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Math 419: Historical Perspectives of Mathematics

Cissoid of Diocles

Eponymic Curve Paper

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The ground trembles as an Athenian man humbly stands before the Oracle at Delos. “Our people are dying, and this death has persisted for many years. What can we do to appease the gods and end this punishment that has fallen on our people?” The Oracle thought but a short moment before responding: “Double the size of the altar to the god Apollo; then, and only then, will the plague be lifted and the people made well again. Be cautious not spoil the form of the altar, and be exact with your geometric interpretation, Athenian.” The Athenian man quickly thanked the Oracle and departed from the temple. After returning home, the man delivered the words of the Oracle to the people of Athens. They quickly found themselves in disagreement over what it meant to “double the size of the altar.” The man who had traveled to the temple, speaking on behalf of his representative authority for the people of Athens, commanded that the builders double the length of each side; “This will satisfy the gods!” he declared. However, some months later when the men had completed the new alter, the plague did not lift ... [1/2]

What happened? Well, it turns out that the initial interpretation of the problem was wrong and that by doubling the length of each side, the new altar had a volume that was actually 8 times larger! While the scene depicted above is somewhat exaggerated for illustrative purposes and is even challenged by alternative legends, this is one of the more popular and widely accepted accounts of how the *doubling the cube* problem, also known as the Delian problem, was first introduced to the Greeks in about 450 B.C..

As an aside, when Plato was asked to interpret what the Oracle meant by doubling the size of the altar, he believed that the gods never wanted an altar of double the size, but rather the gods wanted to pose the problem “to reproach the Greeks for their neglect of mathematics and their contempt for geometry.” [2]

It was later determined that the actual problem before them was that, given a cube whose side was of length a, they were to find a line of length x, so that $x^3 = 2a^3$. [5] Upon defining the problem and attempting to devise a solution, they discovered that their tools would never be able to accomplish what they needed. In fact, the *doubling the cube* problem is recognized as one of the three most famous geometric problems from classical mathematics that cannot be solved with the Euclidean tools of a compass and straightedge. [4]

Little is known about the Greek mathematician, Diocles of Carystus (who lived from about 240 B.C. – about 180 B.C.). We do know that he was not the first to solve the *doubling the cube* problem. In fact, due to the restriction of Euclidean tools, Diocles never actually solved the *double the cube* problem at all! However, he did invent a rather unique curve on the way to exploring a solution to the problem, and determined that if the proper tools existed, this curve, which is now recognized as the Cissoid of Diocles, could solve the problem.

Up to this point it seemed appropriate to show off one of the more notable extensions and applications of the Cissoid of Diocles as well as some of the historical context of how the curve came into existence. The solution to the *double the cube* problem is merely an application of the Cissoid of Diocles, requiring a bit more vigor to yield the desired results. This is beyond the scope of this paper and from here on we will limit our exploration specifically to the characteristics and significance of the Cissoid of Diocles.

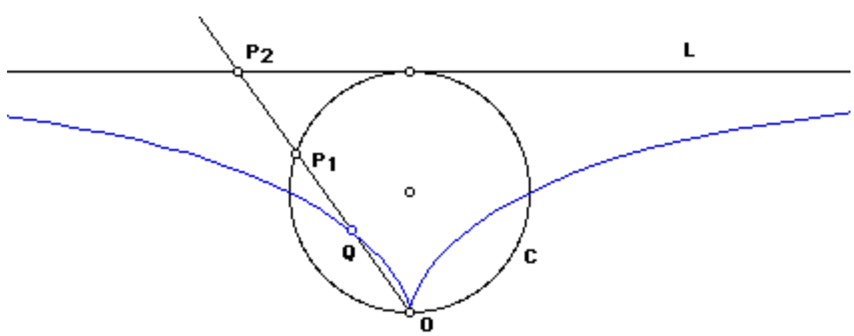

Figure 1: Cissoid of Diocles

The Cissoid (meaning 'ivy'), in general, is the method of defining a new curve based on two other curves, and some fixed point, call it *O*. So what is special about the Cissoid of Diocles? For starters, it should be noted that the two curves used to define a new curve are a circle and a tangent line, denoted by C and L, respectively, in Figure 1. Also note that the point O that shows up in the generalized Cissoid definition is the point on the circle exactly opposite where the circle and the tangent line touch. In order to show how the new curve is defined, we must first note a few more special points that can be seen in Figure 1. Define a point, P_1, and let it be a fixed point that is arbitrarily placed on the circle. Define another point, P_2, which is the intersection of the tangent line L and the line constructed by connecting the points O and P_1. Finally, define a point, call it Q (not a fixed point), on the line segment between O and P_1 such that the distance between points O and Q is the same as the distance between the points P_1 and P_2. Now, to see how the new curve is defined imagine that you begin to roll the circle along the tangent line. As P_1 gets closer to/further from P_2, the point Q gets closer to/ further from the fixed point O. If you were to trace the location of the point Q as you rolled the circle along the tangent line, you would see what we know as the Cissoid of Diocles!

Now you might be saying to yourself "okay…now what? I can't do any math with this!" But I would kindly disagree! As it turns out, the Cissoid of Diocles had nice properties which allow you to use this as a foundation for further exploration. One such property is that points P_1 and Q are symmetric about the midpoint of the line constructed between points O and P_2. There is also usefulness in more modern mathematics which has enabled us to represent various aspects of the curve with equations and coordinate systems as opposed to purely geometrically. Generally speaking, having defined the circle to have a radius of a:

$$(x^2 + y^2)x = 2ay^2 \qquad [3]$$

The derivation of this utilizes concepts beyond that of pre-calculus (conversions from parametric equations to polar coordinates and finally to Cartesian coordinates). Fast forward hundreds of years and we see that the true significance and even the existence of the curve couldn't even be determined until almost 2000 years after being discovered/defined by mathematicians like Descartes and Newton.

Works Cited

[1] Fauvel, John, and Jeremy Gray. *The History of Mathematics: A Reader*. Basingstoke: Macmillan Education in Association with the Open University, 1987. Print.

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[3] Weisstein, Eric. "Cissoid of Diocles." *CRC Concise Encyclopedia of Mathematics*. N.p.: CRC, n.d. 260-61. Print.

[4] Weisstein, Eric. "Cube Duplication." *MathWorld*. Wolfram, Jan.-Feb. 2012. Web. 25 June 2012. <http://mathworld.wolfram.com/CubeDuplication.html>.

[5] Young, Robyn V. *Notable Mathematicians: From Ancient times to the Present*. Detroit, MI: Gale, 1998. Print.