TRIGONOMETRIC IDENTITIES AND TRIGONOMETRIC RATIOS

1) $\sin^2 A + \cos^2 A = 1$
2) $1 + \tan^2 A = \sec^2 A$
3) $1 + \cot^2 A = \mathrm{cosec}^2 A$
4) **Trigonometric Ratios**

θ	0	30^0 $\frac{\pi}{6}$	45^0 $\frac{\pi}{4}$	60^0 $\frac{\pi}{3}$	90^o $\frac{\pi}{2}$	180^o π	270^o $\frac{3\pi}{2}$	360^o 2π
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0
Cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
cot θ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞

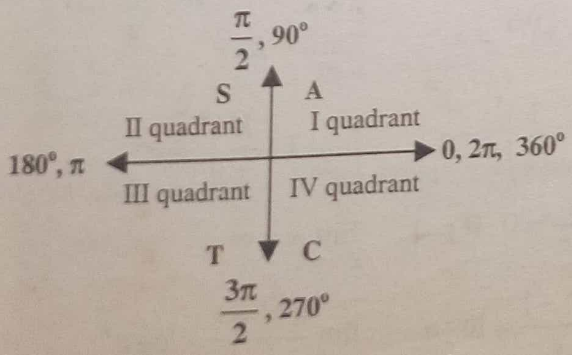
5) COMPOUND ANGLE FORMULAE

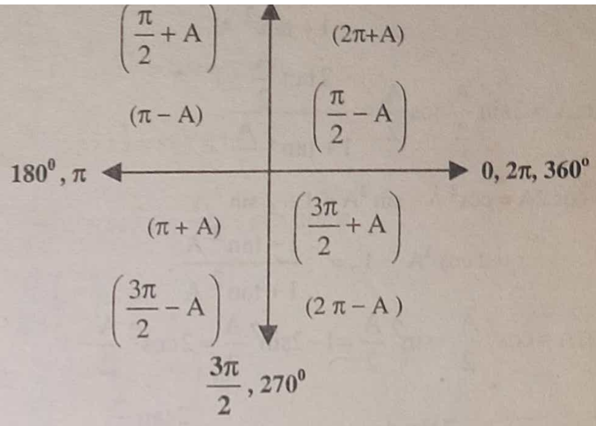
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \mp B) = \cos A \cos B \pm \sin A \sin B$

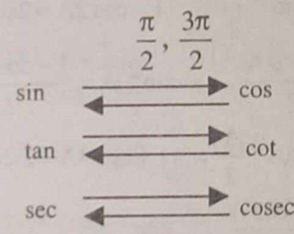
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$, $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot A \pm \cot B}$

6) TRIGONOMETRIC CONVERSIONS USING COMPOUND ANGLES





Following changes take place when compound angle consist of $\frac{\pi}{2}, \frac{3\pi}{2}$.



Under $\pi, 2\pi$ there will be no change in function, but there will be change of sign depending upon quadrant.

$\sin(90^0 - A) = \cos A$, $\cos(90^0 - A) = \sin A$,

$\sin(90^0 + A) = \cos A$, $\cos(90^0 + A) = -\sin A$,

$\sin(180^0 - A) = \sin A$, $\cos(180^0 - A) = -\cos A$,

$\sin(180^0 + A) = -\sin A$, $\cos(180^0 + A) = -\cos A$,

$\sin(360^0 - A) = -\sin A$, $\cos(360^0 - A) = \cos A$,

$\sin(360^0 + A) = \sin A$, $\cos(360^0 + A) = \cos A$

$\sin(-A) = -\sin A$, $\cos(-A) = \cos A$, $\tan(-A) = -\tan A$

7) FACTORISATION FORMULAE

$$\sin C + \sin D = 2\sin\left[\frac{C+D}{2}\right]\cos\left[\frac{C-D}{2}\right]$$

$$\sin C - \sin D = 2\cos\left[\frac{C+D}{2}\right]\sin\left[\frac{C-D}{2}\right]$$

$$\cos C + \cos D = 2\cos\left[\frac{C+D}{2}\right]\cos\left[\frac{C-D}{2}\right]$$

$$\cos C - \cos D = 2\sin\left[\frac{C+D}{2}\right]\sin\left[\frac{D-C}{2}\right]$$

OR $\cos C - \cos D = -2\sin\left[\frac{C+D}{2}\right]\sin\left[\frac{C-D}{2}\right]$

8) DEFACTORISATION FORMULAE

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

9) MULTIPLE ANGLE FORMULAE

i) $\sin 2A = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A}$,

$$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$$

ii) $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

$= 2\cos^2 A - 1$, $= \dfrac{1-\tan^2 A}{1+\tan^2 A}$,

$$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = 1 - 2\sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1$$

iii) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$, $\quad \tan A = \dfrac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

iv) $1-\cos A = 2\sin^2\dfrac{A}{2}$, $\quad 1-\cos 2A = 2\sin^2 A$

v) $\sin^2\dfrac{A}{2} = \dfrac{1-\cos A}{2}$, $\quad \sin^2 A = \dfrac{1-\cos 2A}{2}$

vi) $1+\cos A = 2\cos^2\dfrac{A}{2}$, $\quad 1+\cos 4A = 2\cos^2 2A$

vii) $\cos^2\dfrac{A}{2} = \dfrac{1+\cos A}{2}$, $\quad \cos^2 2A = \dfrac{1+\cos 4A}{2}$

viii) $1-\sin A = \left(\cos\dfrac{A}{2} - \sin\dfrac{A}{2}\right)^2$ **ix)** $1+\sin 2A = (\cos A + \sin A)^2$

x) $\sin 3A = 3\sin A - 4\sin^3 A$, $\quad \sin^3 A = \dfrac{3\sin A - \sin 3A}{4}$

xi) $\cos 3A = 4\cos^3 A - 3\cos A$, $\quad \cos^3 A = \dfrac{3\cos A + \cos 3A}{4}$

xii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

10) SINE RULE: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

11) COSINE RULE: $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$,

$$\cos B = \frac{c^2+a^2-b^2}{2ac},\quad \cos C = \frac{a^2+b^2-c^2}{2ab}.$$

12) PROJECTION RULE, $a = b\cos C + c\cos B$

$b = a\cos C + c\cos A$

$c = a\cos B + b\cos A$

13) HALF ANGLE FORMULAE

i) $\sin\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$

ii) $\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$

iii) $\tan\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

iv) $A(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$, where $2s = a$-

14) LAWS OF INVERSE FUNCTIONS

i) $\sin^{-1}(\sin A) = A$, for $-\dfrac{\pi}{2} \le A \le \dfrac{\pi}{2}$ - 90 to 90

ii) $\cos^{-1}(\cos A) = A$, for $0 \le A \le \pi$ 0 to 180

iii) $\tan^{-1}(\tan A) = A$, for $-\dfrac{\pi}{2} < A < \dfrac{\pi}{2}$ - 90 to 9

iv) $\sin(\sin^{-1} A) = A$, for $-1 \le A \le 1$

v) $\cos(\cos^{-1} A) = A$, for $-1 \le A \le 1$

vi) $\tan(\tan^{-1} A) = A$, for $A \in R$

vii) $\sin^{-1}(-A) = -\sin^{-1} A$

viii) $\cos^{-1}(-A) = \pi - \cos^{-1} A$

ix) $\sin^{-1} A + \cos^{-1} A = \dfrac{\pi}{2}$

x) $\tan^{-1} A + \cot^{-1} A = \dfrac{\pi}{2}$

xi) $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left[\dfrac{A+B}{1-AB}\right]$

xii) $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left[\dfrac{A-B}{1+AB}\right]$

xiii) $\sec^{-1} A = \cos^{-1}\dfrac{1}{A}$; $A \ne 0$

xiv) $\operatorname{cosec}^{-1} A = \sin^{-1}\left(\dfrac{1}{A}\right)$; $A \ne 0$

xv) $\cot^{-1} A = \tan^{-1}\dfrac{1}{A}$; $A \ne 0$

Important formulae from Limits

$\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$ $\quad \lim\limits_{x\to 0}\dfrac{\sin^{-1} x}{x} = 1$

$\lim\limits_{x\to 0}\dfrac{\tan x}{x} = 1$ $\quad \lim\limits_{x\to 0}\dfrac{\tan^{-1} x}{x} = 1$

$\lim\limits_{x\to 0}\cos x = 1$ $\quad \lim\limits_{x\to 0}\cos^{-1} x = \pi/2$

$\lim\limits_{x\to 0}(1+x)^{1/x} = e$ $\quad \lim\limits_{x\to 0}(1+Kx)^{1/x} = e^K$

$\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x = e$ $\quad \lim\limits_{x\to\infty}\left(1+\dfrac{K}{x}\right)^x = e^K$

$\lim\limits_{x\to\infty}\dfrac{1}{x^p} = 0$, if p>1 $\quad \lim\limits_{x\to\infty} x = \infty$

$\lim\limits_{x\to 0}\dfrac{a^x - 1}{x} = \log a$ $\quad \lim\limits_{x\to 0}\dfrac{a^{Kx} - 1}{x} = \log a^K$

$$\lim_{x\to 0}\frac{x^n-a^n}{x-a}=na^{n-1}$$

Important formulae from Logarithms

$\log_x x=1,\ \log_n 1=0,\ \log_y x=\dfrac{\log x}{\log y}$

$\log x^n = n\log x$

$\log xy=\log x+\log y$

$\log\left(\dfrac{x}{y}\right)=\log x-\log y$

$\log\left(\dfrac{xy}{z}\right)=\log x+\log y-\log z$

Important formulae from sequence and series

If $T_1, T_2, T_3, \ldots T_n \ldots$
represents arithmetic progression then nth term of A.P. is given by $T_n = a + (n-1)d$

The sum of n terms of arithmetic progression are given by $S_n=\frac{n}{2}[2a+(n-1)d]$.

If $a, ar, ar^2, ar^3, \ldots ar^{n-1}$ represents G.P. then $T_n = ar^{n-1}$ is the nth term of G.P.
The sum of n terms of G.P. are given by

$$S_n=\frac{a(r^n-1)}{r-1}\quad \text{if } r>1$$

$$S_n=\frac{a(1-r^n)}{1-r}\quad \text{if } r<1$$

Summation formulae for natural numbers

1) The sum of the 1st 'n' natural numbers is $\dfrac{n(n+1)}{2}$

i.e. $\sum_{r=1}^{n} r_i=1+2+3+\cdots+n=\dfrac{n(n+1)}{2}$.

2) The sum of the squares of the 1st 'n' natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$

i.e. $\sum_{r=1}^{n} r_i^2=1^2+2^2+3^2+\cdots+n^2=\dfrac{n(n+1)(2n+1)}{6}$.

3) The sum of the cubes of the 1st 'n' natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$

i.e. $\sum_{r=1}^{n} r_i^3=1^3+2^3+3^3+\cdots+n^3=\dfrac{n^2(n+1)^2}{4}$.

STANDARD FORMS FROM DERIVATIVES

1. $\dfrac{d}{dx}(C)=0$
2. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
3. $\dfrac{d}{dx}(\sin x)=\cos x$
4. $\dfrac{d}{dx}(\cos x)=-\sin x$
5. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
6. $\dfrac{d}{dx}(\cot x)=-\mathrm{cosec}^2 x$
7. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
8. $\dfrac{d}{dx}(\mathrm{cosec}\, x)=-\mathrm{cosec}\, x\cot x$
9. $\dfrac{d}{dx}(e^x)=e^x$
10. $\dfrac{d}{dx}(a^x)=a^x\cdot\log_e a$
11. $\dfrac{d}{dx}(\log_e x)=\dfrac{1}{x}$
12. $\dfrac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}\cdot\dfrac{d}{dx}[f(x)]$
13. $\dfrac{d}{dx}\left[\sqrt{f(x)}\right]=\dfrac{1}{2\sqrt{f(x)}}\cdot\dfrac{d}{dx}[f(x)]$
14. $\dfrac{d}{dx}[\sin f(x)]=\cos f(x)\cdot\dfrac{d}{dx}f(x)$
15. $\dfrac{d}{dx}[\cos f(x)]=-\sin f(x)\cdot\dfrac{d}{dx}f(x)$
16. $\dfrac{d}{dx}[\tan f(x)]=\sec^2 f(x)\cdot\dfrac{d}{dx}f(x)$
17. $\dfrac{d}{dx}[\cot f(x)]=-\mathrm{cosec}^2 f(x)\cdot\dfrac{d}{dx}f(x)$
18. $\dfrac{d}{dx}[\sec f(x)]=\sec f(x)\tan f(x)\cdot\dfrac{d}{dx}f(x)$
19. $\dfrac{d}{dx}[\mathrm{cosec} f(x)]=-\mathrm{cosec} f(x)\cot f(x)\cdot\dfrac{d}{dx}f(x)$
20. $\dfrac{d}{dx}\log[f(x)]=\dfrac{1}{f(x)}\cdot\dfrac{d}{dx}f(x)$
21. $\dfrac{d}{dx}\left[\dfrac{1}{\log f(x)}\right]=\dfrac{d}{dx}[\log f(x)]^{-1}=(-1)[\log f(x)]^{-2}\cdot\dfrac{1}{f(x)}\cdot\dfrac{d}{dx}f(x)$
22. $\dfrac{d}{dx}\left[\sin^n f(x)\right]=n\sin^{n-1}f(x)\cdot\dfrac{d}{dx}[\sin f(x)]$

    $=n\sin^{n-1}f(x)\cdot\cos f(x)\cdot\dfrac{d}{dx}f(x)$
23. $\dfrac{d}{dx}\left[\cos^n f(x)\right]=n\cos^{n-1}f(x)\cdot\dfrac{d}{dx}[\cos f(x)]$

    $=n\cos^{n-1}f(x)\cdot(-\sin f(x))\cdot\dfrac{d}{dx}f(x)$

24. $\frac{d}{dx}\left[\tan^n f(x)\right] = n\tan^{n-1} f(x)\cdot\frac{d}{dx}[\tan f(x)]$

$= n\tan^{n-1} f(x)\cdot\sec^2 f(x)\cdot\frac{d}{dx} f(x)$

25. $\frac{d}{dx}\left[\cot^n f(x)\right] = n\cot^{n-1} f(x)\cdot\frac{d}{dx}[\cot f(x)]$

$= n\cot^{n-1} f(x)\cdot\left(-\operatorname{cosec}^2 f(x)\right)\cdot\frac{d}{dx} f(x)$

26. $\frac{d}{dx}\left[\operatorname{cosec}^n f(x)\right] = n\operatorname{cosec}^{n-1} f(x)\cdot\frac{d}{dx}[\operatorname{cosec} f(x)]$

$= n\operatorname{cosec}^{n-1} f(x)\cdot[(-\operatorname{cosec} f(x)\cot f(x))]\cdot\frac{d}{dx} f(x)$

$= -n\ \operatorname{cosec}^n f(x)\ \cot f(x)\cdot\ \frac{d}{dx} f(x).$

27. $\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)}\cdot\frac{d}{dx} f(x)$

28. $\frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)}\cdot\log a\ \cdot\frac{d}{dx} f(x)$

29. $\frac{d}{dx}[\log f(x)] = \frac{1}{f(x)}\cdot\frac{d}{dx}[f(x)]$

30. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$

31. $\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$

32. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

33. $\frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}$

34. $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}}$

35. $\frac{d}{dx}\left(\operatorname{cosec}^{-1} x\right) = -\frac{1}{x\sqrt{x^2-1}}$

36. $\frac{d}{dx}\sin^{-1}[f(x)] = \frac{1}{\sqrt{1-[f(x)]^2}}\cdot f'(x)$

37. $\frac{d}{dx}\cos^{-1}[f(x)] = -\frac{1}{\sqrt{1-[f(x)]^2}}\cdot f'(x)$

38. $\frac{d}{dx}\tan^{-1}[f(x)] = \frac{1}{1+[f(x)]^2}\cdot f'(x)$

39. $\frac{d}{dx}\cot^{-1}[f(x)] = -\frac{1}{1+[f(x)]^2}\cdot f'(x)$

40. $\frac{d}{dx}\sec^{-1}[f(x)] = \frac{1}{f(x)\sqrt{[f(x)]^2-1}}\cdot f'(x)$

41. $\frac{d}{dx}\operatorname{cosec}^{-1}[f(x)] = -\frac{1}{f(x)\sqrt{[f(x)]^2-1}}\cdot f'(x)$