Homework 4

Problem 1:

a) $x_t = [p_x, v_x, p_y, v_y]$ $\quad x_{t+1} = g_t(x_t, u_t) + \epsilon_t$

$z_t = h_t(x_t) + \delta_t$

$x_{t+1} = A x_t + w_t$ ← Gaussian noise

$$\boxed{\begin{bmatrix} P_x[t+1] \\ V_x[t+1] \\ P_y[t+1] \\ V_y[t+1] \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_x[t] \\ V_x[t] \\ P_y[t] \\ V_y[t] \end{bmatrix} + \begin{bmatrix} w_{px} \\ w_{vx} \\ w_{py} \\ w_{vy} \end{bmatrix}}$$

Δt accounts for position change given velocity. Otherwise velocity stays constant

$x_{t+1} \in \mathbb{R}^{4\times1}$ $\quad A \in \mathbb{R}^{4\times4}$ $\quad x_t \in \mathbb{R}^{4\times1}$ $\quad w_t \in \mathbb{R}^{4\times1}$

b) Landmarks: $l_i = [l^i_x, l^i_y]$

Euclidean distance $d = [(p_x - l_x)^2 + (p_y - l_y)^2]^{1/2}$

$$\boxed{\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} ((p_x - l^1_x)^2 + (p_y - l^1_y)^2)^{1/2} \\ ((p_x - l^2_x)^2 + (p_y - l^2_y)^2)^{1/2} \end{bmatrix}}$$

c)

$$J = \begin{bmatrix} \frac{\partial d_1}{\partial p_x} & \frac{\partial d_1}{\partial v_x} & \frac{\partial d_1}{\partial p_y} & \frac{\partial d_1}{\partial v_y} \\ \frac{\partial d_2}{\partial p_x} & \frac{\partial d_2}{\partial v_x} & \frac{\partial d_2}{\partial p_y} & \frac{\partial d_2}{\partial v_y} \end{bmatrix}$$

→ because velocity is constant, $\frac{\partial d_i}{\partial v_i} = 0$

$$\frac{\partial d_1}{\partial p_x} = \cancel{\tfrac{1}{2}}((p_x - l^1_x)^2 + (p_y - l^1_y)^2)^{-1/2} \cdot \cancel{2}(p_x - l^1_x)$$

$$\frac{\partial d_1}{\partial p_y} = [(p_x - l^1_x)^2 + (p_y - l^1_y)^2]^{-1/2} \cdot (p_y - l^1_y)$$

$$\frac{\partial d_2}{\partial p_x} = [(p_x - l^2_x)^2 + (p_y - l^2_y)^2]^{-1/2} \cdot (p_x - l^2_x)$$

$$\frac{\partial d_2}{\partial p_y} = [(p_x - l^2_x)^2 + (p_y - l^2_y)^2]^{-1/2} \cdot (p_y - l^2_y)$$

$$J = \begin{bmatrix} \dfrac{P_x - l^1_x}{((P_x - l^1_x)^2 + (P_y - l^1_y)^2)^{1/2}} & 0 & \dfrac{P_y - l^1_y}{((P_x - l^1_x)^2 + (P_y - l^1_y)^2)^{1/2}} & 0 \\ \dfrac{P_x - l^2_x}{((P_x - l^2_x)^2 + (P_y - l^2_y)^2)^{1/2}} & 0 & \dfrac{P_y - l^2_y}{((P_x - l^2_x)^2 + (P_y - l^2_y)^2)^{1/2}} & 0 \end{bmatrix}$$

d) $\Sigma_{t+1} = G_t \Sigma_t G_t^T + R_t \qquad G_t = \dfrac{\partial g_t}{\partial x}(x, u)$

Problem 3

a) $\gamma(t) = X_0 \exp(t\dot{\Omega}(\dot{\varphi}_l, \dot{\varphi}_r))$

$$\boxed{X_{t_2} = X_{t_1} \cdot \exp\left((t_2 - t_1)\begin{pmatrix} 0, & -\frac{\text{radius}}{\text{width}}\cdot((\dot{\varphi}_l+\varepsilon_l)-(\dot{\varphi}_r+\varepsilon_r)), & \frac{\text{radius}}{2}\cdot((\dot{\varphi}_l+\varepsilon_l)+(\dot{\varphi}_r+\varepsilon_r)) \\ \frac{\text{radius}}{\text{width}}\cdot((\dot{\varphi}_r+\varepsilon_r)-(\dot{\varphi}_l+\varepsilon_l)), & 0, & 0 \\ 0, & 0, & 0 \end{pmatrix}\right)}$$

b) $g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)$ $\quad z_t = l_t + \varepsilon_p \quad \mu = 0$

$$\boxed{p(\varepsilon_p) = \frac{1}{\sigma_p\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(\varepsilon_p)^2}{\sigma_p^2}\right)}$$

$p(z_t | x_t) \sim p(\varepsilon_p)$