16-0: **Graph Traversals**

- Visit every vertex, in an order defined by the topololgy of the graph.
- Two major traversals:
  - Depth First Search
  - Breadth First Search

16-1: **Depth First Search**

- Starting from a specific node (pseudo-code):

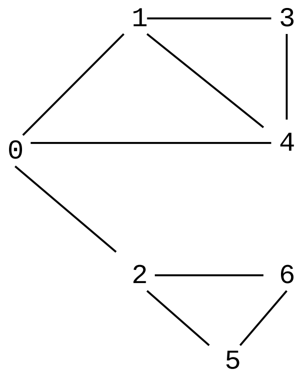
```
DFS(Edge G[], int vertex, boolean Visited[]) {
  Visited[vertex] = true;
  for each node w adajcent to vertex:
    if (!Visited[w])
      DFS(G, w, Visited);
}
```

16-2: **Depth First Search**

```
class Edge {
   public int neighbor;
   public Edge next;
}

void DFS(Edge G[], int vertex, boolean Visited[]) {
  Edge tmp;
  Visited[vertex] = true;
  for (tmp = G[vertex]; tmp != null; tmp = tmp.next) {
    if (!Visited[tmp.neighbor])
      DFS(G, tmp.neighbor, Visited);
  }
}
```

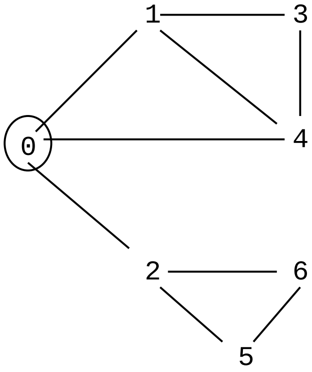
16-3: **Depth First Search**

- Example
  - Visited nodes cicrled in red



16-4: **Depth First Search**

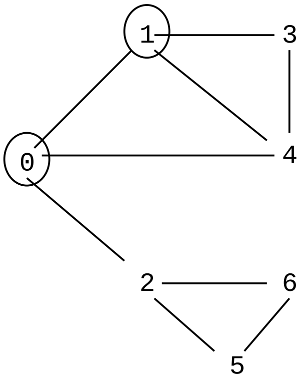
- Example
  - Visited nodes cicrled in red



```
DFS(0)
```

16-5: **Depth First Search**

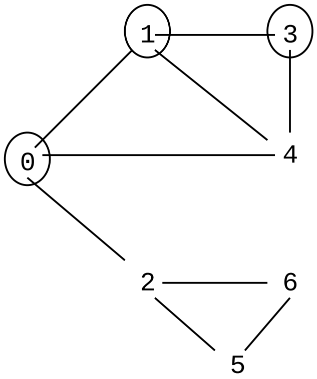
- Example
  - Visited nodes cicrled in red



```
DFS(0)
  DFS(1)
```

16-6: **Depth First Search**

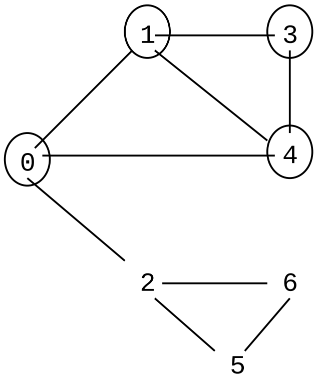
- Example
  - Visited nodes cicrled in red



```
DFS(0)
  DFS(1)
    DFS(3)
```

16-7: **Depth First Search**

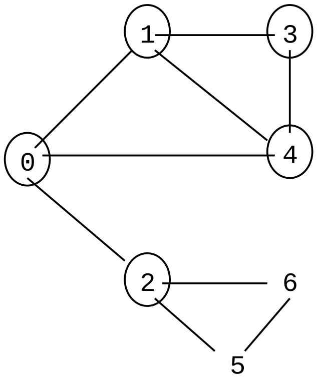
- Example
  - Visited nodes cicrled in red



```
DFS(0)
  DFS(1)
    DFS(3)
      DFS(4)
```

16-8: **Depth First Search**

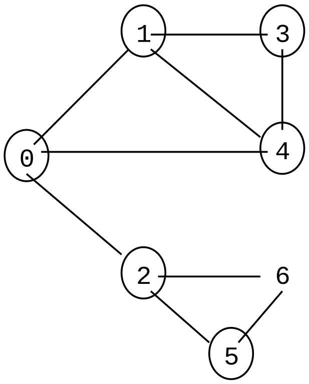
- Example
  - Visited nodes cicrled in red



```
DFS(0)
  DFS(1)
    DFS(3)
      DFS(4)
  DFS(2)
```

16-9: **Depth First Search**

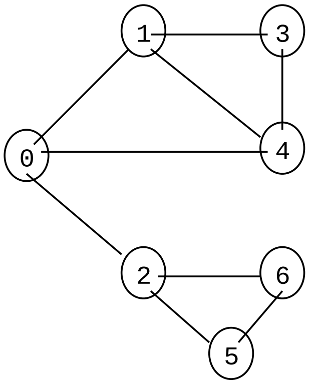
- Example
  - Visited nodes cicrled in red



```
DFS(0)
  DFS(1)
    DFS(3)
      DFS(4)
  DFS(2)
    DFS(5)
```

16-10: **Depth First Search**

- Example
  - Visited nodes cicrled in red



```
DFS(0)
  DFS(1)
    DFS(3)
      DFS(4)
  DFS(2)
    DFS(5)
      DFS(6)
```

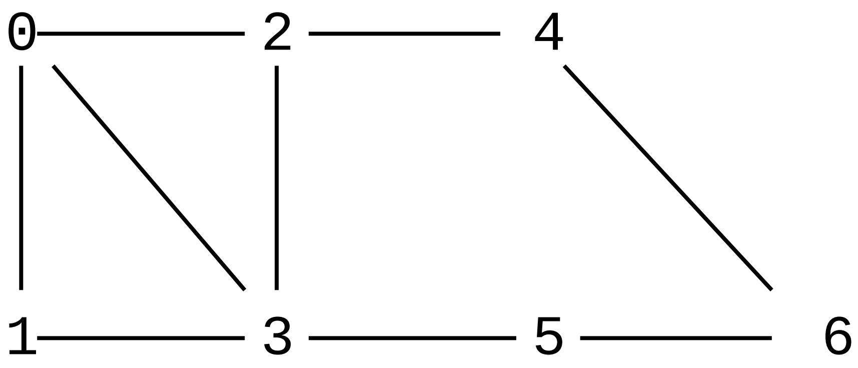
16-11: **Depth First Search**

- To visit every node in the graph:

```
TraverseDFS(Edge G[]) {
  int i;
  boolean Visited = new Edge[G.length];
  for (i=0; i<G.length; i++)
    Visited[i] = false;
  for (i=0; i<G.length; i++)
    if (!Visited[i])
      DFS(G, i, Visited);
}
```

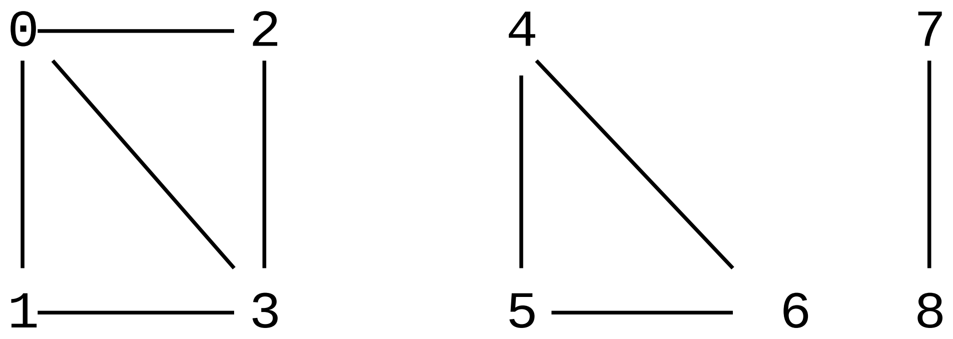
16-12: **Depth First Search**

- Examples



16-13: **Depth First Search**

- Examples



16-14: **DFS & Stacks**

- Keep track of what nodes we have left using a stack
- Recursive version implicitly uses the system stack
- Can write DFS non-recursively, using our own stack

16-15: **DFS & Stacks**

- DFS, using recursion

```
void DFS(Edge G[], int vertex, boolean Visited[]) {
  Edge tmp;
  Visited[vertex] = true;
  for (tmp = G[vertex]; tmp != null; tmp = tmp.next) {
    if (!Visited[tmp.neighbor])
      DFS(G, tmp.neighbor, Visited);
  }
}
```

16-16: **DFS & Stacks**

- DFS, using stack

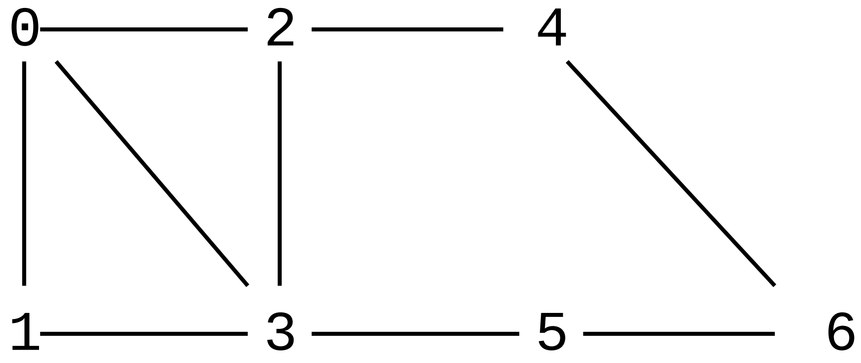
```
void DFS(Edge G[], int vertex, boolean Visited[]) {
  Edge tmp;
  int nextV;
  Stack S = new Stack();

  S.push(new Integer(vertex));
  while (!S.empty()) {
    nextV = ((Integer) S.pop()).intValue();
    if (!Visited[nextV]) {
      Visited[nextV] = true;
      for (tmp = G[nextV]; tmp != null; tmp = tmp.next) {
        S.push(new Integer(tmp.neighbor));
      }
    }
  }
}
```

16-17: **Breadth First Search**

- DFS: Look as *Deep* as possible, before looking wide
    - Examine all descendants of a node, before looking at siblings
- BFS: Look as *Wide* as possible, before looking deep
    - Visit all nodes 1 away, then 2 away, then three away, and so on

16-18: **Breadth First Search**

- Examples



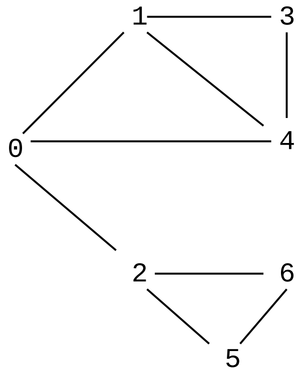
16-19: **Breadth First Search**

- Coding BFS:
    - Use a queue instead of a stack

```
void BFS(Edge G[], int vertex, boolean Visited[]) {
  Edge tmp;
  int nextV;
  Queue Q = new Queue();

  Q.enquque(new Integer(vertex));
  while (!Q.empty()) {
    nextV = ((Integer) Q.dequeue()).intValue();
    if (!Visited[nextV]) {
      Visited[next] = true;
      for (tmp = G[nextV]; tmp != null; tmp = tmp.next) {
        Q.enqueue(new Integer(tmp.neighbor()));
      }
    }
  }
}
```

16-20: **Breadth First Search**

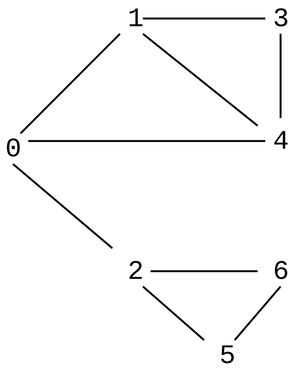
- Example
    - Visited nodes cicrled



16-21: **Breadth First Search**

- Example

- Visited nodes cicrled

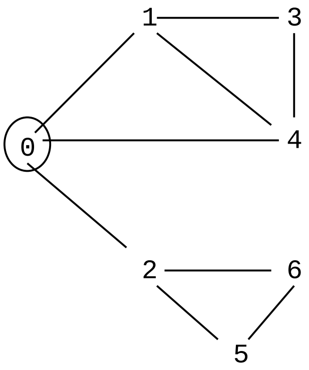


Queue:

0

16-22: **Breadth First Search**

- Example
  - Visited nodes cicrled

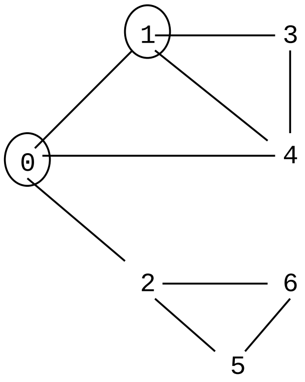


Queue:

124

16-23: **Breadth First Search**

- Example
  - Visited nodes cicrled



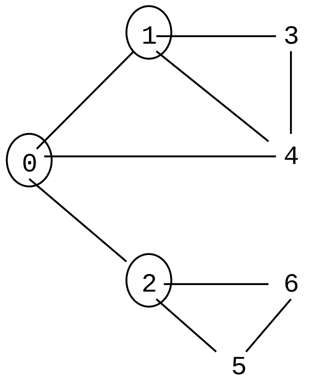
Queue:

24034

16-24: **Breadth First Search**

- Example

- Visited nodes cicrled

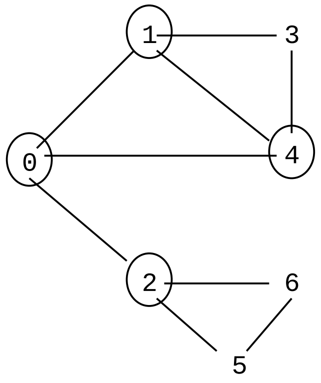


Queue:

4034056

16-25: **Breadth First Search**

- Example
  - Visited nodes cicrled

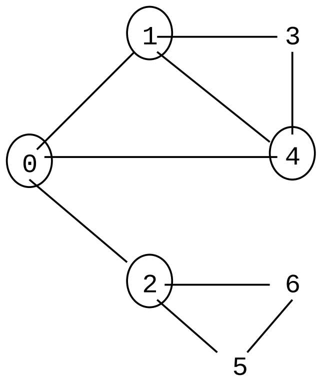


Queue:

034056013

16-26: **Breadth First Search**

- Example
  - Visited nodes cicrled



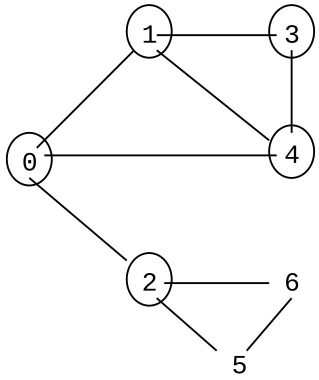
Queue:

34056013

16-27: **Breadth First Search**

- Example

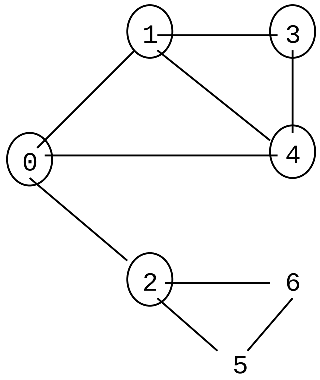
- Visited nodes cicrled



Queue:
405601314

16-28: **Breadth First Search**

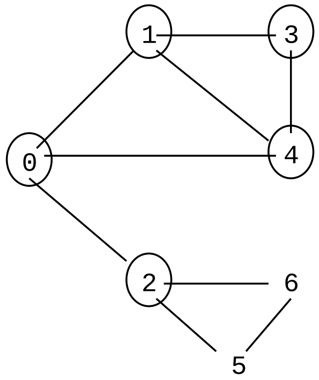
- Example
    - Visited nodes cicrled



Queue:
05601314

16-29: **Breadth First Search**

- Example
    - Visited nodes cicrled

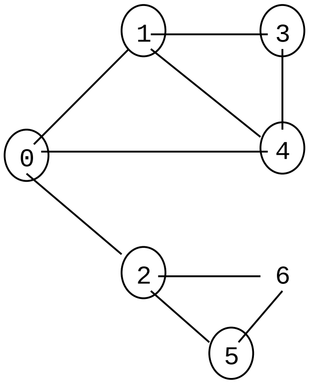


Queue:
5601314

16-30: **Breadth First Search**

- Example

- Visited nodes cicrled

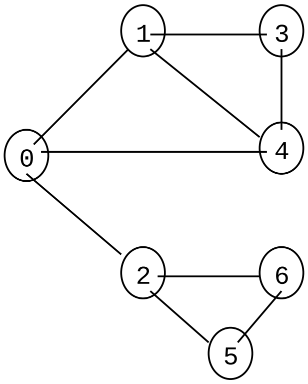


Queue:

60131426

16-31: **Breadth First Search**

- Example
    - Visited nodes cicrled



Queue:

013142625

16-32: **Breadth First Search**

- Alternate version of BFS
    - Previous code marks nodes as VISITED as they are removed from the queue
    - We could also mark nodes as VISITED when they are placed on the queue

16-33: **Breadth First Search**

- Coding BFS (Alternate version):

```
void BFS(Edge G[], int vertex, boolean Visited[]) {
  Edge tmp;
  int nextV;
  Queue Q = new Queue();

  Viisited[vertex] = true;
  Q.enququе(new Integer(vertex));
  while (!Q.empty()) {
    nextV = ((Integer) Q.dequeue()).intValue();
    for (tmp = G[nextV]; tmp != null; tmp = tmp.next) {
      if (!Visited[tmp.neighbor]) {
        Visited[tmp.neighbor] = true;
        Q.enqueue(new Integer(tmp.neighbor));
      }
    }
  }
}
```

16-34: **Breadth First Search**

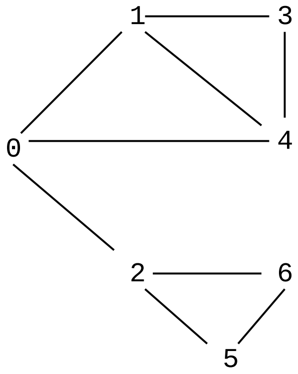
- Alternate version of BFS
  - Previous code marks nodes as VISITED as they are removed from the queue
  - We could also mark nodes as VISITED when they are placed on the queue
- How does execution differ?

16-35: **Breadth First Search**

- Alternate version of BFS
  - Previous code marks nodes as VISITED as they are removed from the queue
  - We could also mark nodes as VISITED when they are placed on the queue
- How does execution differ?
- How does execution differ?
  - Version I: A vertex is added to the queue for each edge in the graph (so the same vertex can be added to the queue more than once
  - Version II: Each vertex is added to the queue at most once

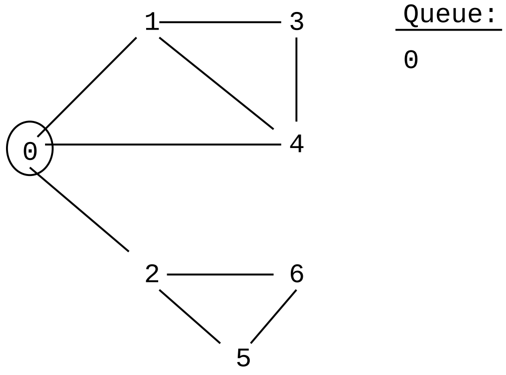
16-36: **Breadth First Search**

- Example
  - Visited nodes cicrled



16-37: **Breadth First Search**

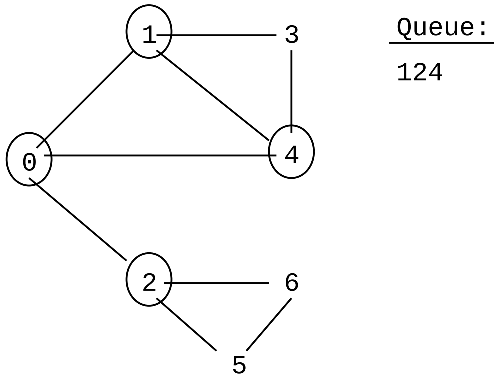
- Example
  - Visited nodes cicrled



```
Queue:
0
```

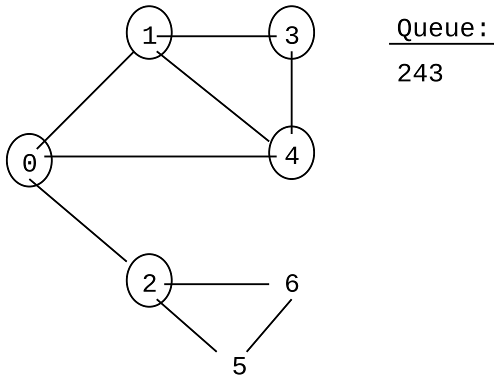
16-38: **Breadth First Search**

- Example
  - Visited nodes cicrled



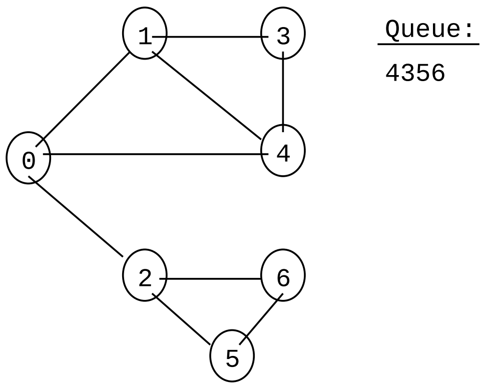
16-39: **Breadth First Search**

- Example
  - Visited nodes cicrled



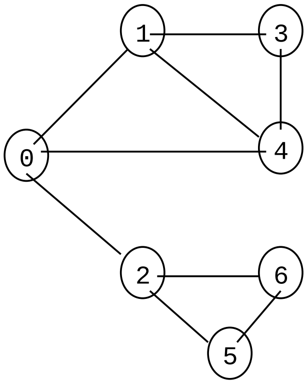
16-40: **Breadth First Search**

- Example
  - Visited nodes cicrled



16-41: **Breadth First Search**

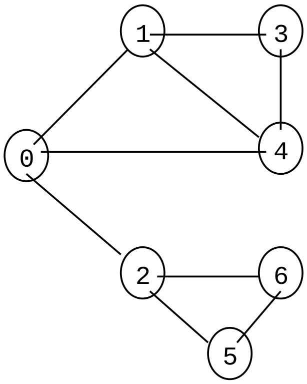
- Example
  - Visited nodes cicrled



Queue:
356

16-42: **Breadth First Search**

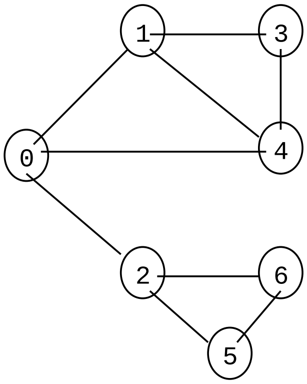
- Example
  - Visited nodes cicrled



Queue:
56

16-43: **Breadth First Search**

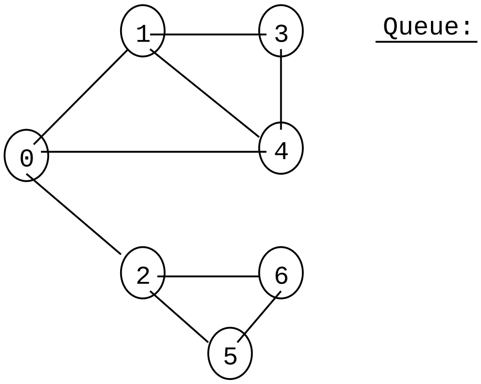
- Example
  - Visited nodes cicrled



Queue:
6

16-44: **Breadth First Search**

- Example
  - Visited nodes cicrled

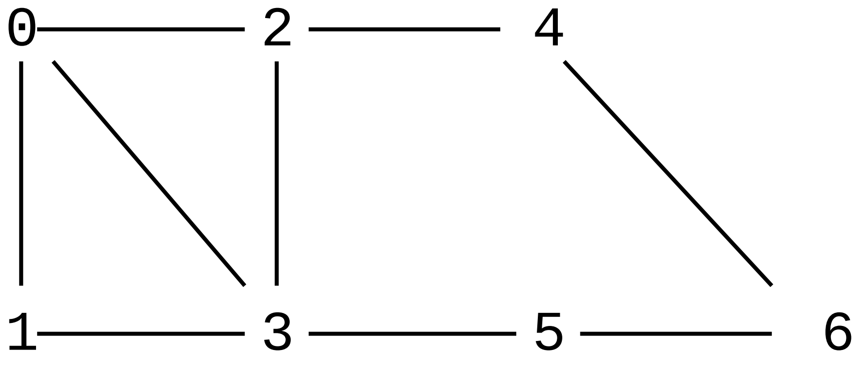


16-45: **Search Trees**

- Describes the order that nodes are examined in a traversal
- Directed Tree
  - Directed edge from v_1 to v_2 if the edge (v_1, v_2) was followed during the traversal

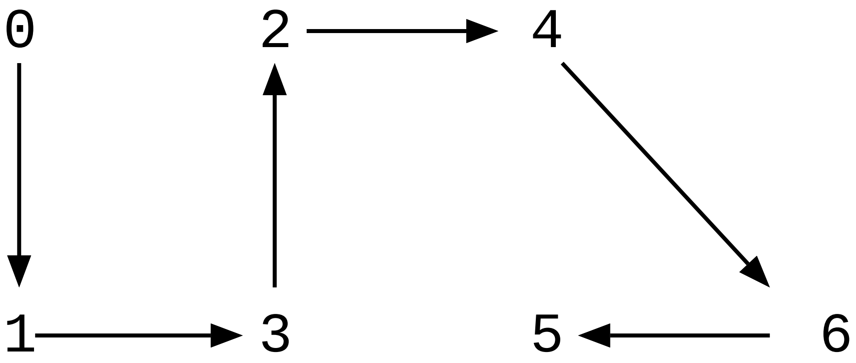
16-46: **DFS Search Trees**

- Starting from node 0, adjacency list sorted by vertex number:



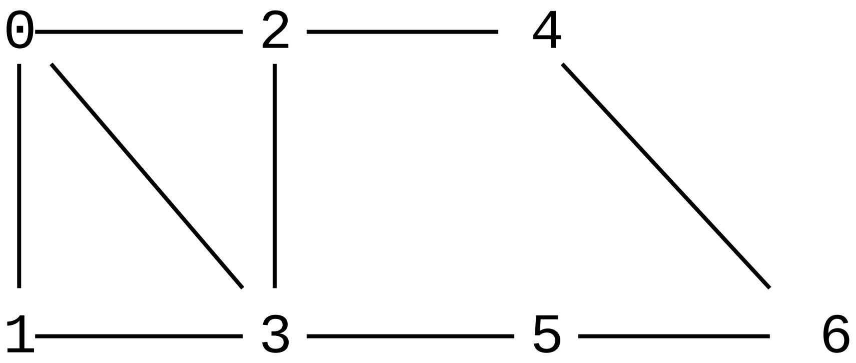
16-47: **DFS Search Trees**

- Starting from node 0, adjacency list sorted by vertex number:



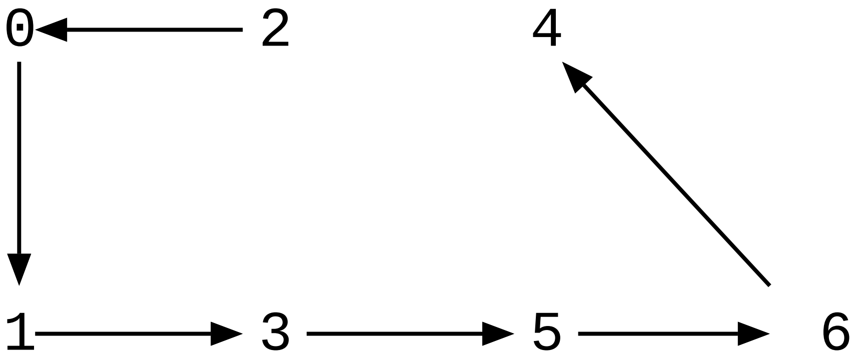
16-48: **DFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



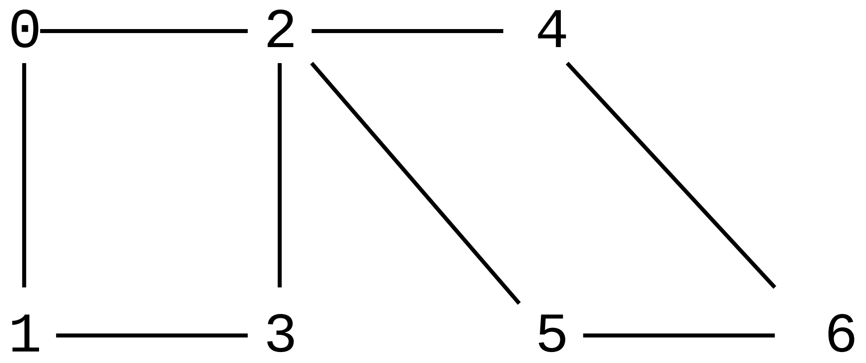
16-49: **DFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



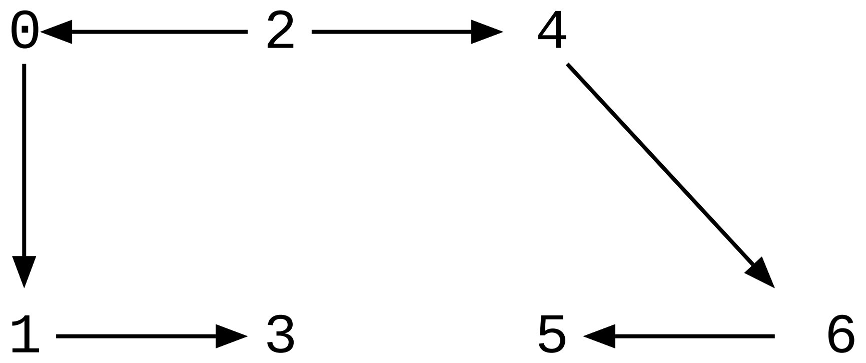
16-50: **DFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



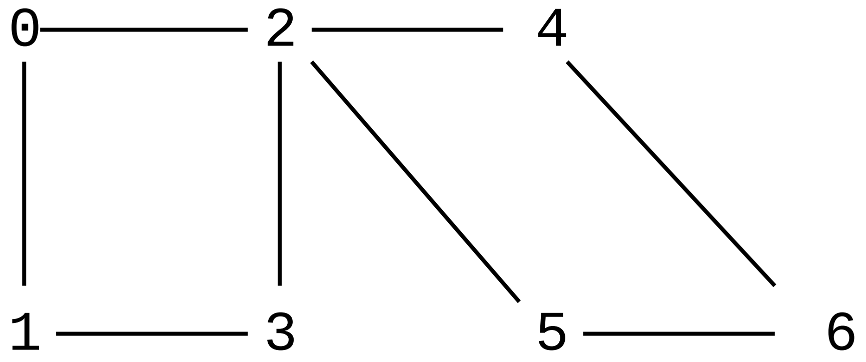
16-51: **DFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



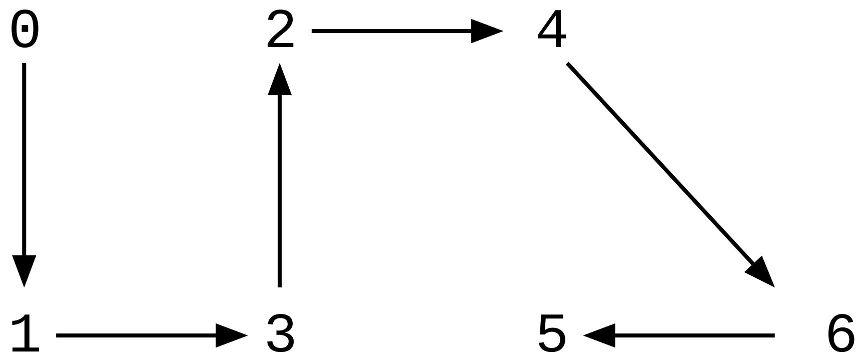
16-52: **DFS Search Trees**

- Starting from node 0, adjacency list sorted by vertex number:



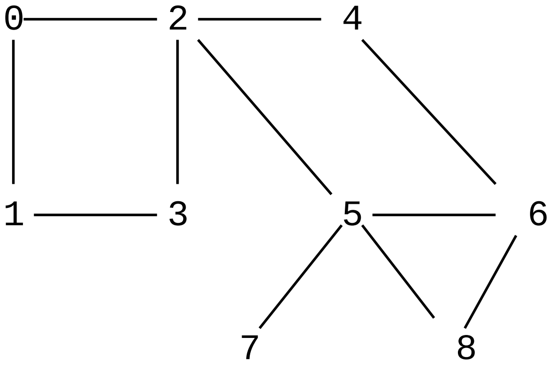
16-53: **DFS Search Trees**

- Starting from node 0, adjacency list sorted by vertex number:



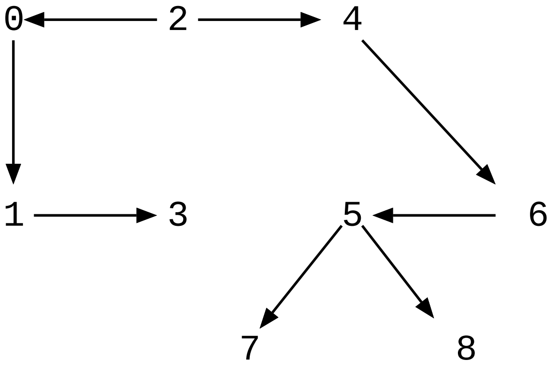
16-54: **DFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



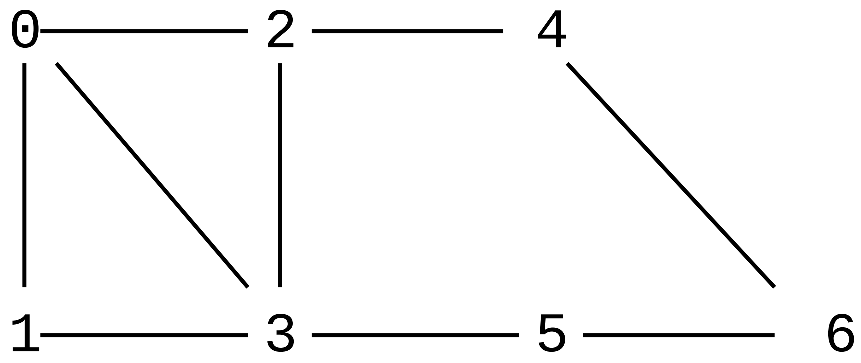
16-55: **DFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



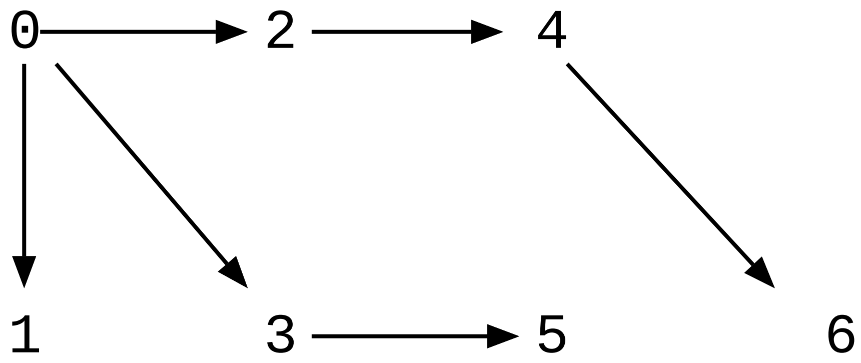
16-56: **BFS Search Trees**

- Starting from node 0, adjacency list sorted by vertex number:



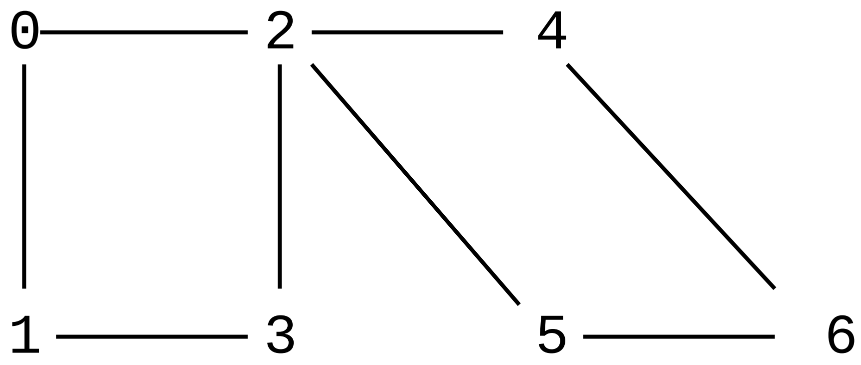
16-57: **BFS Search Trees**

- Starting from node 0, adjacency list sorted by vertex number:



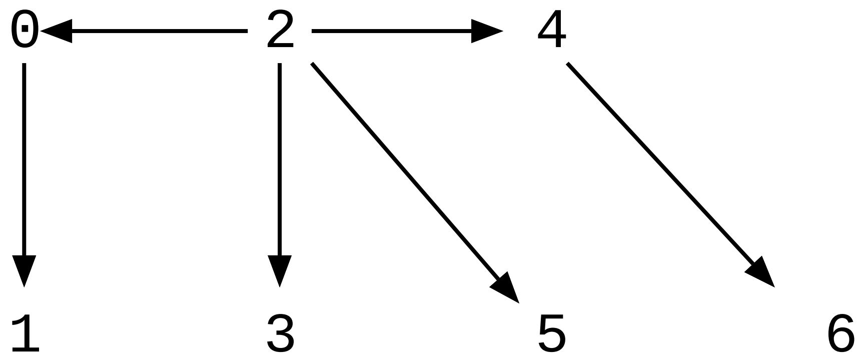
16-58: **BFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



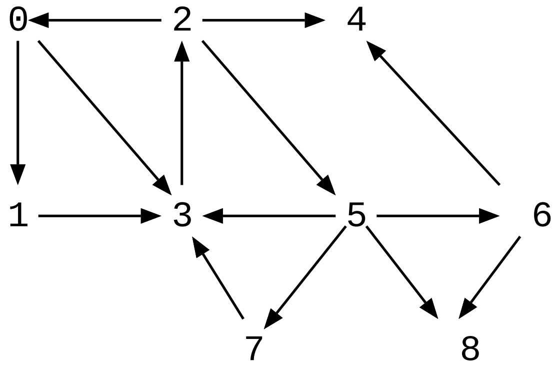
16-59: **BFS Search Trees**

- Starting from node 2, adjacency list sorted by vertex number:



16-60: **DFS in Directed Graphs**

- Starting from node 0, adjacency list sorted by vertex number:



16-61: **DFS in Directed Graphs**

- Starting from node 0, adjacency list sorted by vertex number: