The corresponding m-coupled discrete nonlinear Schrödinger equations (DNLSEs) can be written as

$$\begin{cases} \mathbf{A}\mathbf{u}_j - \lambda_j \mathbf{u}_j + \mu_j \mathbf{u}_j^{②} \circ \mathbf{u}_j + \sum_{i \neq j, i=1}^{m} \beta_{ij} \mathbf{u}_i^{②} \circ \mathbf{u}_j = \mathbf{0}, \\ \mathbf{u}_j > 0, \ \mathbf{u}_j \in \mathbb{R}^N, \ \text{for } j = 1, \ldots, m, \end{cases} \tag{1}$$

where $\mathbf{u}_j \in \mathbb{R}^N$ denotes the approximation of $\phi_j(\mathbf{x})$, for $j = 1, \ldots, m$.

$\mathbf{u} \circ \mathbf{v} = (u_1 v_1, \ldots, u_N v_N)^\top$ denotes the Hadamard product of $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u}^{ⓡ} = \mathbf{u} \circ \cdots \circ \mathbf{u}$ denotes the r-time Hadamard product of $\mathbf{u}$.