

Consider $f(n) = 1000n^2$.

I claim that this function is $\theta(n^2)$.

Since we can choose $c_1 = 1000$ and $c_2 = 1000$. Thus,

$$c_1 n^2 \leq 1000n^2 \leq c_2 n^2$$

$$1000n^2 \leq 1000n^2 \leq 1000n^2$$

Consider $f(n) = n^2 + 1000n$.

I claim that this function is $\theta(n^2)$.

If I choose $c_1 = 1$, then I have $n^2 + 1000n \geq c_1 n^2$, and so this side of the inequality is taken care of.

The other side is a bit tougher: Need to find a constant c_2 s.t. for sufficiently large n , I'll get that $n^2 + 1000n \leq c_2 n^2$.

Subtracting n^2 from both sides gives $1000n \leq c_2 n^2 - n^2 = (c_2 - 1)n^2$.

Dividing both sides by $(c_2 - 1)n$ gives $\frac{1000}{c_2 - 1} \leq n$.

Now, I pick $c_2 = 2$, so that the inequality becomes $\frac{1000}{2-1} \leq n$, or $1000 \leq n$.

Now I'm in good shape, because I have shown that if I choose $n_0 = 1000$ and $c_2 = 2$, then for all $n \geq n_0$, I have $1000 \leq n$, which we saw is equivalent to $n^2 + 1000n \leq c_2 n^2$.

In combination, constant factors and low-order terms don't matter. If we consider a function like $1000n^2 - 200n$, we can ignore the low-order term $200n$ and the constant factor 1000, and therefore we can say that $1000n^2 - 200n$ is $\theta(n^2)$.