

CS 662
Artificial Intelligence Programming Homework #4
First Order Logic, Probabilities and NLP

1.

- a. $\forall x \text{ mortal}(x) \wedge \text{holding}(x, \text{Ring}) \implies \text{tempted}(x)$
- b. $\text{hobbit}(\text{Frodo})$
- c. $\forall x \text{ hobbit}(x) \implies \text{mortal}(x)$
- d. $\forall x \text{ tempted}(x) \implies \text{putOn}(x, \text{Ring})$
- e. $\sim \text{holding}(\text{Frodo}, \text{Ring}) \implies \text{holding}(\text{Gandalf}, \text{Ring})$
- f. $\sim \text{holding}(\text{Gandalf}, \text{Ring})$

2.

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--> use: {x/Frodo} && b. hobbit(x) && c.  $\forall x \text{ hobbit}(x) \implies \text{mortal}(x)$ 
      get: g. mortal(Frodo)
--> use: e.  $\sim \text{holding}(\text{Frodo}, \text{Ring}) \implies \text{holding}(\text{Gandalf}, \text{Ring})$ 
      get: h.  $\sim \text{holding}(\text{Gandalf}, \text{Ring}) \implies \text{holding}(\text{Frodo}, \text{Ring})$ 
--> use: f.  $\sim \text{holding}(\text{Gandalf}, \text{Ring})$  &&
      h.  $\sim \text{holding}(\text{Gandalf}, \text{Ring}) \implies \text{holding}(\text{Frodo}, \text{Ring})$ 
      get: i. holding(Frodo, Ring)
--> use: {x/Frodo} && g. mortal(Frodo) && i. holding(Frodo, Ring)
      && a.  $\forall x \text{ mortal}(x) \wedge \text{holding}(x, \text{Ring}) \implies \text{tempted}(x)$ 
      get: j. mortal(Frodo)  $\wedge \text{holding}(\text{Frodo}, \text{Ring}) \implies \text{tempted}(\text{Frodo})$ 
--> use: {x/Frodo} && j. tempted(Frodo) && d.  $\forall x \text{ tempted}(x) \implies \text{putOn}(x, \text{Ring})$ 
      get: k. tempted(Frodo)  $\implies \text{putOn}(\text{Frodo}, \text{Ring})$ 
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3.

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--> To prove: 1. putOn(Frodo, Ring)
--> 1 unifiers with d. To prove:
    2. tempted(Frodo)
--> 2 unifiers with a. To prove:
    3. mortal(Frodo), 4. holding(Frodo, Ring)
--> 3 unifiers with c. To prove:
    5. hobbit(Frodo), 4. holding(Frodo, Ring)
--> 5 unifiers with b. To prove:
    4. holding(Frodo, Ring)
--> 4 unifiers with e. To prove:
    6.  $\sim \text{holding}(\text{Gandalf}, \text{Ring})$ 
--> 6 unifiers with f
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The list of goals is empty, so we are done.

4.

- 1. $\sim \text{mortal}(x) \vee \sim \text{holding}(x, \text{Ring}) \vee \text{tempted}(x)$
- 2. $\text{hobbit}(\text{Frodo})$
- 3. $\sim \text{hobbit}(x) \vee \text{mortal}(x)$
- 4. $\sim \text{tempted}(x) \vee \text{putOn}(x, \text{Ring})$
- 5. $\text{holding}(\text{Frodo}, \text{Ring}) \vee \text{holding}(\text{Gandalf}, \text{Ring})$
- 6. $\sim \text{holding}(\text{Gandalf}, \text{Ring})$
- 7. $\sim \text{putOn}(\text{Frodo}, \text{Ring})$

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--> Resolve 4 && 7. Add:
    8.  $\sim \text{tempted}(\text{Frodo})$ 
--> Resolve 1 && 8. Add:
    9.  $\sim \text{mortal}(x) \vee \sim \text{holding}(x, \text{Ring})$ 
--> Resolve 2 && 3. Add:
    10. mortal(Frodo)
--> Resolve 5 && 6. Add:
    11. holding(Frodo, Ring)
--> Resolve 9 && 10. Add:
    12.  $\sim \text{holding}(\text{Frodo}, \text{Ring})$ 
--> Resolve 11 && 12. Contradiction!
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So $\text{putOn}(\text{Frodo}, \text{Ring})$ is true.

5.

- a. $P(\text{toothache}) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$
- b. $P(\text{Cavity}) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$
- c. $P(\text{Toothache} | \text{cavity}) = P(\text{Toothache}, \text{cavity}) / P(\text{cavity})$
 $= (0.108 + 0.012) / 0.2 = 0.6$
- d. $P(\text{Cavity} | \text{toothache} \vee \text{catch}) = P(\text{Cavity} \wedge (\text{toothache} \vee \text{catch})) / P(\text{toothache} \vee \text{catch})$
 Since:
 $P(\text{Cavity} \wedge (\text{toothache} \vee \text{catch})) = 0.108 + 0.012 + 0.072 = 0.192$
 $P(\text{toothache} \vee \text{catch}) = 0.108 + 0.016 + 0.012 + 0.064 + 0.072 + 0.144 = 0.416$

So $P(\text{Cavity} | \text{toothache} \vee \text{catch}) = 0.192 / 0.416 = 0.462$

6.

We know:

$P(\text{Forget} | \text{Complete}) = 0.01$

$P(\text{Forget} | \sim \text{Complete}) = 0.5$

$P(\text{Complete}) = 0.9$

We want to know: $P(\text{Complete} | \text{Forget})$

$P(\text{Complete} | \text{Forget}) = (P(\text{Forget} | \text{Complete}) * P(\text{Complete})) / P(\text{Forget})$

From $P(\text{Forget} | \text{Complete}) = 0.01$

From $P(\text{Forget} | \sim \text{Complete}) = 0.5$

And $P(\text{Forget}) = P(\text{Forget} | \text{Complete}) * P(\text{Complete}) + P(\text{Forget} | \sim \text{Complete}) * P(\sim \text{Complete})$
 $= 0.01 * 0.9 + 0.5 * 0.1 = 0.059$

So $P(\text{Complete} | \text{Forget}) = (0.01 * 0.9) / 0.059 = 0.15$

7.

0.1: VP → Verb

0.2: VP → Copula Adjective

0.5: VP → Verb the Noun

0.2: VP → VP Adverb

0.5: Verb → is

0.5: Verb → shoots

0.8: Copula → is

0.2: Copula → seems

0.5: Adjective → unwell

0.5: Adjective → well

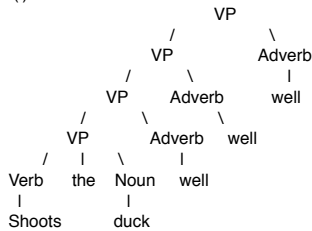
0.5: Adverb → well

0.5: Adverb → badly

0.6: Noun → duck

0.4: Noun → well

a. (i) shoots the duck well well well :



So it's nonzero probability.

(ii) seems the well well:

Copula → seems, so the only VP is VP → Copula Adjective, cannot be followed by "the".

So cannot parse.

(iii) shoots the unwell well badly:

VP → Verb the Noun is the only VP that within "the", but Adjective → unwell, not Noun.

So cannot parse.

b. 1. $P(\text{"is well well"}) = P(\text{VP} \rightarrow \text{VP Adverb}) * P(\text{VP} \rightarrow \text{VP Adverb}) * P(\text{VP} \rightarrow \text{Verb}) * P(\text{Verb} \rightarrow \text{is}) *$

$P(\text{Adverb} \rightarrow \text{well}) * P(\text{Adverb} \rightarrow \text{well})$

$= 0.2 * 0.2 * 0.1 * 0.5 * 0.5 * 0.5 = 0.0005$

2. $P(\text{"is well well"}) = P(\text{VP} \rightarrow \text{VP Adverb}) * P(\text{VP} \rightarrow \text{Copula Adjective}) * P(\text{Copula} \rightarrow \text{is}) *$

$P(\text{Adjective} \rightarrow \text{well}) * P(\text{Adverb} \rightarrow \text{well})$

$= 0.2 * 0.2 * 0.8 * 0.5 * 0.5 = 0.008$

So $P(\text{"is well well"}) = 0.008 + 0.0005 = 0.0085$

c. It's syntactic. Since we have two parse tree for the phrase, so it's syntactic ambiguity.

d. Yes, it's possible. We can find all the trees that with 10-word sentences and then add up the probability.

8.

Pronoun → someone

Adv → slowly

Article → the

Verb → walked

Prep → to

Noun → supermarket

(A):

S → NP VP

NP → Pronoun

NP → Article Noun

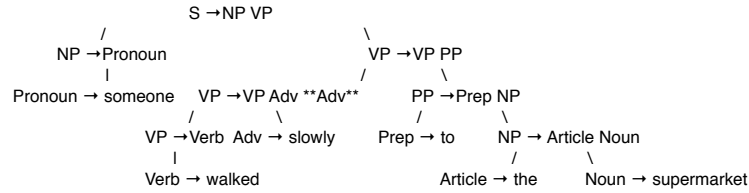
VP → VP PP

VP → VP Adv Adv

VP → Verb

PP → Prep NP

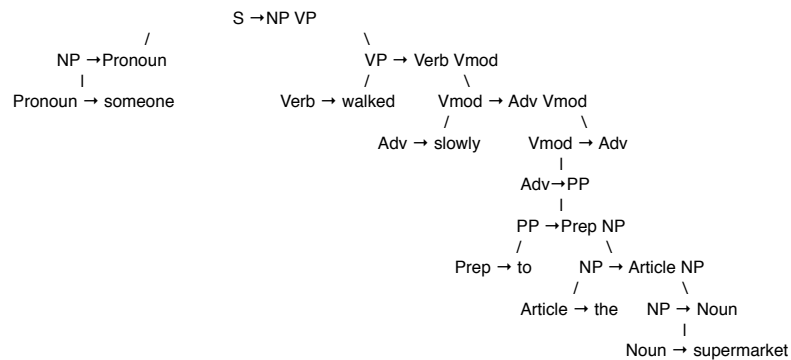
NP → Noun



In the place where is marked with "**", there is one more Adv. So it cannot generate the given sentence.

(B):

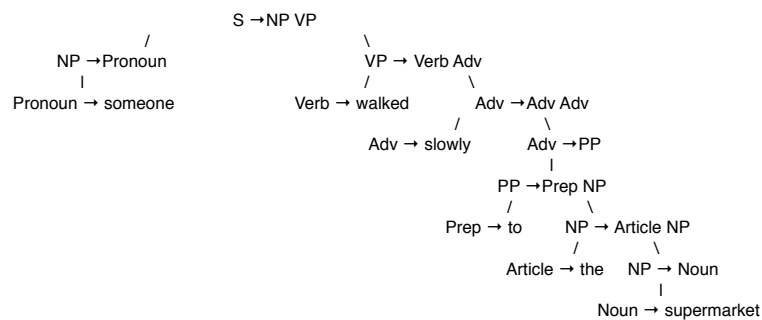
S → NP VP
 NP → Pronoun
 NP → Noun
 NP → Article NP
 VP → Verb Vmod
 Vmod → Adv Vmod
 Vmod → Adv
 Adv → PP
 PP → Prep NP



So it can generate the given sentence.

(C):

S → NP VP
 NP → Pronoun
 NP → Article NP
 VP → Verb Adv
 Adv → Adv Adv
 Adv → PP
 PP → Prep NP
 NP → Noun



So it can generate the given sentence.

(d) Writing sentence:

Pronoun → someone
 Adv → happily, early, very, well
 Article → the
 Verb → played, goes, flied, barks
 Prep → in, to, into, on
 Noun → dog, yard, morning, bird, sky, beach, work

(A) English:

The dog played very happily in the yard.
Someone goes to the beach very early.
The bird flew into the sky.

Non-English:

The dog goes happily early very well in the beach.
The bird flew to morning into work.
Someone played in the yard very happily well early to the morning.

(B)English:

The dog goes to the yard early.
Someone goes to work very happily.
The bird flew very early into the yard.

Non-English:

The someone goes early happily well very.
The the the morning played happily.
The the the the work flew well very happily early.

(C)English:

The bird played very happily on the beach.
Someone goes very early to the beach in the morning.
The dog barks early in the morning.

Non-English:

The sky flew well early very happily in the work.
The work goes very well early happily in the yard on the beach into the sky.
The Someone barks to the dog into the work on the bird in the sky.

Suggestion:

All of these three grammars are easy goes into endless loop. For example:

- (A) VP → VP PP and VP → VP Adv Adv
 - (B) VP → Verb Vmod with Vmod → Adv Vmod
 - (C) VP → Verb Adv with Adv → Adv Adv and VP → Verb Adv with Adv → Adv Adv, Adv → PP
- Also, in (B) and (C), there is easy to have NP → Article Pronoun, which is wrong grammar.

So we can do:

- (A) Change VP → VP PP to VP1 → Verb PP
Change VP → VP Adv Adv to VP2 → Verb Adv Adv
Add: VP → VP1
VP → VP2
- (B) Change: NP → Pronoun to NP1 → Pronoun
Change: NP → Noun to NP2 → Noun
Change: NP → Article NP to NP → Article NP2
Add: NP → NP1

Change: Vmod → Adv Vmod to Vmod → Adv Vmod1
Change: Vmod → Adv to Vmod1 → Adv
- (C) Change: NP → Pronoun to NP1 → Pronoun
Change: NP → Noun to NP2 → Noun
Change: NP → Article NP to NP → Article NP2
Add: NP → NP1

Change: Adv → Adv Adv to Adv → Adv1 Adv1
Change: Adv → PP to Adv1 → PP
Add: Adv → Adverb