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## Abstract

Time series data from scientific fields as diverse as astrophysics, economics, human movement science, and neuroscience all exhibit fractal properties. That is, these time series often exhibit self-similarity and long-range correlations. This **fractalRegression** package implements a number of univariate and bivariate time series tools appropriate for analyzing noisy data exhibiting these properties. These methods, especially the bivariate tools (Kristoufek, 2015; Likens, Amazeen, West, & Gibbons, 2019) have yet to be implemented in a complete package for the R Statistical Software environment. As both practitioners and developers of these methods, we expect these tools will be of interest to a wide audience of R users, especially those from fields such as the human movement, cognitive, and other behavioral sciences. The algorithms have been developed in C++ using the popular Rcpp (Eddelbuettel & Francois, 2011) and RcppArmadillo (Eddelbuettel & Sanderson, 2014) packages. The result is a collection of efficient functions that perform well even on long time series (e.g.,  $\geq 10,000$  data points).

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## Introduction

Fractal analysis, in its many forms, has become an important framework in virtually every area of science, often serving as an indicator of system health (Goldberger et al., 2002), adaptability (Bak, Tang, & Wiesenfeld, 1987), control (Likens, Fine, Amazeen, & Amazeen, 2015), cognitive function (Euler, Wiltshire, Niermeyer, & Butner, 2016), and multi-scale interactions (Kelty-Stephen, 2017). In particular, various methods related to Detrended Fluctuation Analysis (DFA) (Peng et al., 1994) have rose to prominence due to their ease of understanding and broad applicability to stationary and nonstationary time series, alike. The basic DFA algorithm has been implemented in numerous packages and software programs. However, advanced methods such as Multifractal Detrended Fluctuation Analysis (MFDFA) (Kantelhardt et al., 2002), Detrended Cross Correlation (DCCA) (Podobnik, Jiang, Zhou, & Stanley, 2011; Zebende, 2011), and, in particular, fractal regression techniques such as Multiscale Regression Analysis (MRA) (Kristoufek, 2015; Likens, Amazeen, West, & Gibbons, 2019) have not yet been implemented in a comprehensive CRAN Package for the R Statistical Software Environment. Thus, there is a clear need for a package that incorporates this functionality in order to advance theoretical research focused on understanding the time varying properties of natural phenomena and applied research that uses those insights in important areas such as healthcare (Cavanaugh, Kelty-Stephen, & Stergiou, 2017) and education (Snow, Likens, Allen, & McNamara, 2016).

## Methods

### Comparison to other Packages

Some foundational efforts in fractal analyses, which partially overlap with the functionality of this package, have been implemented elsewhere. For example, a number of

univariate fractal and multifractal analyses have been implemented in the ‘fracLab’ library for MATLAB (Legrand & Véhel, 2003) and other toolboxes that are mainly targeted at multifractal analysis (Ihlen et al., 2012; Ihlen & Vereijken, 2010). In terms of open access packages, there are other packages that implement some, but not all of the same functions such as the `fathon` package (Bianchi, 2020) that has been implemented in Python as well as the R packages: `fractal` [1], `nonlinearTseries` (Garcia, 2020), and `MF DFA` (Laib, Golay, Telesca, & Kanevski, 2018). However, none of the above packages incorporate monofractal and multifractal DFA with DCCA and MRA and run on a C++ architecture. Our `fractalRegression` package is unique in this combination of analyses, efficiency. For instance, we are not aware of any other packages that feature MRA and Multiscale Lagged Regression (MLRA).

## Results

## Discussion

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