07-0: **Binary Tree Definition**

```
class Node {
  Node() { }
  Node(Comparable elem) {
    element_ = element;
  }
  Node(Object element, Node left, Node right) {
    element_ = element;
    left_ = left;
    right_ = right;
  }
   /* Access methods on next slide */
   private Node left_;
   private Node right_;
   private Comparable element_;
}
```

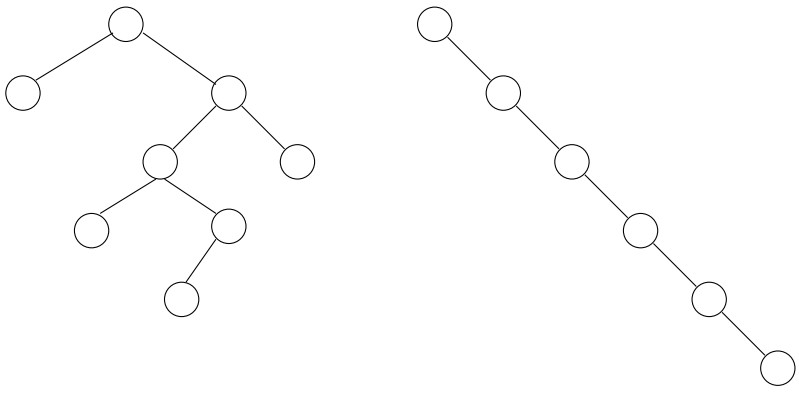
07-1: **Binary Tree Access Methods**

```
Node left() {
  return left_;
}

Node right() {
  return right_;
}

Comparable element() {
  return element_;
}

void setElement(Comparable element) {
  element_ = element;
}
```

```
void setLeft(Node left) {
  left_ = left;
}

void setRight(Node right) {
  right_ = right;
}
```

07-2: **Tree Operations – Height**

- Returns the height of the tree
  - (Length of the path to the deepest leaf) + 1



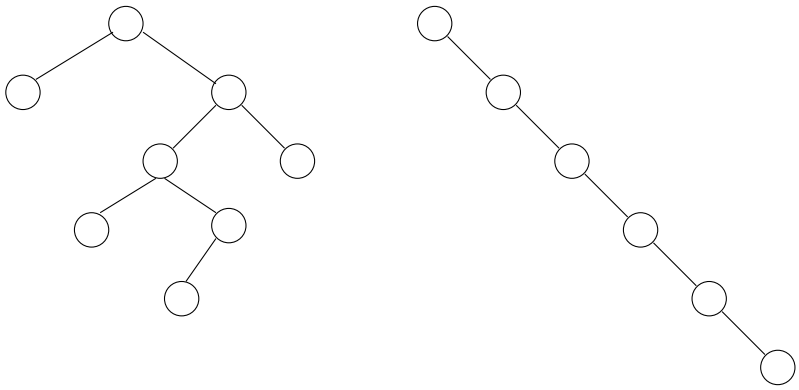
Height = 5 Height = 6

07-3: **Tree Operations – Height**

```
int height(Node tree) {
  if (tree == null)
    return 0;
  return 1 + MAX(height(tree.left()),
                 height(tree.right()));
}
```

07-4: **Tree Operations – NumNodes**

- Returns the number of nodes in a tree

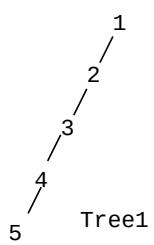


Number of Nodes = 8 Number of Nodes = 6

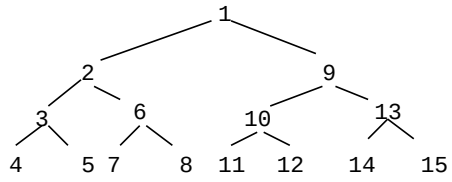
07-5: **Tree Operations – NumNodes**

```
int numNodes(Node tree) {
  if (tree == null)
    return 0;
  return 1 + numNodes(tree.left(),
                      tree.right());
```

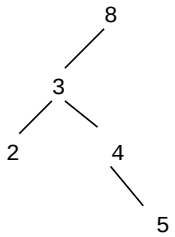
07-6: **Writing Tree Functions**



Tree1



Tree2

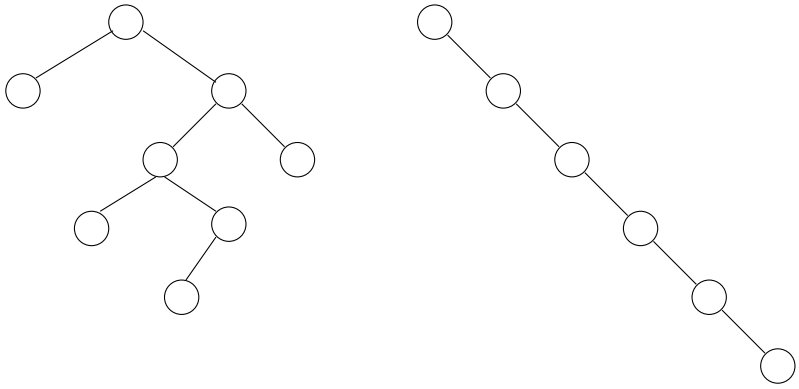


Tree3

Write `find, numLeaves, shallowestleaf`

07-7: **Tree Operations – NumLeaves**

- Returns the number of leaves in a tree



Number of Leaves = 4 Number of Leaves = 1

07-8: **Tree Operations – NumLeaves**

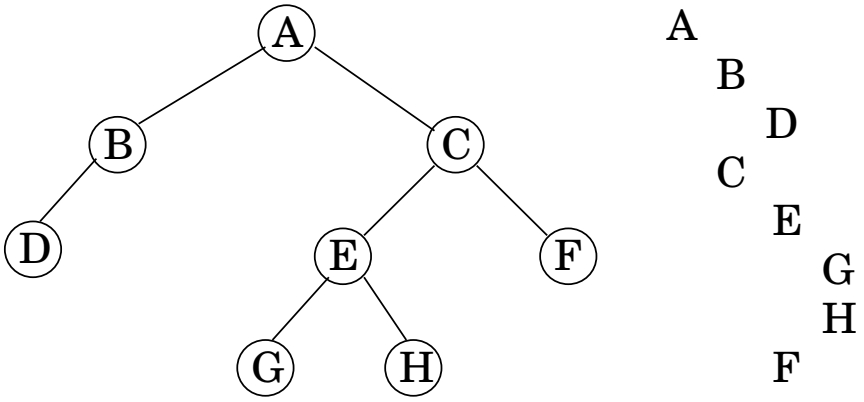
```
int numLeaves(Node tree) {
  if (tree == null)
    return 0;
  if ((tree.left() == null) &&
      (tree.right() == null))
    return 1;
  return numLeaves(tree.left()) +
         numLeaves(tree.right());
}
```

07-9: **Tree Traversals**

- PREORDER Traversal
  - Do operation on root of the tree
  - Traverse left subtree
  - Traverse right subtree
- INORDER Traversal
  - Traverse left subtree
  - Do operation on root of the tree
  - Traverse right subtree
- POSTORDER Traversal
  - Traverse left subtree
  - Traverse right subtree
  - Do operation on root of the tree

07-10: **PREORDER Traversal**

Printing out trees (Showing the shape of the tree in the printout)



07-11: **PREORDER Traversal**

Printing out trees (Showing the shape of the tree in the printout)

- First print the root at current indent level
  - Print the left subtree with larger indentation
  - Print the right subtree with larger indentation

07-12: **Printing Binary Trees**

```
void print(Node tree, int indent) {
  if (tree != null) {
    for(int i=0; i<indent; i++) {
      System.out.print("\t");
    System.out.println(tree.element().toString());
  print(tree.left(), indent + 1);
  print(tree.right(), indent + 1);
}
```

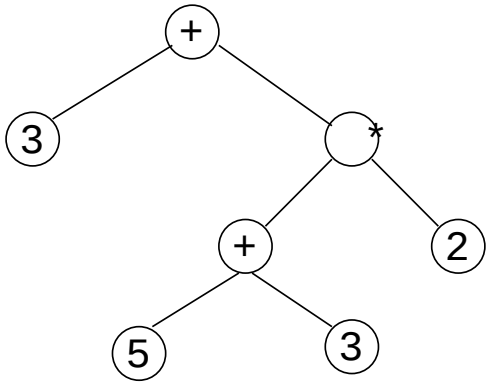
07-13: **INORDER Traversal**

Printing all elements in a Binary Search Tree in order

- (Already covered in previous slides)

07-14: **POSTORDER Traversal**

Calculating the Value of an expression tree



07-15: **POSTORDER Traversal**

Calculating the Value of an expression tree

- Base case:
  - Return value stored at leaf
- Recursive case:
  - Calculate value of left subtree
  - Calculate value of right subtree
  - Calculate expression value

07-16: **Expression Tree Value**

```
int value(Node tree) {
  if (tree.left() == null && tree.right() == null)
    return ((Integer) tree.element()).intValue();
  int left = value(tree.left());
  int right = value (tree.right());
  char op = ((Character) tree.element()).charValue();
  switch (op) {
    case '+':
      return left + right;
    case '*':
      return left * right;
      ...
  }
}
```