Understanding Problems In Concurrency Using Directed Topology and Homotopy Theory

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Recall deadlock from class…

Necessary conditions:

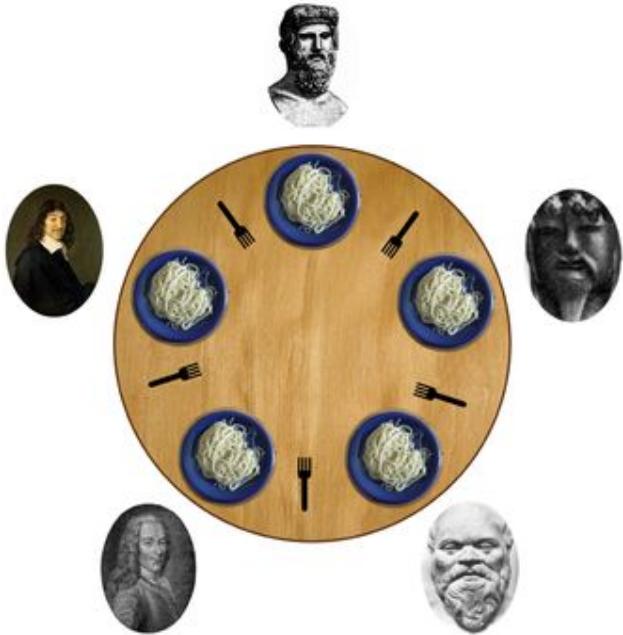- *Hold and wait* or *resource holding*
- *Circular wait*
- *Mutual exclusion*
- *No preemption*

INTERVIEWER: EXPLAIN DEADLOCK AND I'LL HIRE YOU

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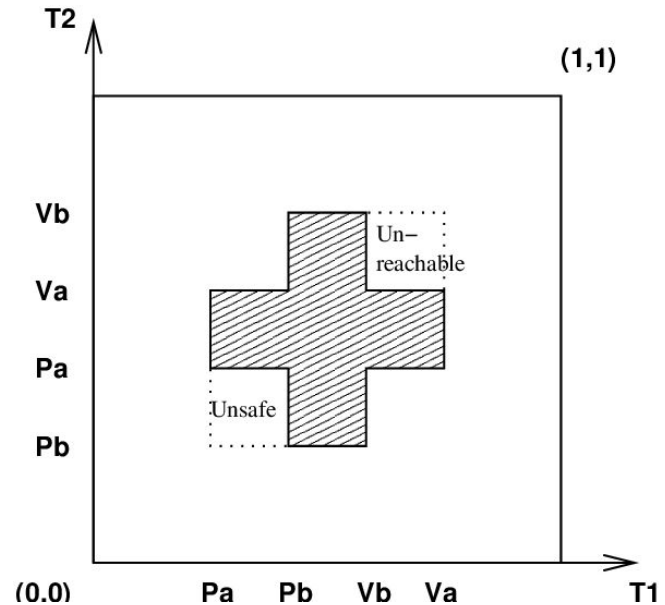
PROGRAMMER: HIRE ME AND I'LL EXPLAIN IT TO YOU

deadlock is a state in which each member of a group is waiting for another member, including itself, to take action
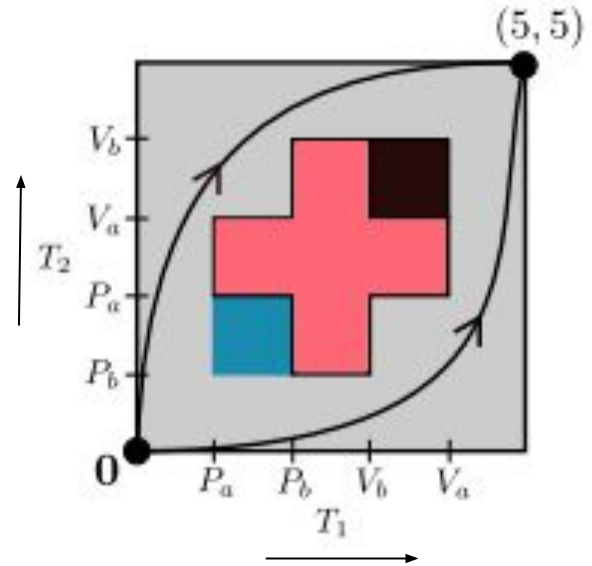
Deadlock

Dining philosophers!

State space and fatal region as a geometric object:
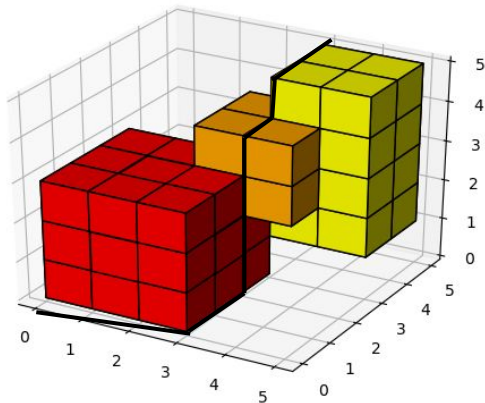
In Geometric Terms...

- The complement of the shaded region produces a *Euclidean cubical complex* (union of cubes) aka a *po-space* (partially ordered)
- Two possible paths through grey region
- These two paths are *dihomotopic* (same output given same input)
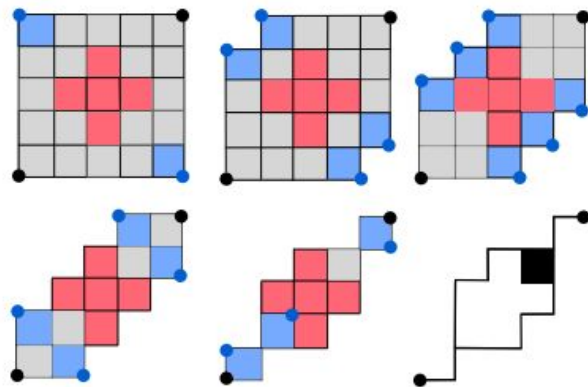
More complexity...

3 dimensions, 3 philosophers

Generating paths gets a lot more complicated!

The goal is to produce classes of paths that result in successful concurrent execution (verification)
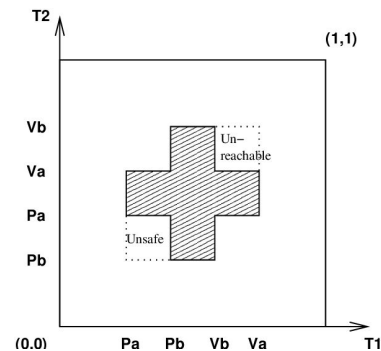
Verification

For any set of processes, does a sequence of acquiring and releasing resources result in deadlock?

Infinite number of sets of inputs
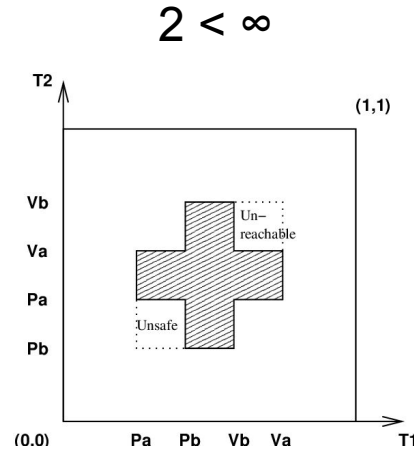
P acquires a resource:

- one second later
- .5 seconds later
- .25 seconds later
- ...

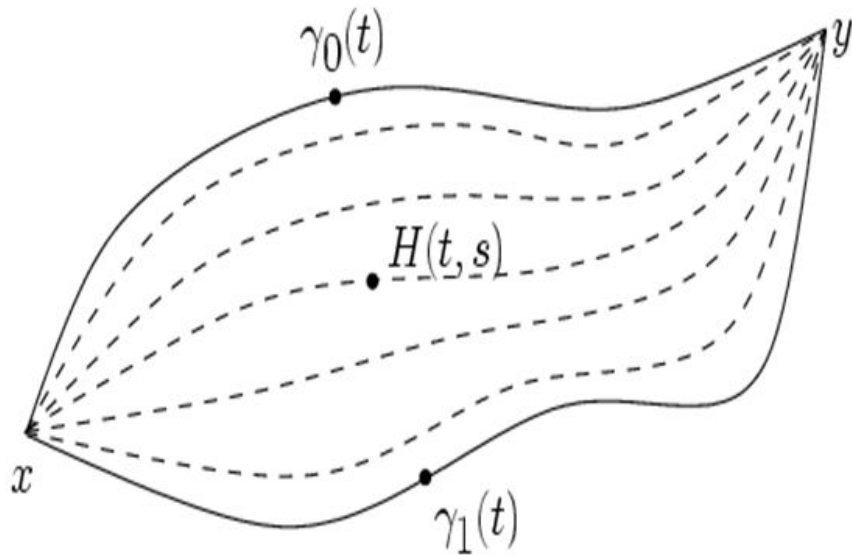Impossible to give a description of the executions that result in deadlock

Solution: Equivalence classes on executions

The payoff: Need to check fewer executions in order to verify that deadlock does not occur.

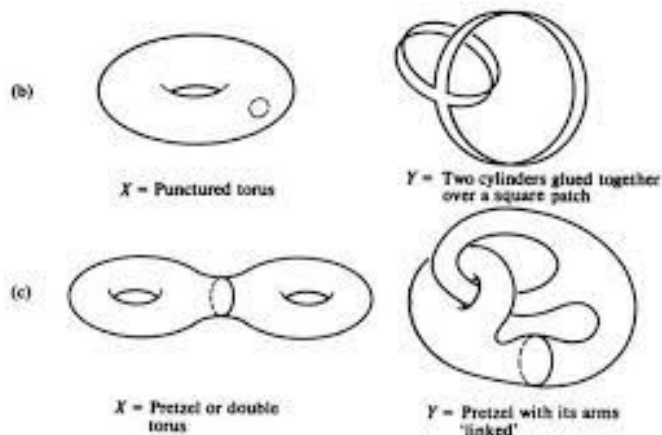$$2 < \infty$$

Equivalence on paths is a homotopy
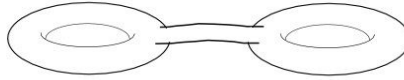
Homotopy - a movie, one path continuously turns
into another.

Application of a tool that was developed to answer theoretical questions in mathematics.
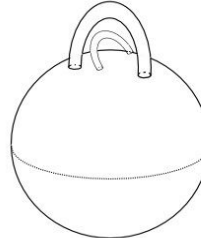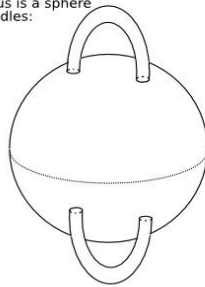
Are two topological spaces the same?

(b)

X = Punctured torus

Y = Two cylinders glued together over a square patch

(c)

X = Pretzel or double torus

Y = Pretzel with its arms 'linked'
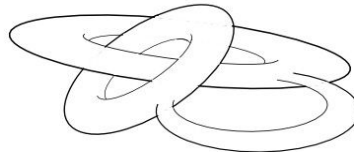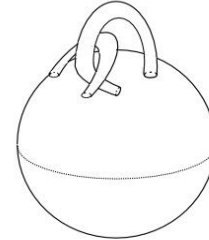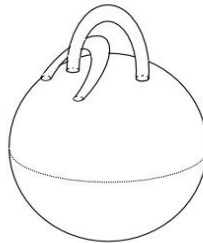
Linked and unlinked double torus

This is an unlinked double torus...

A double torus is a sphere with two handles:

We can slide the handles around however we like without changing the topological properties!

...and this is a linked double torus...

To conclude:

Homotopy is useful in giving a description of executions that will result in deadlock.

Bibliography

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