General Notes

- You will submit a minimum of two files, the core files must conform to the following naming conventions (including capitalization and underscores). 123456789 is a placeholder, please replace these nine digits with your nine-digit Bruin ID. The files you must submit are:

  1. *123456789_stats102c_hw1.Rmd*: Your markdown file which generates the output file of your submission.
  2. *123456789_stats102c_hw1.html/pdf*: Your output file, either a PDF or an HTML file depending on the output you choose to generate.

  If you fail to submit any of the required core files you will receive ZERO points for the assignment. If you submit any files which do not conform to the specified naming convention, you will receive (at most) half credit for the assignment.

- **Your .Rmd file must knit**. If your .Rmd file does not knit you will receive (at most) half credit for the assignment.
  The two most common reason files fail to knit are because of workspace/directory structure issues and missing include files. To remedy the first, ensure all of the file paths in your document are relative paths pointing at the current working directory. To remedy the second, simply make sure you upload any and all files you source or include in your .Rmd file.

- Your coding should adhere to the tidyverse style guide: https://style.tidyverse.org/.

- Any functions/classes you write should have the corresponding comments as the following format.

```
my_function <- function(x, y, ...){
#A short description of the function
#Args:
#x: Variable type and dimension
#y: Variable type and dimension
#Return:
#Variable type and dimension
Your codes begin here
}
```

NOTE: *Everything* you need to do this assignment is here, in your class notes, or was covered in discussion or lecture.

- Please **DO NOT** look for solutions online.
- Please **DO NOT** collaborate with anyone inside (or outside) of this class.
- Please work **INDEPENDENTLY** on this assignment.
- **EVERYTHING** you submit **MUST** be 100% your, original, work. Any student suspected of plagiarizing, in whole or in part, any portion of this assignment, will be **immediately** referred to the Dean of Student's office without warning.

Problem 1: If two random variables, X and Y, are independent, show Var(XY) in terms of the expected values and variances of X and Y.

Problem 2: Suppose X is a geometric random variable with p = .6, determine the value of k such that $P(X \leq k) \simeq 0.9$

Problem 3: At a call center in Los Angeles County, calls come in at an average rate of six calls per minute. Assume that the wait time from one call to the next follows the exponential distribution. Here we concern only with the rate of calling in and ignore the time spent on the phone. Also, we need to assume that the waiting time between calls is independent. That is, a long delay between two calls doesn't make a shorter waiting period for the next call.

(a) What the average time between two successive calls?

(b) What is the probability that exactly five calls occur within a minute?

(c) What is the probability that less than five calls occur within a minute?

(d) After a call is received, what is the probability that the next call occurs in less than 5 seconds?

P (e) Use **rgexp** function in R to generate 1000 samples with the λ you derived from (d), and then draw a EDF plot with **ecdf** function. Finally, add confidence interval to your EDF plot.

Problem 4: Let X be a continuous random variable with cumulative density function $F(x)$. Let $U \sim$ Unif[0,1].

(a) Let $Y = F^{-1}$. Show that $P(Y \leq x) = F(x)$.

(b) Show that $F(x) \sim Unif[0, 1]$.

Problem 5: Let $X \sim$ two-parameter Exponential(λ, η) with probability density function

$$f(x) = \lambda e^{-\lambda(x-\eta)}, x \geq \eta$$

, where λ and η are positive constant.

(a) Calculate the cumulative density function F(x)

(b) Calculate $x = F^{-1}(u)$, i.e., solve the equation $F(x) = u$ for a given u, where u is a random variable from Unif[0,1].

P (c) Write a function to generate random numbers from Exponential(λ, η)

P (d) Generate 10,000 random numbers and plot the histogram. You may set $\lambda =$ the last digit of your UID and $\eta =$ the second last digit.

P (e) Compare the sample quantiles with the theoretical quantiles.