904971914_stats102c_hw1

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Question 1

Since X and Y are independent, X^2 and Y^2 are also independent:

$$\begin{aligned} Var(XY) &= E((XY)^2) - E^2(XY) \\ &= E(X^2) \times E(Y^2) - E(XY) \times E(XY) \\ &= (E^2(X) + Var(X)) \times (E^2(Y) + Var(Y)) - E^2(X)E^2(Y) \\ &= E^2(X) \times Var(Y) + E^2(Y) \times Var(X) + Var(X) \times Var(Y) \end{aligned}$$

Question 2

[

$$\begin{aligned} X &\sim Geo(0.6) \\ P(X \le k) &\approx 0.9 \\ F_X(k) &\approx 0.9 \\ 1 - (1-0.6)^{k+1} &\approx 0.9 \\ k &\approx 1.5 \\ k &= 1 \Rightarrow P(X \le k) = 0.84 \\ k &= 2 \Rightarrow P(X \le k) = 0.93 \\ k &= 2 \end{aligned}$$

]

Question 3

(a) Let T be the time between successive calls. [

$$\begin{aligned} T &\sim exp(\lambda), \lambda = 6 \\ E(T) &= 1/\lambda = 1/6 \end{aligned}$$

]

The average times is 1/6 minute.

(b) [

$$\begin{aligned} N_1 &\sim Poi(\lambda), \lambda = 6 \\ P(N_1 = 5) &= \frac{6^5 e^{-6}}{5!} \\ &= 0.1606 \end{aligned}$$

]

(c) [

$$\begin{aligned} P(N_1 < 5) &= \sum_{k=0}^{4} \frac{6^k e^{-6}}{k!} \\ P(N_1 < 5) &= 0.2851 \end{aligned}$$

]

(d)) Let T be the waiting time in seconds. [

$$\begin{aligned} T &\sim poi(\lambda'), \lambda' = 6/60 = 0.1 \\ P(T \le 5) &= F_T(5) = 1 - e^{-0.1*5} \\ &= 0.3935 \end{aligned}$$

]

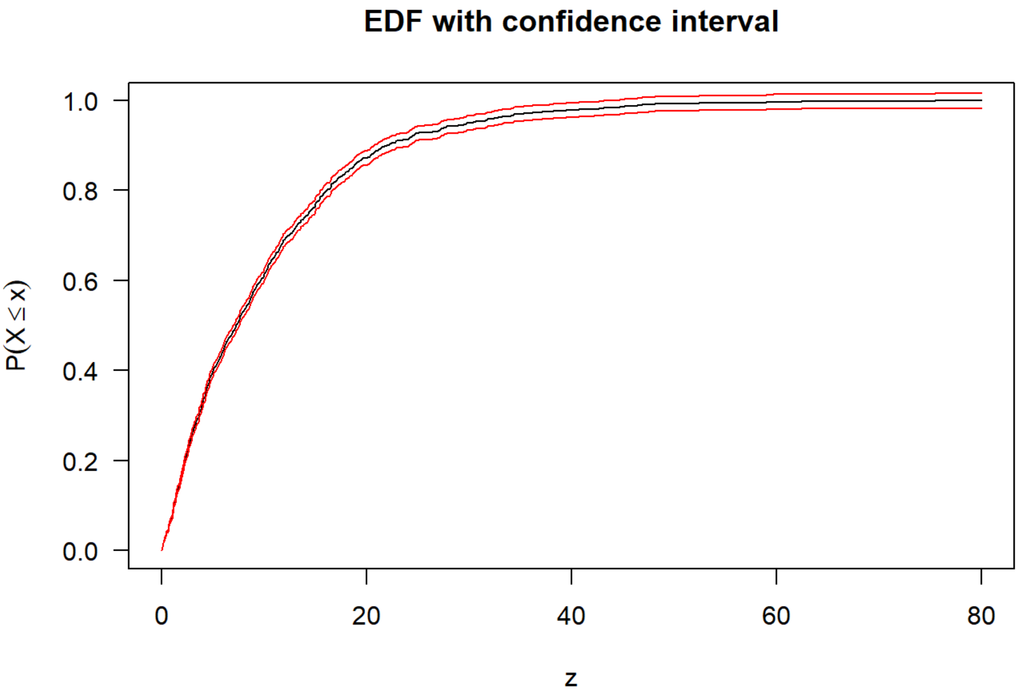
(e)

```
set.seed(904971914)
x<-rexp(n=1000,rate =0.1)
q3e<-ecdf(x)

z <-seq(0,80,0.1)
zy <-q3e(z)
zci<-sqrt((q3e(zy)*(1-q3e(zy)))/1000)*1.96

plot(x=z,y=zy, ylab = expression(P(X <= x)), main = "EDF with confidence interval",type='l',las = 1)

U<- zy+zci
L<-zy-zci
lines(z,U,col='red')
lines(z,L,col='red')
```



The red lines are the confidence intervals.

Question 4

(a)

$$\begin{aligned} Y &= F^{-1}(u) \\ P(Y \leq x) &= P(F^{-1}(u) \leq x) \\ &= P(u \leq F(x)) \\ &= F_U(F(x)) \\ &= \int_0^{F(x)} 1 du \\ &= F_X(x) \end{aligned}$$

(b) [

$$\begin{aligned} Let \quad Y &= F(x), 0 \leq Y \leq 1 \\ P(Y \leq y) &= P(F(x) \leq y) \\ &= P(X \leq F^{-1}(y)) \\ &= F(F^{-1}(y)) \\ &= y, \ \ y \in [0, 1] \end{aligned}$$

] Therefore, Y has the same CDF as U ~ [0,1],so F(x) ~ [0,1].

Question 5

(a) [

$$
\begin{aligned}
F(x) &= \int_{-\infty}^{x} f(t)dt \\
&= \int_{\eta}^{x} \lambda e^{-\lambda(t-\eta)} dt \\
&= -e^{\lambda\eta}(e^{-\lambda x} - e^{-\lambda\eta}) \\
&= 1 - e^{\lambda(\eta - x)}
\end{aligned}
$$

]

(b) [

$$
\begin{aligned}
F(x) &= u \\
1 - e^{\lambda(\eta-x)} &= u \\
\lambda\eta - \lambda x &= log(1-u) \\
x = F^{-1}(u) &= \eta - \frac{log(1-u)}{\lambda}
\end{aligned}
$$

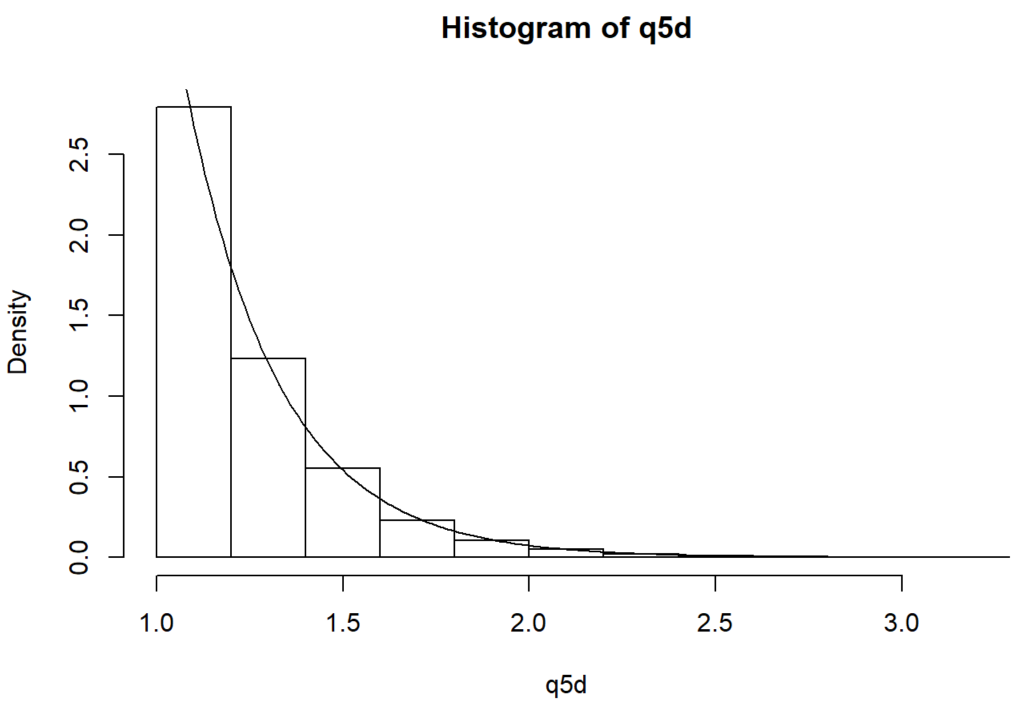
]

(c)

```
q5c <-function(lambda,eta,n){
#This function generates random numbers from the two-parameter exponential distribution.
#lambda and eta are parameters and n specifies how many numbers to generate.
#The function gives a number or a vector of numbers generated from the distribution.
  x <- runif(n)
  y <- eta-(log(1-x)/lambda)
  y
}
```

(d)

```
q5d <-q5c(lambda=4,eta=1,n=10000)
hist(q5d,prob=T)
y<-seq(0,10,0.01)
lines(y,4*exp(-4*(y-1)))
```



Processing math: 100% ...rtiles is very similar to the theoretical exponential quartiles.