Artificial Intelligence Programming

Uninformed Search

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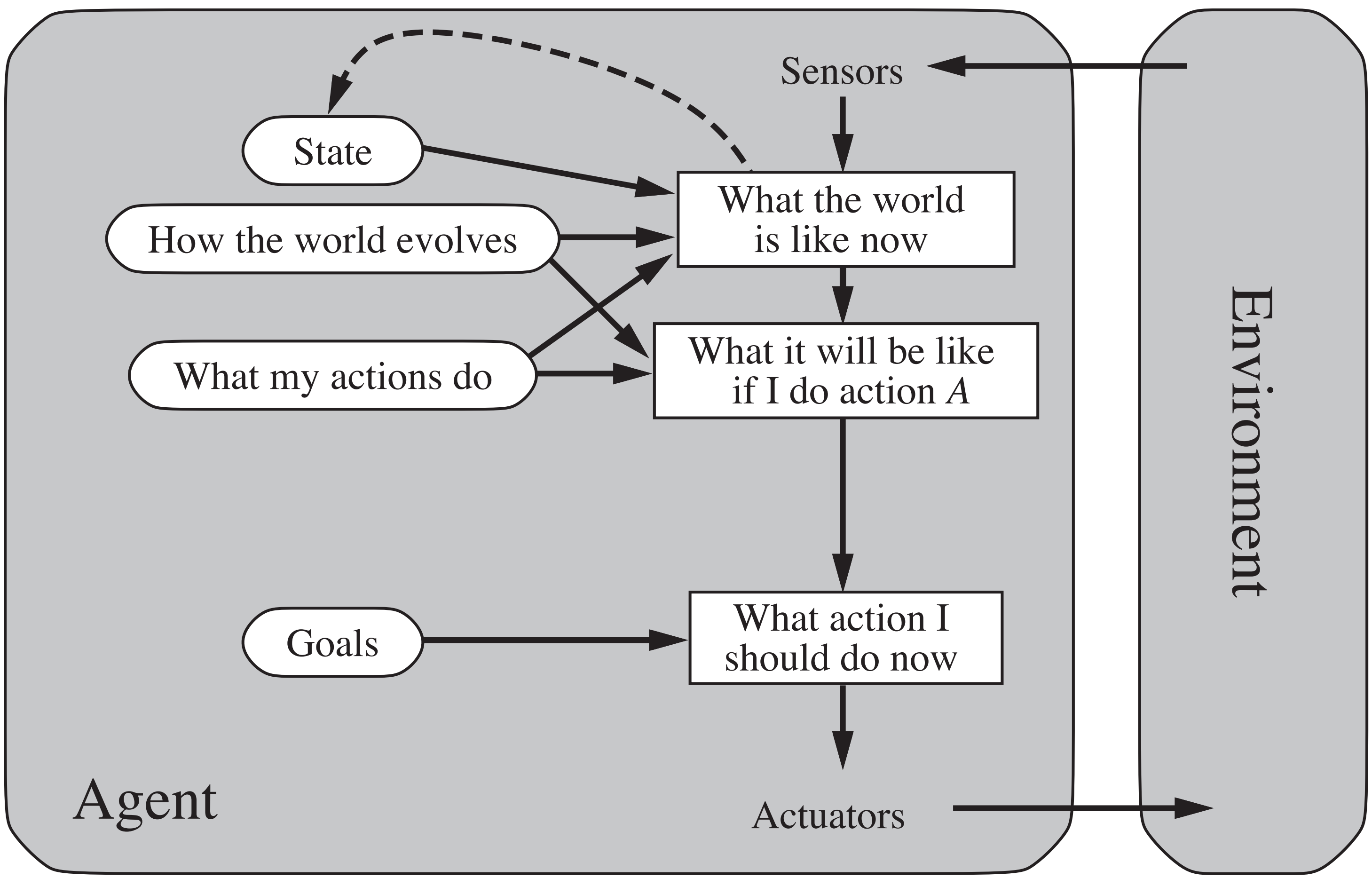
Preview of today

- Many realistic environments are sequential versus episodic
- Recall: A *goal-based* agent is able to consider what it is trying to do and select actions that achieve that goal.
- We'll look at a particular type of goal-based agent called a *problem-solving* agent.
- Agent program uses percepts and goal as input.

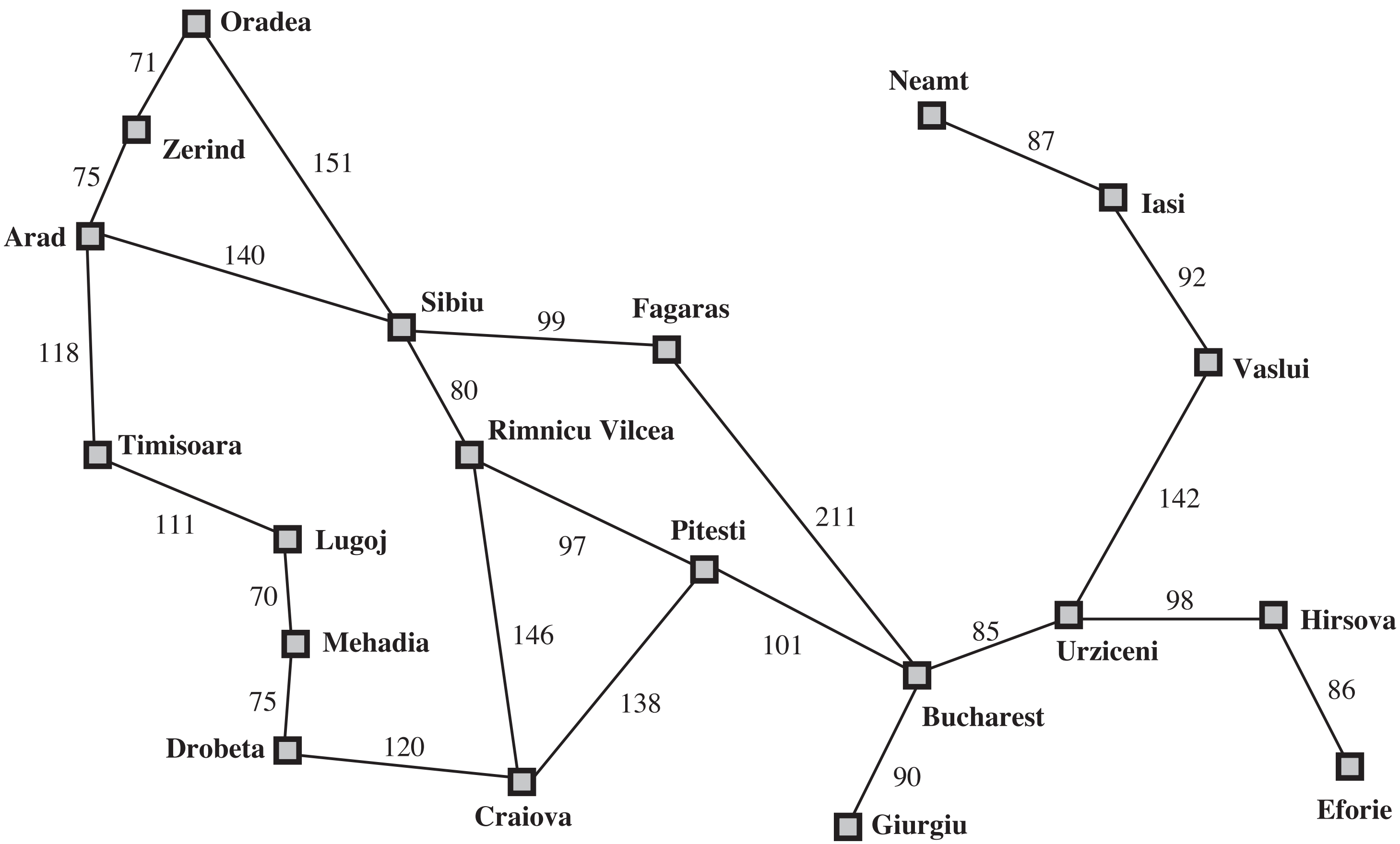
Goal-based Agent



Problem-solving agents

- A Problem-solving agent tries to find a sequence of actions that will lead to a goal.
  - What series of moves will solve a Rubik's cube?
  - How do I drive from USF to the San Francisco airport?
  - How can I arrange components on a chip?
  - What sequence of actions will move a robot across a room?

Example: Romania map

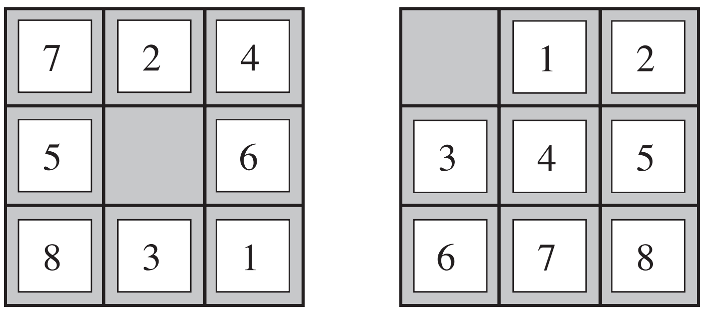


Search

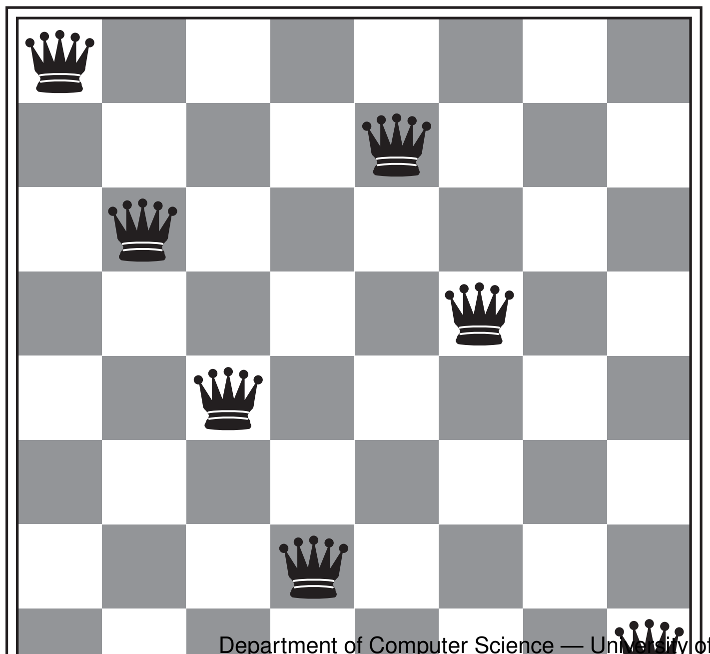
- The process of sequentially considering actions in order to find a sequence of actions that lead from start to goal is called *search*.
- A search algorithm returns an action sequence that is then executed by the agent.
- Sometimes we want a sequence, sometimes just the final state.
  - Search typically happens “offline.”
- Note: (today) the environment is treated as
  - static,
  - observable,
  - discrete,
  - single-agent,
  - and deterministic.

Some classic search problems

- Toy problems: useful to study as examples or to compare algorithms
  - 8-puzzle
  - Vacuum world
  - Rubik’s cube
  - N-queens



Start State Goal State



Classic search Problems

- Real-world problems: typically more messy, but the answer is actually interesting
  - Route finding and logistics
  - Traveling salesman
  - VLSI layout
  - Searching the Internet

State

- We'll often talk about the *state* an agent is in.
- This refers to the values of relevant variables describing the environment and agent.
    - Vacuum World: (0,0), 'Clean'
    - Romania: t = 0, in(Bucharest)
    - Rubik's cube: current arrangement of the cube.
- This is an *abstraction* of our problem.
- Focus only on the details relevant to the problem.

Formulating a Search Problem

- Initial State
- Goal Test
- Actions
- Successor Function
- Path cost
- Solution

Initial State

- Initial State: The state that the agent starts in.
  - Vacuum cleaner world: (0,0), 'Clean'
  - Romania: In(Arad)

Actions

- Actions: What actions is the agent able to take?
  - Vacuum: Left, Right, Up, Down, Suck, Noop
  - Romania: Go(<City>)

Successor Function

- For a given state, returns a set of action/new-state pairs.
  - This tells us, for a given state, what actions we're allowed to take and where they'll lead.
- In a deterministic world, each action will be paired with a single state.
  - Vacuum-cleaner world: (In(0,0)) $\rightarrow$ ('Left', In(0,0)), ('Right', In(0,0)), ('Suck', In(0,0), 'Clean')
  - Romania: In(Arad) $\rightarrow$ ((Go(Timisoara), In(Timisoara)), (Go(Sibiu), In(Sibiu)), (Go(Zerind), In(Zerind))
- In stochastic worlds an action may be paired with many states.

Goal Test

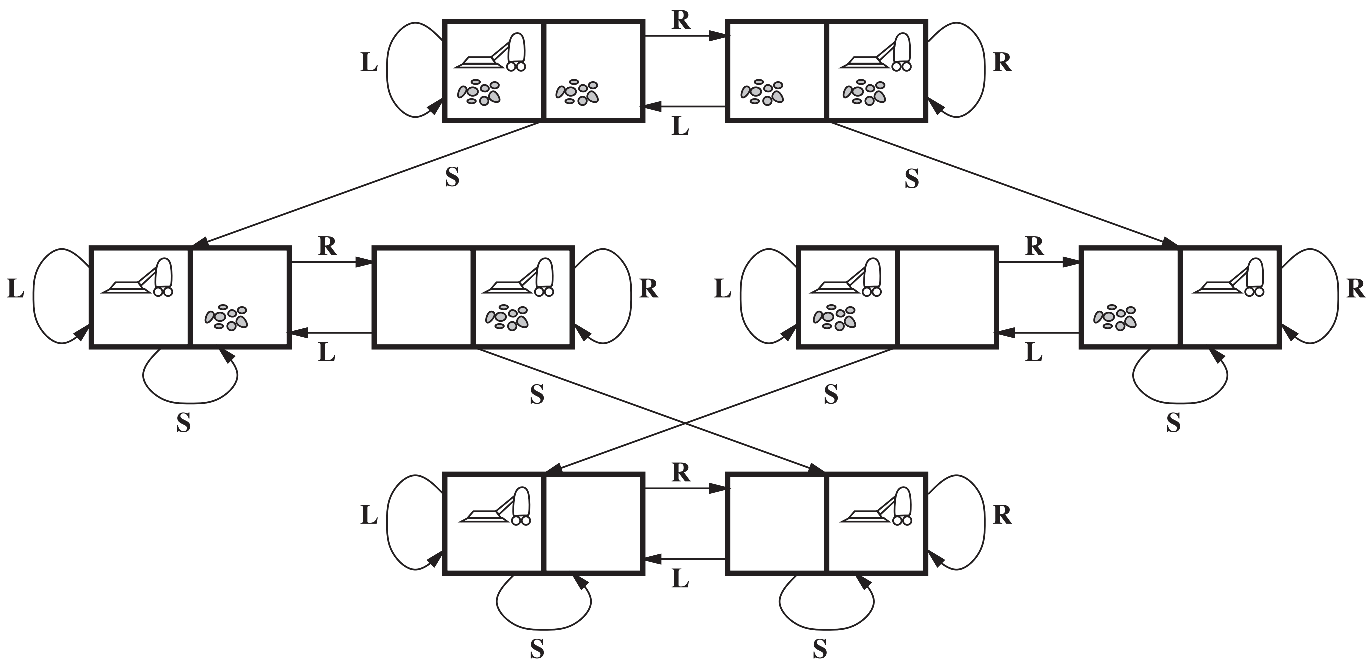
- Determines if a gives state is a goal state.
  - There may be a unique goal state, or many.
  - Vacuum World: every room clean.
  - Chess - checkmate
  - Romania: in(Bucharest)

State space

- The combination of problem states (arrangements of variables of interest) and and successor functions (ways to reach states) leads to the notion of a *state space*.
- This is a graph representing all the possible world states, and the transitions between them.
- Finding a solution to a search problem is reduced to finding a path from the start state to the goal state.
- This framework lets us easily compare different search strategies.

State space

- State space for simple vacuum cleaner world



Types of Solutions

- Depending on the problem, we might want different sorts of solutions
  - Maybe shortest or least-cost
  - Maybe just a path
  - Maybe any path that meets solution criteria (satisficing)
- We'll often talk about the size of these spaces as a measure of problem difficulty.
  - 8-puzzle: $\frac{9!}{2} = 181,000$ states (easy)
  - 15-puzzle: ~ 1.3 trillion states (pretty easy)
  - 24-puzzle: $\sim 10^{25}$ states (hard)
  - TSP, 20 cities: $20! = 2.43 \times 10^{18}$ states (hard)

Path cost

- The *path cost* is the cost an agent must incur to go from the initial state to the currently-examined state.
- Often, this is the sum of the cost for each action
  - This is called the *step cost*
- We'll assume that step costs are nonnegative.
  - What if they could be negative?

Examples

What are the states/operators/path costs for:

- 8-puzzle
- Rubic's cube
- 8-queens

8 Queens Possible Approach

Incremental: Place queens one by one

- States: arrangement of 0-8 Queens
- Operators: Add a queen to the board somewhere
- OR
- States: arrangement of 0-8 Queens, no attacks
- Operators: Place a queen in the leftmost empty column

What if you get stuck?

8 Queens Alternative Approach

Complete: Place all queens, move until no attacks

- States: arrangement of 8 queens
- Operators: Move any attacked queen to another square
- OR
- States: arrangement of 8 queens, one per column
- Operators: Move any queen to another square in same column

Can't get stuck

Shortest-path graph problems

- The state space is a graph, with states as vertices and the successor function defining edges.
- Finding the shortest path through a graph is a well-known problem.
  - Floyd's algorithm, Djikstra's algorithm, Prim's algorithm, Max-flow, All-pairs shortest-path
- Given this, why are we talking about search? Isn't this problem solved?

Shortest-path graph problems

- Djikstra's algorithm is quadratic in the number of vertices, in both time and space.
  - Will this scale to millions of vertices?
- The number of vertices in most search problems is an exponential function of the number of state variables.
  - Vacuum cleaner: 3 binary variables: 8 states.
  - TSP, 15 cities: 15 variables, 15! = 1307674368000 states.

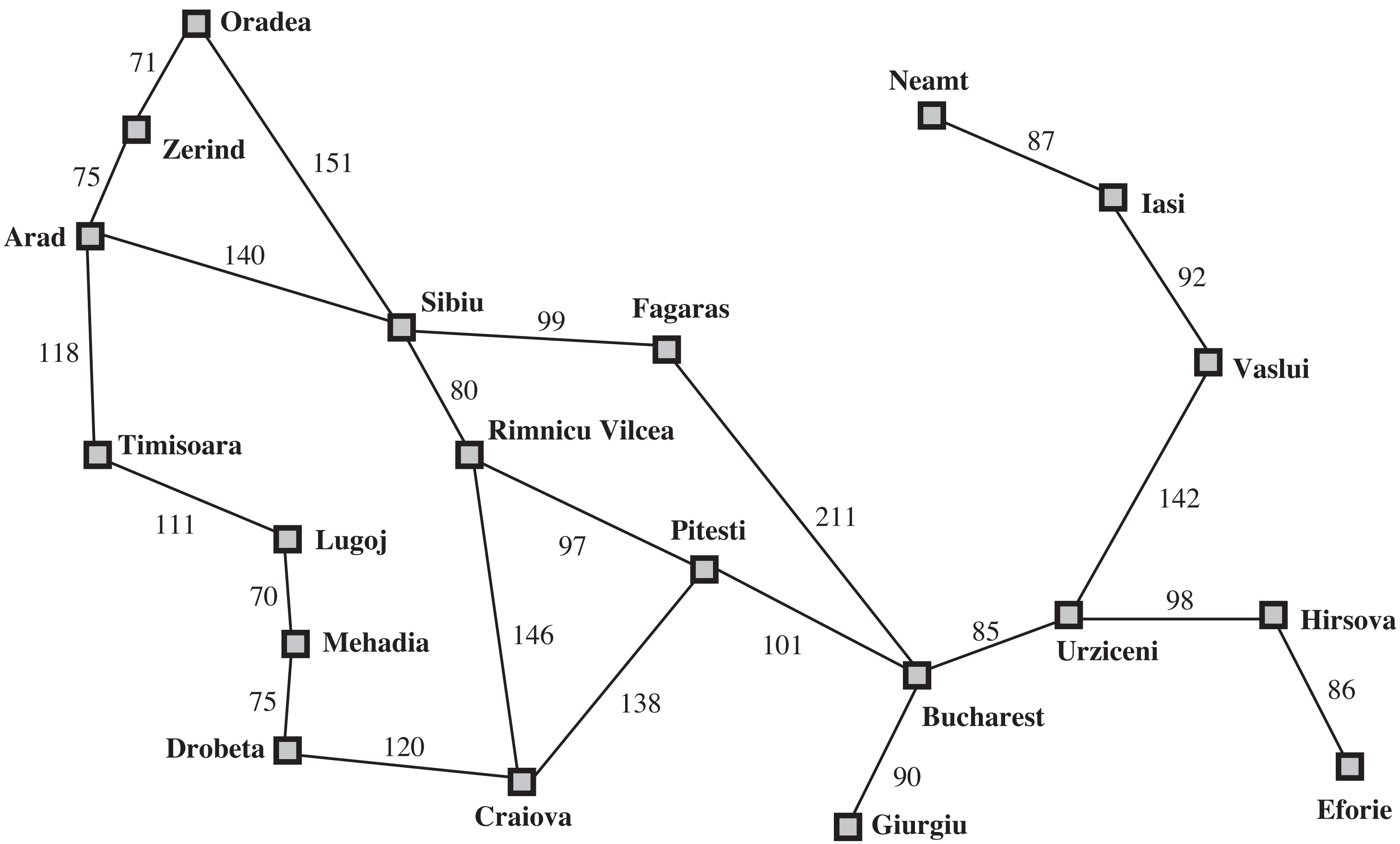
Searching the state space

- Most search problems are too large to hold in memory
  - We need to dynamically instantiate portions of the search space
- We construct a *search tree* by starting at the initial state and repeatedly applying the successor function.
- Basic idea: from a state, consider what can be done. Then consider what can be done from each of those states.

State Space Search

- Some questions we'll be interested in:
  - Are we guaranteed to find a solution?
  - Are we guaranteed to find the optimal solution?
  - How long will the search take?
  - How much space will it require?

Example: Romania map



Search algorithms

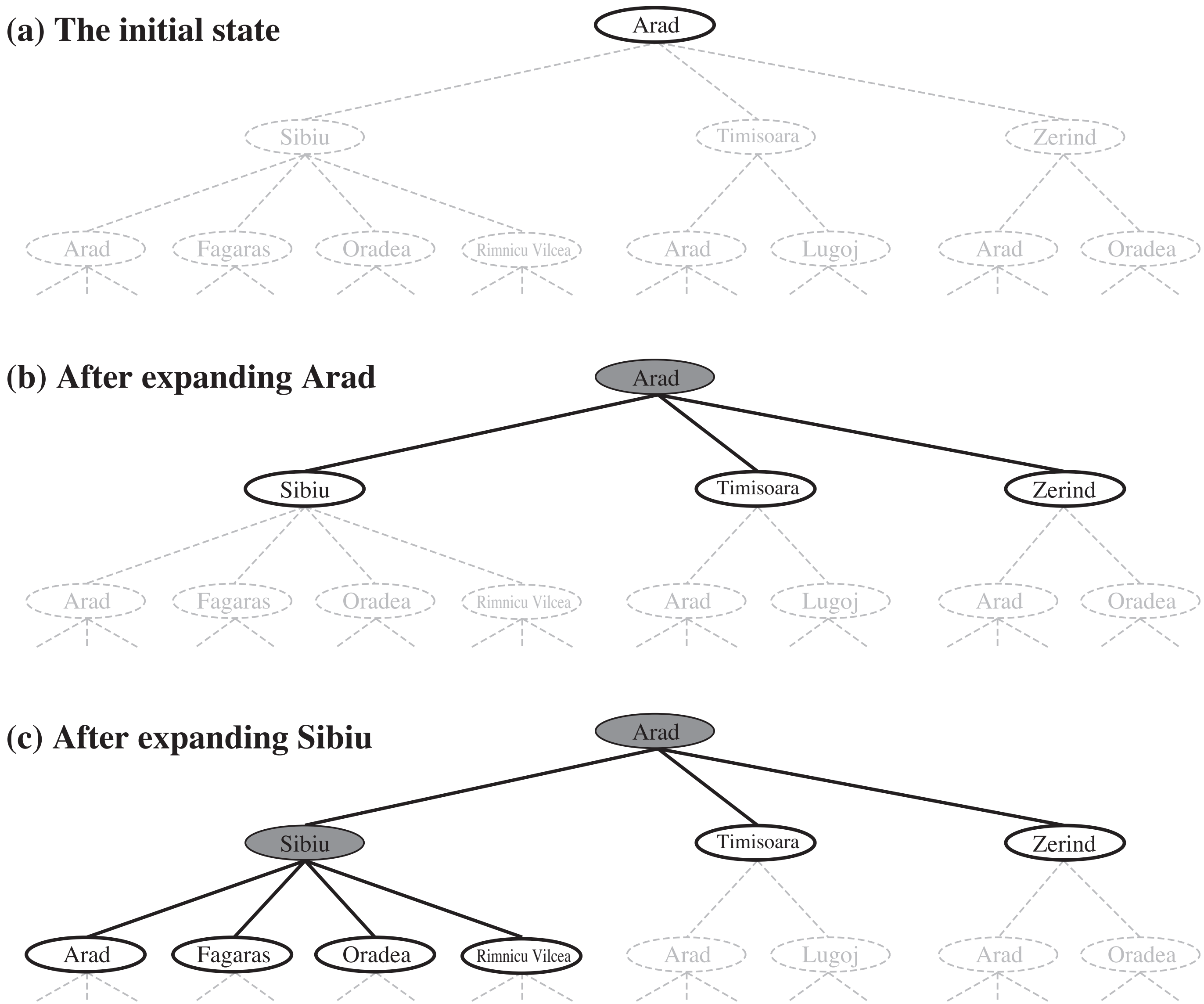
- The basic search algorithm is surprisingly simple:

```
fringe <- initialState
do
   select node from fringe
   if node is not goal
      generate successors of node
      add successors to fringe
```

- We call this list of nodes generated but not yet expanded the *fringe*.
- Question: How do we select a node from the fringe?
  - Differentiates search algorithms

Example Search Tree

- The beginnings of a Romania search tree:



Uninformed Search

- The simplest sort of search algorithms are those that use no additional information beyond what is in the problem description.
- We call this *uninformed search*.
  - Sometimes these are called weak methods.
- If we have additional information about how promising a nongoal state is, we can perform *heuristic search*.

Breadth-first search

- Breadth-first search works by expanding a node, then expanding all of its children, then all of their children, etc.
- All nodes at depth n are visited before a node at depth $n+1$ is visited.
- We can implement BFS using a Queue.

Breadth-first search

- BFS Python-ish code

```
queue.enqueue(initialState)
while not done :
   node = queue.dequeue()
   if goalTest(node) :
     return node
   else :
     children = successor-fn(node)
     for child in children
         queue.enqueue(child)
```

BFS example: Arad to Bucharest

- dequeue Arad
- enqueue Sibiu, Timisoara, Zerind
- dequeue and test Sibiu
- enqueue Oradea, Fagaras, Rimnciu Viclea
- dequeue and test Timisoara
- enqueue Lugoj
- ...

Some subtle points

- How do we avoid revisiting Arad?

Some subtle points

- How do we avoid revisiting Arad?
  - Closed-list: keep a list of expanded states.
  - Do we want a closed-list here? Our solution is a *path*, not a city.
- How do we avoid inserting Oradea twice?

Some subtle points

- How do we avoid revisiting Arad?
  - Closed-list: keep a list of expanded states.
  - Do we want a closed-list here? Our solution is a *path*, not a city.
- How do we avoid inserting Oradea twice?
  - Open-list (our queue, actually): a list of generated but unexpanded states.
- Why don't we apply the goal test when we generate children?
  - Not really any different. Nodes are visited and tested in the same order either way. Same number of goal tests are performed.

Analyzing BFS

- Completeness: Is BFS guaranteed to find a solution?
  - Yes. Assume the solution is at depth n. Since all nodes at or above n are visited before anything at $n + 1$, a solution will be found.
- Optimality: If there are multiple solutions, will BFS find the best one?
  - BFS will find the shallowest solution in the search tree. If *step costs* are uniform, this will be optimal. Otherwise, not necessarily.
  - Arad -> Sibiu -> Fagaras -> Bucharest will be found first. (dist = 450)
  - Arad -> Sibiu -> Rimnicu Vilcea -> Pitesti -> Bucharest is shorter. (dist = 418)

Analyzing BFS

- Time complexity: BFS will require $O(b^{d+1})$ running time.
  - b is the branching factor: average number of children
  - d is the depth of the (shallowest) solution.
  - BFS will visit $b + b^2 + b^3 + ... + b^d + b^{d+1} - (b-1) = O(b^{d+1})$ nodes
- Space complexity: BFS must keep the whole (visited) search tree in memory (since we want to know the sequence of actions to get to the goal).
- This is also $O(b^{d+1})$.

Analyzing BFS

- Assume b = 10, 1kb/node, 10000 nodes/sec
- depth 2: 1100 nodes, 0.11 seconds, 1 megabyte
- depth 4: 111,000 nodes, 11 seconds, 106 megabytes
- depth 6: 10^7 nodes, 19 minutes, 10 gigabytes
- depth 8: 10^9 nodes, 31 hours, 1 terabyte
- depth 10: 10^{11} nodes, 129 days, 101 terabytes
- depth 12: 10^{13} nodes, 35 years, 10 petabytes
- depth 14: 10^{15} nodes, 3523 years, 1 exabyte
- In general, the space requirements of BFS are a bigger problem than the time requirements.

Uniform cost search

- Recall that BFS is nonoptimal when step costs are nonuniform.
- How might we correct this?

Uniform cost search

- Recall that BFS is nonoptimal when step costs are nonuniform.
- We can correct this by expanding the shortest paths first.
- Add a path cost to expanded nodes.
- Use a priority queue to order them in order of increasing path cost.
- Guaranteed to find the shortest path.
- If step costs are uniform, this is identical to BFS.
  - This is how Djikstra's algorithm works

Depth-first Search

- Depth-first search takes the opposite approach to search from BFS.
  - Always expand the deepest node.
- Expand a child, then expand its left-most child, and so on.
- We can implement DFS using a stack.

Depth-first Search

- DFS python-ish code:

```
stack.push(initialState)
while not done :
    node = pop()
    if goalTest(node) :
      return node
    else :
      children = successor-fn(node)
      for child in children :
          stack.push(child)
```

DFS example: Arad to Bucharest

- pop Arad
- push Sibiu, Timisoara, Zerind
- pop and test Sibiu
- push Oradea, Fagaras, Rimnciu Viclea
- pop and test Oradea
- pop and test Fagaras
- push Bucharest
- ...

Analyzing DFS

- Completeness: no. We can potentially wander down an infinitely long path that does not lead to a solution.
- Optimality: no. We might find a solution at depth n under one child without ever seeing a shorter solution under another child. (what if we popped Rimnciu Viclea first?)
- Time requirements: $O(b^m)$, where m is the maximum depth of the tree.
  - m may be much larger than d (the solution depth)
  - In some cases, m may be infinite.

Analyzing DFS

- Space requirements: $O(bm)$
  - We only need to store the currently-searched branch.
  - This is DFS' strong point.
  - In our previous figure, searching to depth 12 would require 118 KB, rather than 10 petabytes for BFS.

Avoiding Infinite Search

- There are several approaches to avoiding DFS' infinite search.
- Closed-list
  - May not always help.
  - Now we have to keep exponentially many nodes in memory.
- Depth-limited search
- Iterative deepening DFS

Depth-limited Search

- Depth-limited search works by giving DFS an upper limit l.
- Search stops at this depth.
- Solves the problem of infinite search down one branch.
- Adds another potential problem
  - What if the solution is deeper than l?
  - How do we pick a reasonable l?
- In the Romania problem, we know there are 20 cities, so $l = 19$ is a reasonable choice.
- What about 8-puzzle?

Depth-limited Search

- DLS pseudocode

```
stack.push(initialState)
while not done :
    node = pop()
    if goalTest(node) :
        return node
    else :
        if depth(node) < limit :
            children = successor-fn(node)
            for child in children:
               push(child)
         else :
            return None
```

Iterative Deepening DFS (IDS)

- Expand on the idea of depth-limited search.
- Do DLS with $l = 1$, then $l = 2$, then $l = 3$, etc.
- Eventually, $l = d$, the depth of the goal.
  - This means that IDS is complete.
- Drawback: Some nodes are generated and expanded multiple times.

Iterative Deepening DFS (IDS)

- Due to the exponential growth of the tree, this is not as much of a problem as we might think.
  - Level 1: b nodes generated d times
  - Level 2: b^2 nodes generated $d-1$ times
  - ...
  - Level d: b^d nodes generated once.
  - Total running time: $O(b^d)$. Slightly fewer nodes generated than BFS.
  - Still has linear memory requirements.

Iterative Deepening DFS (IDS)

- IDS pseudocode:

```
d  = 0
while True :
    result = depth-limited-search(d)
    if result == goal
       return result
    else
      d = d + 1
```

Iterative Deepening DFS (IDS)

- IDS is actually similar to BFS in that all nodes at depth n are examined before any node at depth $n+1$ is examined.
- As with BFS, we can get optimality in non-uniform step cost worlds by expanding according to path cost, rather than depth.
- This is called *iterative lengthening search*
- Search all paths with cost less than p. Increase p by δ

Summary

- Formalizing a search problem
  - Initial State
  - Goal Test
  - Actions to be taken
  - Successor function
  - Path cost
- Leads to search through a *state space* using a *search tree*.

Summary

- Algorithms
  - Breadth First Search
  - Depth First Search
  - Uniform Cost Search
  - Depth-limited Search
  - Iterative Deepening Search

Example Problems

8-puzzle

- What is a state?
- What is a solution
- What is the path cost?
- What are the legal actions?
- What is the successor function?

Example Problems

8-puzzle

- Let's say the start state is [1 3 2 B 6 4 5 8 7]
- Goal state?

Example Problems

8-puzzle

- Let's say the start state is [1 3 2 B 6 4 5 8 7]
- Goal state [B 1 2 3 4 5 6 7 8]
- First steps of BFS, DFS, IDS

Example Problems

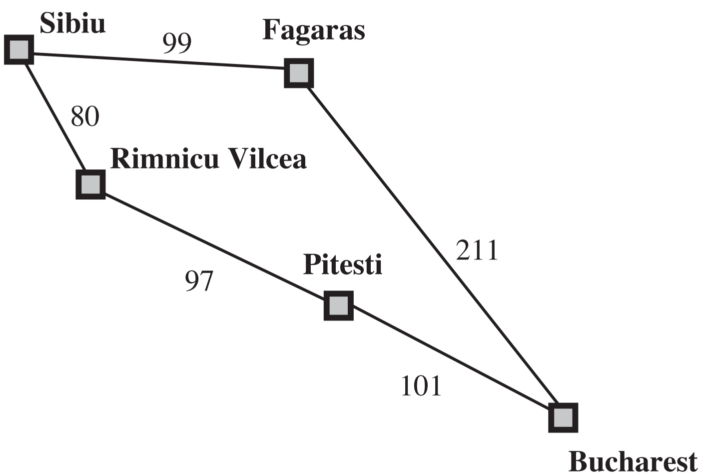
Traveling Salesman

- What is a state?
- What is a solution
- What is the path cost?
- What are the legal actions?
- What is the successor function?

Example Problems

TSP on the reduced Romania map

- Start in Sibiu
- Visit S, F, RV, C, P, B
- First steps of BFS, DFS, IDS



Example Problems

Tower of Hanoi

- Another Toy problem
- What are the problem characteristics?

Example Problems

Tower of Hanoi

- Start: [[5 4 3 2 1][][]]
- Start: [[][][5 4 3 2 1]]
- First steps of BFS, DFS, IDS

Example Problems

Cryptography: given a string of characters, find the mapping that decrypts the message

- How to formulate this?
- Goal test?
- Successor functions?
- Failure states?

Coming Attractions

- Heuristic Search - speeding things up
- Evaluating the “goodness” of nongoal nodes.
- Greedy Search
- A* search.
- Developing heuristics