1. Operator 1 – Observational Projection (your 4D shadow backbone)

We already nailed this, but in “library index” form:

Spaces & objects

: 4D real inner-product space (your “true state space”).

Observer state , with .

Observable hyperplane:

$$H_n := \{ x \in V : \langle n,x\rangle = 0 \}.$$

Operator definition

$$P_n : V \to H_n,\quad P_n(x) = x - \langle n,x\rangle\, n.$$

Key properties

Linear: .

Idempotent: (projection).

Self-adjoint (orthogonal projection).

, .

Equivalence classes (observer indistinguishability)

$$x \sim_n x' \iff P_n(x)=P_n(x')$$

$$\iff \exists \alpha\in\mathbb{R} \text{ s.t. } x' = x+\alpha n.$$

So each class is a line : same 3D shadow, different hidden 4D altitude.

Interpretation

Structural backbone of the 4D Shadow Hypothesis:

true state → observed slice , with whole equivalence classes invisible to a given observer.