Table 1

Summary of the elements of the statistical investigative cycle (Bargagliotti et al., 2020)

Statistical Investigative Process Element	GAISE II Description
Formulate Statistical Investigative Questions	Formulating statistical investigative questions that anticipate variability leads to productive investigations. (p. 13)
Collect/Consider the Data	Data collection designs must acknowledge variability in data…After the data are available—whether they were collected first-hand or acquired from another source— they need to be interrogated…The data collection design impacts the scope of generalizability and the possible limitations in analysis and interpretation. (p. 14)
Analyze the Data	When we analyze data, we seek to understand its variability. Reasoning about distributions is key to accounting for and describing variability at all developmental levels. Graphical displays and numerical summaries are used to explore, describe, and compare variability in distributions. (p. 14)
Interpret the Results	Statistical interpretations are made in the presence of variability and must take variability into account…When we generalize the results and look beyond the study data collected, we must take into account these sources of variability. (p. 14)

Table 2

Example of coding of CCSSM LEs based on the GAISE II Framework

	Process				Developmental level		
Learning Expectancy (LE)	Question	Collect	Analyze	Interpret	A	B	C
Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	0	0	1	0	0	1	0
Summarize numerical data sets in relation to their context, such as by: Reporting the number of observations.	0	0	1	1	1	0	0
Summarize numerical data sets in relation to their context, such as by: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	0	1	0	1	1	1	0

Table 3.

Analytic Approaches to investigating the OTL relative to the highlights of the GAISE II report.

GAISE II Highlights	Analytic Approach
1. The importance of asking questions throughout the statistical problem-solving process and how this process remains at the forefront of statistical reasoning for all studies involving data	Because we already captured standards related to questions in our first analysis, in our second analysis we focused on looking for evidence of questions being connected across the elements of investigative process by identifying standards that were coded for questions and at least one other element to identify the OTL the importance of asking questions throughout the process.
2. The consideration of different data and variable types, the importance of carefully planning how to collect data or how to consider data to help answer statistical investigative questions, and the process of collecting, cleaning, interrogating, and analyzing the data	This highlight specifically targets aspects of collecting and analyzing data, so we began by only looking at standards in those two elements. Within the identified standards, we looked for explicit mention of variables other than categorical or quantitative (i.e., spatial, pictorial, textual, and dynamic). We also looked for explicit mention of data cleaning or structuring.
3. The inclusion of multivariate thinking throughout all Pre-K–12 educational levels	For this highlight, we looked across all the K-8 standards and searched specifically for mention of using more than two variables in an investigation.
4. The role of probabilistic thinking in quantifying randomness throughout all levels	Part of the process of drawing boundaries for our data was parsing out mathematical probability from statistical probability. To investigate the role of probabilistic thinking throughout the levels we then looked for standards that mention randomness, simulation, uncertainty, or probability.
5. The recognition that modern statistical practice is intertwined with technology, and the importance of incorporating technology as feasible	To investigate the process of this aspect, we looked for the explicit mention of technology-related terms used in standards.
6. The enhanced importance of clearly and accurately communicating statistical information	The communication of statistical information falls firmly into the interpretation element of the investigative process, so we began by isolating those standards. Then we looked for standards that related to communicating information to others (beyond a teacher who is assessing a student's understanding of statistical interpretation).

Table 4

Comparison of Kentucky Grade 3 LEs and Comparative CCSSM Grade 3 LEs

<table>
<tr><th>Kentucky Grade 3 LE</th><th>Comparative CCSSM Grade 3 LE</th></tr>
<tr><td>KY.3.MD.3b: Create a scaled pictograph and a scaled bar graph to represent a data set (using technology or by hand)</td><td rowspan="2">CCSS.MATH.CONTENT.3.MD.B.3
Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</td></tr>
<tr><td>KY.3.MD.3c: Make observations from the graph about the question posed, including “how many more” and “how many less” questions.</td></tr>
<tr><td>KY.3.MD.4b: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch</td><td rowspan="2">CCSS.MATH.CONTENT.3.MD.B.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.</td></tr>
<tr><td>KY.3.MD.4c: Show the data by making a dot plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters</td></tr>
<tr><td>KY.3.MD.3a: Identify a statistical question focused on categorical data and gather data</td><td>N/A</td></tr>
<tr><td>KY.3.MD.4a: Identify a statistical question focused on numerical data</td><td>N/A</td></tr>
<tr><td>KY.3.MD.4d: Make observations from the graph about the question posed, including questions about the shape of the data and compare responses.</td><td>N/A</td></tr>
</table>

Table 5

Descriptive statistics for the number of LEs in each state by grade level for all states (n=52) and for states that are not CCSSM verbatim (n=31) in parentheses when different.

Grade	Min	Q1	Median	Q3	Max
K	1	2	2	2	3
1	0	1	1	1 (2)	5
2	1	2	2	2 (3)	6
3	1	2	2	2	7
4	1	1	1	1 (2.25)	3
5	0	1	1	1 (2)	7
6	2	8 (7)	8	8 (8.25)	13
7	3	7 (6.75)	7	8 (9)	14
8	2	4	4	4 (5)	11
Total	15	28	28 (29)	29.25 (33.5)	47

Table 6

A descriptive summary of the common LEs for grades K-5 with counts of the numbers of states that have the same wording (same), similar wording, or did not have the LE (absent).

Grade & LE #	Common Learning Expectancies	Same	Similar	Absent
K-1 *CCSSM*	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	30	17	5
K-2 *CCSSM*	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.	24	26	2
K-3	Create and interpret real-object and picture graphs	0	5	47
1-1 *CCSSM*	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	37	13	2
1-2	Create and interpret tables, pictograms, and bar graphs	0	11	41
2-1 *CCSSM*	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	28	14	10
2-2 *CCSSM*	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	33	18	1
2-3	Draw conclusions/interpret graphs (i.e., pictograph, bar graphs, line plots)	0	10	42
3-1 *CCSSM*	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	31	19	2
3-2 *CCSSM*	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	32	12	8
4-1 *CCSSM*	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using	29	19	4

	information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.			
4-2	Represent and interpret data in bar graphs	0	9	43
5-1 *CCSSM*	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	31	12	9
5-2	Find and use descriptive statistics (i.e., mean, median, mode, range) to describe data.	0	6	46

Table 7

A descriptive summary of the common LEs for grades 6-8 with counts of the numbers of states that have the same wording (same), similar wording, or did not have the LE (absent).

Grade & LE #	Common Learning Expectancies	Same	Similar	Absent
6-1 *CCSSM*	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.	30	17	5
6-2 *CCSSM*	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	30	15	7
6-3 *CCSSM*	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	32	11	9
6-4 *CCSSM*	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	29	20	3
6-5 *CCSSM*	Summarize numerical data sets in relation to their context, such as by: Reporting the number of observations.	32	13	7
6-6 *CCSSM*	Summarize numerical data sets in relation to their context, such as by: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	33	10	9
6-7 *CCSSM*	Summarize numerical data sets in relation to their context, such as by: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	23	27	2
6-8 *CCSSM*	Summarize numerical data sets in relation to their context, such as by: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	29	16	7
7-1 *CCSSM*	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is	32	13	7

	representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.			
7-2 *CCSSM*	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.	31	16	5
7-3 *CCSSM*	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	26	19	7
7-4 *CCSSM*	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	30	14	8
7-5 *CCSSM*	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.	31	19	2
7-6 *CCSSM*	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy: Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	35	8	9

7-7 *CCSSM*	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation: Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?	33	10	9
7-8	Create and interpret circle graphs	0	7	45
8-1 *CCSSM*	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	34	18	0
8-2 *CCSSM*	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	34	14	4
8-3 *CCSSM*	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	32	15	5
8-4 *CCSSM*	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?	28	14	10