Chaos

week 11

4. Chaos versus randomness

4. Chaos versus randomness

In a **dynamical system** with differential equations or discrete-time system → easy to diagnose chaos

(1) Is it behaving irregularly? Yes.
(2) Is there any random input into the system? No.

→ Then it is chaotic

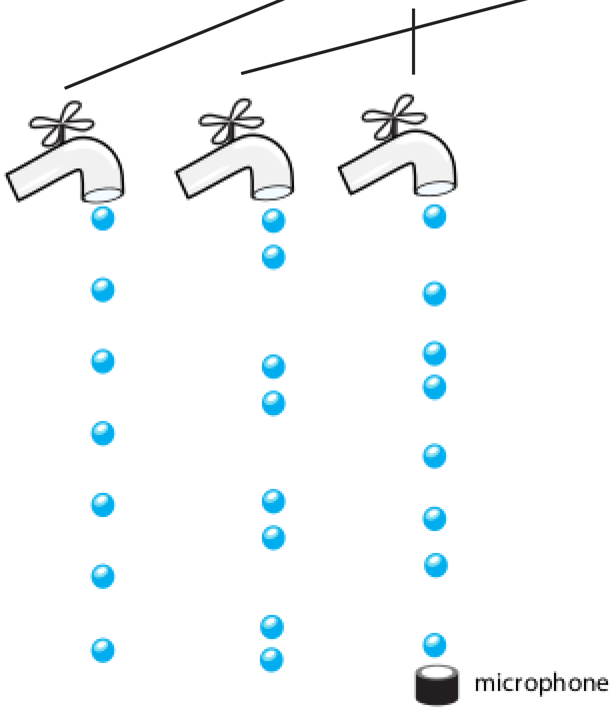
But what about real systems?

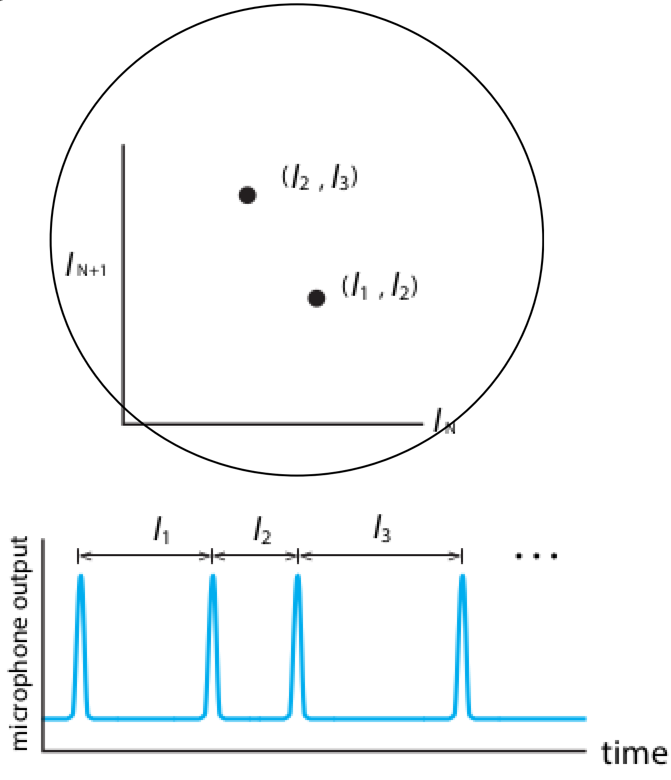
How can we differentiate between randomness and chaos?

Exercise 7.3.1

4. Chaos versus randomness - dripping faucet

- At slow dripping rate: faucets drip regularly: drip-drip-drip
- Make it a bit faster: drip-drip .... drip-drip = period 2 doubling!
- You can get more complex behavior (period 4)
- Eventually you get chaos



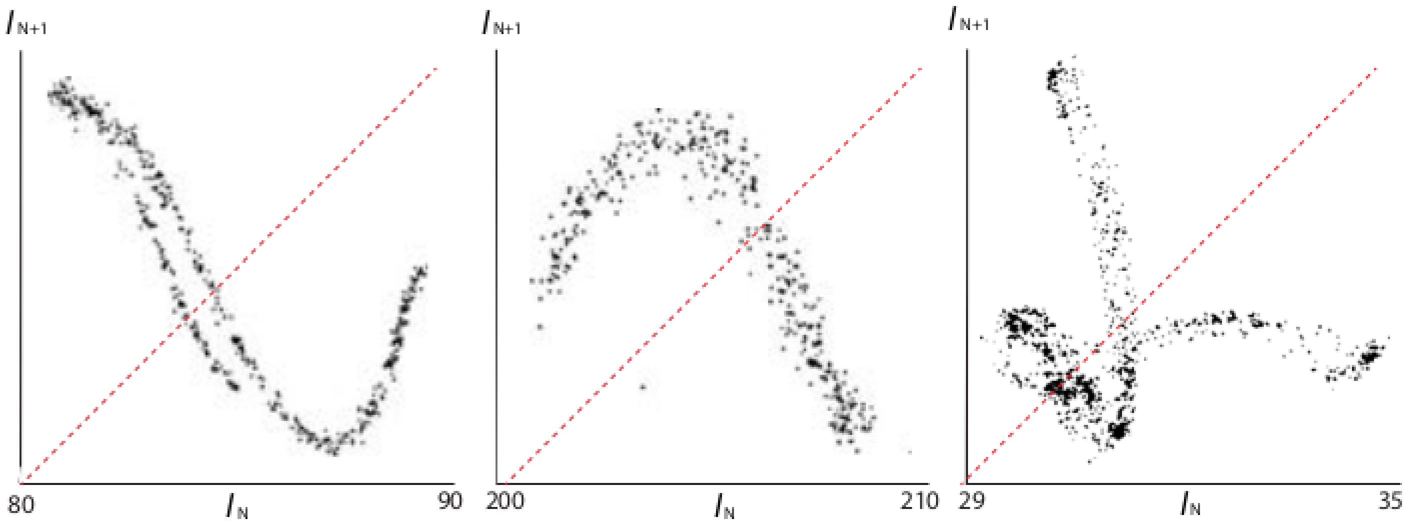


Study by Rob Shaw - Santa Cruz

4. Chaos versus randomness - dripping faucet

Set I_{N+1} out against I_N = Pointcare plot

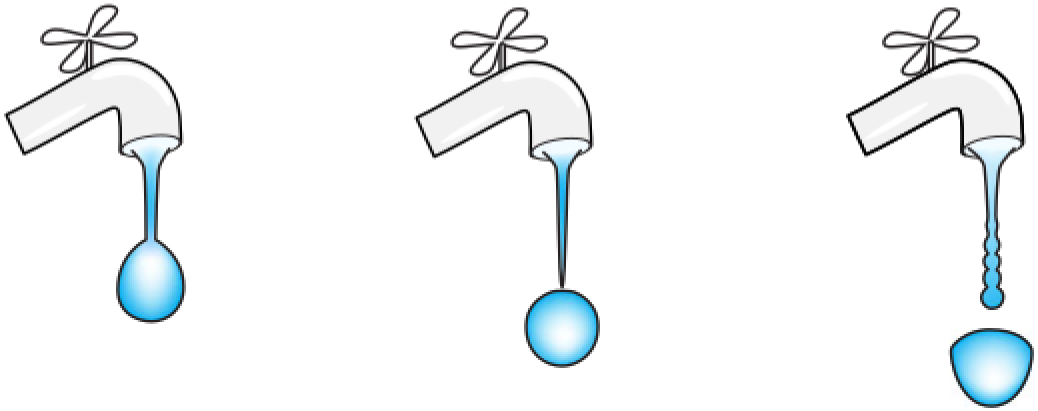
chaos



the slope of the “function” at the intersection is steeper than -1, which is the requirement for an unstable equilibrium point.

4. Chaos versus randomness - dripping faucet

Why?

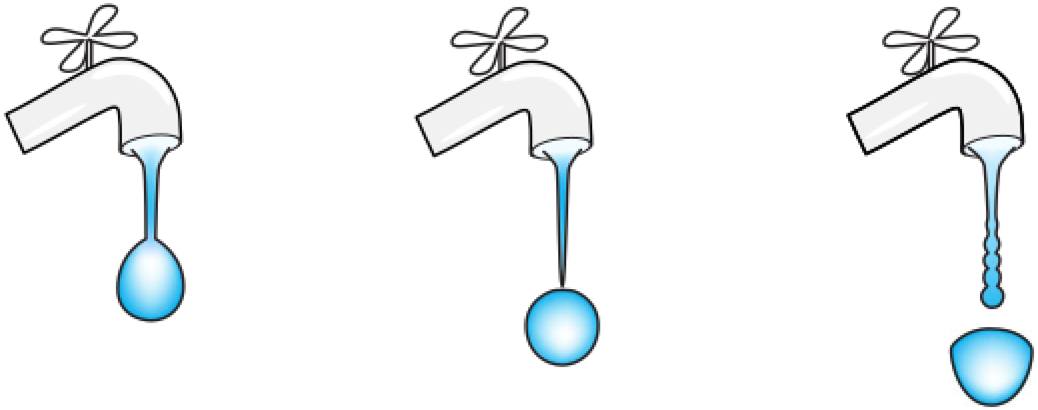


A simple and general mechanism for generating chaos:

- When a drop forms at the mouth of the faucet, it begins to balloon outward and downward due to its growing mass. The descending droplet pinches in, and then the neck separates and the detached drop falls downward
Slow dripping ≈ 1 sec
- When the drop separates, there is a small undropped part that snaps back (due to surface tension) and gives a small elastic oscillation as it retracts ≈ **0.1 sec**

4. Chaos versus randomness - dripping faucet

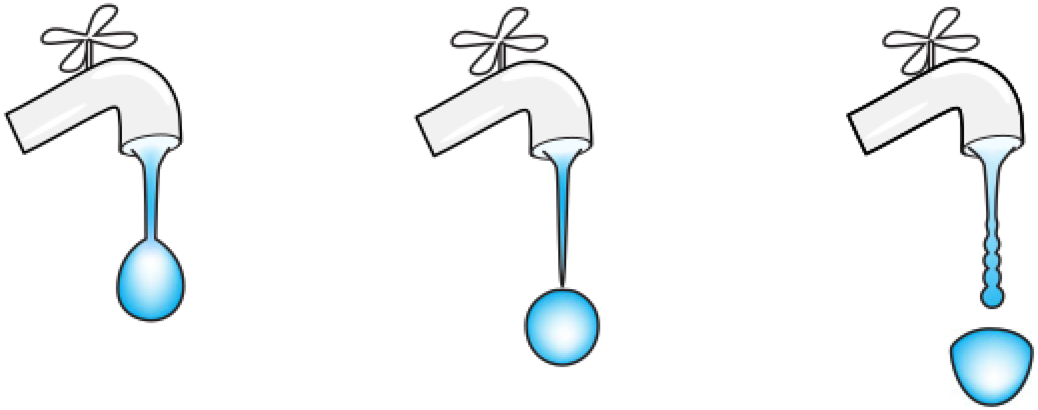
Why?



- Low flow rates: two processes do not interact
- Faster dripping rates:
  the (N +1)st drop begins to separate, the system has not fully recovered from the Nth drop.
  A: if the little oscillation in the undropped recoiling part is in its downward phase when the next drop is near separation, that slightly retards the separation,
  B: whereas if it is in its upward movement, separation comes faster.

4. Chaos versus randomness - dripping faucet

Why?



If you have an action phase and a recovery phase:

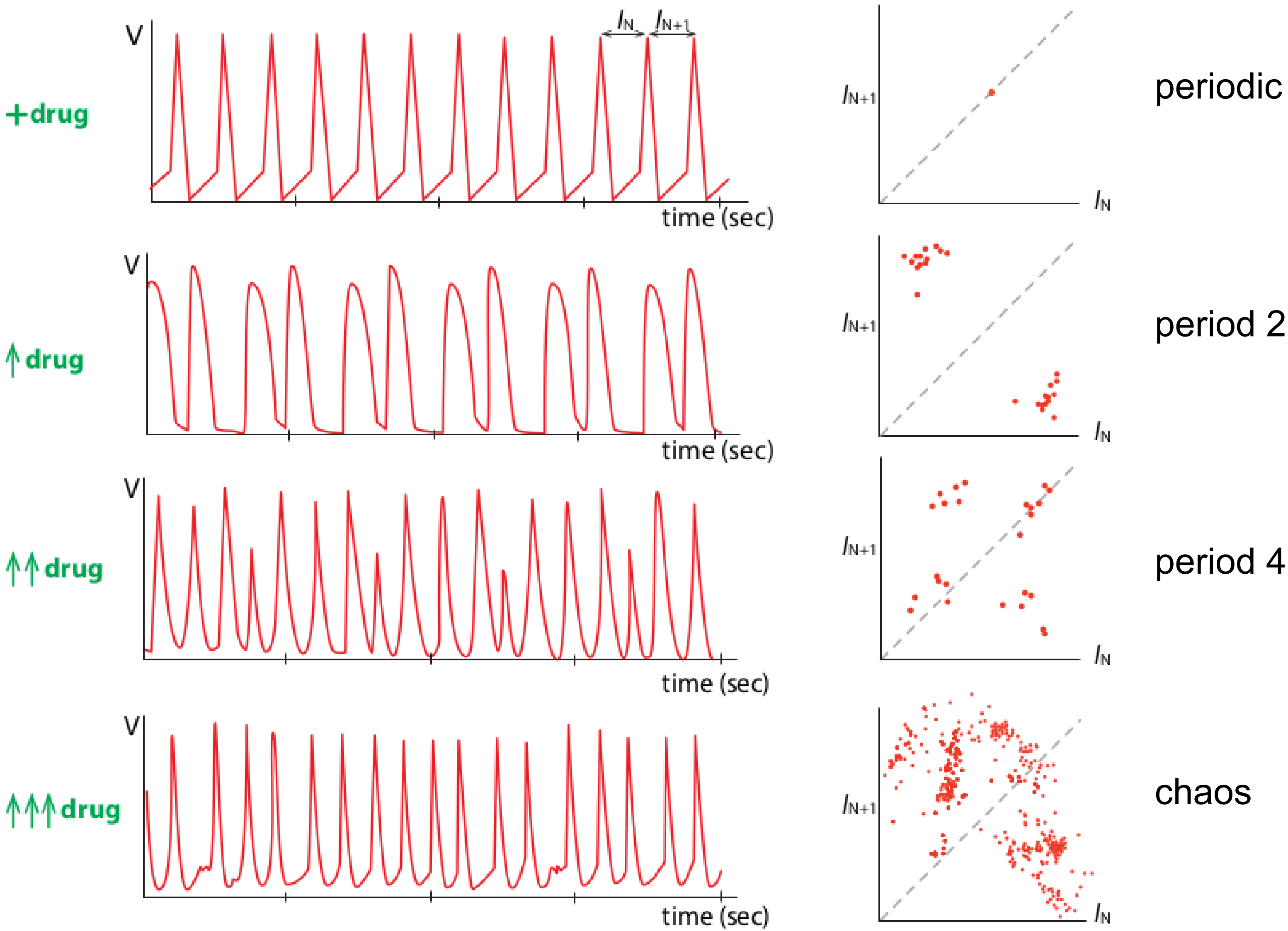
- At slow rate wrt to recovery phase:
- At fast rate: you will get complex behavior if the action phase is in the region of the recovery phase!

4. Chaos versus randomness - cardiac arrhythmia

In a piece of heart tissue:

drug was added and the rhythm of the beats was measured

4. Chaos versus randomness - cardiac arrhythmia



4. Chaos versus randomness

There are 3 different ways to diagnose chaos in a continuous-time series, by extracting a discrete-time series X1 , X2 , X3 , . . . from the continuous data and then plotting X_N+1 against X_N in a Poincaré plot.

(1) plotting the duration of the (N + 1)st active phase against the duration of the Nth active phase

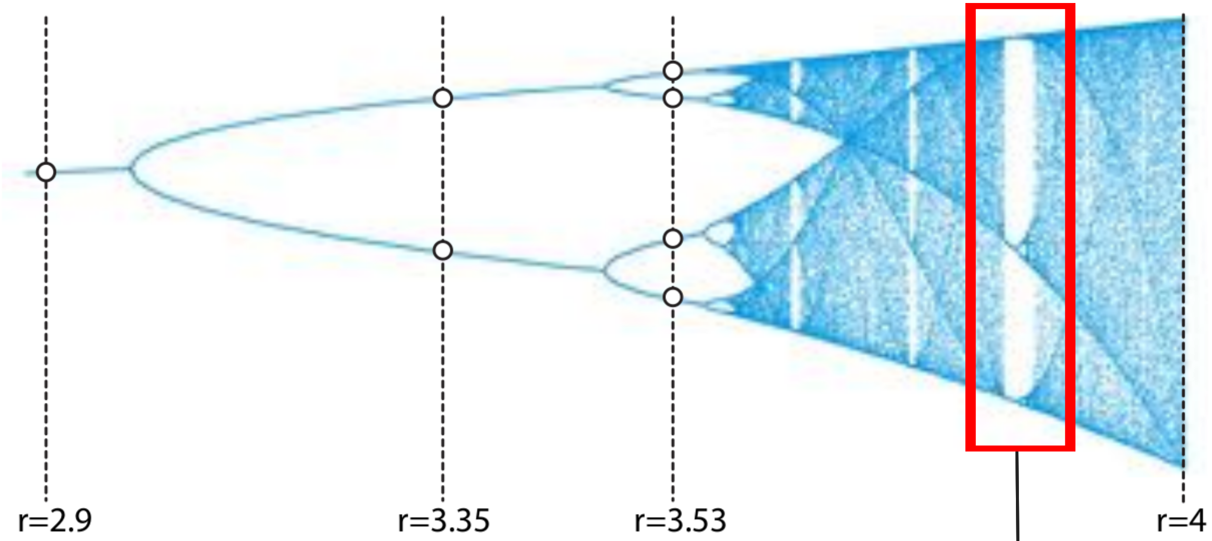
(2) plotting the maximum amplitude of the (N + 1)st phase against the amplitude of the Nth phase

(3) stroboscopic plot in which we take the value of the variable at times t = 1, 2, 3, . . . and then plot the value at time t + 1 against the value at time t

Exercise 7.3.2

5. Fractals

5. Fractals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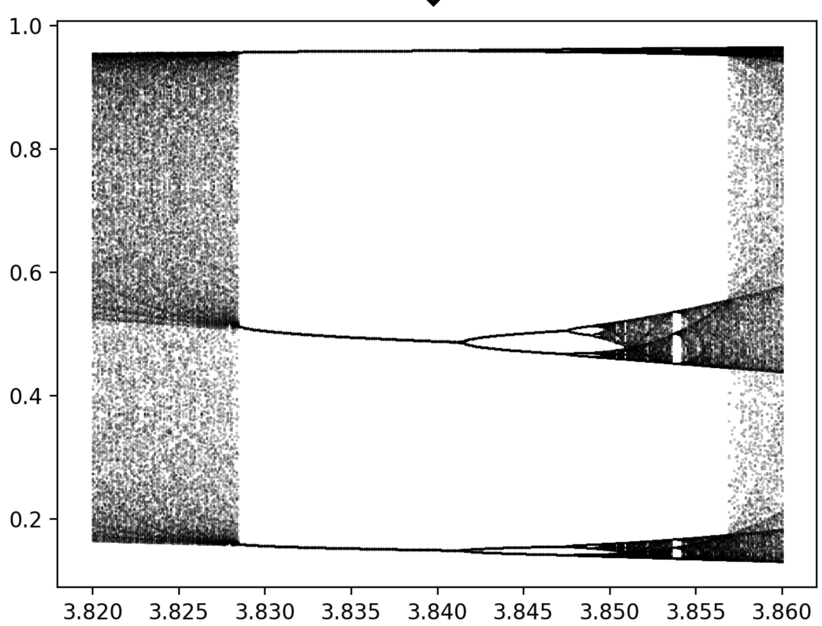
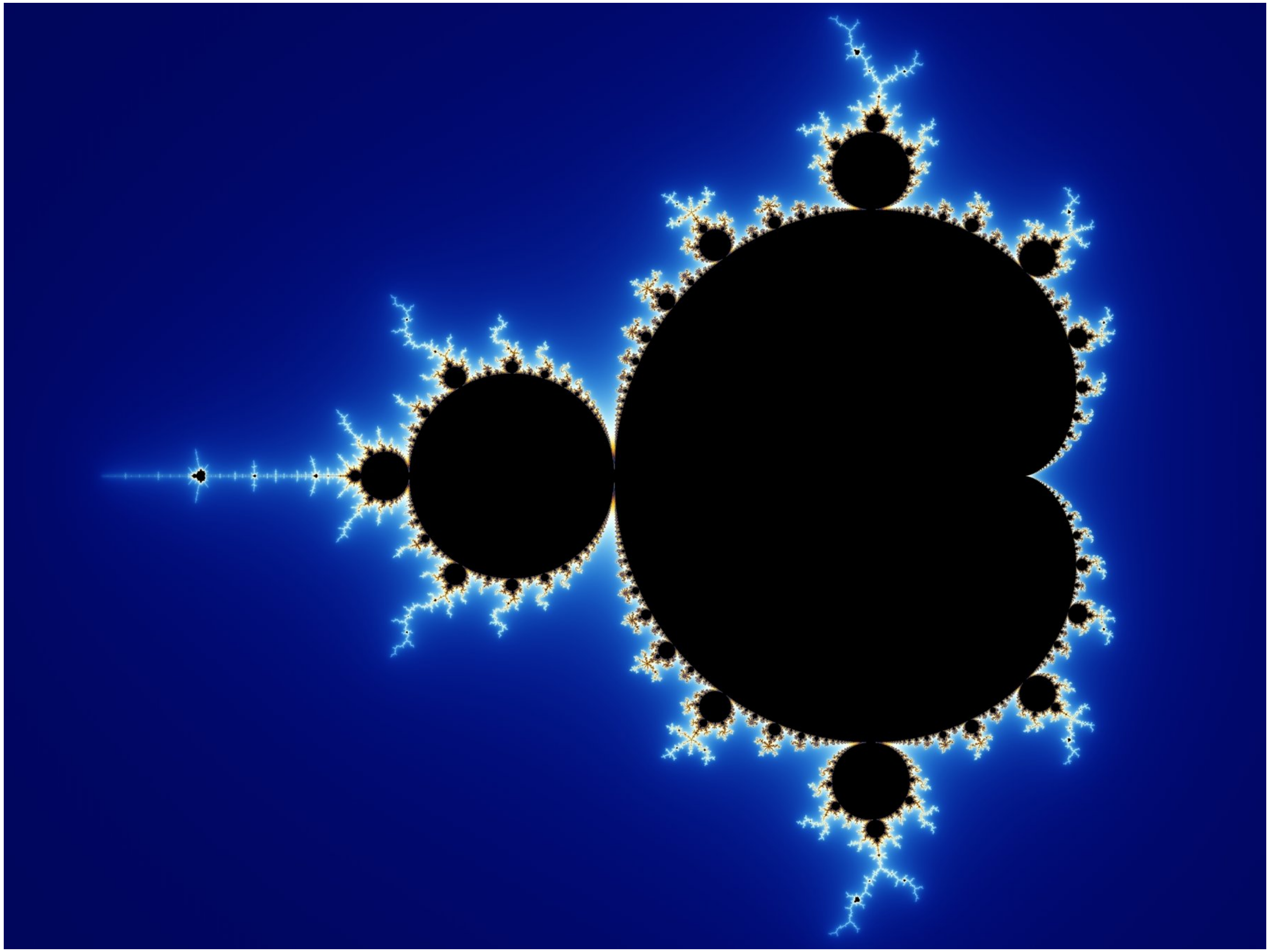
r=3.53
r=4


1.0
0.8
0.6
0.4
0.2
3.820
3.825
3.830
3.835
3.840
3.845
3.850
3.855
3.860

5. Fractals

Mandelbrot set



5. Fractals

Mandelbrot set

https://www.youtube.com/embed/NGMRB4O922I?enablejsapi=1

Exercise 7.4