Operator 3 – Surprise / Black-Swan Operator

“How violently reality deviates from what was expected.”

Now we build the operator that captures your black-swan / blind-spot talk: where the model’s expectation fails hard.

3.1 Spaces & objects

Same state space .

A trajectory .

A predictive model (or expectation map):

$$M : \mathbb{R} \to V,\quad t \mapsto m(t),$$

You can think of as:

An external forecast,

An internal pattern learned by your framework,

Or even a simple moving average.

3.2 Parameters

The model .

A scale parameter (expected variability or “noise level”). Could be constant or time-dependent.

3.3 Operator definition

Define the normalized surprise field:

$$
S_{m,\sigma}(x)(t)
:= \frac{\| x(t) – m(t) \|}{\sigma(t)}.
$$

You now have a scalar time-series

$$
S_{m,\sigma}(x) : \mathbb{R} \to [0,\infty)
$$

If you want a binary black-swan indicator, you can compose with a threshold:

$$
\mathcal{B}_\theta(x)(t) :=
\begin{cases}
1, & S_{m,\sigma}(x)(t) \ge \theta,\\[3pt]
0, & \text{otherwise.}
\end{cases}
$$

Where is your “this is insane” level (e.g. 3σ, 5σ).

3.4 Key properties

Affine invariance in the error:

Surprise depends only on the difference .

Scale normalization:

If you double both the typical noise and the deviations, the normalized surprise doesn’t change.

Nonlinearity:

Is not linear in because of the norm; it’s more like “distance from the model.”

3.5 Equivalence classes & blind spots

Define:

$$X \sim^{(m,\sigma)}_\theta y$$

$$\iff \mathcal{B}_\theta(x)(t) = \mathcal{B}_\theta(y)(t)\ \forall t.$$

If two trajectories always cross the black-swan threshold at the same times, they are equivalent at the black-swan level.

This isolates a structure purely in terms of where expectation catastrophically fails, not how it wiggles in between.

This is exactly the “macro-scale blind spot” story:

A world where your model almost never fires can feel “stable,” even if the underlying 4D dynamics are wild.

Conversely, a world where spikes constantly is one where your model is fundamentally misaligned with reality.