Fin 395 4 Lecture 1: Overview of Empirical Asset Pricing

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Handouts: Syllabus, Lecture Notes, papers for next time

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Course objective

This course is an in-depth study of empirical work in asset pricing, including econometric and statistical methods. The focus is on the fundamental economic questions in asset pricing, how to answer them empirically, what answers are in existing research, and what we still don't know. The course should prepare you to understand and produce cutting-edge empirical asset pricing research.

Course materials

- I'll provide printed copy of lecture notes in each class
- Textbooks:
  - *The Econometrics of Financial Markets* by John Campbell, Andrew Lo, and Craig MacKinley, ISBN 0691043019
  - *Empirical Dynamic Asset Pricing: Model Specification and Econometric Assessment* by Kenneth Singleton, ISBN 0691122970
  - *Asset Pricing* by John Cochrane, ISBN 0691121370
- You will need access to the data available via Wharton Research Data Services (WRDS)

Homework assignments

30% of your grade is based on four homework assignments, each due three weeks after I assign them

- Mix of problem solving, data analysis, and some replication of existing research
- Your solutions should be typed up (ideally in LaTeX) and should explain your answers like you would in a paper – including tables and figures when appropriate
- Will require coding and estimation, I recommend Matlab, R, or Python (SAS and STATA won't work for many problems)
- Use of AI tools is **strongly encouraged**, the scope of these assignments is huge
  - They will take approximately 15–20 hours, assuming AI assistance
  - Questions written to be intentionally ambiguous and require you to think through what methodologies should be used, what data you might need, etc.

Participation, presentation, exams

10% of your grade is based on contribution to class discussions

- Read required papers and book chapters – I cold call!
- Ask questions!

10% is based on an in-class presentation

- Present one of the reading list papers that is **not** required
- 30 minutes, including questions and discussion

20% is based on an in-class midterm exam

- Hand-written, one page of notes only

30% is based on an interview-style exam

- 30 minutes per student, held during finals week
- More details on Syllabus and later in semester

Discussion

What do regression standard errors mean?

Asset pricing

What determines the price today of an asset promising future financial payments that may be risky?

$$\text{Future payments: } x_{t+1}, x_{t+2}, \cdots, \infty$$
$$\text{Price: } p_t(\text{moments of } x_{t+s}\ \forall\ s, \text{time } t \text{ information})$$

1. What is the distribution of future payments?
   - Depends on type of contract (equity? debt? etc) as well as economics of the promising entity (firm, government, etc)

2. What is present value of these payments?
   - Discount for time value of money and risk

Stochastic discount factor

What is the price at time t of an asset paying x_T at time $T > t$?

- I_t: Investors' information set at $t \equiv$ set of all random variables investors know the realization of at t
- Securities and portfolios with payoffs $x_T \in X_T$, $X_T \in I_T$
- Prices $p_t(x_T) \in I_t$

Definition

A **stochastic discount factor** (SDF, or pricing kernel) at time t for payoffs at time T is a random variable m_T that satisfies $p_t = \mathbb{E}\left(m_T x_T | I_t\right) \; \forall \; x_T \in X_T$

SDFs discount future cash flows, are random because investors value payoffs differently in different states of the world

Power of SDFs

SDFs, if we can find them, are **extremely** powerful

- Tells you the price of any asset given distribution of x_T
- Tells you expected return $\mathbb{E}_t(R_T) = \mathbb{E}_t\left(\frac{x_T}{p_t}\right)$

$$
\begin{aligned}
p_t &= \mathbb{E}_t\left(m_T x_T\right) \\
1 &= \mathbb{E}_t\left(m_T \frac{x_T}{p_t}\right) \\
&= \mathbb{E}_t\left(m_T R_T\right) = \mathbb{E}_t\left(m_T\right) \mathbb{E}_t\left(R_T\right) + \mathsf{cov}_t\left(m_T, R_T\right)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \mathbb{E}_t\left(R_T\right) - R_f &= -R_f \mathsf{cov}_t\left(m_T, R_T\right) \\
R_f &\equiv \frac{1}{\mathbb{E}_t\left(m_T\right)}
\end{aligned}
$$

Existence of SDF and LOOP

We now consider for what pairs of payoff spaces X_T and prices p_t there exists a stochastic discount factor

Definition

Law of one price (LOOP): $p_t(a_t x_{1,T} + b_t x_{2,T}) = a_t p_t(x_{1,T}) + b_t p_t(x_{2,T})$, for any $a_t, b_t \in I_t$ and $x_{1,T}, x_{2,T} \in X_T$

Theorem

Law of one price $\Leftrightarrow$ *there exists a SDF* $m_T \in I_T$

Existence of SDF and No arbitrage

Definition

No arbitrage: For any $x_T \in X_T$ with $\mathbb{P}(x_T \geq 0) = 1$, we have $\mathbb{P}(p_t(x_T) \leq 0 \;\&\; x_T > 0) = 0$

Theorem

No arbitrage $\Leftrightarrow$ *there exists a SDF* $m_T \in I_T$ *such that* $\mathbb{P}(m_T > 0) = 1$

Unique SDF in space of payoffs

- Let payoff space be generated by $N \times 1$ vector of basis payoffs Y_T: $X_T = \{c_t' Y_T\}$, $c_t \in I_t$, $Y_T \in I_T$
  - Y_T are payoffs of all individual (non-redundant) investments, $x_T \in X_T$ is payoff of specific a portfolio of investments
  - By LOOP, if we can price Y_T we can price all $x_T \in X_T$ as well
- Look for SDF $m_T^* \in X_T$, i.e. $m_T^* = w_t' Y_T$ s.t.

$$\begin{aligned} p_t(Y_T) &= \mathbb{E}\left(m_T^* Y_T \mid I_t\right) \\ &= \mathbb{E}\left(w_t' Y_T Y_T' \mid I_t\right) = \mathbb{E}\left(Y_T Y_T' \mid I_t\right) w_t \end{aligned}$$

- This implies $w_t = \left(\mathbb{E}\left(Y_T Y_T' \mid I_t\right)\right)^{-1} p_t(Y_T)$ and so

$$m_T^* = p_t(Y_T)' \left(\mathbb{E}\left(Y_T Y_T' \mid I_t\right)\right)^{-1} Y_T$$

- Note that $m_T^* = \mathbb{E}\left(m_T Y_T \mid I_t\right)' \mathbb{E}\left(Y_T Y_T' \mid I_t\right)^{-1} Y_T = \text{proj}_t(m_T | X_T)$ for any SDF m_T

SDF portfolio properties

The m_T^* derived on the previous page has the following properties:

1. Unique regardless of how many SDFs m_T there are
2. Can be written in return space as:

$$
\begin{aligned}
m_T^* &= \dot{w}_t' R_T \\
\dot{w}_t &\equiv \left(\mathbb{E}\left(R_T R_T' \mid I_t\right)\right)^{-1} \mathbf{1}_N
\end{aligned}
$$

   where $R_T \equiv \frac{Y_T}{p_t(Y_T)}$ are the returns of the basis payoffs

SDFs and Sharpe Ratio

Define S_T as the returns of the risky basis payoffs (unlike R_T, no risk-free) and v_t as the elements of $\dot{w}_t$ corresponding to S_T

$$m_T^* = \text{constant} + v_t' S_T$$

Portfolio "msr" with weights $\frac{v_t}{\sum v_t}$ has highest possible Sharpe Ratio:

$$\frac{|\mathbb{E}(R_{x,T}) - R_f|}{\sigma(R_{x,T})} \leq \frac{|\mathbb{E}(R_{msr,T}) - R_f|}{\sigma(R_{msr,T})} = R_f \sigma(m_T^*),$$

where $R_{x,T} \equiv \frac{x_T}{p_t(x_T)}, x_T \in X_T$ and $R_{msr,T} = \frac{v_t'}{\sum v_t} S_T$

Single factor beta representation

As long as an SDF exists, $\forall\, x_T \in X_T$,

$$\mathbb{E}(R_{x,T}) = R_f + \beta_{x,msr}\left(\mathbb{E}(R_{msr,T}) - R_f\right)$$

$$\beta_{x,msr} = \frac{\mathsf{cov}(R_{x,T}, R_{msr,T})}{\mathsf{var}(R_{msr,T})}$$

Portfolio with maximum Sharpe Ratio prices all portfolios

- Requires almost no assumptions (just no arbitrage)
- CAPM intuition summarizes all of asset pricing theory

Differentiating prediction of CAPM: market portfolio has maximum Sharpe Ratio

- Can test this!

Differentiating prediction of other AP theories: other portfolio has max SR

- Can also test this!

Discussion

Why does maximum Sharpe Ratio portfolio matter so much even when investors do not have mean-variance preferences?

Number of stochastic discount factors

- **Complete market**: m_T is unique

$$m_T = m_T^*$$

- **Incomplete market**: infinite different m_T

$$m_T = m_T^* + \epsilon_T, \text{ where } \mathbb{E}\left(\epsilon_T x_T \mid I_t\right) = 0 \text{ and } \mathbb{E}\left(\epsilon_T \mid I_t\right) = 0$$

- Economic content of asset pricing models is the restrictions that they place on m_T

Preference-based restrictions

Example

- Consider an economy with an investor having time-separable VNM-utility and consumption c_t. The investor maximizes

$$\mathbb{E}\left(\sum_{i=0}^{\infty}\beta^{i}U\left(c_{t+i}\right)\middle|I_t\right)$$

- In equilibrium, the agent's first-order condition implies

$$m_T = \beta^{T-t}\frac{U'(c_T)}{U'(c_t)}$$

- In words, SDF is intertemporal marginal rate of substitution for *all* agents, even if they have different utility functions
  - Different consumption and portfolio choices allow them all to have same IMRS even when they start with different endowments/preferences

Consumption-based models and factor models

Representative agent consumption-based models

- c_t is aggregate consumption, $U()$ is representative utility function, portfolio choice $=$ market portfolio
- Lecture 2

Factor models

- The fact that

$$m_T \text{ prices all } x_T \in X_T \iff \text{proj}(m_T | X_T) \text{ prices all } x_T \in X_T$$

  can be used to specify the SDF as a function of asset payoffs ("mimicking portfolios")
  - CAPM and related models
  - Ad-hoc factor models
- Lectures 6–7

No-arbitrage restrictions

Motivation

- Modeling preferences is difficult, strong assumptions are needed for existence of representative agent, etc. Can we instead get some mileage just from no arbitrage?
- No-arbitrage follows from minimal assumptions on preferences (increasing utility function)
- We can always "price" asset payoffs using m_T^*, but this is almost a tautology
- Goal: find a more parsimonious specification of the SDF using only a subset of Y_T
  - Maybe one that only prices a subset of X_T

No-arbitrage restrictions – Stocks

Arbitrage Pricing Theory (Ross 1976)

- Assume a factor structure: Payoff on asset i depends on a "small" number of factors $f_T \in X_T$

$$x_{i,T} = a_{i,t} + \beta'_{i,t} f_T + \epsilon_{i,T}$$

  where $\mathbb{E}\left(\epsilon_{i,T} | I_t\right) = \mathbb{E}\left(\epsilon_{i,T} f_T | I_t\right) = \mathbb{E}\left(\epsilon_{i,T}\epsilon_{j,T} | I_t\right) = 0$ for $i \neq j$

- Apply LOOP:

$$p_t(x_{i,T}) = p_t(a_{i,t}) + \beta'_{i,t} p_t(f_T) + p_t(\epsilon_{i,T})$$

- Let $\hat{f}_T = [1 \quad f_T]$. Under the factor structure assumption, $\hat{f}_T$ are the basis payoffs that span the "systematic" (not-$\epsilon_{i,T}$) component of the payoffs in X_T

No-arbitrage restrictions – Stocks

Let $f_T^* = p_t(\hat{f}_T)'\mathbb{E}\left(\hat{f}_T'\hat{f}_T \middle| I_t\right)^{-1}\hat{f}_T$

- Prices f_T and 1 by construction

Perfect factor structure: $\epsilon_{i,T} = 0$

- f_T^* prices all $x_{i,T}$ since $\hat{f}_T$ spans X_T

Approximate factor structure: $\epsilon_{i,T} \neq 0$

- **APT**: consider limit as number of assets $N \to \infty$ and portfolios $w_t' x_t$ satisfying

$$\lim_{N\to\infty} \text{max}(w_t) = 0$$

- Under some conditions, idiosyncratic risk of the portfolio Var $(w_t'\epsilon_T | I_t) \to 0$ and f_T^* prices these portfolios as well

No-arbitrage restrictions – Stocks

Problem: APT only works for assets/portfolios with **no** idiosyncratic risk

- If portfolio i has $\text{Var}\left(\epsilon_{i,T}|I_t\right) > 0$ any SDF $m_T = f_T^* + k\epsilon_{i,T}$, for any k, prices all the factors but predicts different $p_t(x_{i,T})$

$$
\begin{aligned}
p_t(x_{i,T}) &= \mathbb{E}\left((f_T^* + k\epsilon_{i,T})x_{i,T}|I_t\right) \\
&= \mathbb{E}\left(f_T^*(a_{i,t} + \beta_{i,t}' f_T)\middle|I_t\right) + k\mathbb{E}\left(\epsilon_{i,t}^2\middle|I_t\right)
\end{aligned}
$$

- Not true in real world since even huge portfolios have risk unspanned by typical factor structure
- Without any further restrictions, all APT says is we only need to price non-redundant assets – no economic content

No-arbitrage restrictions – Stocks

State of the APT literature for stocks:

- Lots of research in 1980s, but now largely abandoned
- Often cited as informal motivation for ad-hoc linear models in empirical studies of stock returns (Lecture 7)
  - Clear from above argument that APT without preference-based restrictions doesn't make any economic predictions
- Fama has called the APT a "factor fishing license"

Not a focus of this course

No-arbitrage restrictions – Bonds

- Riskless payoff of a zero-coupon bond maturing at T: $x_{i,T} = 1_s$

$$p_t(1_T) = \mathbb{E}\left(m_T | I_t\right)$$

- Combining many different maturities T, the observed zero-coupon bond prices $p_t(1_T)$ tell the entire path of future expected SDFs
  - This works as long as there's no arbitrage
  - Get more mileage out of NA here since there's no cash-flow risk
- Much of the bond pricing literature has searched for a parsimonious statistical specification of the SDF consistent with no arbitrage and then use proj $(m_s|X)$ in pricing
- Recently, focus has shifted somewhat to finding connections between the SDF and macro variables
- Term structure literature – Lecture 11

No-arbitrage restrictions – Derivatives

- **Example**: forward contract to buy an asset that delivers payoff $x_{i,T}$ at time T, where price F is also paid at time T
- No arb: $p_t(x_{i,T} - F) = 0 \Leftrightarrow p_t(x_{i,T}) - p_t(1_T)F = 0$, and so

$$F = \frac{p_t(x_{i,T})}{p_t(1_T)}$$

- Payoff of forward contract is redundant (replicate with spot and riskless bond), and therefore no arb has powerful implications
- Pricing of options and other derivatives follows same logic, but the arbitrage portfolio is more complicated

BREAK

Are returns predictable?

Define (gross) returns as

$$R_T = \frac{x_T}{p_t(x_T)}$$

Lots of research in asset pricing has investigated the null hypothesis

$$\mathbb{E}\left(R_{t+k}|I_t\right) = \mu_k \,\forall\, t, \quad k > 0$$

- One notion of "informational efficiency" (Fama, 1970)
- Different "informational efficiency" concepts depending on specification of information set I_t

Are returns predictable?

Motivations for $\mathbb{E}\left(R_{t+k}|I_t\right) = \mu_k$ hypothesis

- Risk-free rate, quantity of risk, and price of risk all constant

$$\mathbb{E}(R_{t+1}|I_t) = R_{f,t} + \underbrace{\beta_{r,msr}}_{\text{risk}} \underbrace{\left(\mathbb{E}(R_{msr,t+1} \mid I_t) - R_{f,t}\right)}_{\text{risk premium}}, \text{ or}$$

$$\mathbb{E}(R_{t+1}|I_t) = R_{f,t} - R_{f,t} \underbrace{\frac{\text{cov}(R_{t+1}, m_{t+1} \mid I_t)}{\text{var}(m_{t+1} \mid I_t)}}_{\text{risk}} \underbrace{\text{var}(m_{t+1} \mid I_t)}_{\text{risk premium}}$$

- With time-varying risk-free rate, can also consider hypothesis that $\mathbb{E}\left(R_{t+k}|I_t\right) - R_{f,t} = \mu_k$
  - Requires that max. Sharpe Ratio portfolio has constant risk premium even as $R_{f,t}$ varies

Random walk hypothesis

To test $\mathbb{E}\left(r_{t+k} \mid I_t\right) = \mu_k$ hypothesis

- We must specify the information set we want to condition on
  - For example, **random walk** (with drift) hypothesis

$$\mathbb{E}\left(r_{t+1} \mid r_t, r_{t-1}, \ldots\right) = \mu,$$

- We must also specify a parametric model that gives the functional form of $\mathbb{E}\left(r_{t+k} \mid I_t\right)$
  - For example could assume a linear relation between past and future returns, meaning we only need to test $\text{proj}(r_{t+k} | r_t) = \mu$

$$r_{t+k} = \alpha + \rho(k) r_t + \epsilon_{t,k}$$

$$H_0 : \quad \rho(k) = 0$$

  - Can use GMM w/moments $\mathbb{E}\left(\epsilon_{t+k}\right) = 0$ and $\mathbb{E}\left(\epsilon_{t+k} r_t\right) = 0$

Autocorrelation estimator

Let $\dot{r}_t = [1 \;\, r_t]'$, $\delta = [\alpha \;\, \rho(k)]'$, and denote an estimate from sample with size N with a N subscript. Consider the OLS estimate:

$$\delta_N = \mathbb{E}_N \left(\dot{r}_t \dot{r}_t'\right)^{-1} \mathbb{E}_N \left(\dot{r}_t r_{t+k}\right)$$

Standard GMM results tell us this is a consistent estimate with asymptotic distribution

$$\sqrt{N}(\delta_N - \delta) \overset{a}{\sim} N(0, \Omega)$$
$$\Omega = \mathbb{E}_N \left(\dot{r}_t \dot{r}_t'\right)^{-1} \mathbb{E}_N \left(\epsilon_{t+k}^2 \dot{r}_t \dot{r}_t'\right) \mathbb{E}_N \left(\dot{r}_t \dot{r}_t'\right)^{-1}$$

If we assume conditional homoskedasticity, $\mathbb{E}\left(\epsilon_{t+k}^2 \mid r_t\right) = \sigma^2$, we get $\Omega = \mathbb{E}_N \left(\dot{r}_t \dot{r}_t'\right)^{-1} \sigma^2$, which implies

$$\text{Var}\left(\rho(k)_N\right) = \frac{1}{N}$$

Finite-sample bias of the AC estimator

Kendall (1954) shows that under the null hypothesis, we have

$$\mathbb{E}(\rho(k)_N) = -\frac{1}{N} + O(N^{-2})$$

For an AR(1) process,

$$\mathbb{E}(\rho(1)_N) = -\frac{1+3\rho(1)}{N} + O(N^{-2})$$

Unbiasedness of OLS, when mean is unknown, requires strict exogeneity, i.e. Cov $(\epsilon_s, r_t) = 0$ for $s, t = 1, \cdots N$

- Here, mechanical correlation between $\epsilon_{t-k,k}$ and r_t

First-order bias-correction under the null

$$\tilde{\rho}(k)_N = \rho(k)_N + \frac{1}{N}$$

Intuition for bias

In a finite sample, need to estimate mean μ as well as ρ

- $\hat{\rho}$ depends on $\hat{\mu}$
- OLS $\hat{\mu}$ happens to produce the smallest possible $\hat{\rho}$

If you have a panel of data with N periods and M firms (or other independent chunks), letting $M \to \infty$ does **not** help

- Autocorrelation coefficient with firm (or chunk) fixed effects is the average of the within-firm autocorrelation estimates, each having only N observations and so downward biased
- Need $N \to \infty$ to eliminate bias

Implications of bias

Asset pricing:

- Because bias is $\frac{1}{N}$ under zero-correlation null, irrelevant for high-frequency datasets (e.g. intraday or daily) with large N
- When studying lower-frequency returns, e.g. yearly or longer, can be substantial
- Leads to bias in time-series return predictability regressions (Lectures 4–5)

Other fields:

- Biases against finding “hot hand” when studying autocorrelation of outcomes within sporting events (this involves averaging a bunch of low-N autocorrelation estimates)
- Creates small-sample bias that can go either way when using panel data in corporate finance and innovations in RHS variable correlated with LHS variable (Grieser and Hadlock 2019)

Box-Pierce (1970) Q-statistic

Statistic to test the null hypothesis that first j AC are jointly zero

$$Q(j) = N \sum_{k=1}^{j} \rho(k)^2$$
$$H_0 : Q(j) = 0$$

Under null with $\mathbb{E}\left(\epsilon_{t+k}^2 \mid r_t\right) = \sigma^2$, since $\sqrt{N}(\rho(k)_N - \rho(k))$ are asymptotically standard normal,

$$Q_N(j) = N \sum_{k=1}^{j} \rho(k)_N^2 \overset{a}{\sim} \chi_j^2(j),$$

where $\chi_j^2(j)$ means there are N degrees of freedom

- Selection of j?
- Effect of selection of j on power?
- Heteroskedasticity?

Variance ratios

Alternative approach: examine Var $\left(\sum_{k=1}^{j} r_{t+k}\right)$

- Under null: $=$ Var$(r_t) \cdot j$
- Positive ρ: $>$ Var$(r_t) \cdot j$
- Negative ρ: $<$ Var$(r_t) \cdot j$

Variance ratio statistic

$$VR(j) = \frac{1}{j} \frac{\text{Var}\left(\sum_{k=1}^{j} r_{t+k}\right)}{\text{Var}(r_t)} - 1$$

$$H_0 : VR(j) = 0$$

Richardson and Smith (1994) show that

$$VR(j)_N = 2 \sum_{k=1}^{j-1} \frac{j-k}{j} \rho(k)_N$$

Variance ratios

Recall that with $\mathbb{E}\left(\epsilon_{t+k}^2 \mid r_t\right) = \sigma^2$, $\sqrt{N}(\rho(k)_T - \rho(k)) \sim N(0,1)$ under the null. Since $\rho(k)_N$ and $\rho(k+i)_N$ are uncorrelated under the null and $VR(j)_N$ is a linear combination of $\rho(k)_N$,

$$\sqrt{N} VR(j)_N \overset{a}{\sim} N(0, \Omega_{VR})$$
$$\Omega_{VR} = 4 \sum_{k=1}^{j-1} \left(\frac{j-k}{j}\right)^2$$

- Can also stick to the GMM formulas that allow for conditional heteroskedasticity

Long-run return regressions

Jegadeesh (1991), Hodrick (1992):

$$r_{t+1} = \alpha + \delta(j)\left(\sum_{k=0}^{j-1} r_{t-k}\right) + \epsilon_{t+1}$$

Richardson and Smith (1994) show that

$$\delta_N(j) = \frac{1}{j}\sum_{k=1}^{j} \rho_N(k) \frac{\text{Var}_N(r_t)}{\frac{1}{j}\text{Var}_N(\sum_{k=0}^{j-1} r_{t-k})}$$

and, under the null,

$$\delta_N(j) \to \frac{1}{j}\sum_{k=1}^{j} \rho_N(k), \text{ as } N \to \infty$$

Summary of serial correlation tests

- Thus, $Q_N(j)$, $VR_N(j)$, and $\delta_N(j)$ are all sums of ACs weighted in different ways. Their asymptotic distribution depends only on the joint asymmetric distribution of the $\rho_N(k)$
- Power of the tests can differ due to the different weighting of the ACs
- Other variants of long-run return regressions: Fama and French (1988)

You'll find out what the data say for US equities indices in Homework 1

More-recent research (e.g. Gupta and Kelly (2019)) looks at ρ for non-market portfolios

Factor Return Monthly AR(1) Coefficients