Hopf bifurcation

week 7

3. Hopf bifurcation

3. Hopf bifurcation

Lotka-Volterra models: example

$T' = bT - \beta ST$ $\longrightarrow$ N' = r1 N - a N P

$S' = m\,\beta ST - dS$ $\longrightarrow$ P' = b N P - d P

N = prey

P = predator

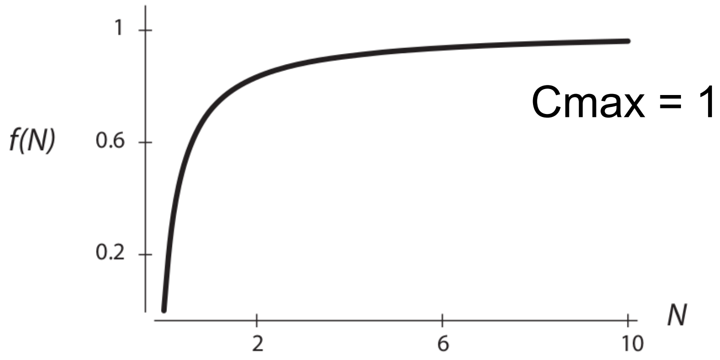
problem 1: in absence of predator: prey can grow exponentially

solution 1: we should add $rN(1 - \frac{N}{K})$

problem 2: term aNP : at every value of P, the amount of prey (N) eaten by predators (P) ~ N → No matter what, the predators never get full

solution 2: change term a NP → f(N)*P

$$f(N) = \frac{C_{max} \cdot N}{N + h}$$



3. Hopf bifurcation

Lotka-Voltera models: example

$T' = bT - \beta ST \longrightarrow$ N' = r1 N - a N P

$S' = m\,\beta ST - dS \longrightarrow$ P' = b N P - d P

N = prey

P = predator

$$N' = r_1 N(1 - \frac{N}{k}) - \frac{wN}{d+N}P$$

w = maximal consumption rate

d = the prey density at which consumption is half the max rate

3. Hopf bifurcation

Lotka-Volterra models: example

$$T' = bT - \beta ST \longrightarrow \text{N' = r1 N - a N P}$$

$$S' = m\,\beta ST - dS \longrightarrow \text{P' = b N P - d P}$$

N = prey

P = predator

Predator equation:

$$P' = r_2 P(1 - \frac{jP}{N})$$

Logistic growth:

- Carrying capacity is set by the prey population!
- j = number of prey needed to support one predator → jP = number of prey necessary to support P predators
- jP < N: P can grow
- jP > N, P exceeded its carrying capacity and must decline

3. Hopf bifurcation

The destruction of a stable equilibrium point and its replacement by an unstable equilibrium point and a stable limit cycle attractor is called Hopf bifurcation.

You can see this in the eigenvalues:

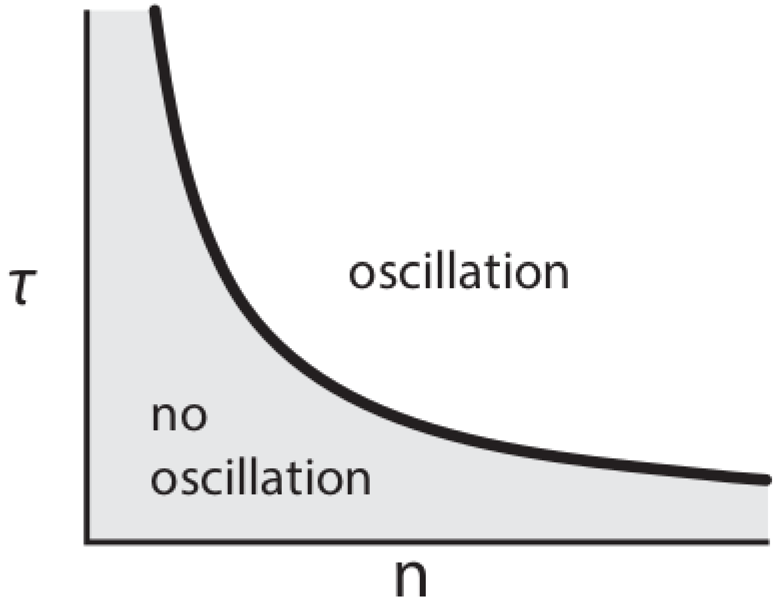
For a complex eigenvalue, the real part of the eigenvalue passes through zero from negative (stable) to positive (unstable)

Exercise 5.3

3. Hopf bifurcation

Oscillations depend on:

- n , which controls the steepness of the feedback,
- τ , which controls the time delay



The chief causes of oscillation in feedback systems are steep negative feedback and time delays