# Biophysics Exam January

Prof. Nele Vandersickel - Sander Hendrickx

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## 1 Degradation of Glucose (6/20)

Consider the following model which represents a simple model for the degradation of glucose:

$$\begin{aligned} X' &= -X + aY + X^2Y \\ Y' &= b - aY - X^2Y \end{aligned} \tag{1}$$

with $a, b > 0$.

1. Start by taking $a = 0.06$ and $b = 0.4$. Plot the nullclines of the system, add the change vectors in the different segments including on top of the nullclines. To guide you, set $X \in [0, 4], Y \in [0, 7]$.
2. Determine the position(s) of the equilibrium (equilibria), and the type of equilibrium (equilibria) you have.
3. Now plot a few trajectories. Close to the equilibrium (equilibria) and further away (closer to the edge the limits we gave in point 1). What do you notice?
4. Now you have to vary $b \in [0, 1.2]$. Find the bifurcations in the system and explain the type of bifurcation.
5. As a final step, you also have to vary $a \in [0, 0.14]$. Make a 2D plot of the stability of your equilibrium (equilibra) and explain the where the bifurcations occur.

## 2 A system with radial symmetry (5/20)

Consider the following radial equation:

$$r' = \mu r + r^3 - r^5 \tag{2}$$

where $\mu$ is the parameter of our system, $\mu$ can be negative as well as positive.

a. Check the type of bifurcation(s) that you can find in this system, when considering this system as a 1D system, with $r$ as your variable.

b. Now consider this system as a two-dimensional system, with $r$ the radius of your system. How does your previous bifurcation translate in 2D, whereby you now consider $(x, y)$ instead of $r$. Think about complete and creative ways to present your solutions. Chose an initial $\omega = \dot{\theta} \neq 0$.

c. Explain what is going on in your system in detail for the different regions that you find.

## 3 The Rossler system (9/20)

The simplest three-dimensional non-linear system of differential equations is the Rossler model:

$$\begin{aligned} X' &= -Y - Z \\ Y' &= X + aY \\ Z' &= b + XZ - cZ \end{aligned} \tag{3}$$

Solve the following questions.

(a) Currently we have seen the Euler equation for integrating systems. However, the Runge-Kutta method is much more precise for integrating dynamical systems. Suppose we have the following differential equation:

$$\begin{aligned} X' &= f(X, Y) \\ Y' &= g(X, Y) \end{aligned} \tag{4}$$

Implement the following integration scheme in your equations:

$$\begin{aligned} X_{n+1} &= X_n + \frac{h}{6}(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}) \\ Y_{n+1} &= Y_n + \frac{h}{6}(\ell_{n1} + 2\ell_{n2} + 2\ell_{n3} + \ell_{n4}) \end{aligned} \tag{5}$$

with

$$\begin{aligned} k_{n1} &= f(X_n, Y_n) \\ \ell_{n1} &= g(X_n, Y_n) \\ k_{n2} &= f(X_n + \frac{h}{2}k_{n1}, Y_n + \frac{h}{2}\ell_{n1}) \\ \ell_{n2} &= g(X_n + \frac{h}{2}k_{n1}, Y_n + \frac{h}{2}\ell_{n1}) \\ k_{n3} &= f(X_n + \frac{h}{2}k_{n2}, Y_n + \frac{h}{2}\ell_{n2}) \\ \ell_{n3} &= g(X_n + \frac{h}{2}k_{n2}, Y_n + \frac{h}{2}\ell_{n2}) \\ k_{n4} &= f(X_n + hk_{n3}, Y_n + h\ell_{n3}) \\ \ell_{n4} &= g(X_n + hk_{n3}, Y_n + h\ell_{n3}) \end{aligned} \tag{6}$$

(b) Compare the Euler method with the Runge-Kutta method. Set the time step ($h$ in the equation above) equal to 0.1 and check which time step you should take for the Euler equation to get the same precision. Do this in an elegant way! What do you notice if you also take 0.1 as time step for the Euler equation?

(c) Show the trajectory to chaos (set $a = b = 0.1$ and vary $c$) by making plots of the behavior in 3D. What do you see?

(d) Show that there is in fact chaos and no randomness in this system in 2 different ways.

(e) Make a bifurcation diagram by plotting the maxima of $X$ in function of $b$ and in function of $c$. Explain what you see.