The neuron

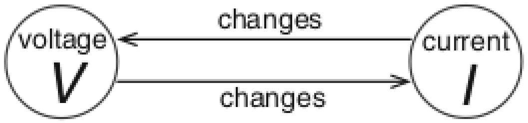
week 8

4. The neuron

4. The neuron

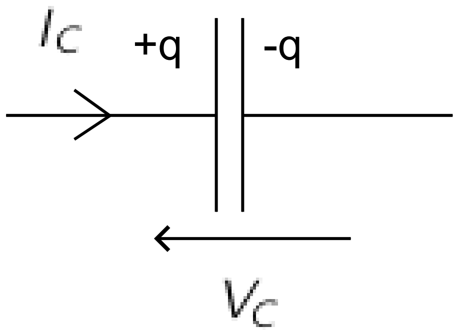
This is the field of electrophysiology

Differential equations will be based on the following principle



Capacitor

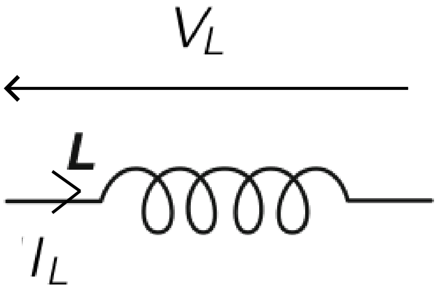
$$\frac{dV_C}{dt} = \frac{1}{C} \cdot I_C$$



C = 1

Inductor

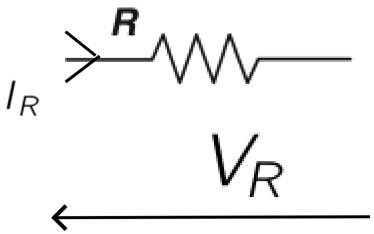
$$\frac{dI_L}{dt} = \frac{1}{L} \cdot V_L$$



L = 1

Resistor

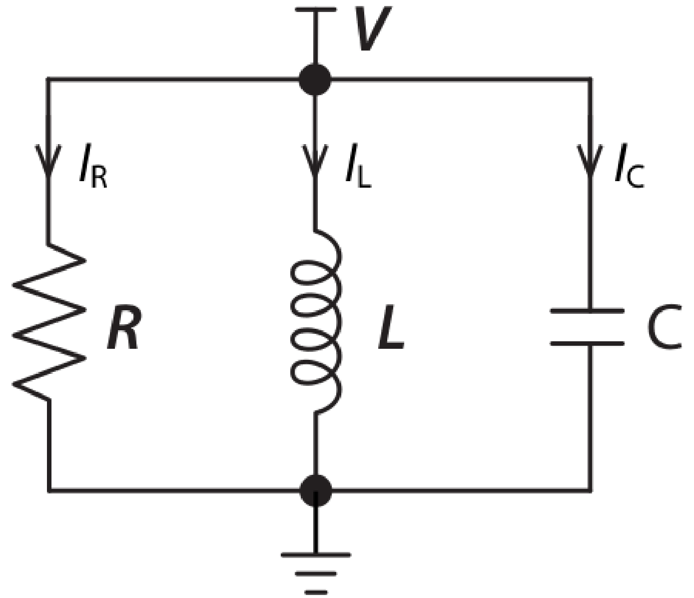
$$I_R = f(V_R)$$



$$I_R = \frac{1}{R} \cdot V_R$$

ohm law

4. The neuron



Kirchhoff's voltage law

$$V_R = V_L = V_C$$

Kirchhoff's current law

$$I_R + I_L + I_C = 0$$

$$\begin{aligned}
I_L' &= V_L && \text{(Faraday's law)} \\
&= V_C && \text{(by KVL)} \\
V_C' &= I_C && \text{(capacitor law)} \\
&= -I_R - I_L && \text{(by KCL)} \\
I_R &= f(V_R) && \text{(generalized Ohm's law)} \\
&= f(V_C) && \text{(by KVL)}
\end{aligned}$$

$\rightarrow$

$I = I_L$ and $V = V_C$

$$\boxed{\begin{aligned} I' &= V \\ V' &= -I - f(V) \end{aligned}}$$

4. The neuron

$$\boxed{\begin{aligned} I' &= V \\ V' &= -I - f(V) \end{aligned}}$$

electrical	mechanical
$I' = V$	$X' = V$
$V' = -I - f(V)$	$V' = -X - f(V)$

electrical V = voltage

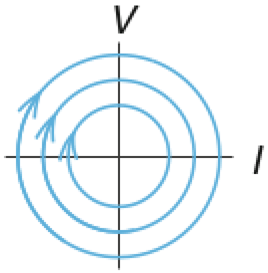
mechanical V = velocity

4. The neuron

$$I' = V$$
$$V' = -I - f(V)$$

Case 0: zero resistance

electrical	mechanical
$I' = V$	$X' = V$
$V' = -I$	$V' = -X$

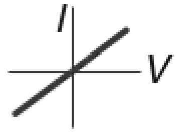


4. The neuron

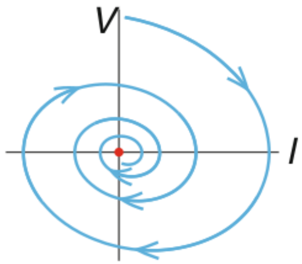
$$I' = V$$
$$V' = -I - f(V)$$

Case 1: ohm's law

$I = g \cdot V$



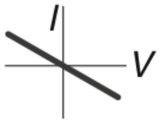
electrical	mechanical
$I' = V$	$X' = V$
$V' = -I - gV$	$V' = -X - kV$



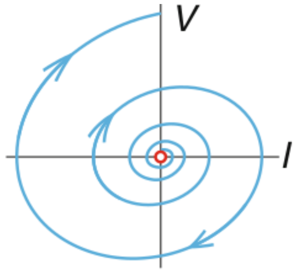
4. The neuron

$$I' = V$$
$$V' = -I - f(V)$$

Case 2: negative resistance $I = -g \cdot V$



electrical	mechanical
$I' = V$	$X' = V$
$V' = -I + gV$	$V' = -X + kV$

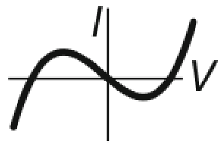


4. The neuron

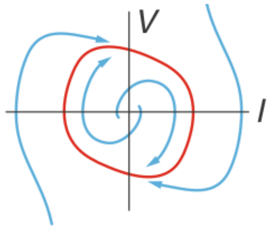
$$I' = V$$

$$V' = -I - f(V)$$

Case 3: N-shaped resistance $I = V^3 - V$

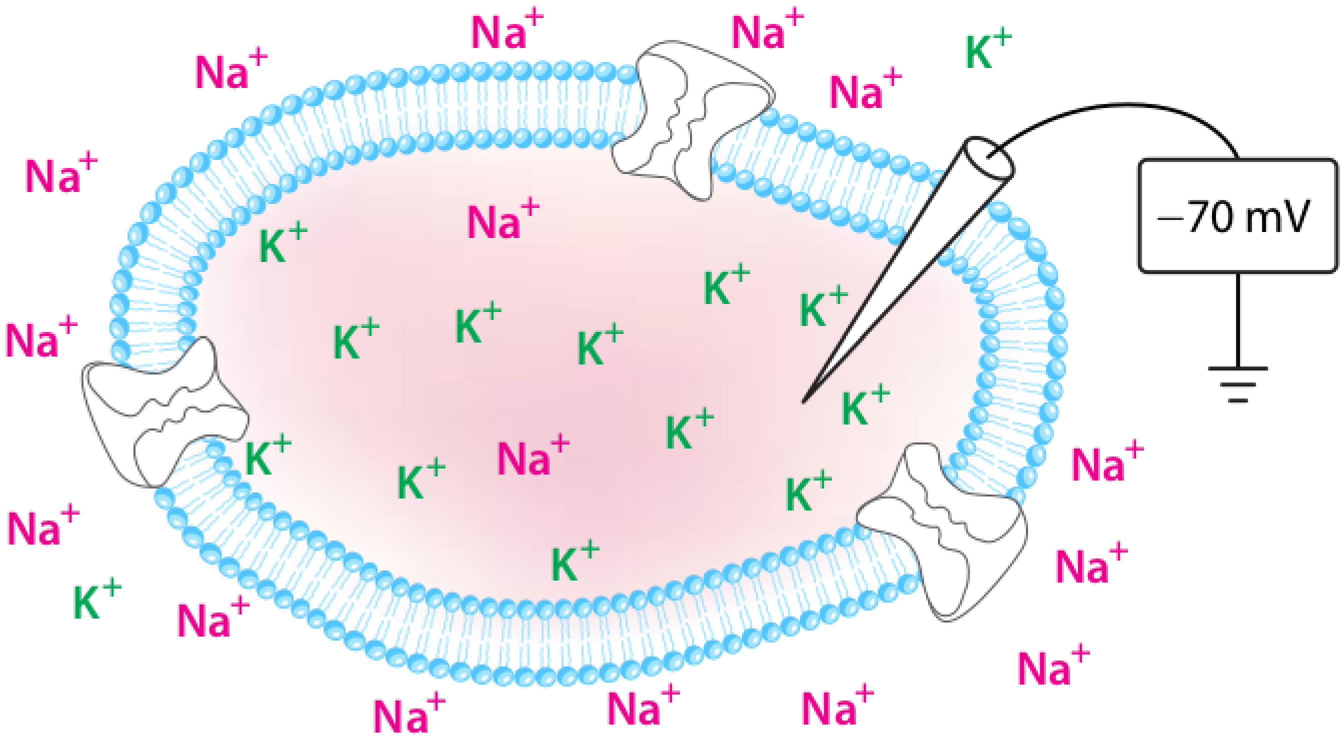


electrical	mechanical
$I' = V$	$X' = V$
$V' = -I - (V^3 - V)$	$V' = -X - (V^3 - V)$



4. The neuron

The membrane: contains channels which can open and close



Outside: sea = many Na

Inside: reverse situation

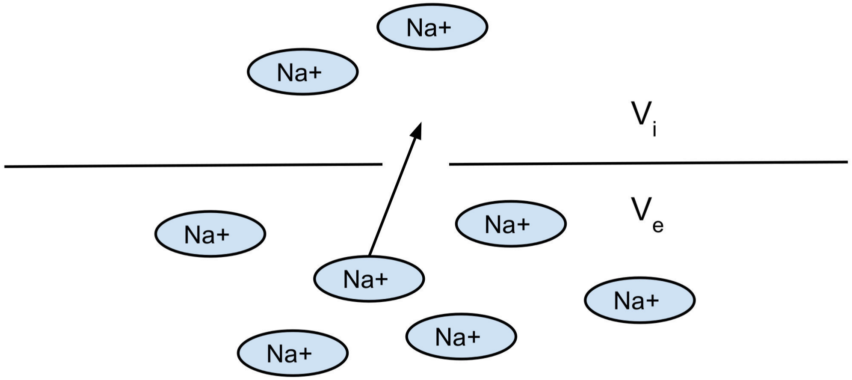
4. The neuron

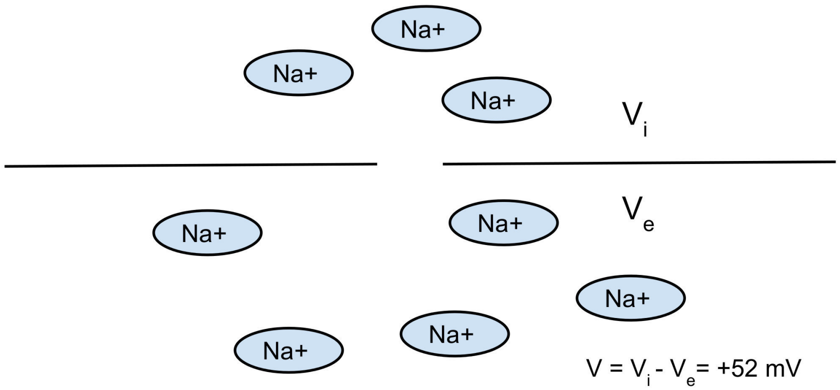
Nerst potential: 2 opposite forces:

- diffusion
- electrical force

You can find the equilibrium which gives you the Nerst potential

$$V = V_i - V_o = -\frac{RT}{zF} \ln \frac{c_i}{c_0}$$





4. The neuron

Nerst potential: 2 opposite forces:

- diffusion
- electrical force

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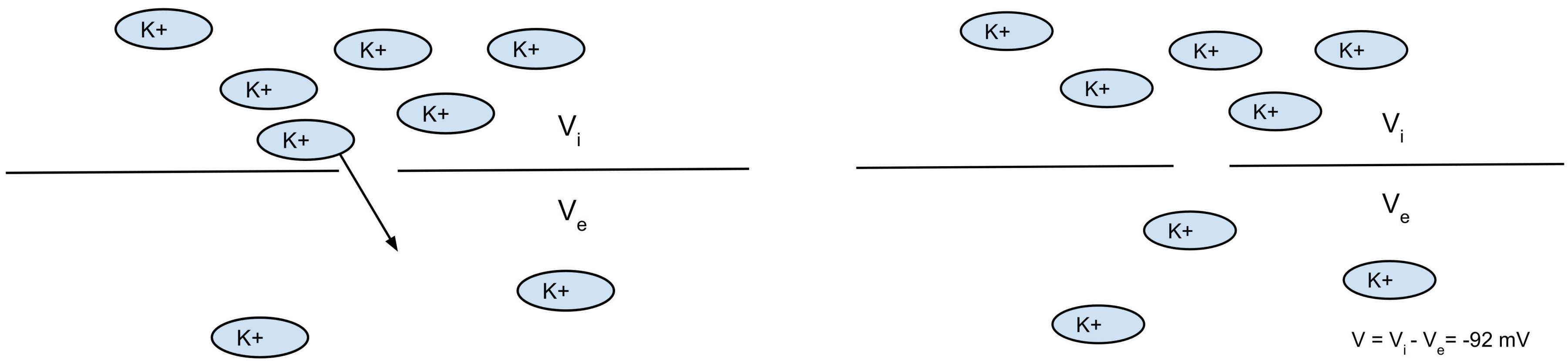
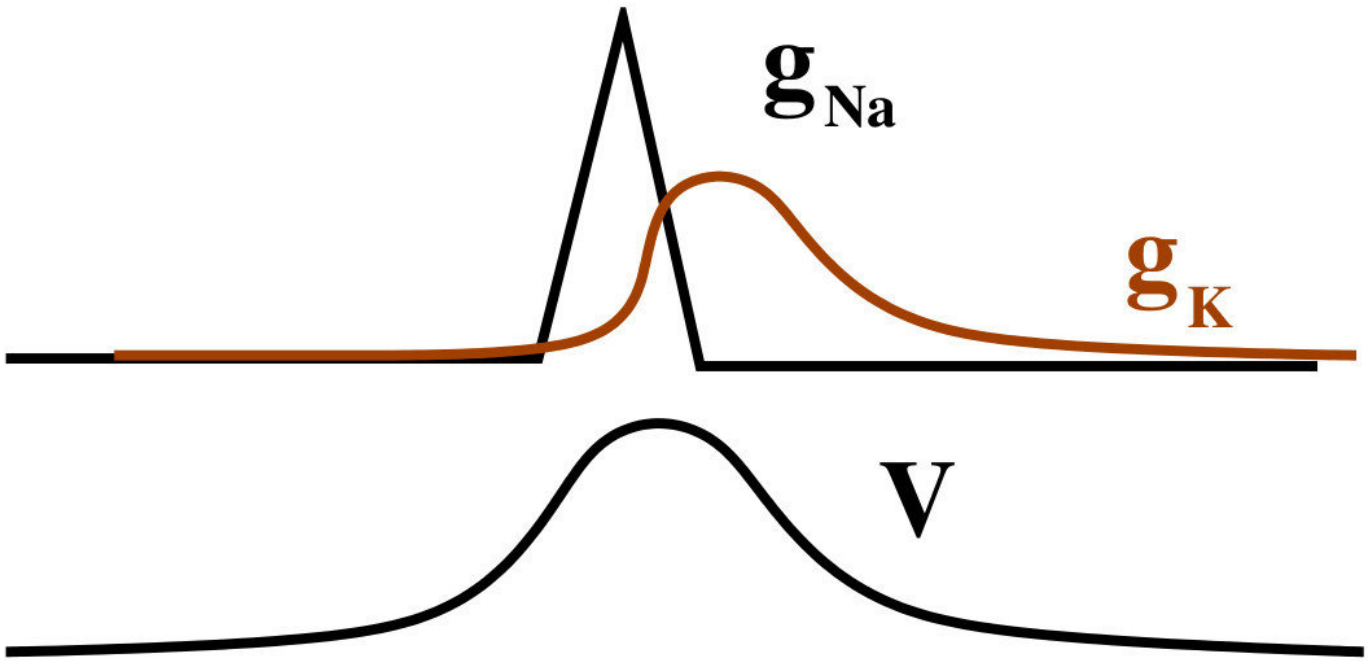
$$V = V_i - V_o = -\frac{RT}{zF} \ln \frac{c_i}{c_0}$$



4. The neuron

When left undisturbed, a cell remains stable at −70 mV

When giving a stimulating currents, it responded with a much larger action and then a return to the resting state.



4. The neuron

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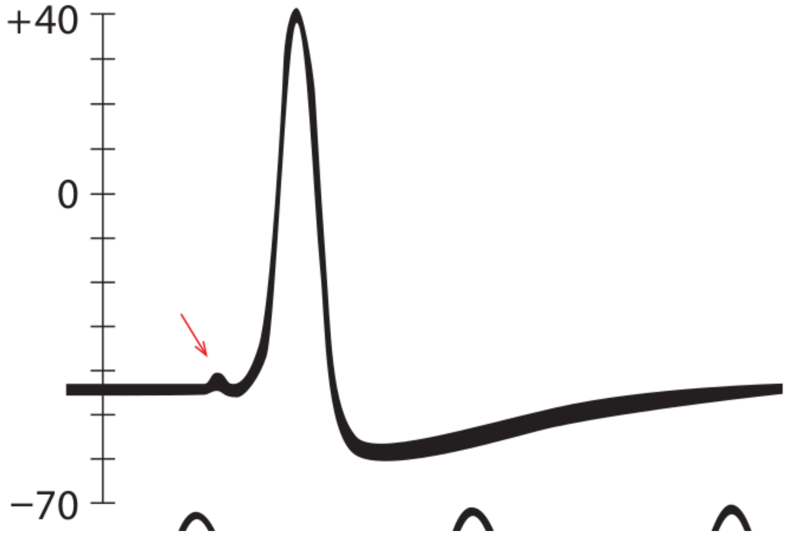
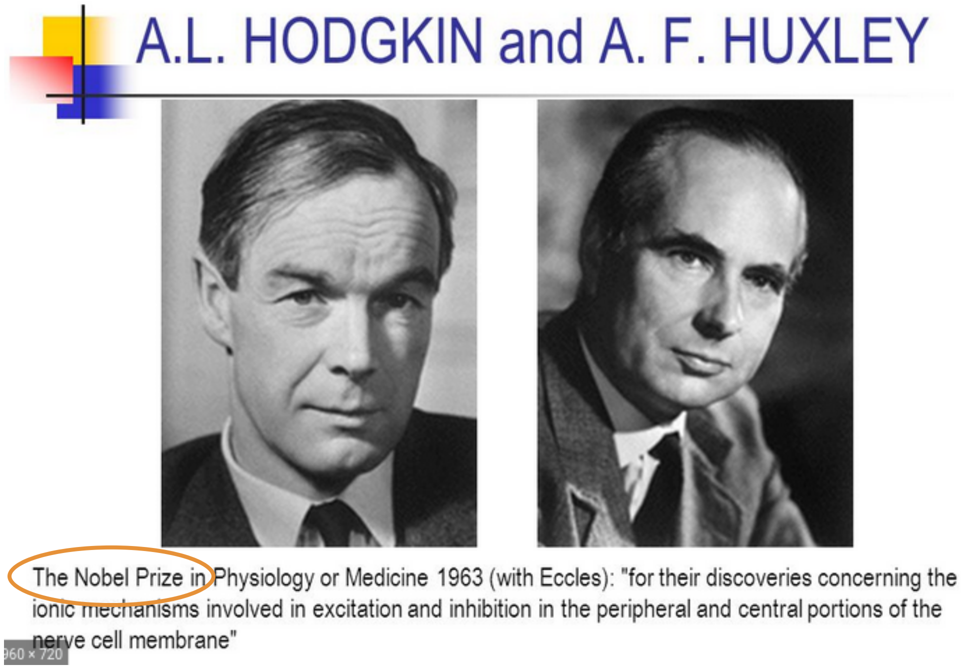


- Action potential is the key of signaling of the neuron
- Basis of neural communication

4. The neuron

- Hodgkin and Huxley developed a set of hypotheses about how the action potential is generated
- No knowledge about ion channels!
- Developed a 4 variable equation = Hodgkin - Huxley equations → computed by hand using a mechanical calculator
- Found the action potential!



4. The neuron

We will develop a two-variable simplification of the Hodgkin - Huxley model that captures the essential dynamics, called the **FitzHugh - Nagumo** (FHN) **model**.

4. The neuron

Fast inward process

- Experiment: fast inward process → sodium-dependent. Removing sodium from the bath water abolished the action potential
- Therefore, the f (V) → Sodium

$$\boxed{\begin{aligned} I' &= V \\ V' &= -I - f(V) \end{aligned}}$$

- Very tiny stimulus current to cell → no action potential
- Stronger stimulus → large response of the action potential

→ **equilibrium point of this system must be stable**.

4. The neuron

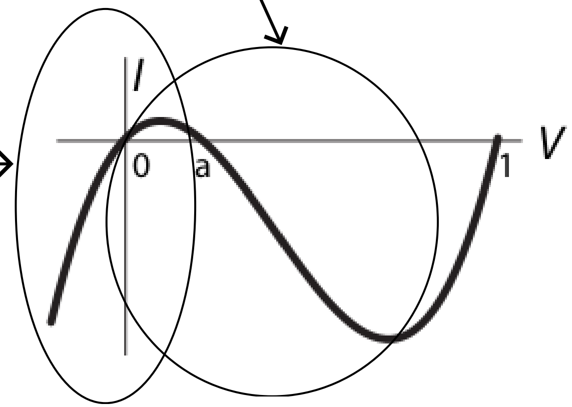
- **equilibrium point of this system must be stable**

$$\boxed{\begin{aligned} I' &= V \\ V' &= -I - f(V) \end{aligned}}$$

- Action potential starts → positive feedback mechanism: Na+ entry into the cell → V ↑ → Na + entry ↑↑

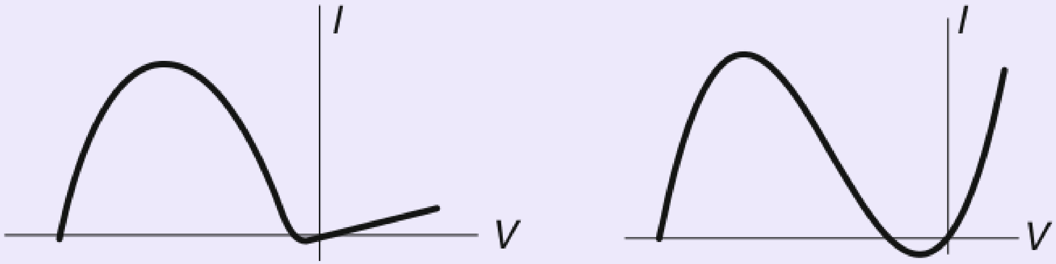
= negative resistance

$$f(V) = V(V-1)(V-a) \qquad \text{with } 0 < a < 1$$



4. The neuron

Hodgkin and Huxley experimentally recorded the I/V curve of the squid neuron, and found that it had exactly such a negatively sloped region.



On the left is the I/V curve of the squid axon, recorded by Hodgkin and Huxley. On the right is the function $f(V)$ we use to model this process. Here we have plotted $f(-V)$, since in their day, what was meant by V is now what we call $-V$.

4. The neuron

$$I' = V$$
$$V' = -I - f(V)$$

$$f(V) = V(V-1)(V-a) \qquad \text{with } 0 < a < 1$$

$$\Downarrow$$

$$V' = -I + f(V)$$

$$f(V) = V(1-V)(V-a) \qquad \text{with } 0 < a < 1$$

$$\Downarrow$$

speed

$$V' = \frac{1}{\epsilon}\Big(-I + f(V)\Big) \qquad \epsilon = 0.01$$

$$f(V) = V(1-V)(V-a) \qquad \text{with } 0 < a < 1$$

4. The neuron

$$V' = \frac{1}{\epsilon}\Big(-I + f(V)\Big) \qquad \epsilon = 0.01$$

$$f(V) = V(1-V)(V-a) \qquad \text{with } 0 < a < 1$$

consider $V' = f(V)$ → logistic equation with allee effect.

3 equilibrium points, $V = 0$, $V = a$, $V = 1$.

- $V = 0$; $V = 1$ are stable,
- $V = a$ is unstable

$V < a \rightarrow V' < 0$: system $\rightarrow V = 0$

$V > a \rightarrow V' > 0$: system $\rightarrow V = 1$

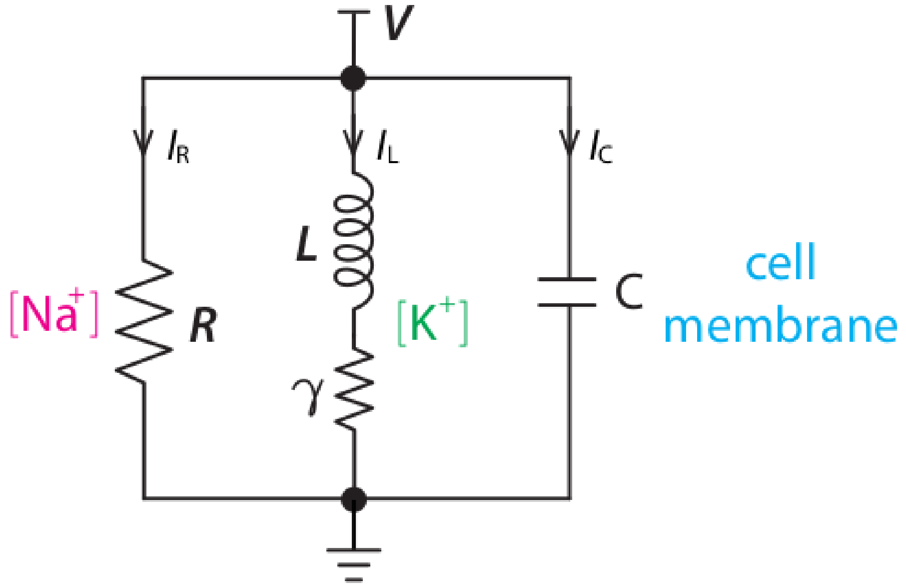
The fast inward dynamics inherits this threshold behavior from the Allee-like character of the resistance curve.

4. The neuron

$$V' = \frac{1}{\epsilon}\Big(-I + f(V)\Big) \qquad \epsilon = 0.01$$

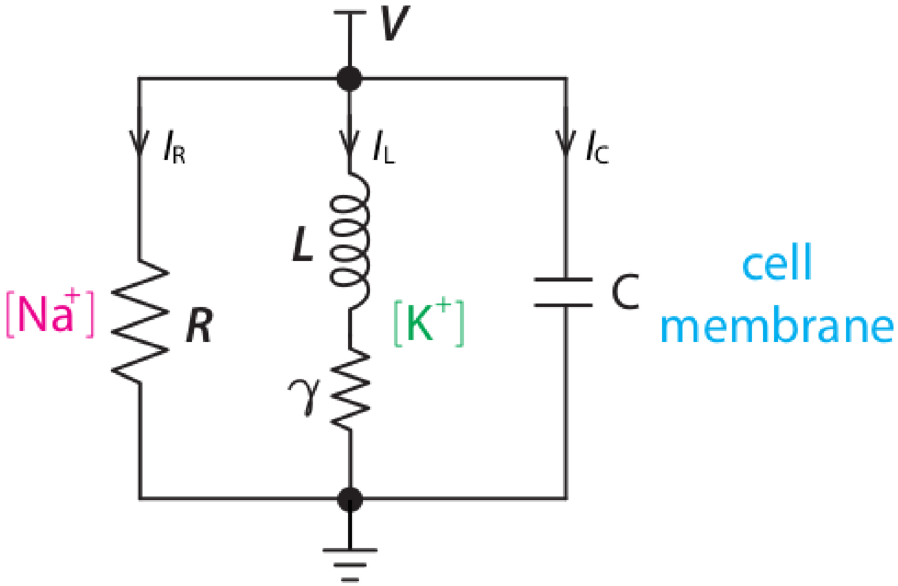
$$f(V) = V(1-V)(V-a) \qquad \text{with } 0 < a < 1$$

The recovery process is dominated by the flow of K+ ions



$$I' = V - \gamma I$$

4. The neuron

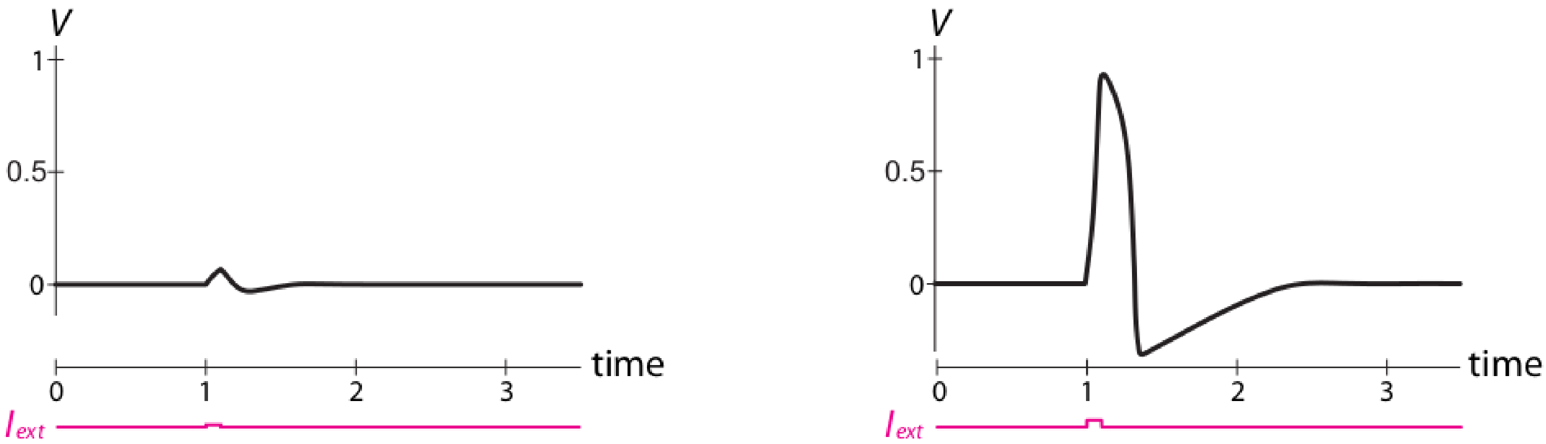


$$V' = \frac{1}{\epsilon}\Big(-w + f(V) + I_{ext}\Big) \qquad f(V) = V(1-V)(V-a)$$

$$w' = V - \gamma w \qquad \epsilon = 0.01 \text{ with } 0 < a < 1$$

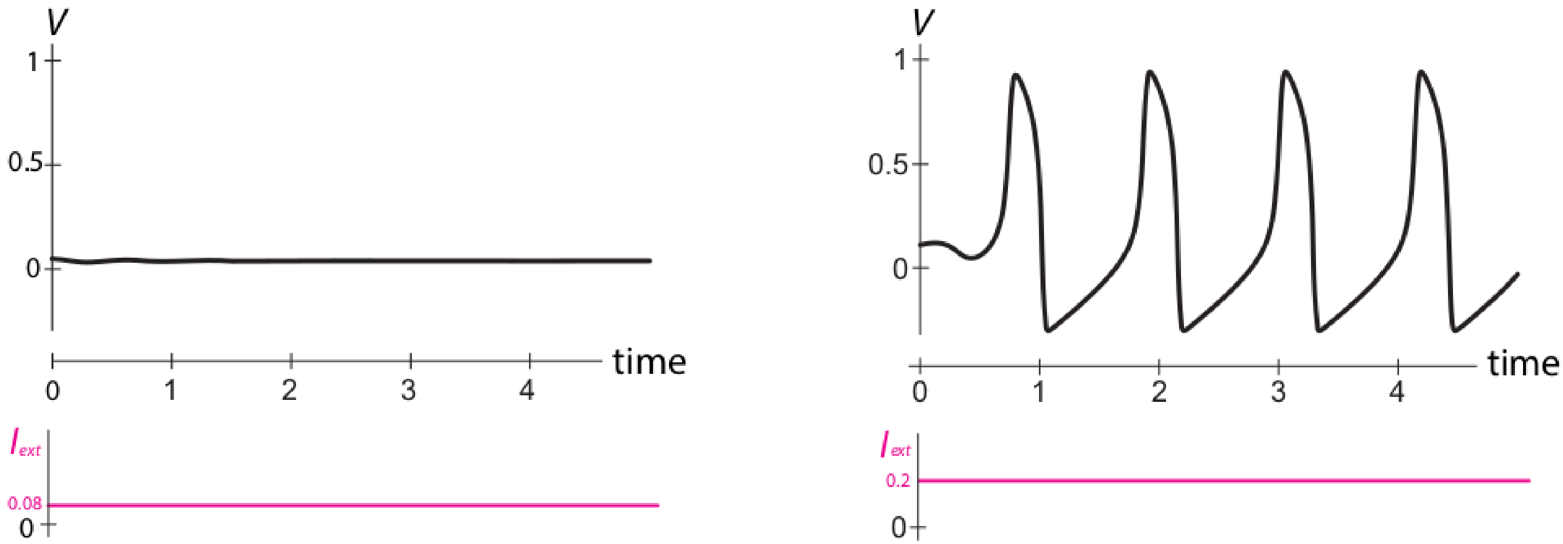
4. The neuron

Give a small stimulus to the cell, and a bit of a larger stimulus to the cell, see what happens



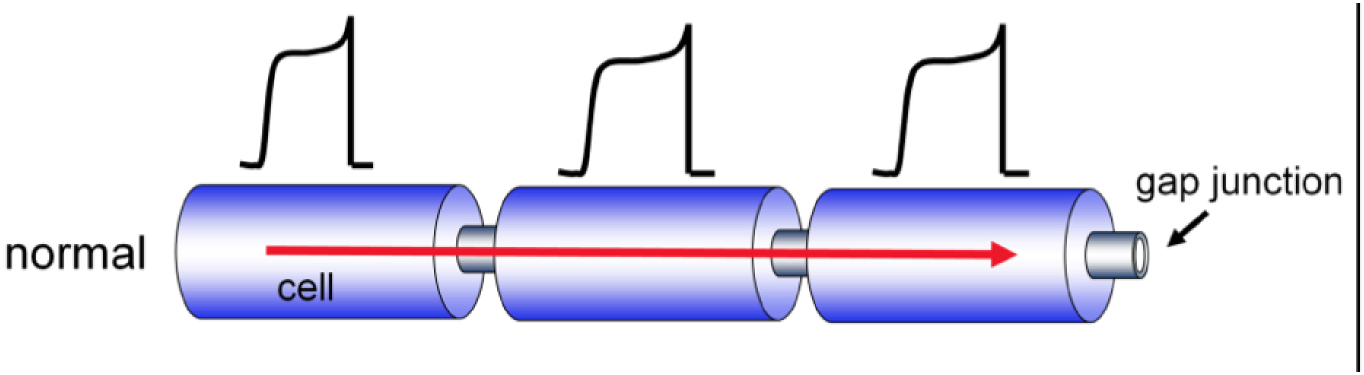
4. The neuron

Give a small constant stimulus to the cell, and a bit of a larger constant stimulus to the cell, see what happens



Exercise 6.1

4. The neuron



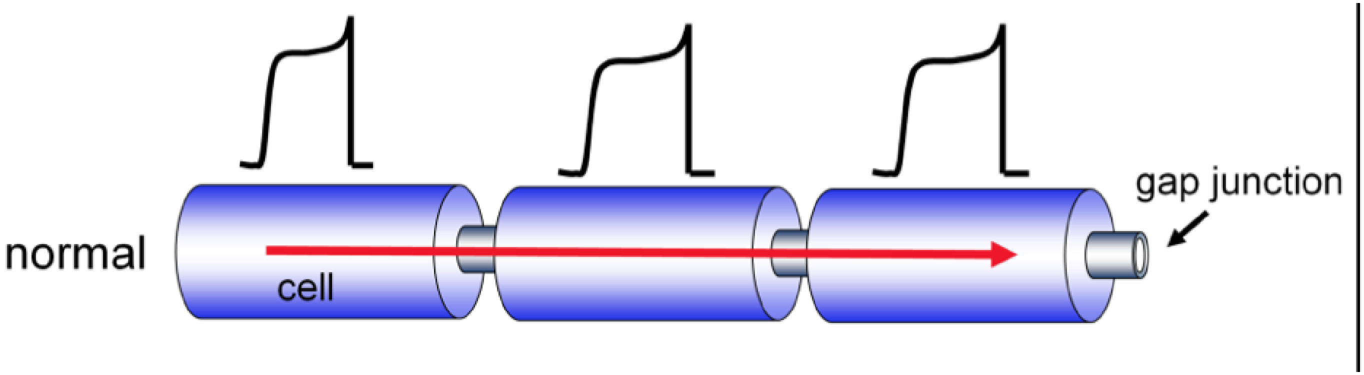
Couple 2 neurons together!

Gap junction = resistor R:

$$I_{coupling\ 2\to 1} = \frac{(V_2 - V_1)}{R}$$

$$I_{coupling\ 1\to 2} = \frac{(V_1 - V_2)}{R}$$

4. The neuron



$$I_{coupling\ 2\rightarrow 1} = \frac{(V_2 - V_1)}{R} \qquad I_{coupling\ 1\rightarrow 2} = \frac{(V_1 - V_2)}{R}$$

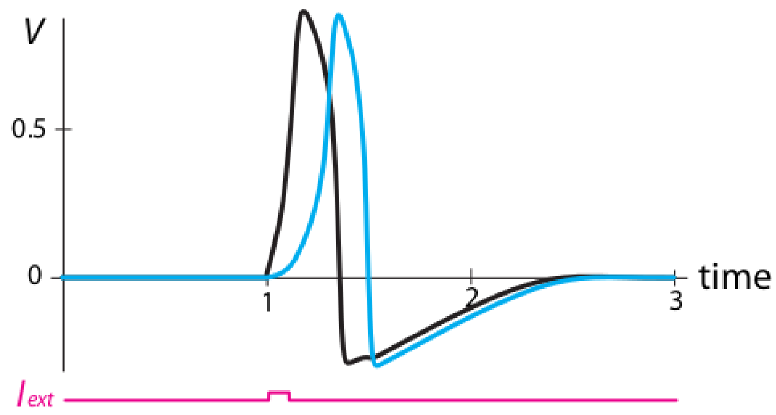
$$V_1' = \frac{1}{\epsilon}\Big(- w_1 + f(V_1) + I_{coupling\ 2\rightarrow 1} + I_{ext}\Big)$$
$$w_1' = V_1 - \gamma w_1$$
$$V_2' = \frac{1}{\epsilon}\Big(- w_2 + f(V_2) + I_{coupling\ 1\rightarrow 2}\Big)$$
$$w_2' = V_2 - \gamma w_2$$

4. The neuron

$$I_{coupling\ 2\to1} = \frac{(V_2 - V_1)}{R}$$

$$I_{coupling\ 1\to2} = \frac{(V_1 - V_2)}{R}$$

$$V_1' = \frac{1}{\epsilon}\Big(- w_1 + f(V_1) + I_{coupling\ 2\to1} + I_{ext}\Big)$$
$$w_1' = V_1 - \gamma w_1$$
$$V_2' = \frac{1}{\epsilon}\Big(- w_2 + f(V_2) + I_{coupling\ 1\to2}\Big)$$
$$w_2' = V_2 - \gamma w_2$$



Exercise 6.2

4. The neuron

Dynamics of the FHN model

V-nullcline:

$$V' = 0 = \frac{1}{\epsilon}\Big(- w + f(V)\Big)$$

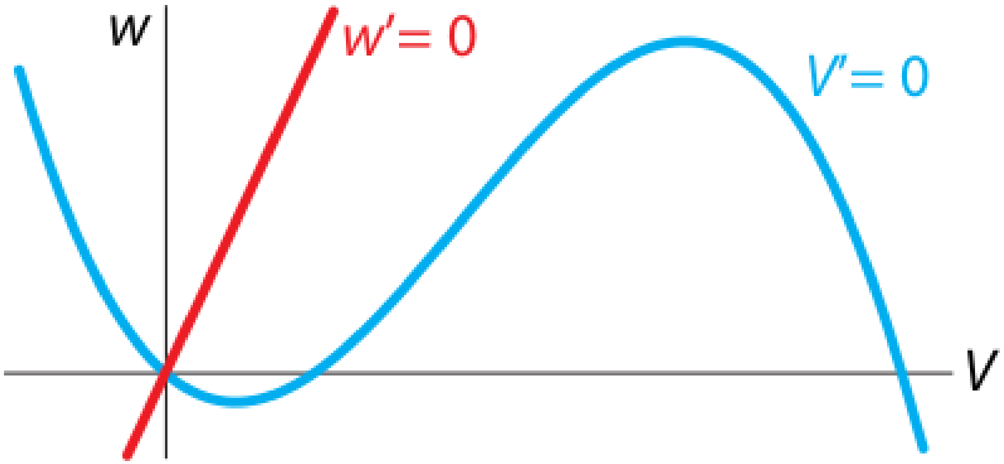
$$w = V(1 - V)(V - a)$$

w-nullcline:

$$w = \frac{1}{\gamma} V$$

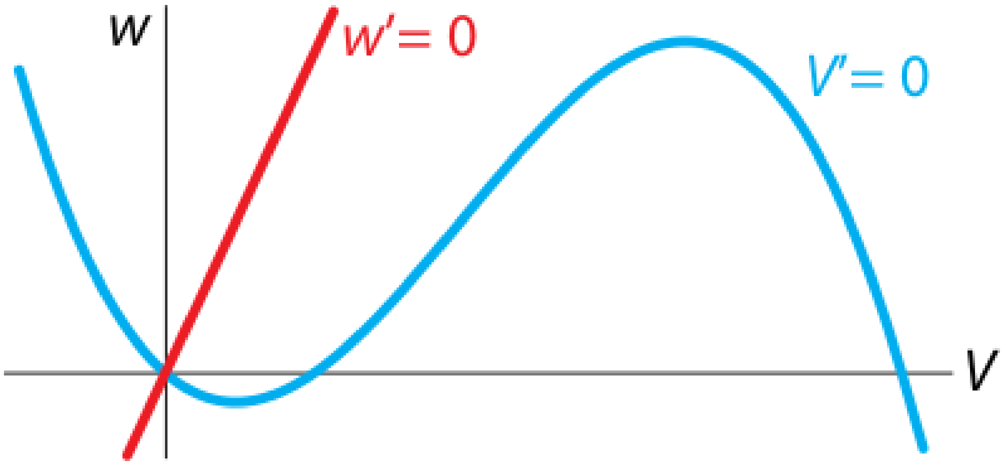
Exercise 6.3

Dynamics of the FHN model



What type of equilibrium do you have?

Dynamics of the FHN model



Draw the change vector in each section and check manually

Draw a few trajectories

Exercise 6.4