Roll No: | | | | | |

PADRE CONCEIÇÃO COLLEGE OF ENGINEERING, VERNA-GOA

Internal Test-I

Semester: II (RC 2019-'20)
Course: FE 210 Mathematics-II
Course Instructor: Ms. Komal Paroolkar/ Dr. A.K Handa

Date: 28/04/2022
Time: 9:30-10:30 am
Max Marks: 25

Instructions: *Attempt all questions. Assume missing data, if any and justify.*

	Question	Marks	CO	CL
Q1.	**Compute** the length of the curve $y = \frac{1}{3}(x^2+2)^{3/2}$ *from* $x = 0$ *to* $x = 3$	[5]	FE210.1	CL3
Q2	**Find** the area of the surface generated by revolving one loop of the leminiscate $r^2 = a^2\cos(2\theta)$ about the initial line.	[5]	FE210.1	CL3
Q3	**Evaluate** $\int_0^1 \int_0^{x^2} e^{y/x}\, dxdy$	[5]	FE210.1	CL3
Q4	**Evaluate** $\iint xy\, dxdy$ over the region bounded by the triangle having vertices $(0,1), (1,1) and (1,2)$.	[5]	FE210.1	CL3
Q5	Write the sum of the following double integrals and one double integral and hence **evaluate** $$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} x\, dxdy + \int_1^4 \int_{-\sqrt{y}}^{2-y} x\, dxdy$$	[5]	FE 210.1	CL3

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28/04/2022 Solution Set of Internal Test No: 01

① $y = \frac{1}{3}(x^2+2)^{3/2}$

$$\text{length} = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \quad \text{—— ①}$$

$x_1 = 0, \quad x_2 = 3, \quad y = \frac{1}{3}(x^2+2)^{3/2}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{3} \cdot \frac{3}{2}(x^2+2)^{1/2} \cdot 2x$$

$$\Rightarrow \frac{dy}{dx} = x(x^2+2)^{1/2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = x^2(x^2+2)$$

$$\therefore 1+\left(\frac{dy}{dx}\right)^2 = 1+(x^4+2x^2)$$

$$= x^4+2x^2+1$$

$$= (x^2+1)^2 \quad \text{—— ②}$$

Substituting ② in ①, we get

$$\text{length} = \int_0^3 \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^3 x^2+1 \, dx$$

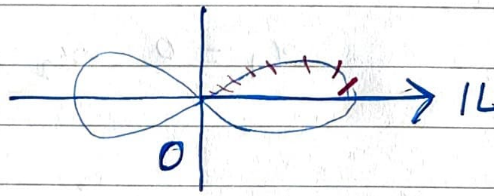
$$= \left.\frac{x^3}{3} + x \right|_0^3$$

$$= \frac{27}{3} + 3 - 0 = 12 \text{ units}$$

② Surface Area about the initial line

$$= \int_{\theta_1}^{\theta_2} 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad \text{——— ①}$$

$$r^2 = a^2 \cos(2\theta) \quad \text{——— ②}$$



$$\Rightarrow r = a\sqrt{\cos(2\theta)} \quad \text{——— ③}$$

$$\Rightarrow \frac{dr}{d\theta} = a \cdot \frac{1}{2\sqrt{\cos(2\theta)}} \cdot (-\sin(2\theta)) \cdot 2$$

$$\Rightarrow \frac{dr}{d\theta} = \frac{-a\sin(2\theta)}{\sqrt{\cos(2\theta)}}$$

$$\therefore r^2 + \left(\frac{dr}{d\theta}\right)^2 = a^2\cos(2\theta) + \frac{a^2 \sin(2\theta)}{\cos(2\theta)}$$

$$\therefore r^2 + \left(\frac{dr}{d\theta}\right)^2 = \frac{a^2}{\cos(2\theta)} \quad \text{——— ④}$$

Subs ②, ③, ④ in ① ; we get

$$S.A = \int_0^{\pi/4} 2\pi\, a \cdot \sqrt{\cos(2\theta)} \cdot \sin\theta \sqrt{\frac{a^2}{\cos(2\theta)}}\, d\theta$$

$$= 2\pi a^2 \int_0^{\pi/4} \sin\theta\, d\theta = 2\pi a^2 \left[-\cos\theta \Big|_0^{\pi/4}\right]$$

$$= 2\pi a^2 \left(1 - \frac{1}{\sqrt{2}}\right) \text{ sq. units}$$

③ Let $I = \int_0^1 \int_0^{x^2} e^{y/x}\, dx\, dy$

$$= \int_0^1 \left[\int_0^{x^2} e^{y/x}\, dy \right] dx$$

$$= \int_0^1 \left[\frac{e^{y/x}}{1/x} \Big|_0^{x^2} \right] dx$$

$$= \int_0^1 x \left[e^{x} - 1 \right] dx$$

$$= \int_0^1 x e^{x}\, dx - \int_0^1 x\, dx$$

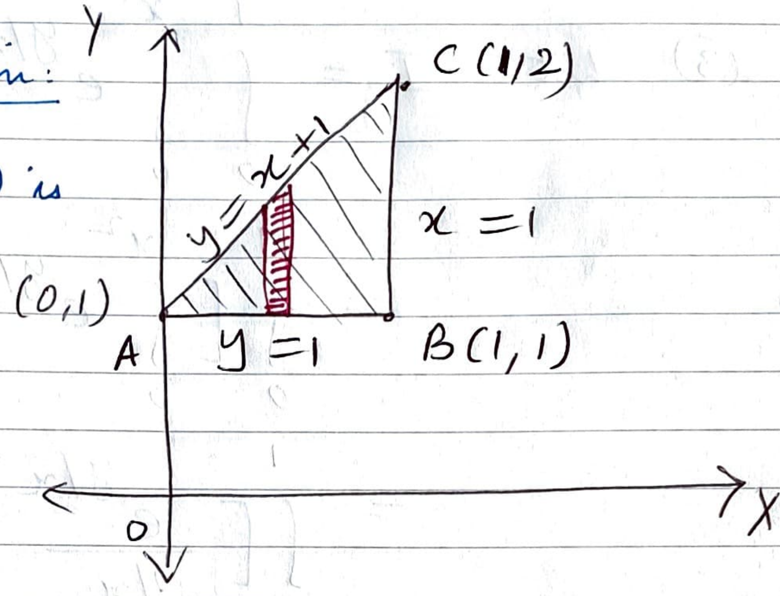
$$= \left[x e^{x} - (1).(e^{x}) \Big|_0^1 \right] - \frac{1}{2}$$

$$= \left[\cancel{e} - \cancel{e} - (0 - 1) \right] - \frac{1}{2}$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

④ Sketching the region:



Eqn of A(0,1)-B(1,2) is

$$\frac{x-0}{0-1} = \frac{y-1}{1-2}$$

$$\Rightarrow \frac{x}{-1} = \frac{y-1}{-1}$$

$$\Rightarrow x = y-1$$

or $y = x+1$

$$I = \int_0^1 \left[\int_1^{x+1} xy\,dy\right] dx$$

$$= \int_0^1 x\left[\frac{y^2}{2}\Big|_1^{x+1}\right] dx$$

$$= \frac{1}{2}\int_0^1 x\left[(x+1)^2 - 1\right] dx$$

$$= \frac{1}{2}\int_0^1 x^3 + 2x^2\,dx$$

$$= \frac{1}{2}\left[\frac{x^4}{4} + 2\cdot\frac{x^3}{3}\Big|_0^1\right]$$

$$= \frac{1}{2}\left[\frac{1}{4} + \frac{2}{3} - 0\right]$$

$$= \frac{1}{2}\left[\frac{11}{12}\right] = \frac{11}{24}$$

(5) Sketching R_1 and R_2 (existing order: 1^{st} w.r.t 'x')

For R_1: $x = -\sqrt{y}$ to $x = \sqrt{y}$ ~~For R_2: $x = -\sqrt{y}$~~

$y = 0$ to $y = 1$

$\therefore$ $x^2 = y$ (left arm to right arm)

For R_2: $x = -\sqrt{y}$ to $x = 2 - y$ $<$ $(0, 2)$, $(2, 0)$

and $y = 1$ to $y = 4$.

Point of intersection of $x^2 = y$ and $x = 2 - y$

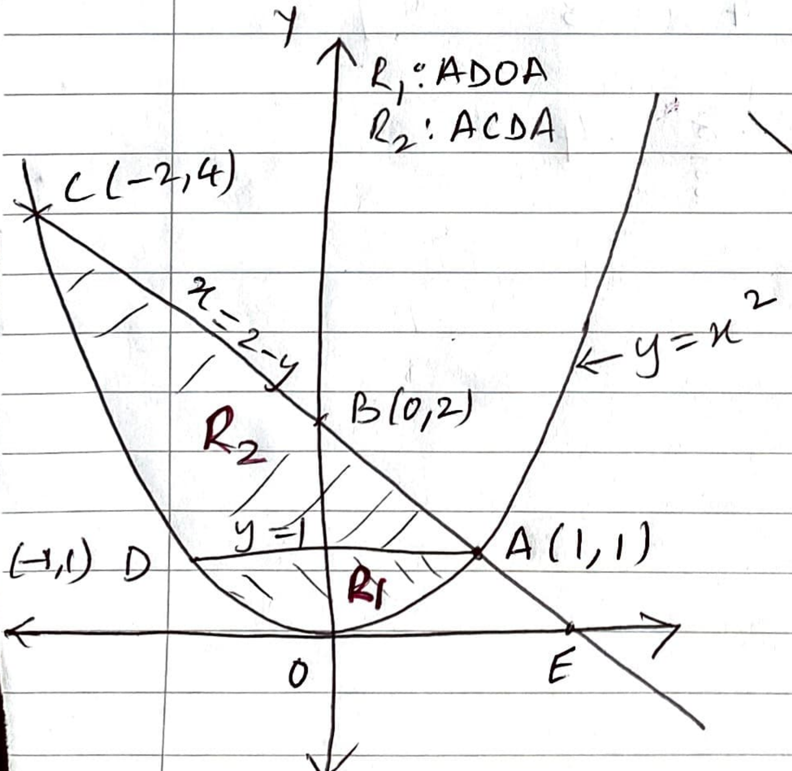
$\because$ $x = 2 - x^2 \Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

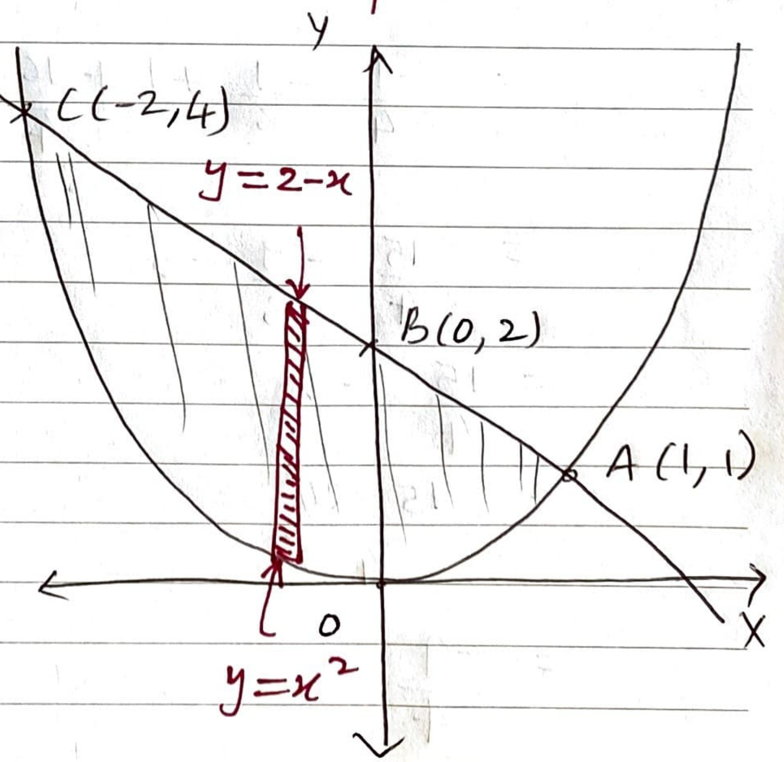
$\Rightarrow x = -2, \; x = 1$

$\therefore$ pts are $(-2, 4)$ and $(1, 1)$.

Also, $(1,1)$ lies on $x = 2 - y$.



Define $R = R_1 \cup R_2$



$$\therefore \quad I_1 + I_2 = \int_{-2}^{1} \left[\int_{x^2}^{2-x} x \, dy \right] dx$$

$$= \int_{-2}^{1} x \left[y \Big|_{x^2}^{2-x} \right] dx$$

$$= \int_{-2}^{1} x \left[2 - x - x^2 \right] dx$$

$$= \int_{-2}^{1} (-x^3 - x^2 + 2x) \, dx$$

$$= \frac{-x^4}{4} - \frac{x^3}{3} + x^2 \Big|_{-2}^{1}$$

$$= \frac{-1}{4} - \frac{1}{3} + 1 - \left[\frac{-16}{4} - \frac{(-8)}{3} + 4 \right]$$

$$= \frac{-1}{4} - \frac{1}{3} + 1 + \frac{16}{4} - \frac{8}{3} - 4$$

$$= \frac{15}{4} - \frac{9}{3} - 3$$

$$= \frac{15}{4} - 6$$

$$= \frac{15 - 24}{4}$$

$$= \frac{-9}{4}$$