904971914_stats102c_hw3

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QUestion 1

(a) [

$$\begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} * \begin{pmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ 0 & a_{22} & a_{32} & a_{42} \\ 0 & 0 & a_{33} & a_{43} \\ 0 & 0 & 0 & a_{44} \end{pmatrix} = \begin{pmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{pmatrix}$$

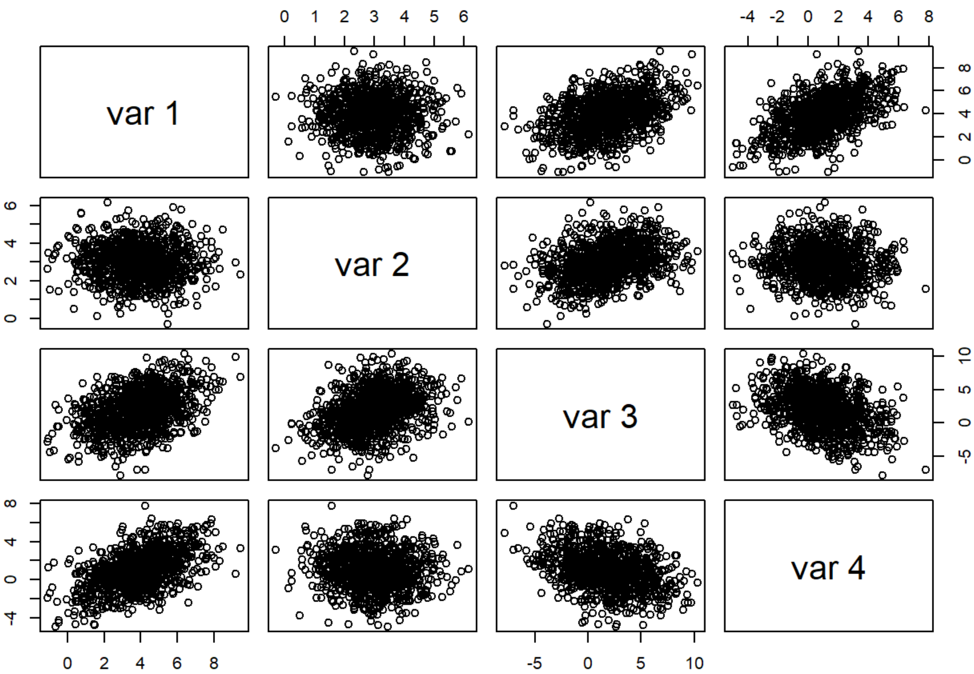
]

```
library(MASS)
set.seed(904971914)
mean1a<-c(4,3,2,1)
sigma<-matrix(c(3,0,2,2,0,1,1,0,2,1,9,-2,2,0,-2,4),4,4)

Z<-mvrnorm(1000,mu=c(0,0,0,0),diag(4))
A<-chol(sigma)

q1a<-matrix(mean1a,1000,4,byrow=T)+Z %*% A
head(q1a)
```

```
##          [,1]     [,2]       [,3]       [,4]
## [1,] 1.411083 1.577421  0.7221179 -2.2063924
## [2,] 1.995770 3.417552  2.7334145 -0.9072104
## [3,] 5.340124 1.618154 -2.3952032  3.9703706
## [4,] 6.617373 3.303557  9.4434047 -2.4225448
## [5,] 5.208262 1.557922 -1.2601343  3.5464972
## [6,] 4.384869 4.962526  4.3684444  2.1400898
```

(b)

```
pairs(q1a)
```



Yes, the plots are consistent

with the covariance matrix.

Question 2

(a) [

$$
\begin{aligned}
X &= \mu + Z * A \\
Z &= (X - \mu) * A^{-1}
\end{aligned}
$$

]

(b)

```
q1b<-function(m,mean_single,sigma){
  A<-chol(sigma)
  A_inverse<-solve(A)
  mean_expanded<-matrix(mean_single,nrow=nrow(m),ncol=ncol(m),byrow=T)
  return ((m-mean_expanded) %*% A_inverse)

}
```

(c)

```
q1c<-q1b(q1a,c(4,3,2,1),sigma)
mean(q1c[,1])
```

```
## [1] -0.05313073
```

```
mean(q1c[,2])
```

```
## [1] -0.03497711
```

```
mean(q1c[,3])
```

```
## [1] -0.04454689
```

```
mean(q1c[,4])
```

```
## [1] 0.001600428
```

```
cov(q1c)
```

```
##              [,1]         [,2]        [,3]        [,4]
## [1,]  0.952852273 -0.006724903  0.03553208 -0.01415288
## [2,] -0.006724903  0.947531817  0.01727272 -0.07473928
## [3,]  0.035532079  0.017272722  0.96385933 -0.05074687
## [4,] -0.014152878 -0.074739284 -0.05074687  1.06417455
```

The mean is very close to zero, and the covariance matrix is very close to the identity matrix.

Question 3

(a) [

$$
\begin{aligned}
E(\hat{\theta}) &= E(\frac{1}{m}\sum_{i=1}^{m}(g(X_i)) \\
&= \frac{1}{m}E(g(X_1) + g(X_2) + ... + g(X_m)) \\
&= \frac{1}{m}(E(g(X_1) + ... + E(g(X_m))) \\
&= \frac{1}{m} * m * \theta
\end{aligned}
$$

$$= \theta$$

]

(b) [

$$
\begin{aligned}
Var(\hat{\theta}) &= Var(\frac{1}{m}\sum_{i=1}^{m}(g(X_i)) \\
&= \frac{1}{m^2} Var(g(X_1) + g(X_2) + \ldots + g(X_m)) \\
&= \frac{1}{m^2} (Var(g(X_1)) + \ldots + Var(g(X_m))) \\
&= \frac{1}{m^2} * m * Var(g(X)) \\
&= \frac{Var(g(X))}{m} \\
\text{To estimate } Var(g(X)): & \\
estimator &= \frac{\sum_{i=1}^{m}(g(X_i) - \overline{g(X)})^2}{m^2}
\end{aligned}
$$

]

(c)

$$
\begin{aligned}
Var(\hat{\theta}) &= \frac{\sum_{i=1}^{m}(g(X_i) - \overline{g(X)})^2}{m^2} \\
SD(\hat{\theta}) &= \sqrt{Var(\hat{\theta})} \\
\text{By CLT, } \frac{\hat{\theta} - \theta}{SD(\hat{\theta})} &\sim N(0, 1) \\
\hat{\theta} &\pm 1.96 SD(\hat{\theta}) \text{ for } 95\% \text{ CI}
\end{aligned}
$$

(d) g(x)=x^2, f(x) is exp(0.5).

```
q3d<-function(m,rate){
  x<-rexp(m,rate=rate)
  g<-x^2
  return(mean(g))

}
```

(e) E(X^2)=E^2(X)+Var(X)=2*2+4=8. So 8 is the true value.

```
set.seed(904971914)
collect<-c()
for(i in 1:1000){
 x<-rexp(1000,0.5)
 g<-x^2
 theta<-sum(g)/1000
 var_theta<-sum((g-mean(g))^2)/(1000^2)
 sd_theta<-sqrt(var_theta)
 CI<-1.96*sd_theta

 if(theta-CI<=8 & 8<=theta+CI){
   collect<-c(collect,T)
 }
 else{
   collect<-c(collect,F)
 }


}
mean(collect)
```

```
## [1] 0.934
```

The frequency is very close to 95%.

Question 4

(a)

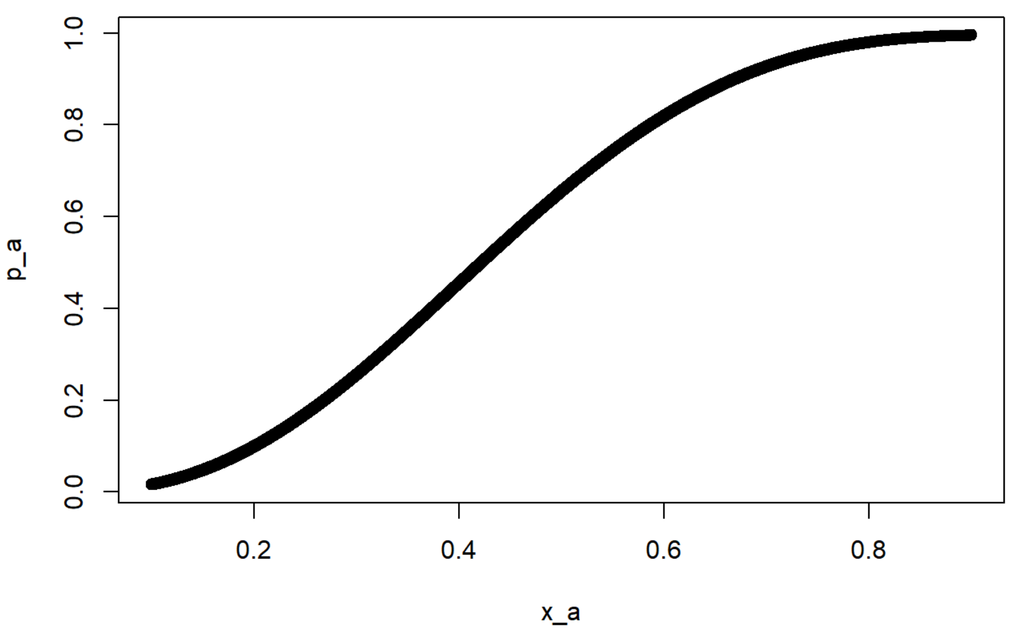
```
q4a<-function(x){

m <- 10000
u <- runif(m)
mc <- numeric(length(x))
for (i in 1:length(x)) {
g <- ((u*x[i])^2)*((1-(u*x[i]))^3)*x[i]
mc[i] <-mean(g) /factorial(2)/factorial(3)*factorial(6)
}
return(mc)


}


x_a <- seq(0.1, 0.9, length = 5000)
p_a<-q4a(x_a)
```

(b)

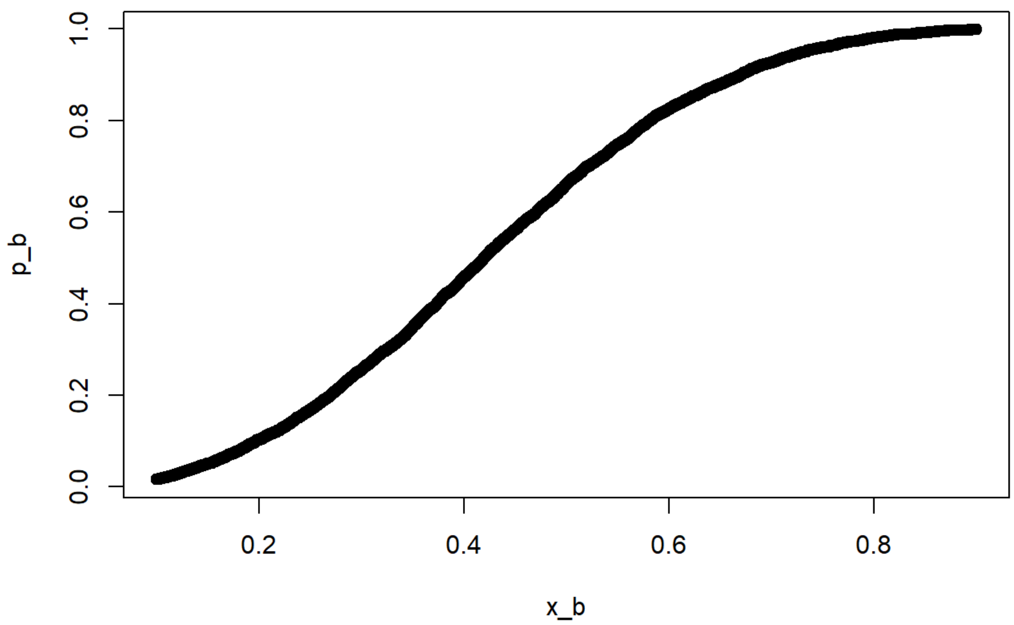
```
x_b<-seq(0.1,0.9,length=5000)
z_b <- rbeta(5000,3,4)
dim(x_b) <- length(x_b)
p_b <- apply(x_b, MARGIN = 1, FUN = function(x_b, z_b){mean(z_b < x_b)}, z_b = z_b)
```

(c)

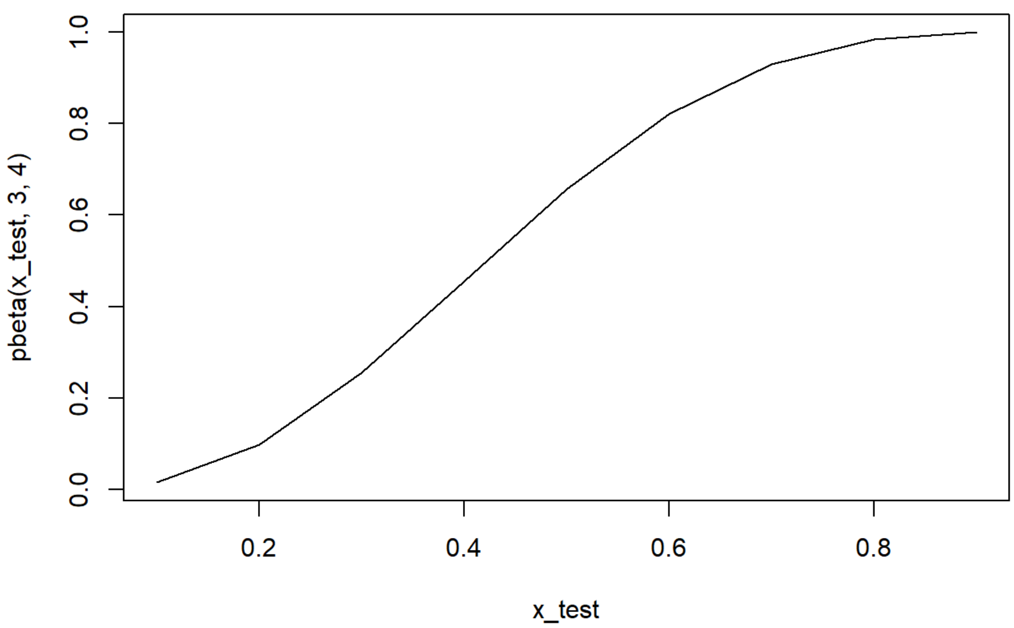
```
plot(x_a,p_a)
```




```
plot(x_b,p_b)
```




```
x_test<-seq(0.1,0.9,length=9)
plot(x_test,pbeta(x_test,3,4),type="l")
```



 very close to the theoretical values.