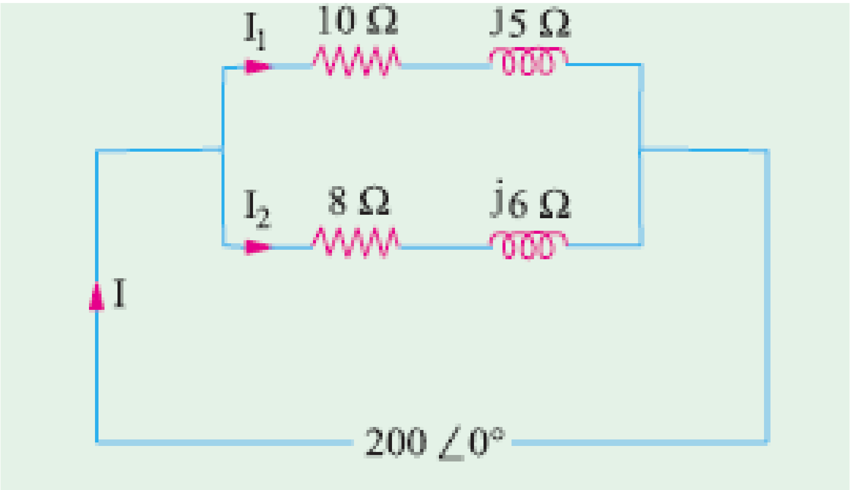
Ans: Z = (2.5-449.26j)Ω; I = (2.85+0.511j)A; P =0.6830W

c) Find the reading of Ammeter if voltmeter reads 45V in the following circuit Also find I_1, I_2 *and* V *and* Draw phasor diagram **12**

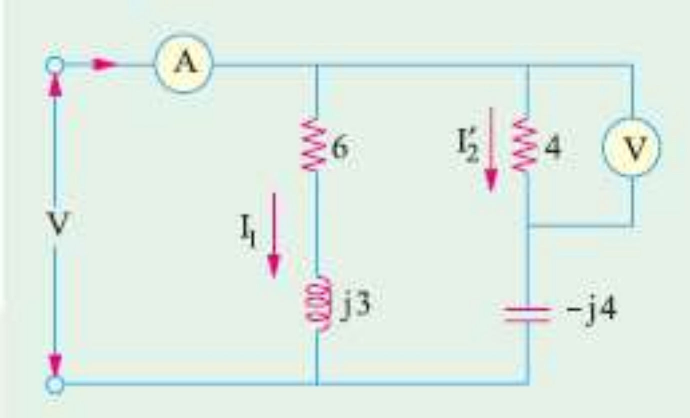


Ans: I1 = (4.65-10.86j) A; I2 = (15+0j)A; I = (19.65-10.85j)A; V = (45-45j)V

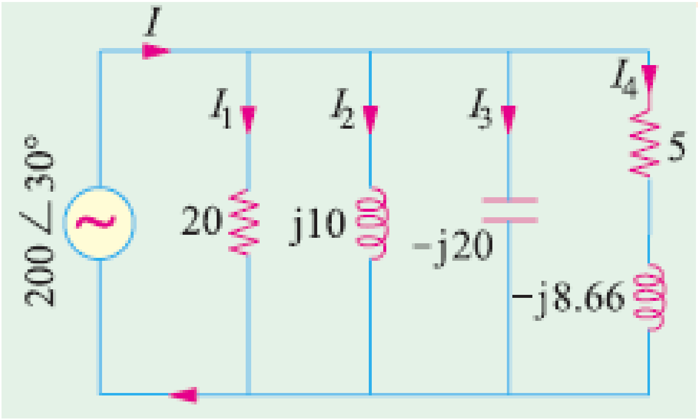
Two impedances given by $Z_1 = (10 + j\,5)$ and $Z_2 = (8 + j\,6)$ are joined in parallel and connected across a voltage of $V = 200 + j0$. Calculate the circuit current, its phase and the branch currents. Draw the vector diagram.



If the voltmeter in Fig. reads 60 V, find the reading of the ammeter.



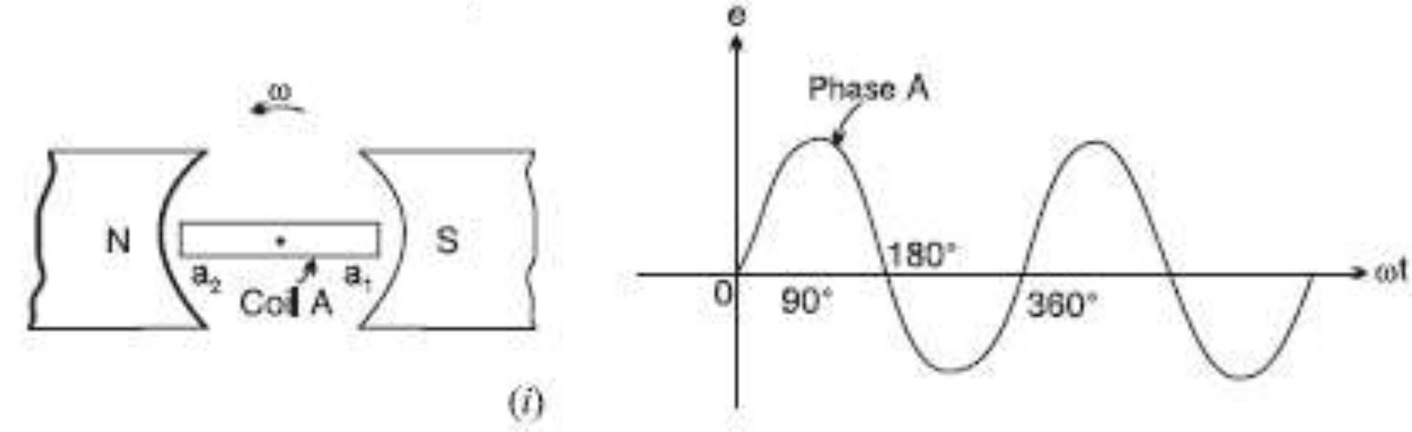
Calculate (i) total current and (ii) equivalent impedance for the four-branched circuit of Fig.



Polyphase Circuits

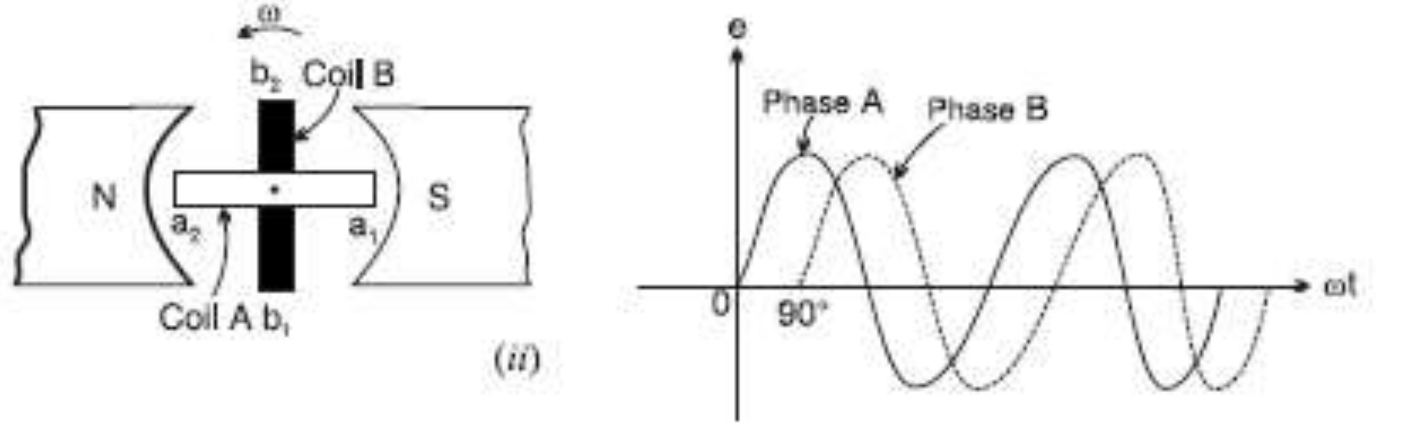
If the generator is arranged to have two or more separate windings displaced from each other by equal electrical angles, it is called a Polyphase generator and will produce as many independent voltages as the number of windings or phases.

The 3-phase system is by far the most popular because it is the most efficient of all the supply systems.



single-phase alternator

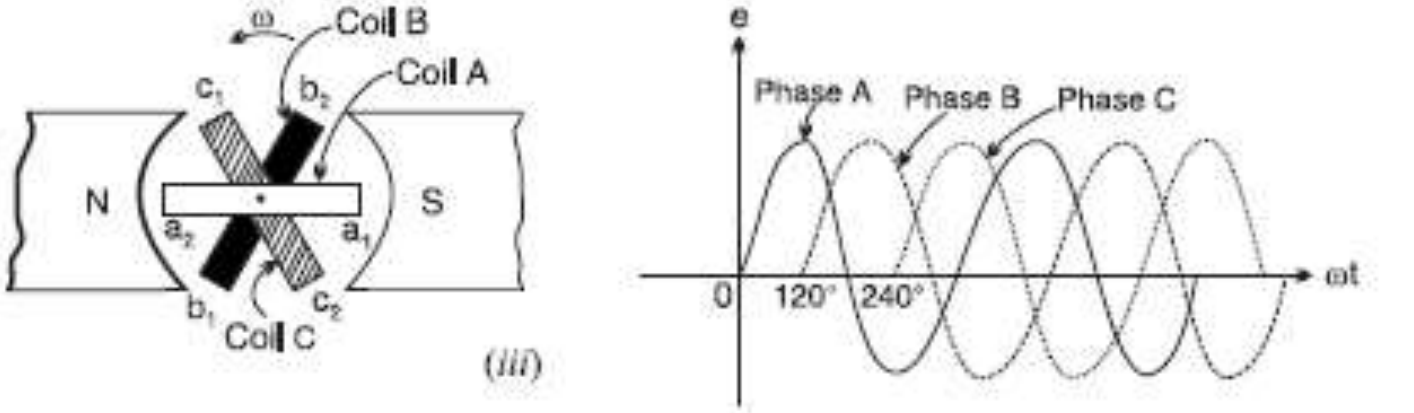
$$e_{a_1a_2} = E_m \sin \omega t$$



two-phase alternator

a_1 and b_1 are 90° apart

$$e_{a_1a_2} = E_m \sin \omega t$$
$$e_{b_1b_2} = E_m \sin (\omega t - 90°)$$



3-phase alternator

a_1, b_1 and c_1 are 120° apart

$$e_{a_1a_2} = E_m \sin \omega t$$
$$e_{b_1b_2} = E_m \sin (\omega t - 120°)$$
$$e_{c_1c_2} = E_m \sin (\omega t - 240°)$$

Reasons for the Use of 3-phase System

1. Constant power

In a single-phase circuit, the instantaneous power varies sinusoidally from zero to a peak value at twice the supply frequency. However, in a balanced 3-phase system, the power supplied at all instants of time is constant. Because of this, the operating characteristics of 3-phase apparatus, in general, are superior to those of similar single-phase apparatus.

2. Greater output

The output of a 3-phase machine is greater than that of a single-phase machine for a given volume and weight of the machine. In other words, a 3-phase machine is smaller than a single-phase machine of the same rating.

3. Cheaper

The three-phase motors are much smaller and less expensive than single-phase motors because less material (copper, iron, insulation) is required. Moreover, 3-phase motors are self-starting *i.e*. they do not require any special provision to get them started.

4. Power transmission economics

Transmission of electric power by 3-phase system is cheaper than that of single-phase system, even though three conductors are required.

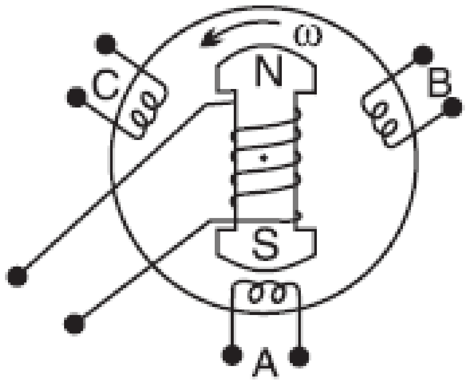
5. Three-phase rectifier service

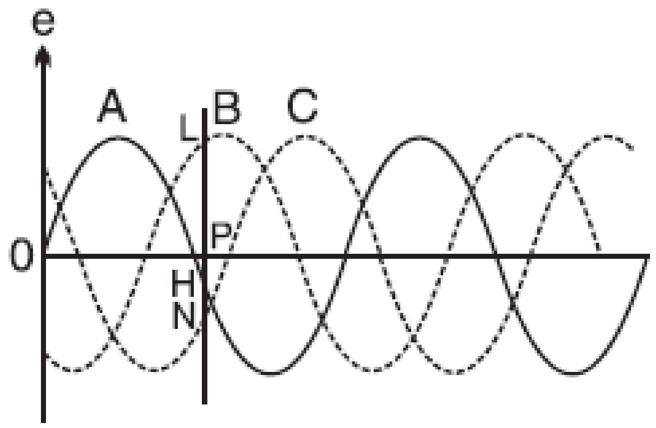
Rectified 3-phase voltage is smoother than rectified single phase voltage. As a result, it is easier to filter out the ripple component of 3-phase voltage than that of a single-phase voltage. This is especially useful where large a.c. power is to be converted into steady d.c. power *e.g.* radio and television transmitters.

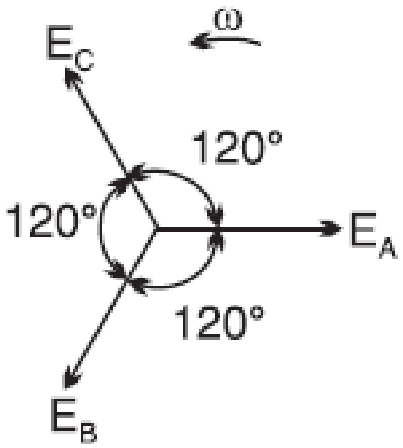
6. Miscellaneous advantages

(*i*) A 3-phase system can set-up a rotating uniform magnetic field in stationary windings. This cannot be done with a single-phase current.
(*ii*) The 3-phase motors are more efficient and have a higher power factor than single phase motors of the same capacity.

Elementary Three-Phase Alternator







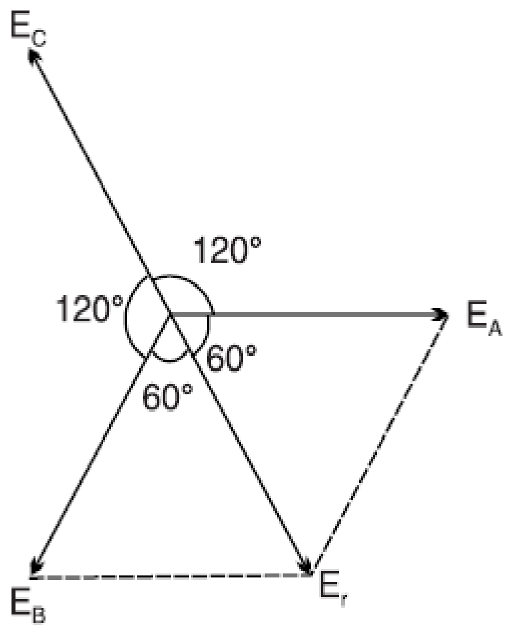
$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin (\omega t - 120°)$$

$$e_C = E_m \sin (\omega t - 240°) = E_m \sin (\omega t + 120°)$$

$$\begin{aligned}
\text{Resultant} &= e_A + e_B + e_C \\
&= E_m [\sin \omega t + \sin (\omega t - 120°) + \sin (\omega t - 240°)] \\
&= E_m [\sin \omega t + 2 \sin (\omega t - 180°) \cos 60°] \\
&= E_m [\sin \omega t - 2 \sin \omega t \cos 60°] \\
&= 0
\end{aligned}$$

the sum of the three e.m.f.s at every instant is zero.

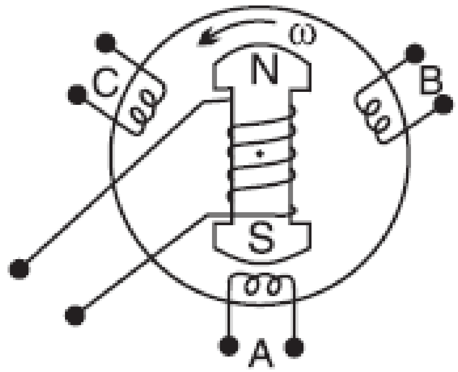


Er= 2*E* cos 60° = *E* = -*Ec*

Phase sequence

The order in which the voltages in the three phases (or coils) of an alternator reach their maximum positive values is called **phase sequence** or **phase order.**

Since the alternator can be rotated in either clockwise or anticlockwise direction, there can be only two possible phase sequences.



Naming the phases

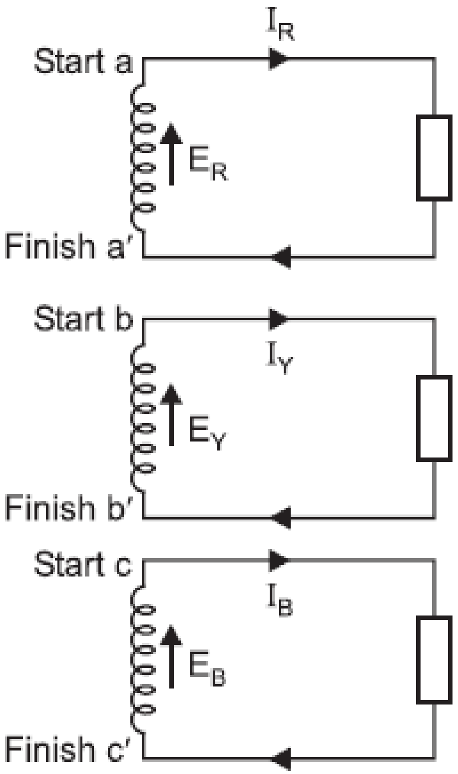
Red (R), yellow (Y) and blue (B)
By convention, sequence RYB is taken as positive and RBY as negative

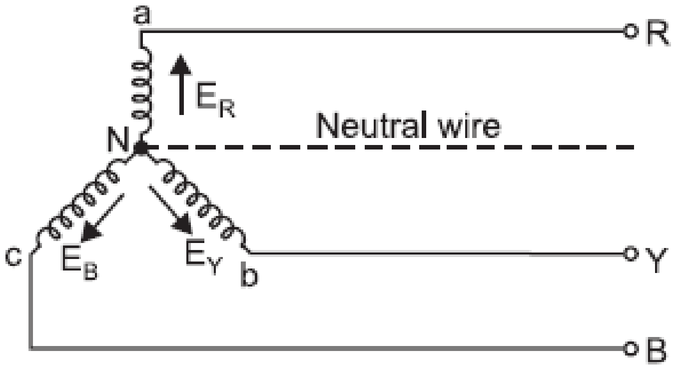
Double-subscript notation

V_{RY} indicates a voltage V between points R and Y with point R being positive w.r.t. point Y

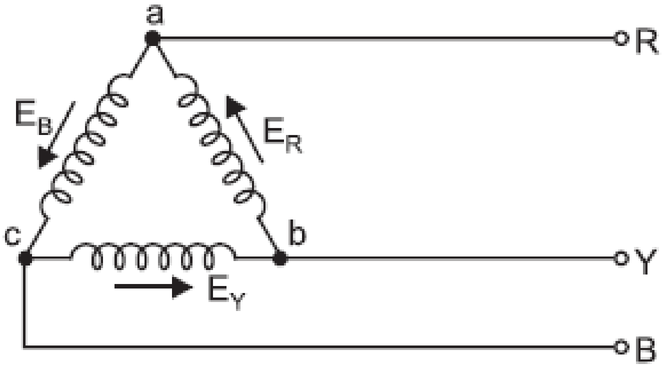
Interconnection of Three Phases

(i) Star or Wye (Y) connection
(*ii*) Mesh or Delta (Δ) connection





(*i*)



In Y-connection

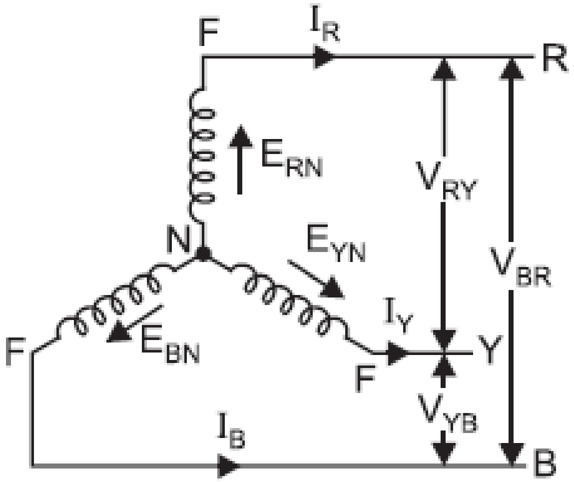
Neutral point/ star point
If a neutral conductor exists, the system is called 3-phase, 4-wire system.
If there is no neutral conductor, it is called 3-phase, 3-wire system.

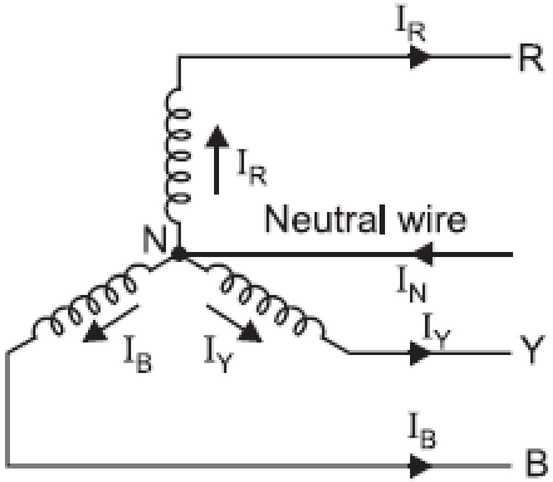
In Δ-connection

No neutral point exists and only 3-phase, 3-wire system can be formed

Star or Wye Connected System

phase voltage
line voltage
phase currents
line currents





Balanced load-load in each phase of the alternator has the same impedance and power factor

The 3-wire star-connected system is used for balanced loads → No neutral conductor needed

4-wire star-connected system- Unbalanced load → neutral conductor provides the return path

The three phase voltages (i.e. ERN, EYN, and EBN) are equal in magnitude but displaced 120°
from each other. The same is true for line voltages (i.e. VRY, VYB and VBR)→**Balanced supply system.**

Line voltage = $\sqrt{3}$ × Phase voltage ... *in magnitude*

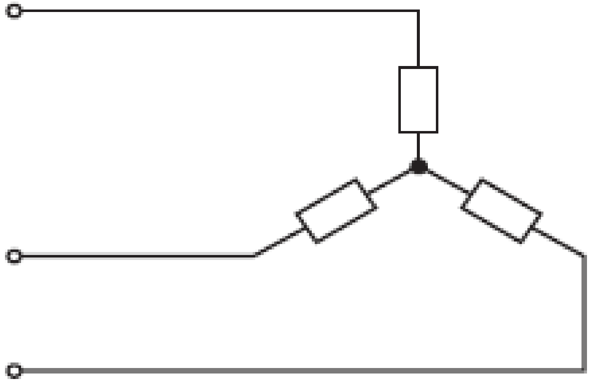
Line current = Phase current ... *in magnitude*

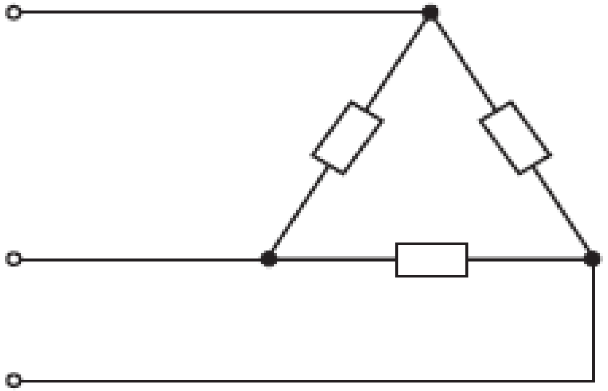
- Thus Y-connected balanced supply system enables us to use two voltages viz. phase voltage and line voltage.
- For balanced loads, all line currents (or phase currents) are equal in magnitude but displaced 120° from each other
- For 3-phase, 4-wire star-connected system, the current IN in the neutral wire is the phasor sum of the three line currents.

3-phase loads

(*i*) **star-connected load**
(*ii*) **delta-connected load**

The 3-phase load (star or delta connected) is said to be **balanced** if load in each phase is the same (*i.e*. load in each phase has the same impedance and power factor).

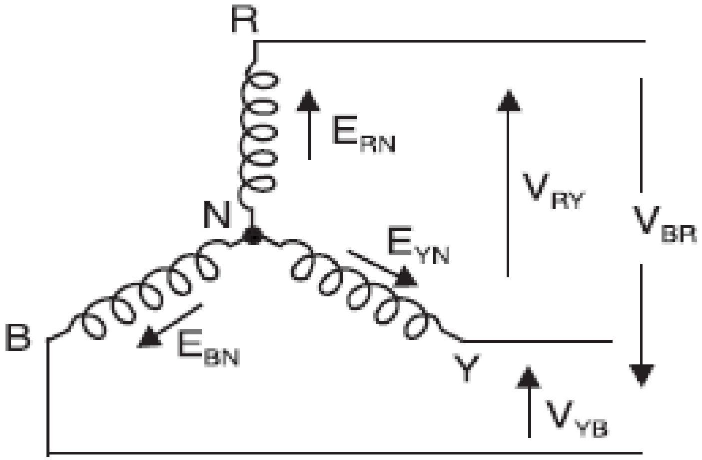


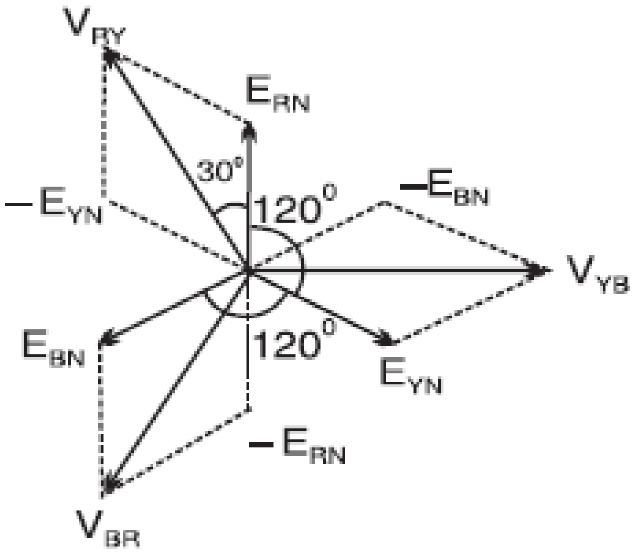


Balanced 3-phase system

(*i*) It should have balanced 3-phase supply.
(*ii*) It should have balanced 3-phase or single phase loads on the balanced 3-phase supply.
(*iii*) It should have equal active power and equal reactive power flow in each phase.

Voltages and Currents in Balanced Y-Connected Supply System





The r.m.s. values of the e.m.f.s generated in the three phases are E_{RN}, E_{YN} and E_{BN}

phasor difference between lines R and Y, $*V_{RY} = E_{RN} - E_{YN}$

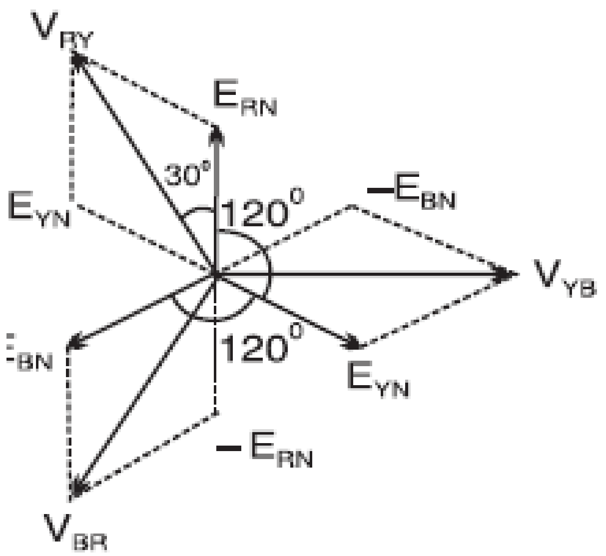
phasor difference between lines Y and B, $V_{YB} = E_{YN} - E_{BN}$.

phasor difference between lines B and R, $V_{BR} = E_{BN} - E_{RN}$.

1. Relation between line voltage and phase voltage

Considering the lines *R* and *Y*, the line voltage V_{RY} is equal to the phasor difference of E_{RN} and E_{YN}.

equal in magnitude (= E_{ph}) and are 60° apart.



$$\therefore \quad V_{RY} = 2\,E_{ph}\cos(60°/2) = 2\,E_{ph}\cos 30° = \sqrt{3}\,E_{ph}$$

Similarly,

$$V_{YB} = E_{YN} - E_{BN} \quad \text{...phasor difference}$$
$$= \sqrt{3}\,E_{ph}$$

and

$$V_{BR} = E_{BN} - E_{RN} \quad \text{...phasor difference}$$
$$= \sqrt{3}\,E_{ph}$$

Hence in a balanced 3-phase Y-connected supply system :

(i) Line voltage, $V_L = \sqrt{3}\,E_{ph}$ ... in magnitude

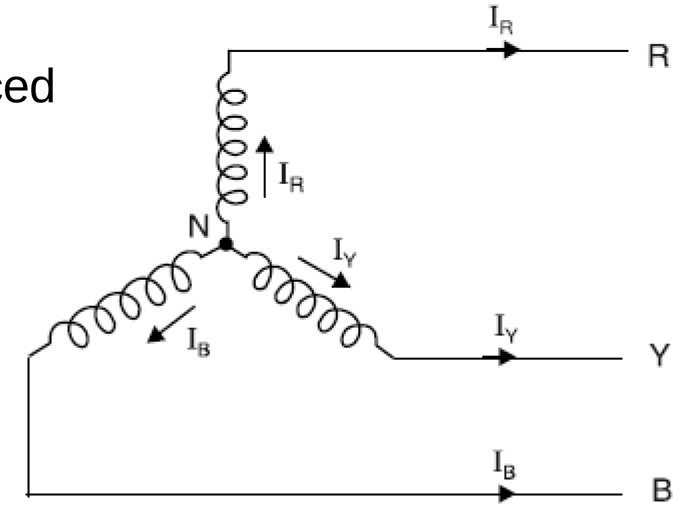
(ii) All line voltages are equal in magnitude (i.e. = $\sqrt{3}E_{ph}$) but displaced 120° from one another

(iii) Line voltages are 30° ahead of their respective phase voltages.

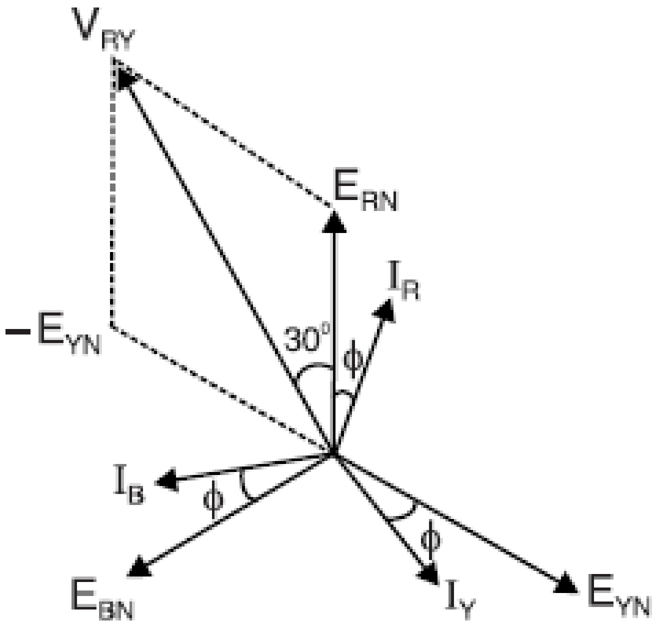
2. Relation between line current and phase current

Current in a line conductor is the same as that in the phase to which the line conductor is connected.

Line current, $I_L = I_{ph}$ ...*in magnitude*



- For a balanced load, all the phase currents are equal in magnitude but displaced 120° from one another
- Φ is the angle between phase voltage and the corresponding phase current.
- The angle between the line current and the corresponding line voltage (IR and VRY) is (30°+Φ)
- If the balanced load has a leading power factor of cos Φ , then the angle between the line current and the corresponding line voltage will be (30°- Φ)



Hence in a balanced 3-phase Y-connected supply system :

(i) Line current, $I_L = I_{ph}$

(ii) All the line currents are equal in magnitude (i.e. = I_{ph}) but displaced 120° from one another.

(iii) The angle between the line currents and the corresponding line voltages is 30° ± Φ ; + if p.f. is lagging and – if it is leading.

3. Power

For a balanced load, the power consumed in each load phase is the same.

$$\therefore \quad \text{Total power, } P = 3 \times \text{Power in each phase} = 3 \times V_{ph} I_{ph} \cos\phi$$

$$\text{For a star connection, } V_{ph} = V_L/\sqrt{3}\ ; \ I_{ph} = I_L$$

$$\therefore \quad P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$$

$$\text{or} \quad P = \sqrt{3}\ V_L I_L \cos\phi$$

$$\text{Also,} \quad \text{Reactive power, } Q = \sqrt{3} V_L I_L \sin\phi$$

The relationship between active power (P), reactive power (Q) and apparent power (S) is the same for balanced 3-phase circuits as for single-phase circuits.

$$\therefore \quad S = \sqrt{P^2 + Q^2} \text{ and power factor } \cos\phi = \frac{P}{S}$$

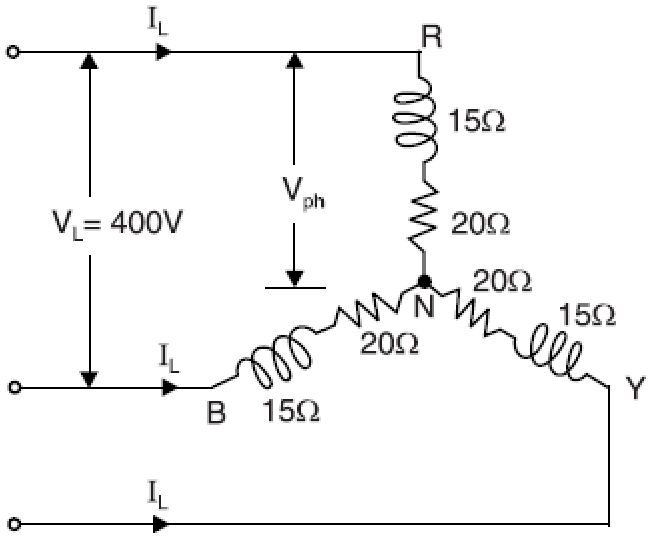
$$\text{Apparent power, } S = \sqrt{3}\ V_L I_L$$

For a balanced load, the three line currents have the same magnitude (say I_L) but differ in phase by 120°.

$$\begin{aligned} I_N &= I_L \angle 0° + I_L \angle -120° + I_L \angle -240° \\ &= I_L (1 + j0) + I_L (-0.5 - j\,0.866) + I_L (-0.5 + j\,0.866) = 0 \end{aligned}$$

Therefore, for balanced loads, current in the neutral wire is zero and no neutral conductor is required, resulting in the saving of conductor material

Three coils, each having a resistance of 20 W and an inductive reactance of 15 W, are connected in star to a 400 V, 3-phase, 50 Hz supply. Calculate (i) the line current (ii) power factor and (iii) power supplied.



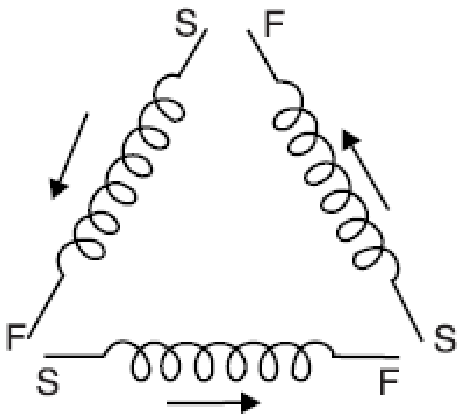
A balanced star connected load is supplied from a symmetrical three phase, 400V and 50Hz. The current in each phase is 30A and lag 30 degree behind the phase voltage. Find:

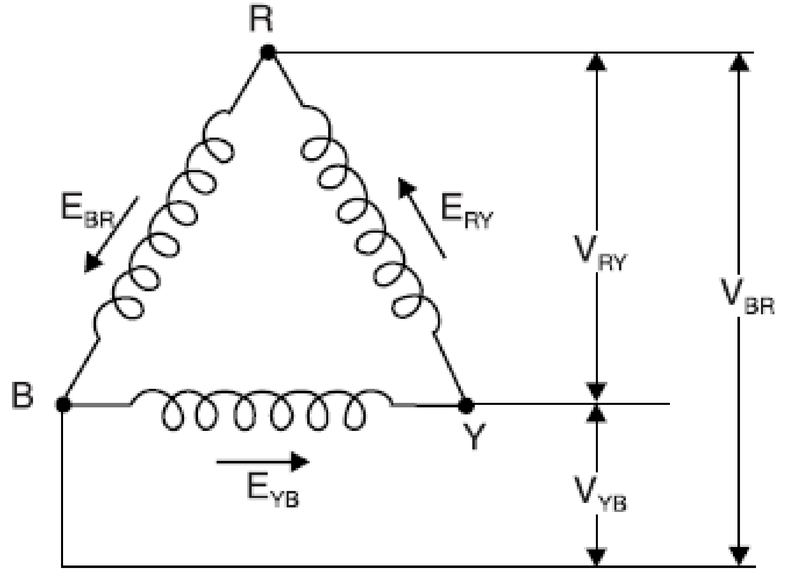
a) Phase voltage
b) Resistance and reactance per phase
c) Load inductance per phase
d) Total power consumed
e) Reactive volt ampere
f) Total volt ampere
g) Draw the phasor diagram showing the current and voltage

A balanced three phase star connected load of 120KW takes a leading current of 100A, when connected across three phase, 3.3KV and 50Hz supply. Determine the impedance, resistance, capacitance and power factor of load.

Delta (Δ) or Mesh Connected System

no neutral exists





Voltages and Currents in Balanced Δ Connected Supply System

(*i*) Line voltage and phase voltage

Line voltage magnitude, V_L = Phase voltage magnitude (E_{ph})

$$V_L = V_{ph} \qquad \text{... in magnitude}$$

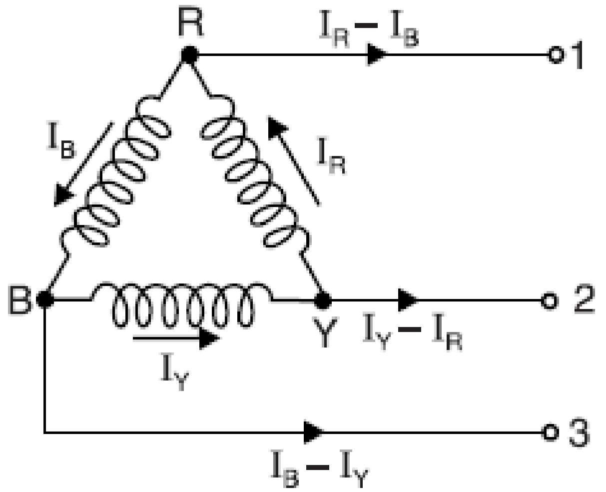
Only one phase is included between any two lines. Hence magnitude of voltage between any two lines (i.e. **line voltage**) is equal to the magnitude of **phase voltage**

The three phase voltages (= line voltages) are equal in magnitude but displaced 120° from one another.

(*ii*) Line current and phase current

Current in any line is equal to the phasor difference of the currents in the two phases connected to that line. Therefore, magnitude of line currents is different from the magnitude of phase currents.

For balanced load, the three phase currents (I_R, I_Y and I_B) are equal in magnitude but displaced 120° from one another



$$\text{Line current} = \sqrt{3} \times \text{Phase current} \qquad \textit{... in magnitude}$$

$$\text{Current in line 1, } **I_1 = I_R - I_B$$

$$\text{Current in line 2, } I_2 = I_Y - I_R$$

$$\text{Current in line 3, } I_3 = I_B - I_Y$$

$\therefore$ $$I_1 = 2\, I_{ph} \cos(60°/2) = 2\, I_{ph} \cos 30° = \sqrt{3}\ I_{ph}$$

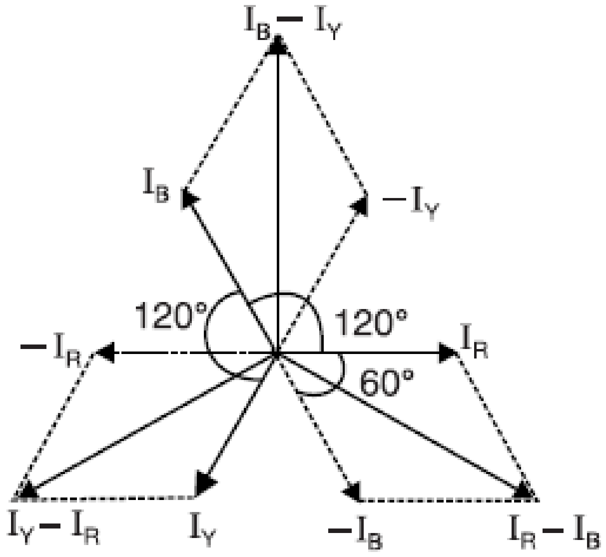
Similarly, $$I_2 = I_Y - I_R \qquad \textit{...phasor difference}$$

$$= \sqrt{3}\ I_{ph}$$

and $$I_3 = I_B - I_Y \qquad \textit{...phasor difference}$$

$$= \sqrt{3}\ I_{ph}$$

$$\text{Line current, } I_L = \sqrt{3}\ I_{ph}$$



Hence in a balanced Δ connected supply system :

(*a*) *Line current,* $I_L = \sqrt{3}\, I_{ph}$

(*b*) *All the line currents are equal in magnitude* ($= \sqrt{3}\, I_{ph}$) *but displaced* 120° *from one another as seen from Fig. 15.30.*

(*c*) *Line currents are 30° behind the respective phase currents.*

(*d*) *The angle between the line currents and the corresponding line voltages is* $30° \pm \phi$; + *if p.f. is lagging and* – *if it is leading.*

(*e*) *Line voltage* (V_L) *is equal to phase voltage* (V_{ph}).

(*iii*) Power

Total power, $P = 3 \times \text{Power per phase} = 3 \times V_{ph}\, I_{ph} \cos\phi = \sqrt{3}\, V_L\, I_L \cos\phi$

Reactive power, $Q = \sqrt{3}\, V_L I_L \sin\phi$

Apparent power, $S = \sqrt{3} V_L I_L$

$$S = \sqrt{P^2 + Q^2}$$

Advantages of Delta Connection

- Most suitable for rotary convertors
- Most of the 3-phase loads are Δ-connected rather than *Y*-connected.
- Most of 3-phase induction motors are delta-connected.

Advantages of Star Connection

- A star-connected alternator will require less number of turns of the winding than a Δ-connected alternator.
- For the same line voltage, a star-connected alternator requires less insulation than a delta connected
- alternator.
- This permits to use two voltages viz, phase voltages as well as line voltages.
- In star connection, the neutral point can be earthed.

Three similar coils each having a resistance of 5Ω and an inductance of 0.02H are connected in delta to a 440V, 3-phase, 50Hz supply. Calculate the line current and total power absorbed.

A delta connection balanced load is connected to a three phase, 400V supply. The load power factor is 0.8 lagging. The line current is 34.64A. Find resistance, total power, total reactive volt ampere, total volt ampere. Draw the phasor diagram.

The three equal impedances, each of 10<60 degree ohms are connected in star across three voltages, 400V and 50Hz supply. Calculate

- Line voltage and phase voltage
- Line current and phase current
- Power factor and active power consumed

Also calculate the active power consumed if the three impedances are connected in delta to the same source of supply.