7.2 Discrete time

One of the simplest examples of chaotic behavior is a difference equation called the discrete logistic equation:

$$X_{N+1} = rX_N(1 - X_N). \tag{44}$$

This is similar to the continuous-time logistic equation with k = 1, but makes some important changes. In the discrete logistic model, time can only take on integer values: 0,1,2,... . Also, in this model, we assume that each generation dies off completely and only their offspring survive to the next time unit. Population sizes in the model range between 0 and 1.

1. What is the maximum value for r, so values of this function stay between 0 and 1?
2. Make a time series for $r = 4$, starting from value $X_0 = 0.01$. You should be able to reproduce Figure 5.4.
3. Write a program to create your own cobweb where you can chose the number of iterations, you can set r to a chosen number, and you can chose your X_0
4. Extend your program, so it also shows the time series of a chosen X_0, next to the cobweb.
5. We will now investigate this system in detail. We start with $0 < r < 1$. Which equilibria do you have? Are they stable or unstable?
6. What happens when you pass r = 1? What type of behavior do the equilibria have for $1 < r < 2$? Which type of bifurcation is this? What happens for $2 < r < 3$?
7. What happens when you pass $r = 3$? Set e.g. $r = 3.1$ and observe what happens.
8. Now set $r = 3.53$. What do you observe now?
9. To understand the long-term behavior and the bifurcation plots, we will create Figure 5.22 from the course. Earlier in this course, we used bifurcation plots to look at how the equilibria of a system changed as we varied a parameter. However, bifurcation plots don't have to just display equilibria – they can display any kind of long-term behavior. For example, we might run a simulation, throw away the first 100 time steps (or more, if necessary) and plot the rest. This shows us how the system's behavior varies with a parameter.
10. Also look at a smaller window: $[3.82, 3.86]$. What do you see, crazy right?
11. One of the hallmarks of chaos is sensitive dependence on initial conditions. This means that time series starting very close together will eventually diverge to the point of being completely uncorrelated. Iterate the discrete logistic 20 times for r = 3.9 and an initial state of X = 0.4. Plot the time series. Then, do the same thing using 0.401 for the initial state. Overlay the plots and observe the relationship between the time series. Also plot the difference between both series.