Computing Along the Big Long River

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Abstract

We develop a model to schedule trips down the Big Long River. The goal is to optimally plan boat trips of varying duration and propulsion so as to maximize the number of trips over the six-month season.

We model the process by which groups travel from campsite to campsite. Subject to the given constraints, our algorithm outputs the optimal daily schedule for each group on the river. By studying the algorithm's long-term behavior, we can compute a maximum number of trips, which we define as the river's carrying capacity.

We apply our algorithm to a case study of the Grand Canyon, which has many attributes in common with the Big Long River.

Finally, we examine the carrying capacity's sensitivity to changes in the distribution of propulsion methods, distribution of trip duration, and the number of campsites on the river.

Introduction

We address scheduling recreational trips down the Big Long River so as to maximize the number of trips. From First Launch to Final Exit (225 mi), participants take either an oar-powered rubber raft or a motorized boat. Trips last between 6 and 18 nights, with participants camping at designated campsites along the river. To ensure an authentic wilderness experience, at most one group at a time may occupy a campsite. This constraint limits the number of possible trips during the park's six-month season.

We model the situation and then compare our results to rivers with similar attributes, thus verifying that our approach yields desirable results.

Our model is easily adaptable to find optimal trip schedules for rivers of varying length, numbers of campsites, trip durations, and boat speeds.

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Defining the Problem

- How should trips of varying length and propulsion be scheduled to maximize the number of trips possible over a six-month season?
- How many new groups can start a river trip on any given day?
- What is the carrying capacity of the river—the maximum number of groups that can be sent down the river during its six-month season?

Model Overview

We design a model that

- can be applied to real-world rivers with similar attributes (i.e., the Grand Canyon);
- is flexible enough to simulate a wide range of feasible inputs; and
- simulates river-trip scheduling as a function of a distribution of trip lengths (either 6, 12, or 18 days), a varying distribution of propulsion speeds, and a varying number of campsites.

The model predicts the number of trips over a six-month season. It also answers questions about the carrying capacity of the river, advantageous distributions of propulsion speeds and trip lengths, how many groups can start a river trip each day, and how to schedule trips.

Constraints

The problem specifies the following constraints:

- Trips begin at First Launch and end at Final Exit, 225 miles downstream.
- There are only two types of boats: oar-powered rubber rafts and motorized boats.
- Oar-powered rubber rafts travel 4 mph on average.
- Motorized boats travel 8 mph on average.
- Group trips range from 6 to 18 nights.
- Trips are scheduled during a six-month period of the year.
- Campsites are distributed uniformly along the river.
- No two groups can occupy the same campsite at the same time.

Assumptions

- We can prescribe the ratio of oar-powered river rafts to motorized boats that go onto the river each day.

  There can be problems if too many oar-powered boats are launched with short trip lengths.

- The duration of a trip is either 12 days or 18 days for oar-powered rafts, and either 6 days or 12 days for motorized boats.

  This simplification still allows our model to produce meaningful results while letting us compare the effect of varying trip lengths.

- There can only be one group per campsite per night.

  This agrees with the desires of the river manager.

- Each day, a group can only move downstream or remain in its current campsite—it cannot move back upstream.

  This restricts the flow of groups to a single direction, greatly simplifying how we can move groups from campsite to campsite.

- Groups can travel only between 8 A.M. and 6 P.M., a maximum of 9 hours of travel per day (one hour is subtracted for breaks/lunch/etc.).

  This implies that per day, oar-powered rafts can travel at most 36 miles, and motorized boats at most 72 miles. This assumption allows us to determine which groups can reasonably reach a given campsite.

- Groups never travel farther than the distance that they can feasibly travel in a single day: 36 miles per day for oar-powered rafts and 72 miles per day for motorized boats.

- We ignore variables that could influence maximum daily travel distance, such as weather and river conditions.

  There is no way of accurately including these in the model.

- Campsites are distributed uniformly so that the distance between campsites is the length of the river divided by the number of campsites.

  We can thus represent the river as an array of equally-spaced campsites.

- A group must reach the end of the river on the final day of its trip:
  - A group will not leave the river early even if able to.
  - A group will not have a finish date past the desired trip length.

  This assumption fits what we believe is an important standard for the river manager and for the quality of the trips.

Table 1.
Notation.

Symbol	Meaning
g_i	group i
t_i	trip length for group i, measured in nights; $6 \leq t_i \leq 18$
d_i	number of nights group i has spent on the river
Y	number of campsites on the river
c_Y	location of campsite Y in miles downstream; $0 < c_Y < 225$
c_0	campsite representing First Launch (used to construct a waitlist of groups)
c_{final}	campsite (which is always "open") representing Final Exit
l_i	location of group i's current campsite in miles down the river; $0 < l_i < 225$
a_i	average distance that group i should move each day to be on schedule; $a_i = 225/t_i$
m_i	maximum distance that group i can travel in a single day
P_i	priority of group i; $P_i = (d_i/t_i)(l_i/225)$
G_c	set of groups that can reach campsite c
R	ratio of oar-powered rafts to motorized boats launched each day
X	current number of trips down Big Long River each year
M	peak carrying capacity of the river (maximum number of groups that can be sent down the river during its six-month season)
D	distribution of trip durations of groups on the river

Methods

We define some terms and phrases:

Open campsite: A campsite is open if there is no group currently occupying it: Campsite c_n is open if no group g_i is assigned to c_n.

Moving to an open campsite: For a group g_i, its campsite c_n, moving to some other open campsite $c_m \neq c_n$ is equivalent to assigning g_i to the new campsite. Since a group can move only downstream, or remain at their current campsite, we must have $m \geq n$.

Waitlist: The waitlist for a given day is composed of the groups that are not yet on the river but will start their trip on the day when their ranking on the waitlist and their ability to reach a campsite c includes them in the set G_c of groups that can reach campsite c, and the groups are deemed "the highest priority." Waitlisted groups are initialized with a current campsite value of c_0 (the zeroth campsite), and are assumed to have priority $P = 1$ until they are moved from the waitlist onto the river.

Off the River: We consider the first space off of the river to be the "final campsite" c_{final}, and it is always an open campsite (so that any number of groups can be assigned to it. This is consistent with the understanding that any number of groups can move off of the river in a single day.

The Farthest Empty Campsite

Our scheduling algorithm uses an array as the data structure to represent the river, with each element of the array being a campsite. The algorithm begins each day by finding the open campsite c that is farthest down the river, then generates a set G_c of all groups that could potentially reach c that night. Thus,

$$G_c = \{g_i \mid l_i + m_i \geq c\},$$

where l_i is the group's current location and m_i is the maximum distance that the group can travel in one day.

- The requirement that $m_i + l_i \geq c$ specifies that group g_i must be able to reach campsite c in one day.
- G_c can consist of groups on the river and groups on the waitlist.
- If $G_c = \emptyset$, then we move to the next farthest empty campsite—located upstream, closer to the start of the river. The algorithm always runs from the end of the river up towards the start of the river.
- If $G_c \neq \emptyset$, then the algorithm attempts to move the group with the highest priority to campsite c.

The scheduling algorithm continues in this fashion until the farthest empty campsite is the zeroth campsite c_0. At this point, every group that was able to move on the river that day has been moved to a campsite, and we start the algorithm again to simulate the next day.

Priority

Once a set G_c has been formed for a specific campsite c, the algorithm must decide which group to move to that campsite. The *priority* P_i is a measure of how far ahead or behind schedule group g_i is:

- $P_i > 1$: group g_i is behind schedule;
- $P_i < 1$: group g_i is ahead of schedule;
- $P_i = 1$: group g_i is precisely on schedule.

We attempt to move the group with the highest priority into c.

Some examples of situations that arise, and how priority is used to resolve them, are outlined in **Figures 1** and **2**.

Priorities and Other Considerations

Our algorithm always tries to move the group that is the most behind schedule, to try to ensure that each group is camped on the river for a

Downstream ⟶

Campsite	1	2	3	4	5	6
Group Priority	A $P_A = 1.1$	B $\mathbf{P_B = 1.5}$	C $P_C = 0.8$	Open	Open	Farthest open campsite

Figure 1. The scheduling algorithm has found that the farthest open campsite is Campsite 6 and Groups A, B, and C can feasibly reach it. Group B has the highest priority, so we move Group B to Campsite 6.

Downstream ⟶

Campsite	1	2	3	4	5	6
Group Priority	A $\mathbf{P_A = 1.1}$	Open	C $P_C = 0.8$	Open	Farthest open campsite	B

Figure 2. As the scheduling algorithm progresses past Campsite 6, it finds that the next farthest open campsite is Campsite 5. The algorithm has calculated that Groups A and C can feasibly reach it; since $P_A > P_C$, Group A is moved to Campsite 5.

number of nights equal to its predetermined trip length. However, in some instances it may not be ideal to move the group with highest priority to the farthest feasible open campsite. Such is the case if the group with the highest priority is *ahead* of schedule ($P < 1$).

We provide the following rules for handling group priorities:

- If g_i is *behind* schedule, i.e. $P_i > 1$, then move g_i to c, its farthest reachable open campsite.
- If g_i is *ahead* of schedule, i.e. $P_i < 1$, then calculate $d_i a_i$, the number of nights that the group has already been on the river times the average distance per day that the group should travel to be on schedule. If the result is greater than or equal (in miles) to the location of campsite c, then move g_i to c. Doing so amounts to moving g_i only in such a way that it is no longer ahead of schedule.
- Regardless of P_i, if the chosen $c = c_{\text{final}}$, then do not move g_i unless $t_i = d_i$. This feature ensures that g_i's trip will not end before its designated end date.

The one case where a group's priority is disregarded is shown in **Figure 3**.

Scheduling Simulation

We now demonstrate how our model could be used to schedule river trips.

In the following example, we assume 50 campsites along the 225-mile river, and we introduce 4 groups to the river each day. We project the trip

Downstream $\longrightarrow$

Campsite	170	171	...	223	224	Off
Group Priority	D $\mathbf{P_D = 1.1}$ $t_D = 12$ $d_D = 11$	Open	Open	Open	Open	Farthest open campsite

Figure 3. The farthest open campsite is the campsite off the river. The algorithm finds that Group D could move there, but Group D has $t_D > d_D$—that is, Group D is supposed to be on the river for 12 nights but so far has spent only 11—so Group D remains on the river, at some campsite between 171 and 224 inclusive.

schedules of the four specific groups that we introduce to the river on day 25. We choose a midseason day to demonstrate our model's stability over time. The characteristics of the four groups are:

- g_1: motorized, $t_1 = 6$;
- g_2: oar-powered, $t_2 = 18$;
- g_3: motorized, $t_3 = 12$;
- g_4: oar-powered, $t_4 = 12$.

Figure 5 shows each group's campsite number and priority value for each night spent on the river. For instance, the column labeled g_2 gives campsite numbers for each of the nights of g_2's trip. We find that each g_i is off the river after spending exactly t_i nights camping, and that $P \rightarrow 1$ as $d_i \rightarrow t_i$, showing that as time passes our algorithm attempts to get (and keep) groups on schedule. **Figures 6** and **7** display our results graphically. These findings are consistent with the intention of our method; we see in this small-scale simulation that our algorithm produces desirable results.

Case Study

The Grand Canyon

The Grand Canyon is an ideal case study for our model, since it shares many characteristics with the Big Long River. The Canyon's primary river rafting stretch is 226 miles, it has 235 campsites, and it is open approximately six months of the year. It allows tourists to travel by motorized boat or by oar-powered river raft for a maximum of 12 or 18 days, respectively [Jalbert et al. 2006].

Using the parameters of the Grand Canyon, we test our model by running a number of simulations. We alter the number of groups placed on the water each day, attempting to find the carrying capacity for the river—the

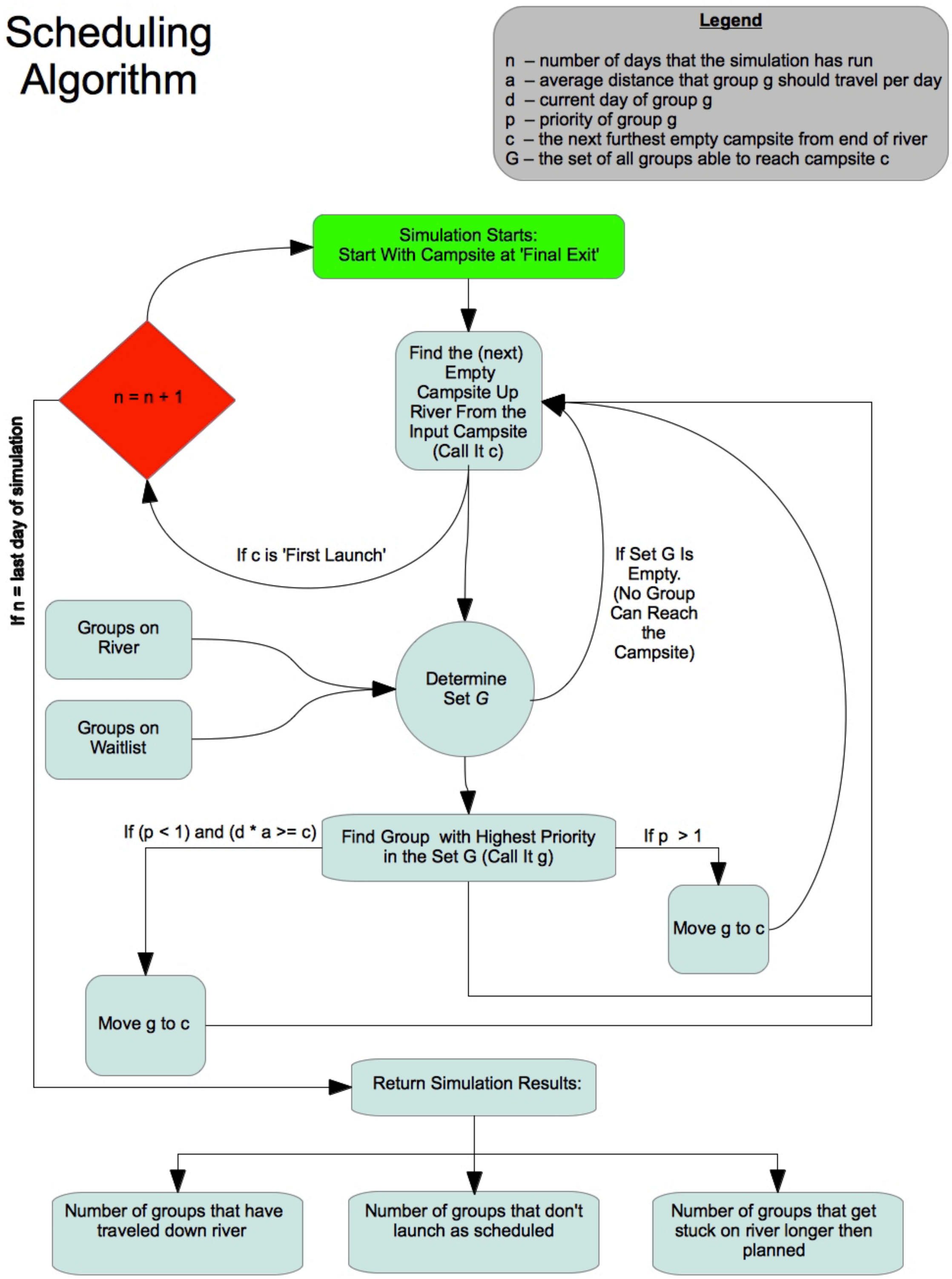

Figure 4. Visual depiction of scheduling algorithm.

Campsite numbers and priority values for each group									
Number of nights spent on river		g_1	P_1	g_2	P_2	g_3	P_3	g_4	P_4
	1	5	1.43	2	1.32	3	1.28	1	3.85
	2	20	0.71	6	0.87	19	0.4	8	0.96
	3	20	1.07	6	1.32	19	0.61	8	1.44
	4	36	0.79	13	0.81	19	0.81	15	1.02
	5	36	0.99	13	1.01	19	1.01	15	1.28
	6	49	0.87	13	1.21	35	0.66	23	1
	7	OFF	1	13	1.42	35	0.77	25	1.08
	8			13	1.66	35	0.88	32	0.96
	9			15	1.58	35	0.99	32	1.08
	11			23	1.14	35	1.1	39	0.99
	12			30	0.96	49	0.86	39	1.08
	13			30	1.05	49	0.94	46	1
	14			30	1.14	OFF	1	OFF	1
	15			36	1.02				
	16			36	1.1				
	17			44	0.96				
	18			44	1.02				
	19			44	1.01				
	20			OFF	1				

Figure 5. Schedule for example of groups launched on Day 25.

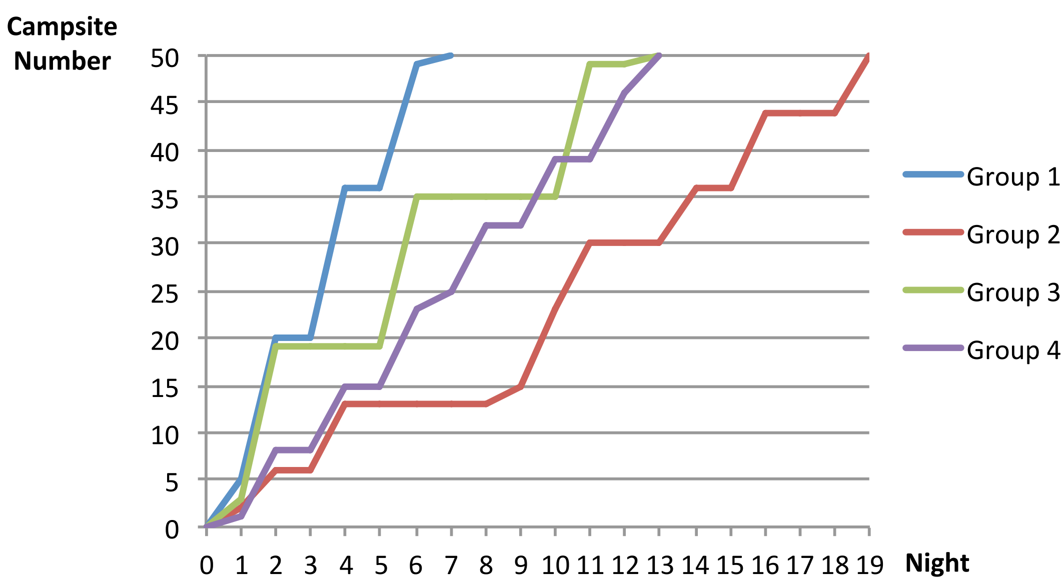

Figure 6. Movement of groups down the river based on **Figure 5**. Groups reach the end of the river on different nights due to varying trip-duration parameters.

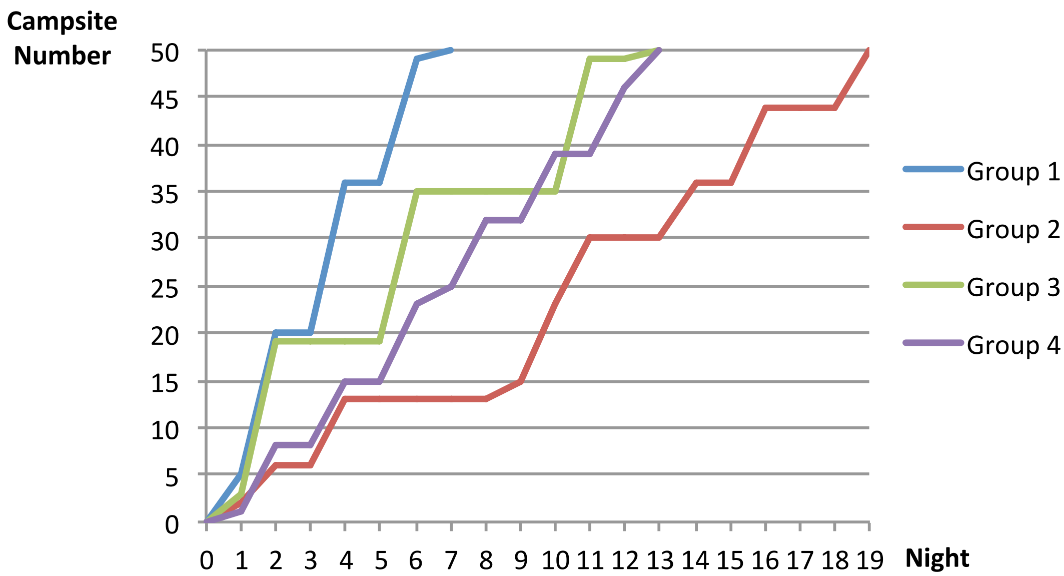

Figure 7. Priority values of groups over the course of each trip. Values converge to $P = 1$ due to the algorithm's attempt to keep groups on schedule.

maximum number of possible trips over a six-month season. The main constraint is that each trip must last the group's planned trip duration. During its summer season, the Grand Canyon typically places six new groups on the water each day [Jalbert et al. 2006], so we use this value for our first simulation. In each simulation, we use an equal number of motorized boats and oar-powered rafts, along with an equal distribution of trip lengths.

Our model predicts the number of groups that make it off the river (completed trips), how many trips arrive past their desired end date (late trips), and the number of groups that did not make it off the waitlist (total left on waitlist). These values change as we vary the number of new groups placed on the water each day (groups/day).

Table 2.

Results of simulations for the number of groups to launch each day.

Simulation	Groups/day	Trips Completed	Late	Left on waitlist
1	6	996		
2	8	1328		
3	10	1660		
4	12	1992		
5	14	2324		
6	16	2656		
7	17	2834		
8	18	2988		
9	19	3154	5	
10	20	3248	10	43
11	21	3306	14	109

Table 1 indicates that a maximum of 18 groups can be sent down the river each day. Over the course of the six-month season, this amounts to nearly 3,000 trips. Increasing groups/day above 18 is likely to cause late trips (some groups are still on the river when our simulation ends) and long waitlists. In Simulation 1, we send 1,080 groups down river (6 groups/day $\times$ 180 days) but only 996 groups make it off; the other groups began near the end of the six-month period and did not reach the end of their trip before the end of the season. These groups have negligible impact on our results and we ignore them.

Sensitivity Analysis of Carrying Capacity

Managers of the Big Long River are faced with a similar task to that of the managers of the Grand Canyon. Therefore, by finding an optimal solution for the Grand Canyon, we may also have found an optimal solution for the Big Long River. However, this optimal solution is based on two key assumptions:

- Each day, we put approximately the same number of groups onto the river; and
- the river has about one campsite per mile.

We can make these assumptions for the Grand Canyon because they are true for the Grand Canyon, but we do not know if they are true for the Big Long River.

To deal with these unknowns, we create **Table 3**. Its values are generated by fixing the number Y of campsites on the river and the ratio R of oar-powered rafts to motorized boats launched each day, and then increasing the number of trips added to the river each day until the river reaches peak carrying capacity.

Table 3.

Capacity of the river as a function of the number of campsites and the ratio of oarboats to motorboats.

		Number of campsites on the river				
		100	150	200	250	300
Ratio oar : motor	1:4	1360	1688	2362	3036	3724
	1:2	1181	1676	2514	3178	3854
	1:1	1169	1837	2505	3173	3984
	2:1	1157	1658	2320	2988	3604
	4:1	990	1652	2308	2803	3402

The peak carrying capacities in **Table 3** can be visualized as points in a three-dimensional space, and we can find a best-fit surface that passes (nearly) through the data points. This best-fit surface allows us to estimate the peak carrying capacity M of the river for interpolated values. Essentially, it gives M as a function of Y and R and shows how sensitive M is to changes in Y and/or R. **Figure 7** is a contour diagram of this surface.

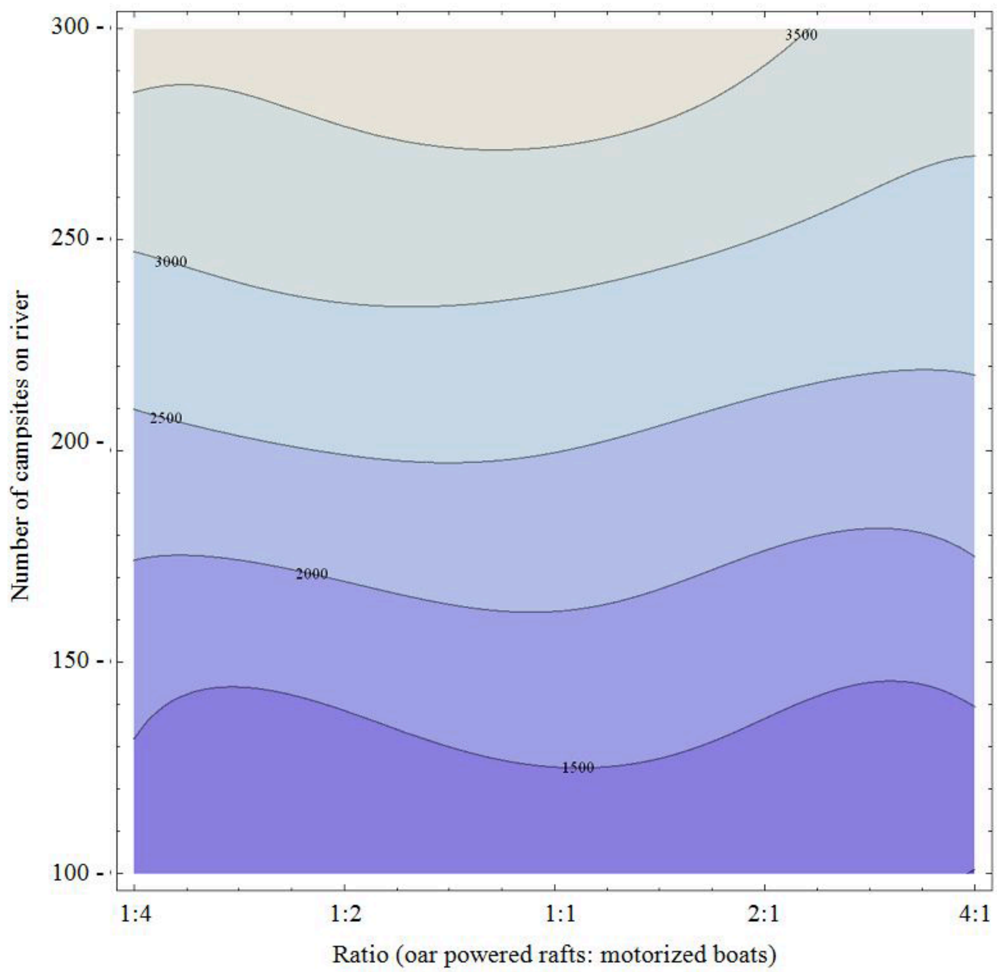

Figure 7. Contour diagram of the best-fit surface to the points of **Table 3**.

The ridge along the vertical line $R = 1 : 1$ predicts that for any given value of Y between 100 and 300, the river will have an optimal value of M when $R = 1 : 1$. Unfortunately, the formula for this best-fit surface is rather complex, and it doesn't do an accurate job of extrapolating beyond the data of **Table 3**; so it is not a particularly useful tool for the peak carrying capacity for other values of R. The best method to predict the peak carrying capacity is just to use our scheduling algorithm.

Sensitivity Analysis of Carrying Capacity re R and D

We have treated M as a function of R and Y, but it is still unknown to us how M is affected by the mix of trip durations of groups on the river (D).

For example, if we scheduled trips of either 6 or 12 days, how would this affect M? The river managers want to know what mix of trips of varying duration and speed will utilize the river in the best way possible.

We use our scheduling algorithm to attempt to answer this question. We fix the number of campsites at 200 and determine the peak carrying capacity for values of R and D. The results of this simulation are displayed in **Table 4**.

Table 4.
Carrying capacity of the river by trip lengths and boat type.

		Distribution of trip lengths			
		12 only	12 or 18	6 or 12	6, 12, or 18
	1:4	2004	1998	2541	2362
Ratio	1:2	2171	1992	2535	2514
oar : motor	1:1	2171	1986	2362	2505
	2:1	1837	2147	2847	2320
	4:1	2505	2141	2851	2308

Table 4 is intended to address the question of what mix of trip durations and speeds will yield a maximum carrying capacity. For example: If the river managers are currently scheduling trips of length

- 6, 12, or 18: Capacity could be increased either by increasing R to be closer to 1:1 or by decreasing D to be closer to "6 or 12."
- 12 or 18: Decrease D to be closer to "6 or 12."
- 6 or 12: Increase R to be closer to 4:1.

Conclusion

The river managers have asked how many more trips can be added to the Big Long River's season. Without knowing the specifics of how the river is currently being managed, we cannot give an exact answer. However, by applying our model to a study of the Grand Canyon, we found results which could be extrapolated to the context of the Big Long River. Specifically, the managers of the Big Long River could add approximately $(3{,}000 - X)$ groups to the rafting season, where X is the current number of trips and 3,000 is the capacity predicted by our scheduling algorithm.

Additionally, we modeled how certain variables are related to each other; M, D, R, and Y. River managers could refer to our figures and tables to see how they could change their current values of D, R, and Y to achieve a greater carrying capacity for the Big Long River.

We also addressed scheduling campsite placement for groups moving down the Big Long River through an algorithm which uses priority values to move groups downstream in an orderly manner.

Limitations and Error Analysis

Carrying Capacity Overestimation

Our model has several limitations. It assumes that the capacity of the river is constrained only by the number of campsites, the trip durations, and the transportation methods. We maximize the river's carrying capacity, even if this means that nearly every campsite is occupied each night. This may not be ideal, potentially leading to congestion or environmental degradation of the river. Because of this, our model may overestimate the maximum number of trips possible over long periods of time.

Environmental Concerns

Our case study of the Grand Canyon is evidence that our model omits variables. We are confident that the Grand Canyon could provide enough campsites for 3,000 trips over a six-month period, as predicted by our algorithm. However, since the actual figure is around 1,000 trips [Jalbert et al. 2006], the error is likely due to factors outside of campsite capacity, perhaps environmental concerns.

Neglect of River Speed

Another variable that our model ignores is the speed of the river. River speed increases with the depth and slope of the river channel, making our assumption of constant maximum daily travel distance impossible [Wikipedia 2012]. When a river experiences high flow, river speeds can double, and entire campsites can end up under water [National Park Service 2008]. Again, the results of our model don't reflect these issues.

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Memo to Managers of the Big Long River

In response to your questions regarding trip scheduling and river capacity, we are writing to inform you of our findings.

Our primary accomplishment is the development of a scheduling algorithm. If implemented at Big Long River, it could advise park rangers on how to optimally schedule trips of varying length and propulsion. The optimal schedule will maximize the number of trips possible over the six-month season.

Our algorithm is flexible, taking a variety of different inputs. These include the number and availability of campsites, and parameters associated with each tour group. Given the necessary inputs, we can output a daily schedule. In essence, our algorithm does this by using the state of the river from the previous day. Schedules consist of campsite assignments for each group on the river, as well those waiting to begin their trip. Given knowledge of future waitlists, our algorithm can output schedules months in advance, allowing management to schedule the precise campsite location of any group on any future date.

Sparing you the mathematical details, allow us to say simply that our algorithm uses a priority system. It prioritizes groups who are behind schedule by allowing them to move to further campsites, and holds back groups who are ahead of schedule. In this way, it ensures that all trips will be completed in precisely the length of time the passenger had planned for.

But scheduling is only part of what our algorithm can do. It can also compute a maximum number of possible trips over the six-month season. We call this the carrying capacity of the river. If we find we are below our carrying capacity, our algorithm can tell us how many more groups we could be adding to the water each day. Conversely, if we are experiencing river congestion, we can determine how many fewer groups we should be adding each day to get things running smoothly again.

An interesting finding of our algorithm is how the ratio of oar-powered river rafts to motorized boats affects the number of trips we can send downstream. When dealing with an even distribution of trip durations (from 6 to 18 days), we recommend a 1:1 ratio to maximize the river's carrying capacity. If the distribution is skewed towards shorter trip durations, then our model predicts that increasing towards a 4:1 ratio will cause the carrying capacity to increase. If the distribution is skewed the opposite way, towards longer trip durations, then the carrying capacity of the river will always be less than in the previous two cases—so this is not recommended.

Our algorithm has been thoroughly tested, and we believe that it is a powerful tool for determining the river's carrying capacity, optimizing daily schedules, and ensuring that people will be able to complete their trip as planned while enjoying a true wilderness experience.

Sincerely yours,

Team 13955

Team members Chip Jackson, Lucas Bourne, and Travis Peters, and team advisor Edoh Amiran.