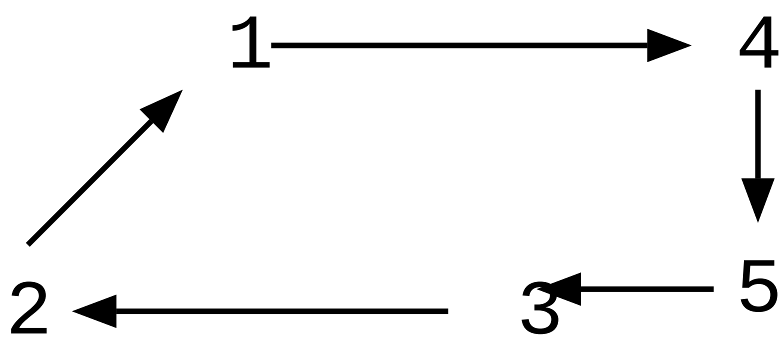
21-0: **Strongly Connected Graph**

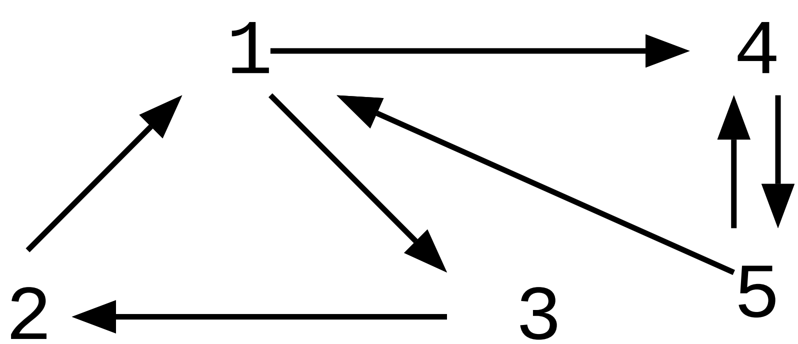
- Directed Path from every node to every other node



- Strongly Connected

21-1: **Strongly Connected Graph**

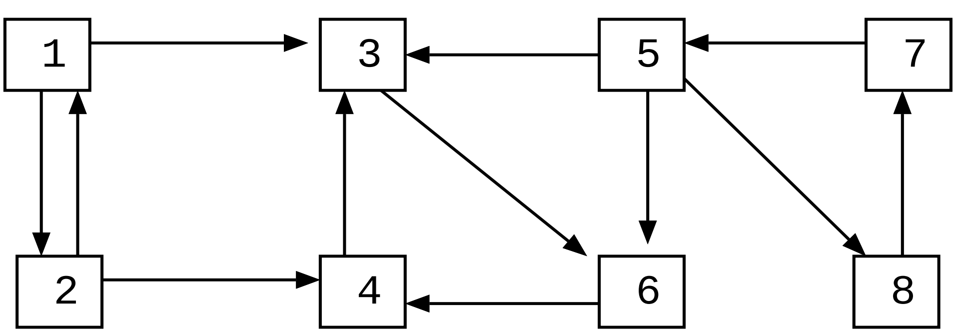
- Directed Path from every node to every other node



- Strongly Connected

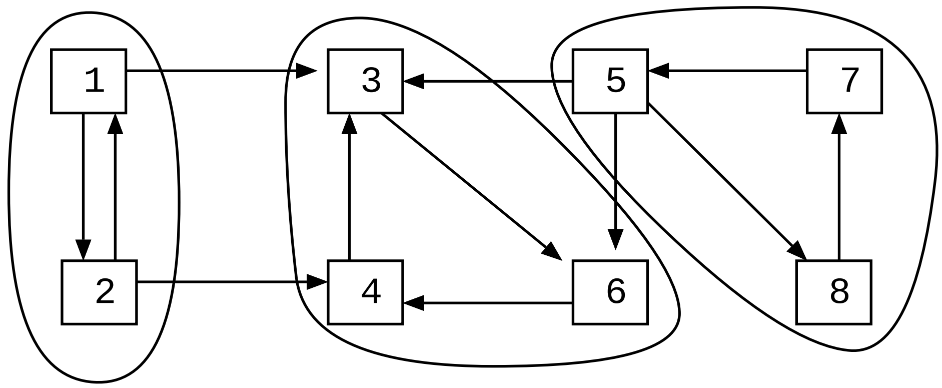
21-2: **Connected Components**

- Subgraph (subset of the vertices) that is strongly connected.



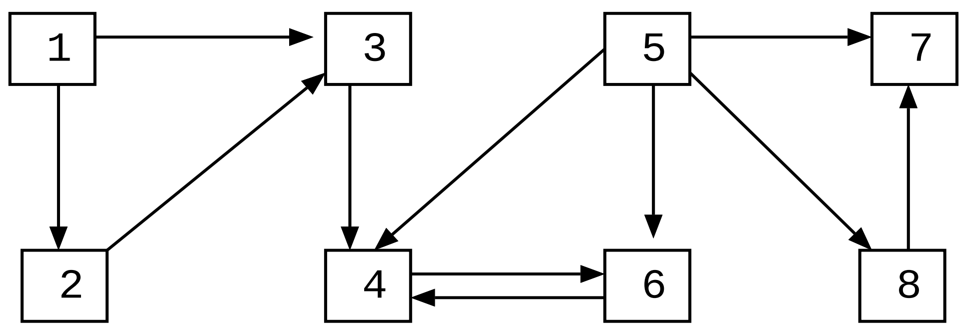
21-3: **Connected Components**

- Subgraph (subset of the vertices) that is strongly connected.



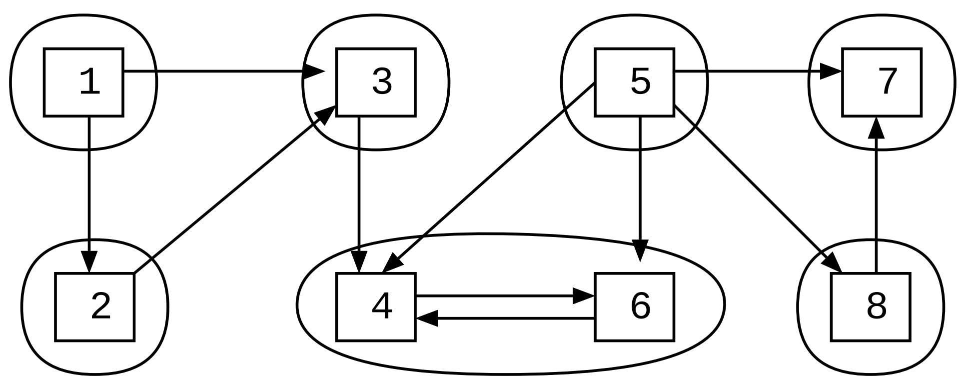
21-4: **Connected Components**

- Subgraph (subset of the vertices) that is strongly connected.



21-5: **Connected Components**

- Subgraph (subset of the vertices) that is strongly connected.



21-6: **Connected Components**

- Connected components of the graph are the *largest possible* strongly connected subgraphs
- If we put each vertex in its own component – each component would be (trivially) strongly connected
    - Those would not be the connected components of the graph – unless there were no larger connected subgraphs

21-7: **Connected Components**

- Calculating Connected Components
  - Two vertices v_1 and v_2 are in the same connected component if and only if:
    - Directed path from v_1 to v_2
    - Directed path from v_2 to v_1
  - To find connected components – find directed paths
    - Use DFS

21-8: **DFS Revisited**

- We can keep track of the order in which we visit the elements in a Depth-First Search
- For any vertex v in a DFS:
  - d[v] = *Discovery* time – when the vertex is first visited
  - f[v] = *Finishing* time – when we have finished with a vertex (and all of its children)

21-9: **DFS Revisited**

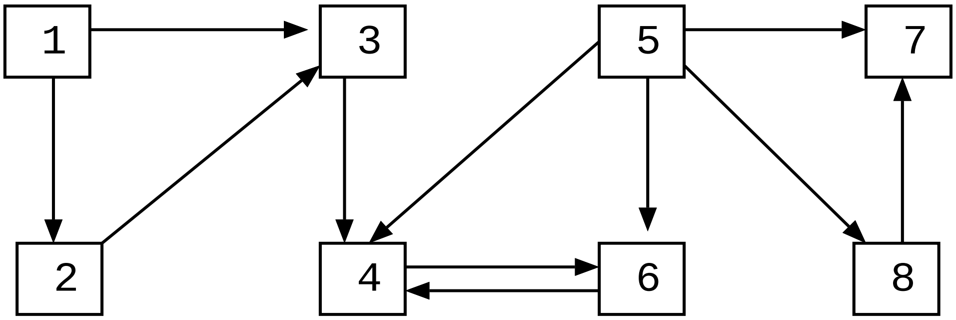
```
class Edge {
   public int neighbor;
   public int next;
}

void DFS(Edge G[], int vertex, boolean Visited[], int d[], int f[]) {
  Edge tmp;
  Visited[vertex] = true;
  d[vertex] = time++;
  for (tmp = G[vertex]; tmp != null; tmp = tmp.next) {
    if (!Visited[tmp.neighbor])
      DFS(G, tmp.neighbor, Visited);
  }
  f[vertex] = time++;
}
```

21-10: **DFS Revisited**

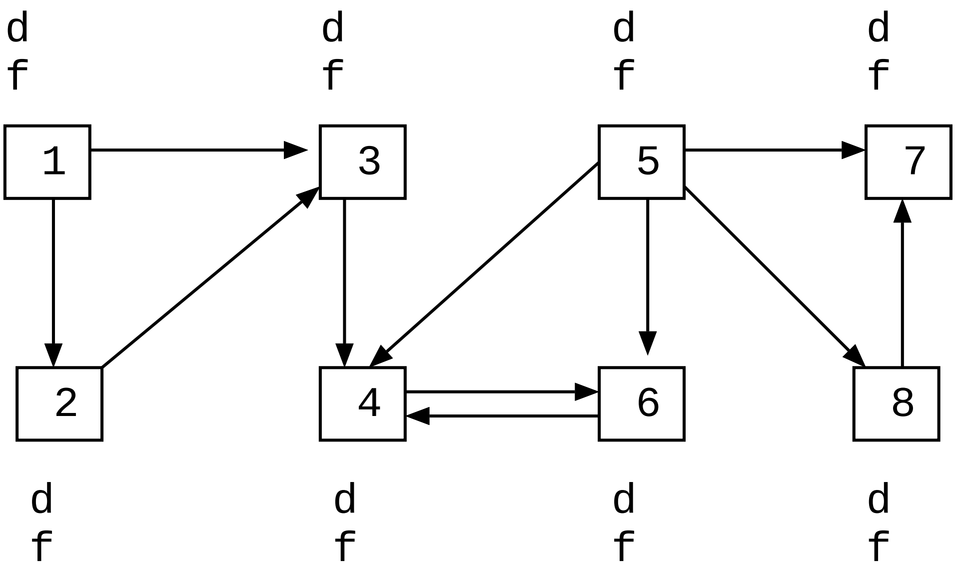
- To visit every node in the graph:

```
TraverseDFS(Edge G[]) {
  int i;
  boolean Visited = new boolean[G.length];
  int d = new int[G.length];
  int v = new int[G.length];
  time = 1;
  for (i=0; i<G.length; i++)
    Visited[i] = false;
  for (i=0; i<G.length; i++)
    if (!Visited[i])
      DFS(G, i, Visited, d, f);
}
```

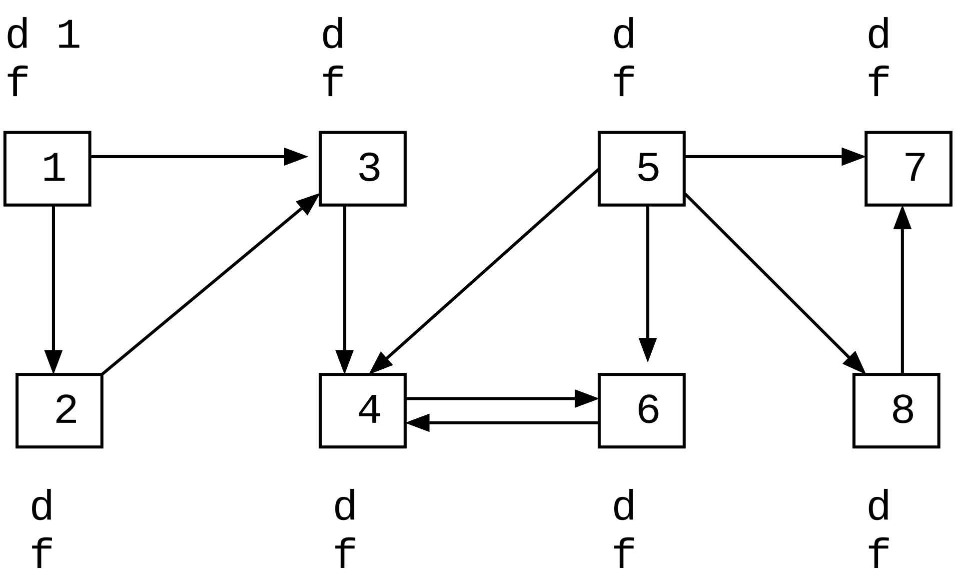
21-11: **DFS Example**



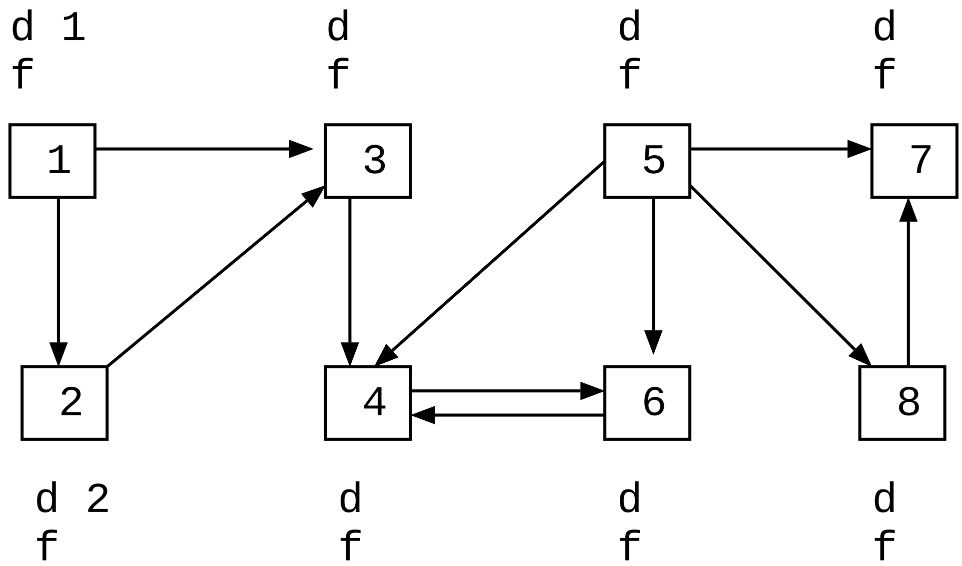
21-12: **DFS Example**



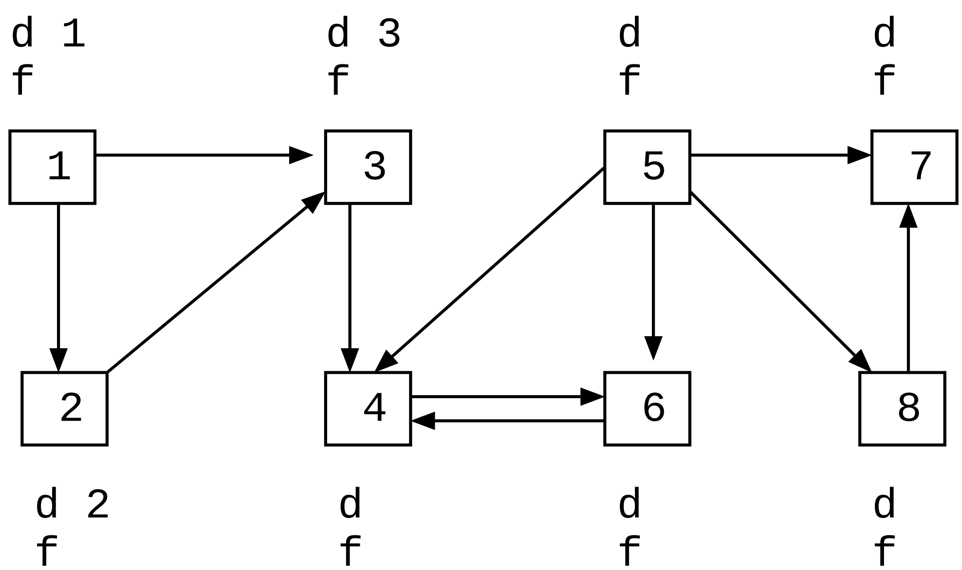
21-13: **DFS Example**



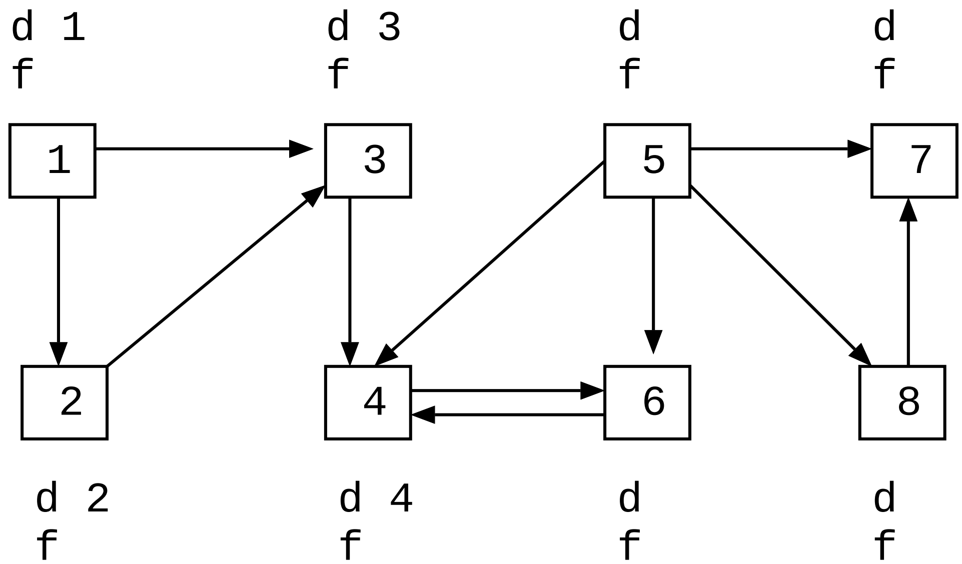
21-14: **DFS Example**



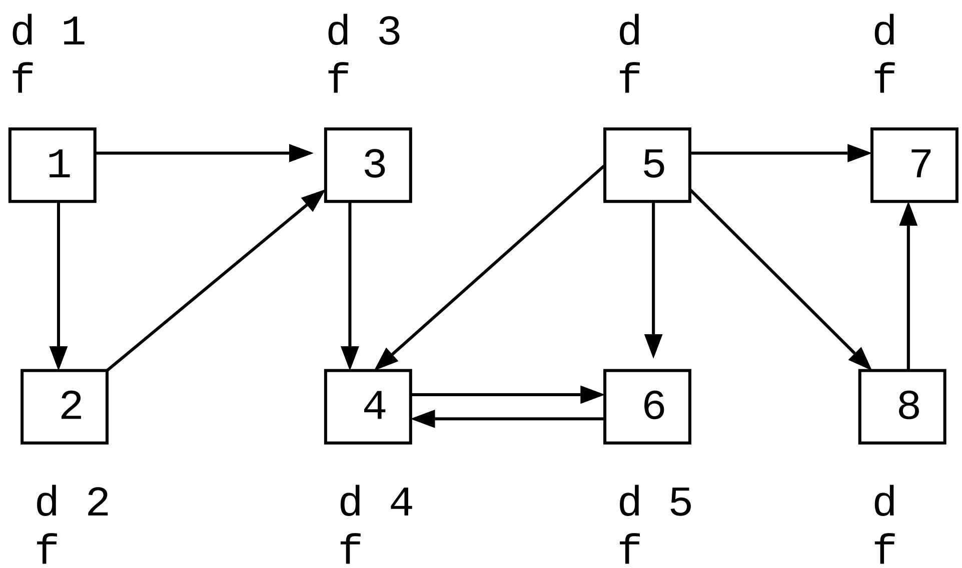
21-15: **DFS Example**



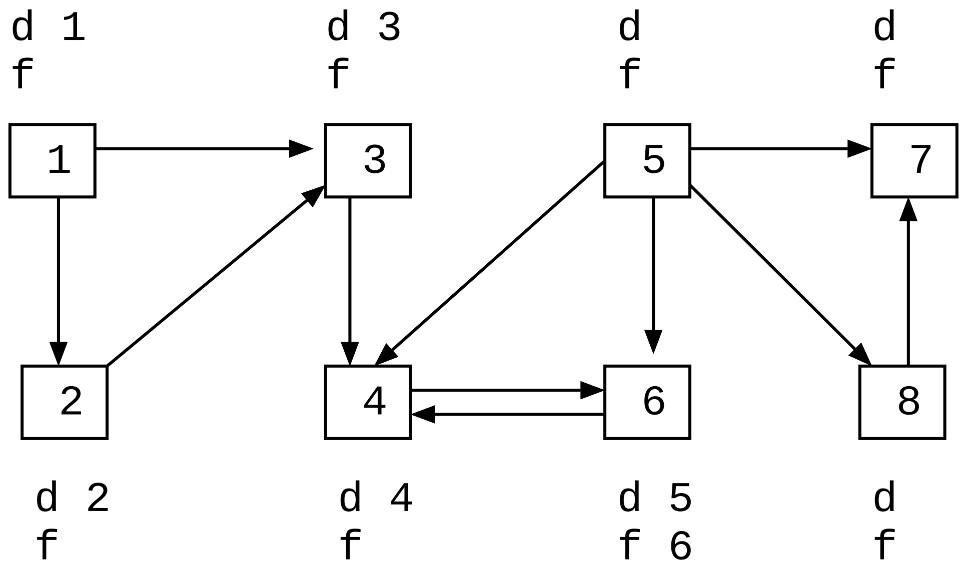
21-16: **DFS Example**



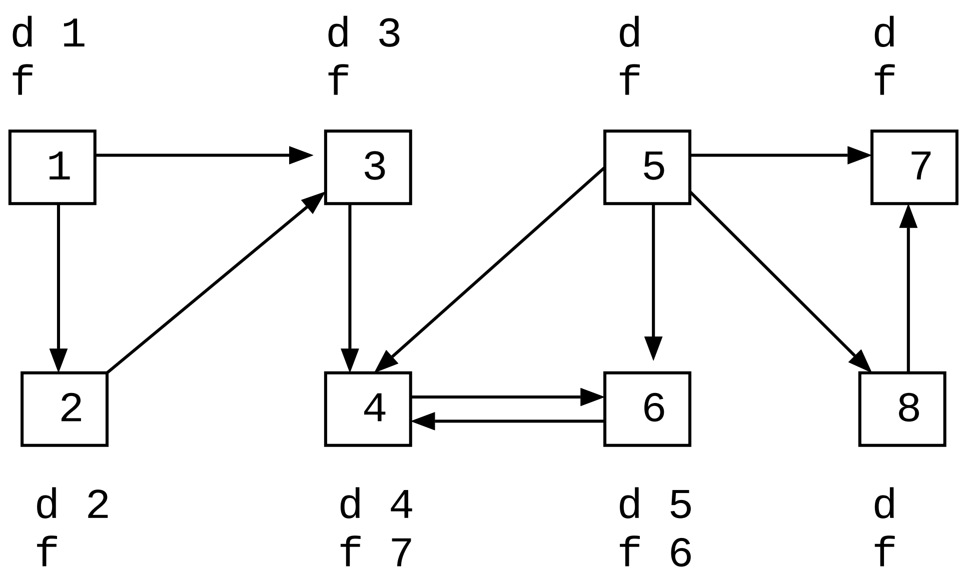
21-17: **DFS Example**



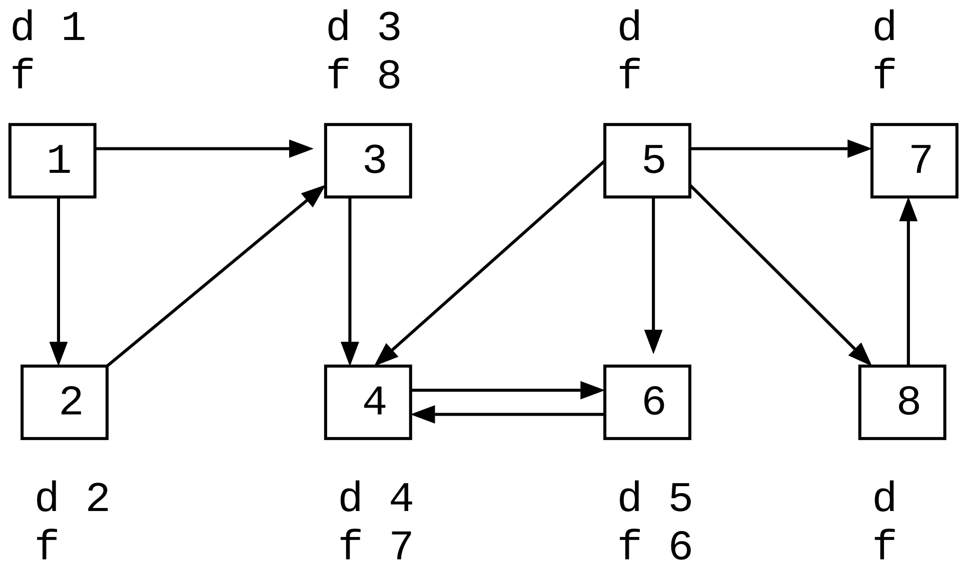
21-18: **DFS Example**



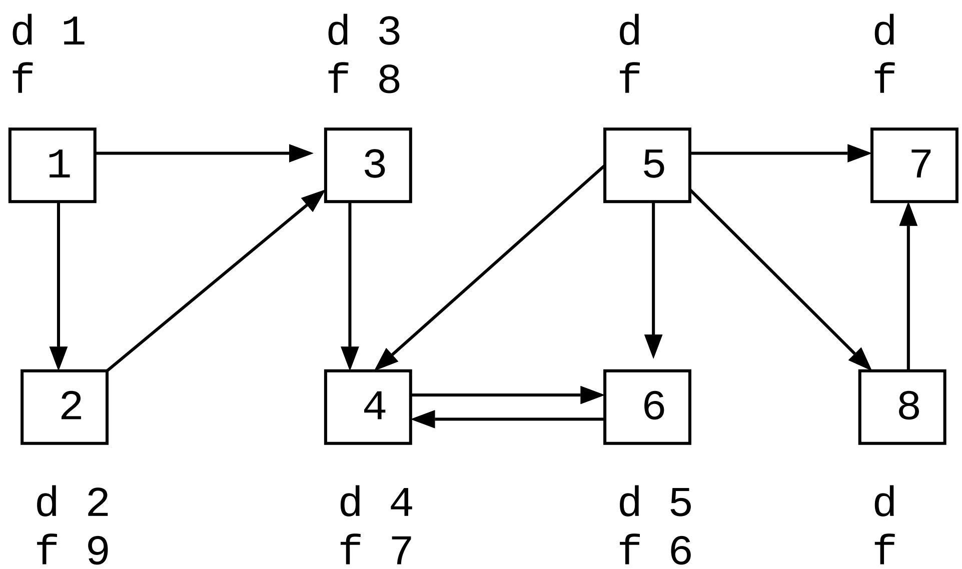
21-19: **DFS Example**



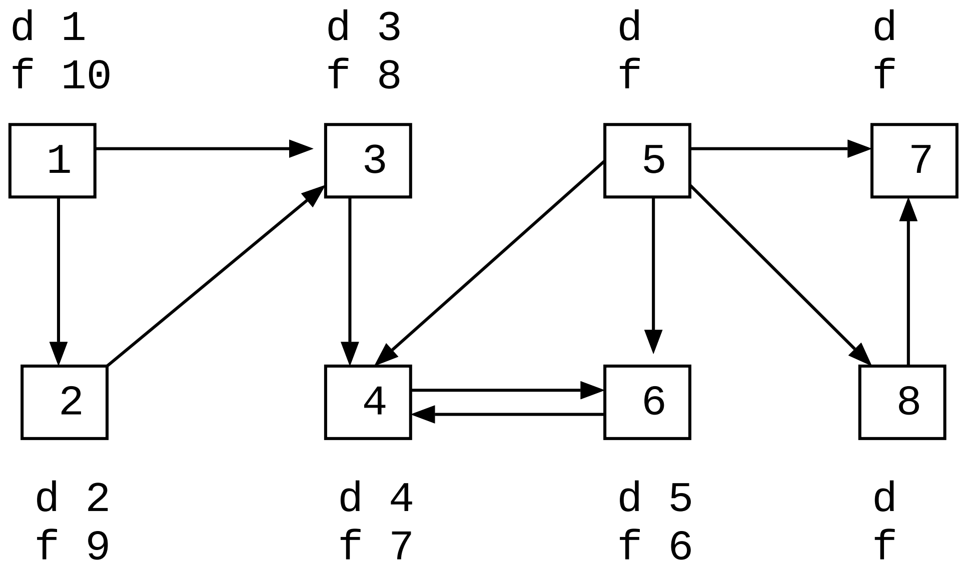
21-20: **DFS Example**



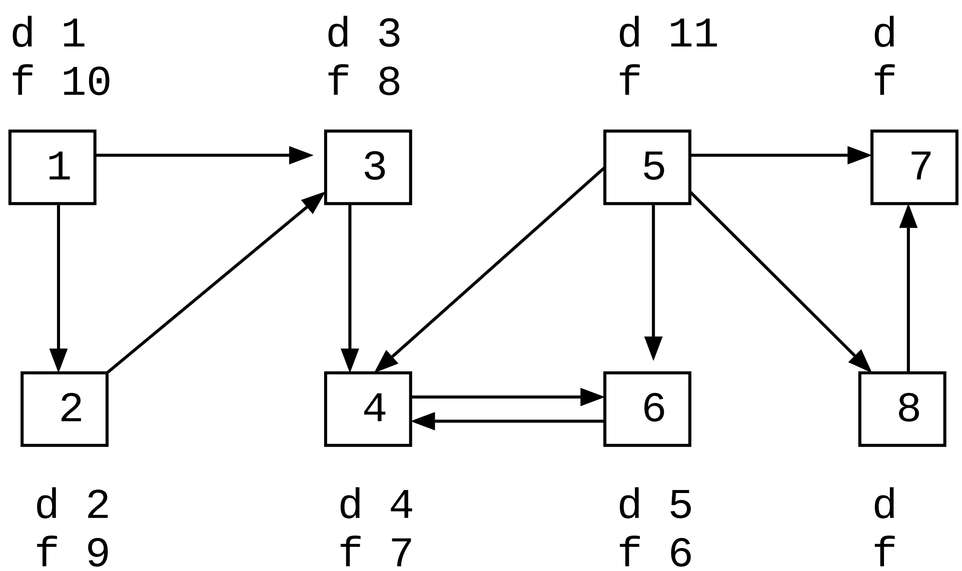
21-21: **DFS Example**



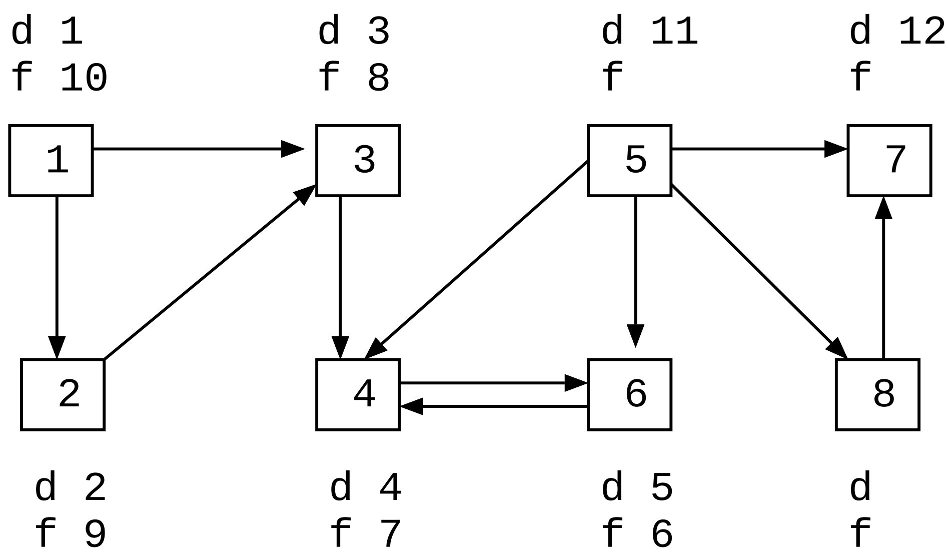
21-22: **DFS Example**



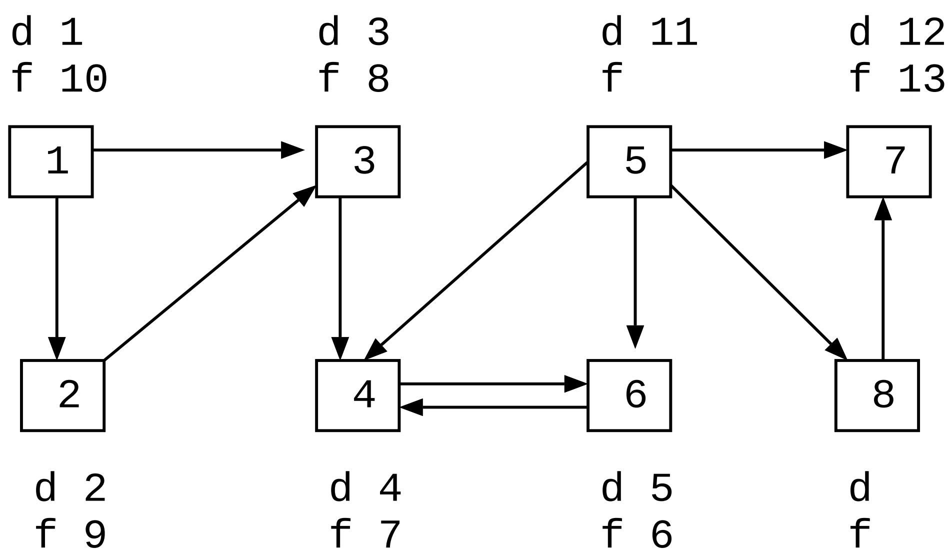
21-23: **DFS Example**



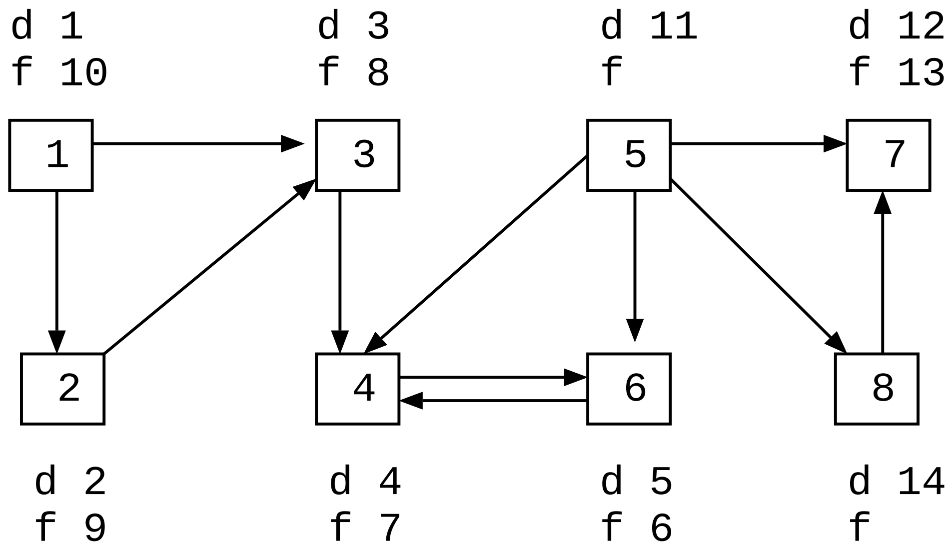
21-24: **DFS Example**



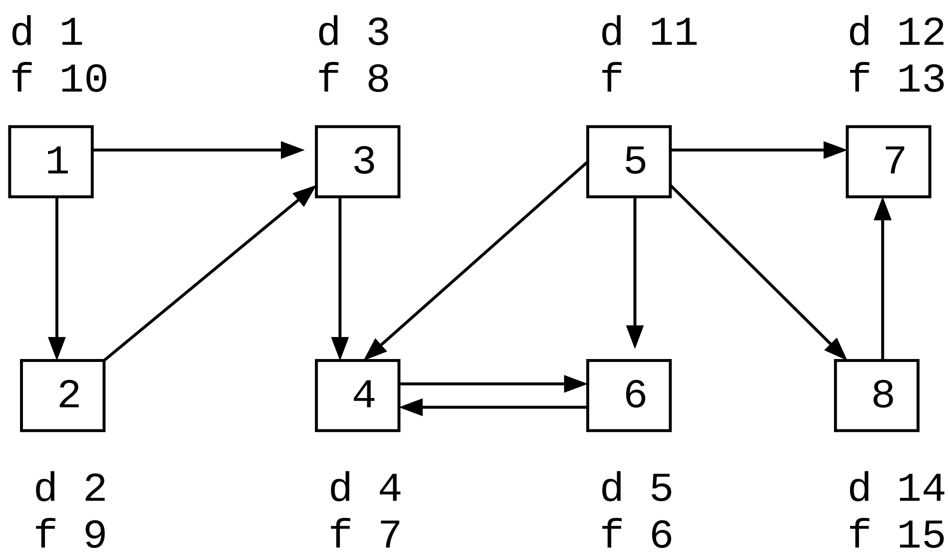
21-25: **DFS Example**



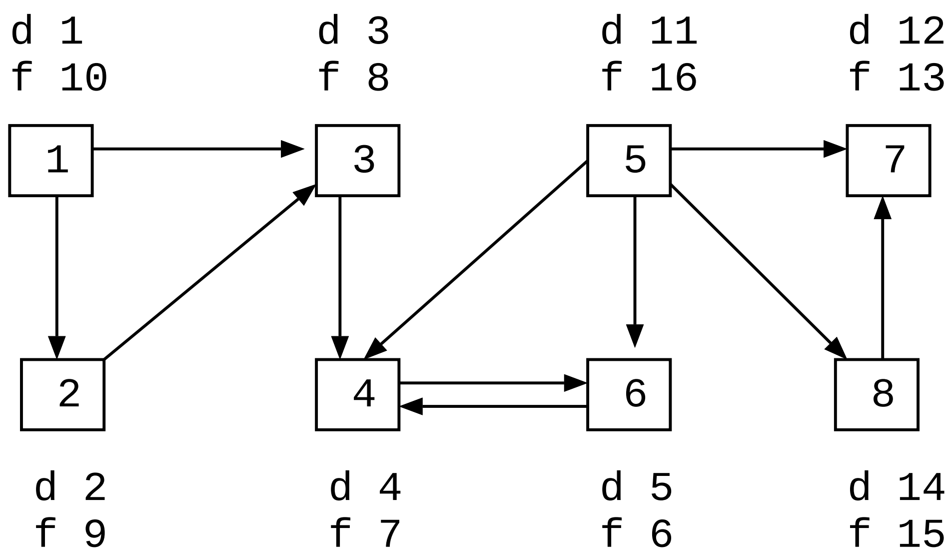
21-26: **DFS Example**



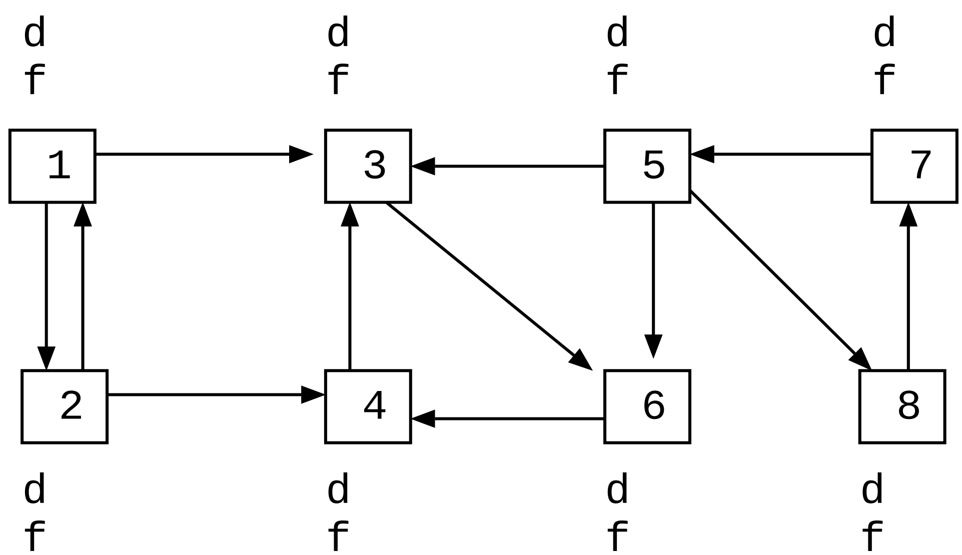
21-27: **DFS Example**



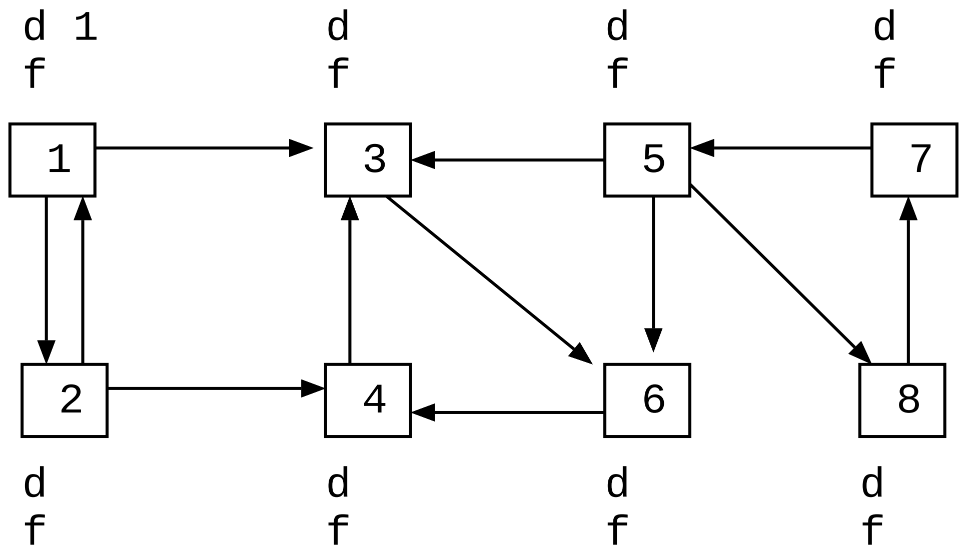
21-28: **DFS Example**



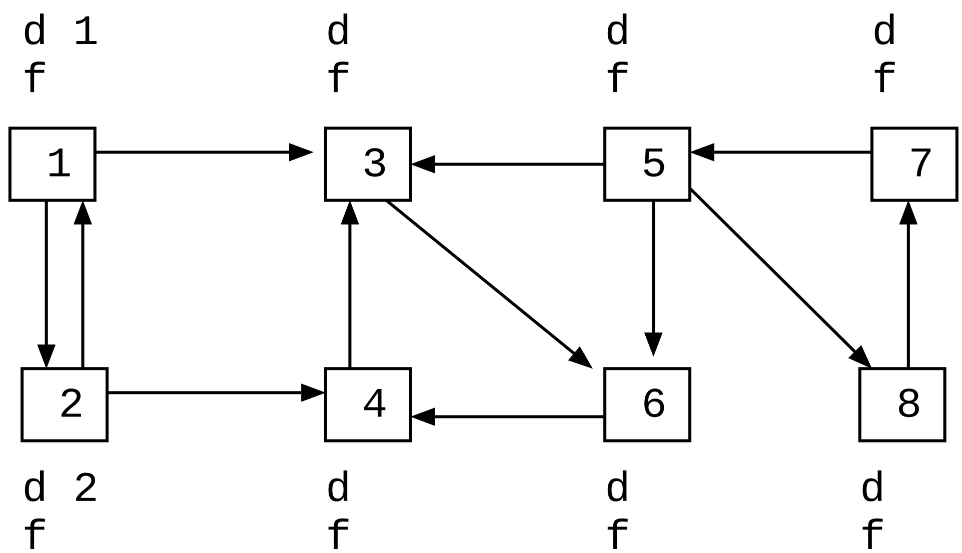
21-29: **DFS Example**



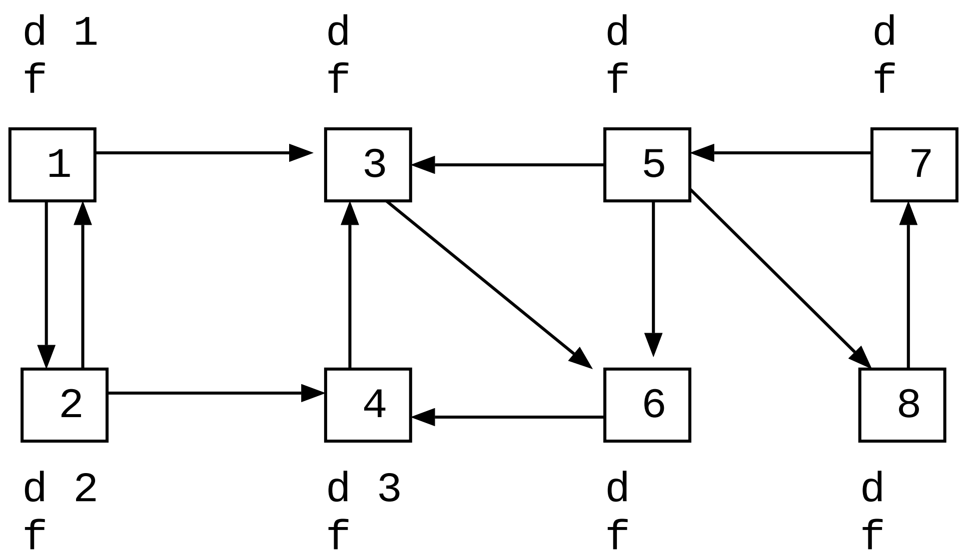
21-30: **DFS Example**



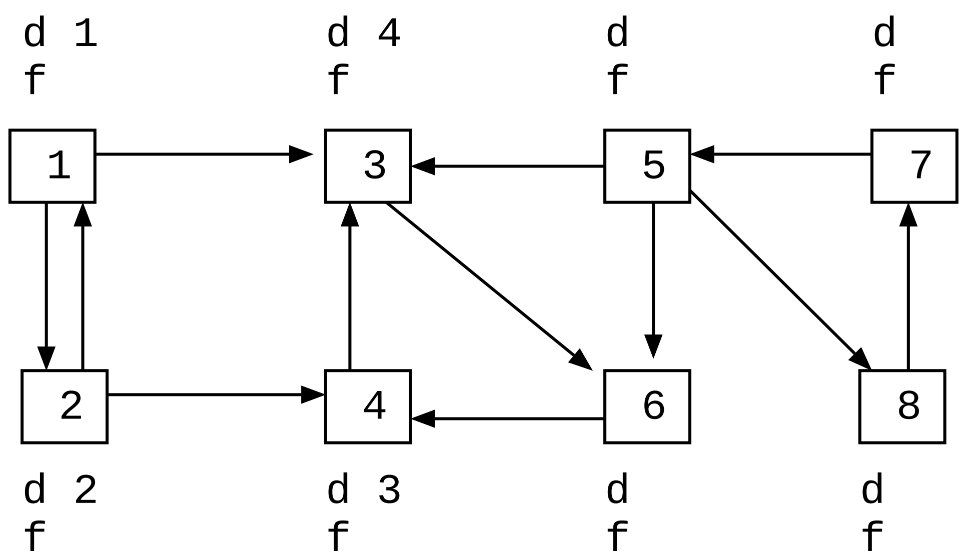
21-31: **DFS Example**



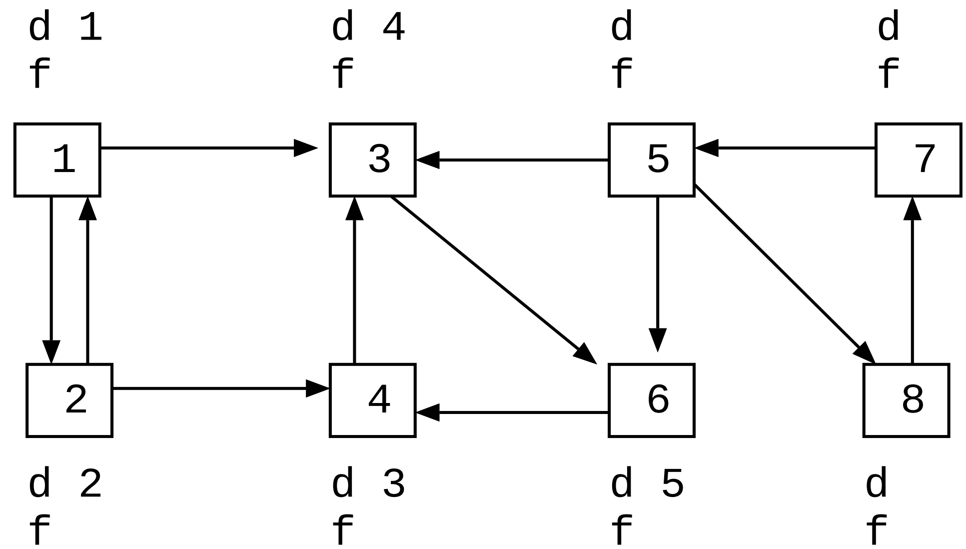
21-32: **DFS Example**



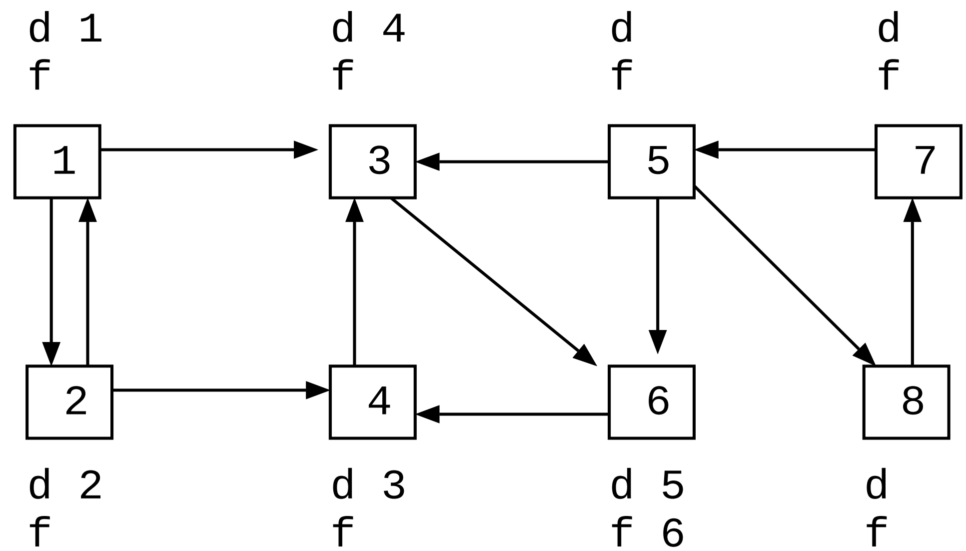
21-33: **DFS Example**



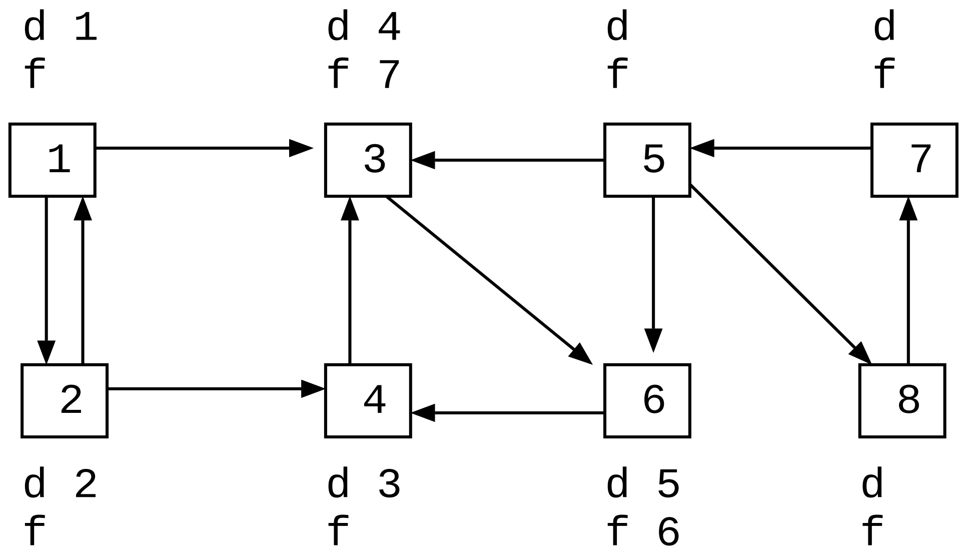
21-34: **DFS Example**



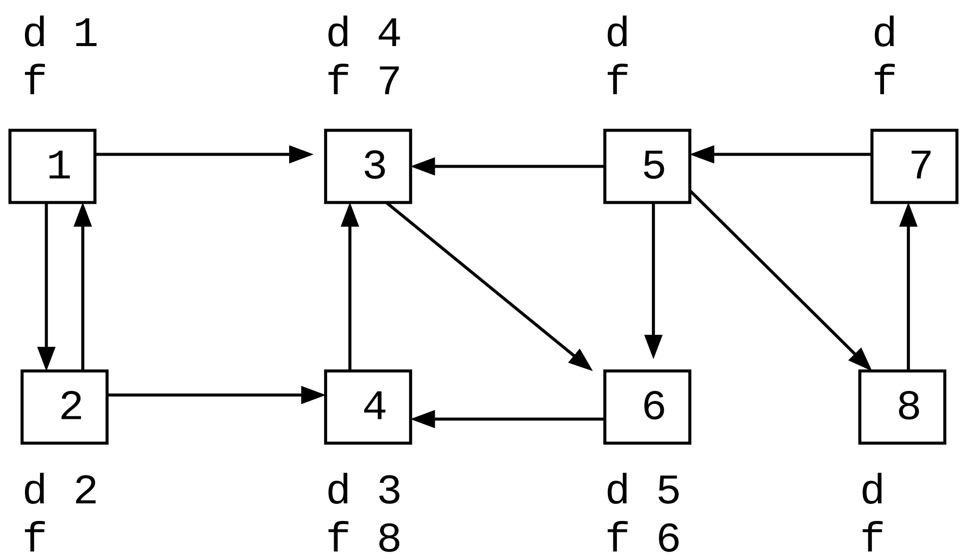
21-35: **DFS Example**



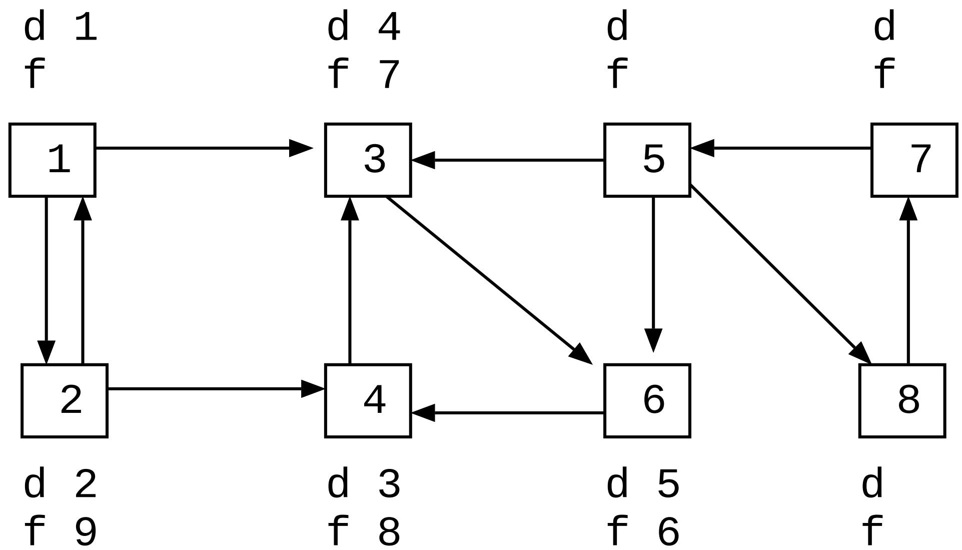
21-36: **DFS Example**



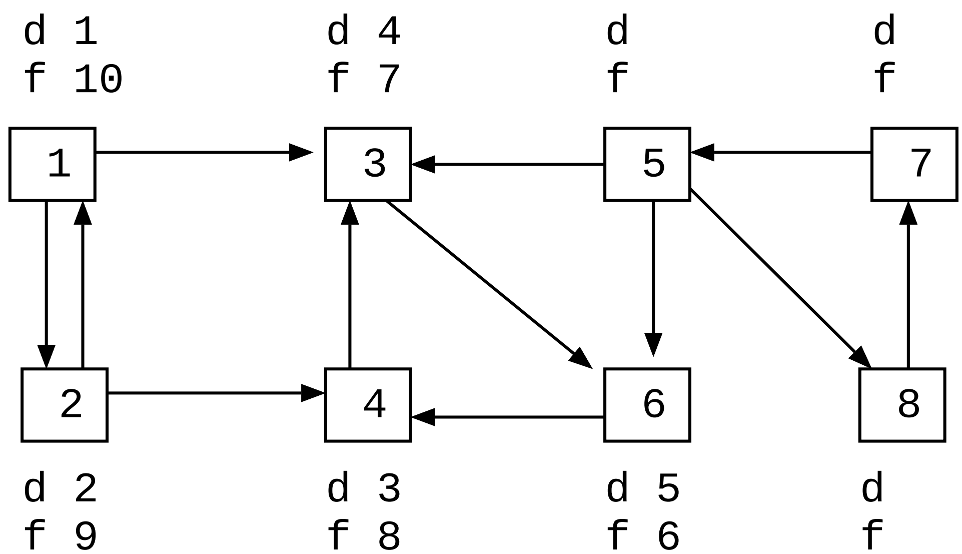
21-37: **DFS Example**



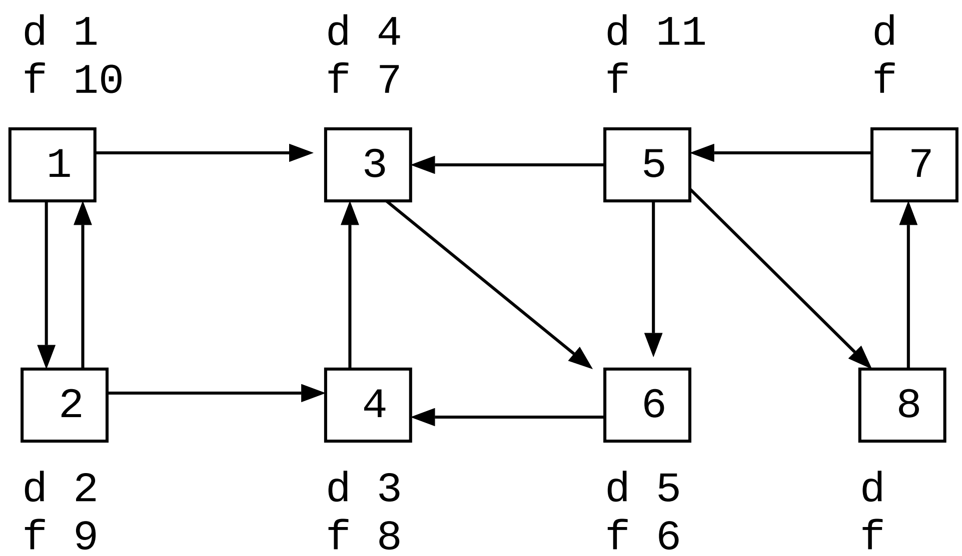
21-38: **DFS Example**



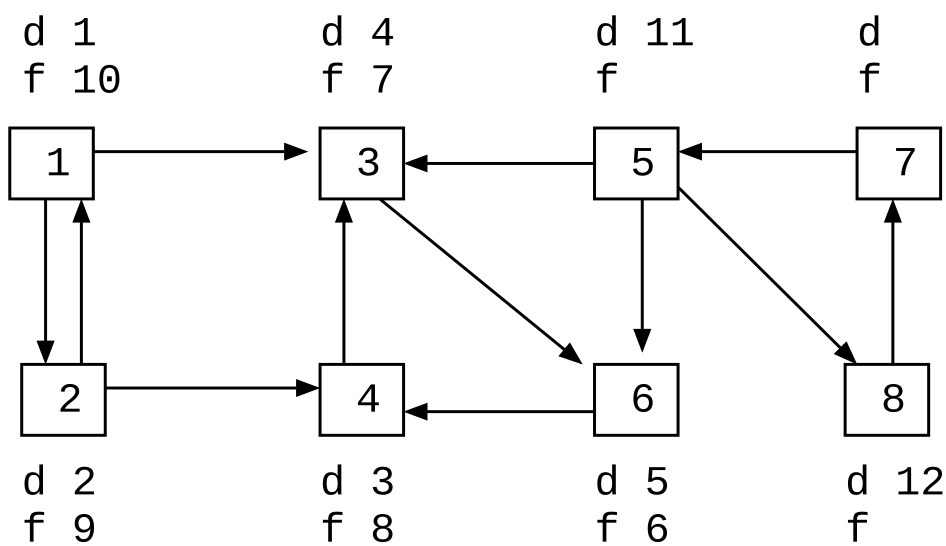
21-39: **DFS Example**



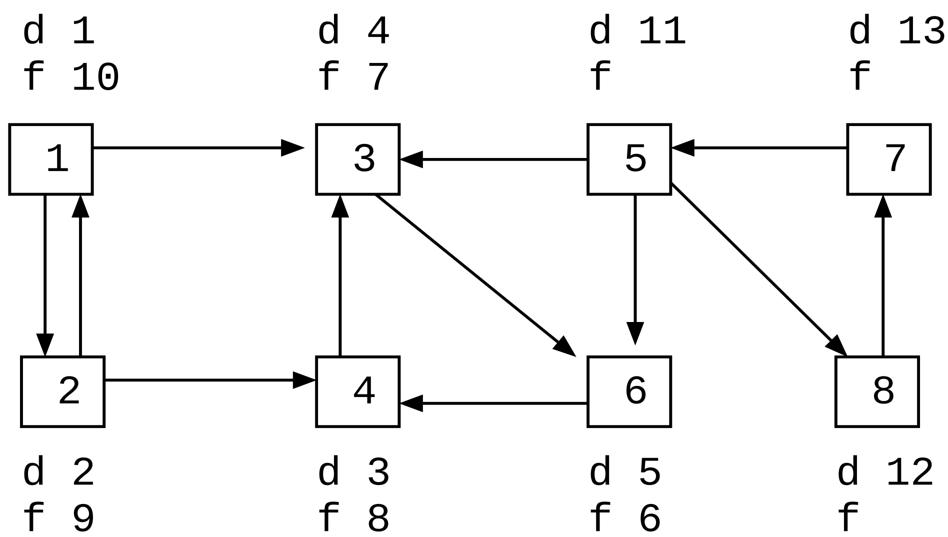
21-40: **DFS Example**



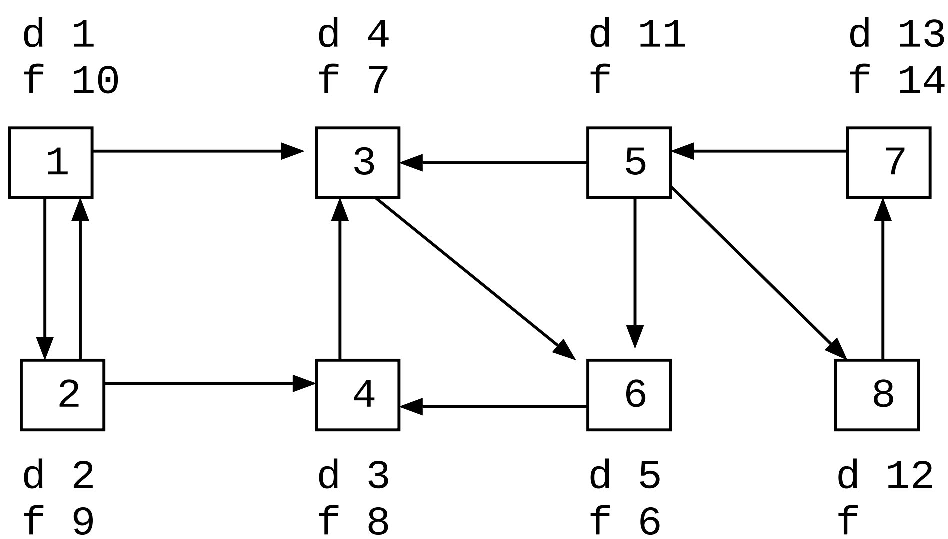
21-41: **DFS Example**



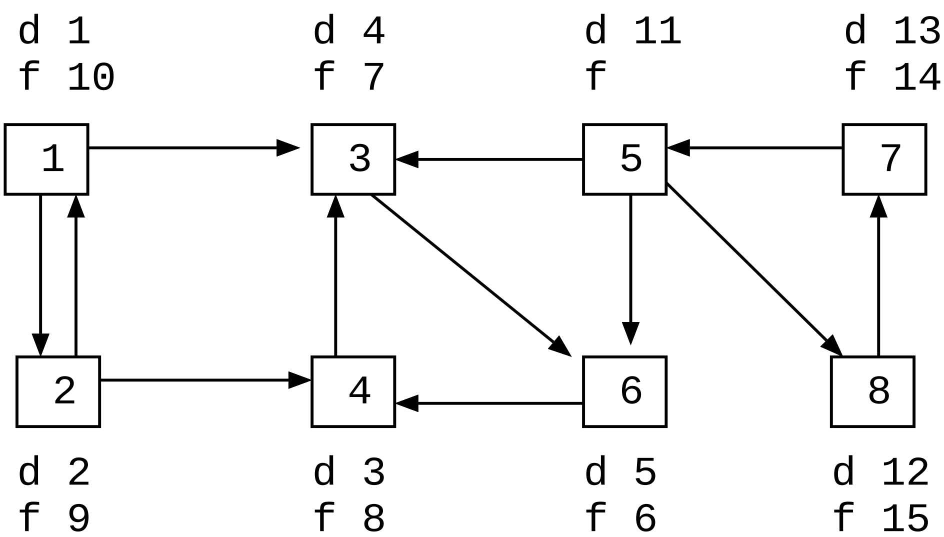
21-42: **DFS Example**



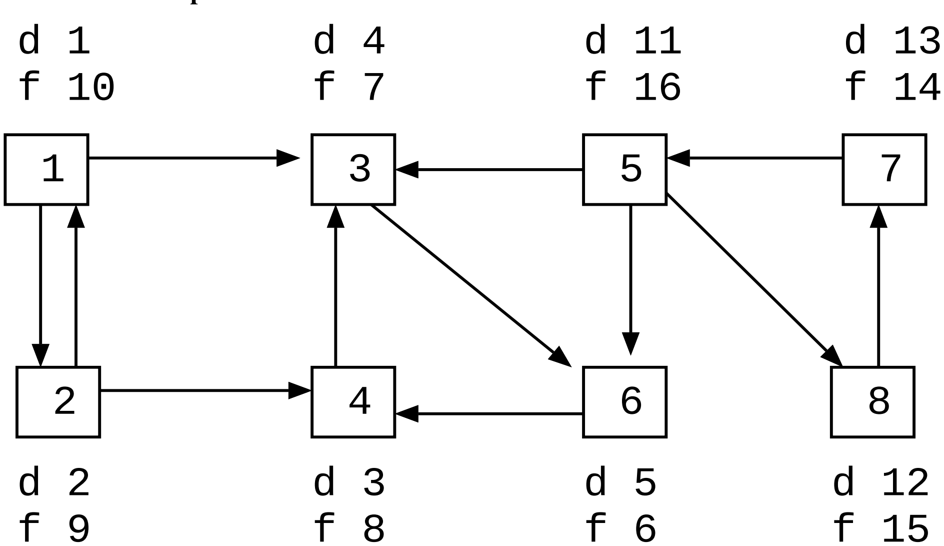
21-43: **DFS Example**



21-44: **DFS Example**



21-45: **DFS Example**



21-46: **Using d[] & f[]**

- Given two vertices v_1 and v_2, what do we know if $f[v_2] < f[v_1]$?

21-47: **Using d[] & f[]**

- Given two vertices v_1 and v_2, what do we know if $f[v_2] < f[v_1]$?
  - Either:
    - Path from v_1 to v_2
      - Start from v_1
      - Eventually visit v_2
      - Finish v_2
      - Finish v_1

21-48: **Using d[] & f[]**

- Given two vertices v_1 and v_2, what do we know if $f[v_2] < f[v_1]$?
  - Either:
    - Path from v_1 to v_2
    - No path from v_2 to v_1
      - Start from v_2
      - Eventually finish v_2
      - Start from v_1
      - Eventually finish v_1

21-49: **Using d[] & f[]**

- If $f[v_2] < f[v_1]$:
  - Either a path from v_1 to v_2, or no path from v_2 to v_1
  - If there is a path from v_2 to v_1, then there must be a path from v_1 to v_2
- $f[v_2] < f[v_1]$ and a path from v_2 to v_1 $\Rightarrow$ v_1 and v_2 are in the same connected component

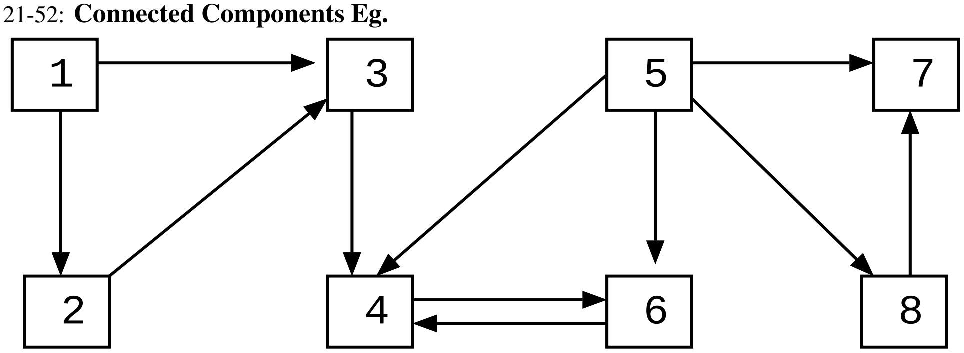
21-50: **Calculating paths**

- Path from v_2 to v_1 in G if and only if there is a path from v_1 to v_2 in G^T
  - G^T is the transpose of G – G with all edges reversed
- If after DFS, $f[v_2] < f[v_1]$
- Run second DFS on G^T, starting from v_1, and v_1 and v_2 are in the same DFS spanning tree
- v_1 and v_2 must be in the same connected component

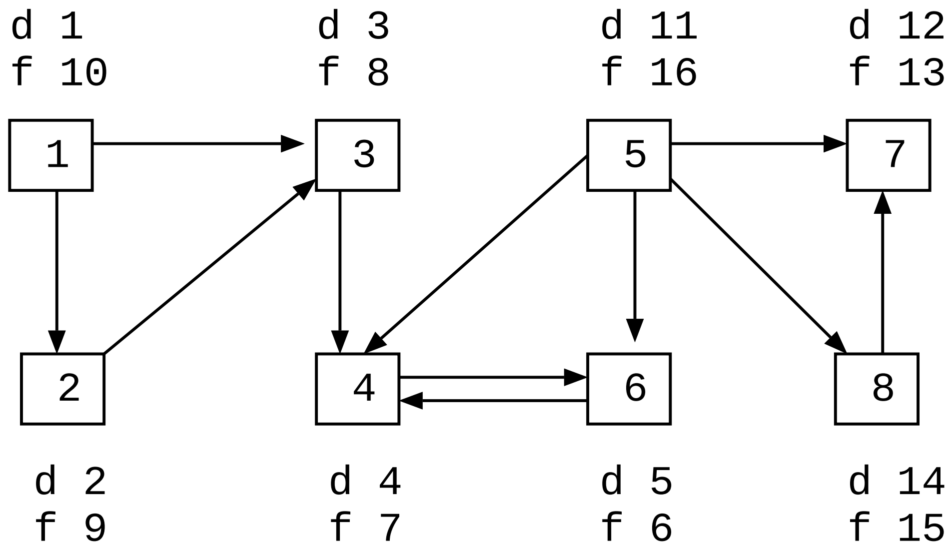
21-51: **Connected Components**

- Run DFS on G, calculating f[] times
- Compute G^T
- Run DFS on G^T – examining nodes in *inverse order of finishing times* from first DFS
- Any nodes that are in the same DFS search tree in G^T must be in the same connected component

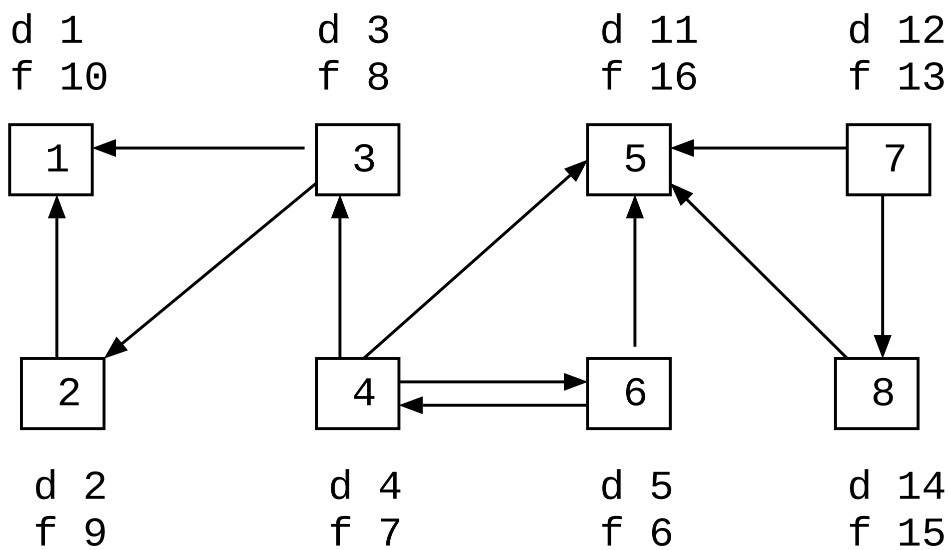
21-52: **Connected Components Eg.**



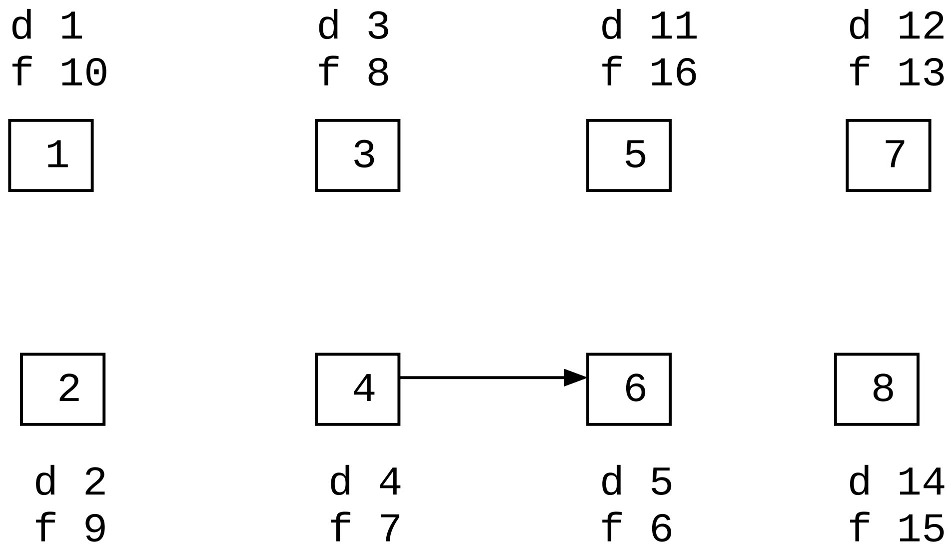
21-53: **Connected Components Eg.**



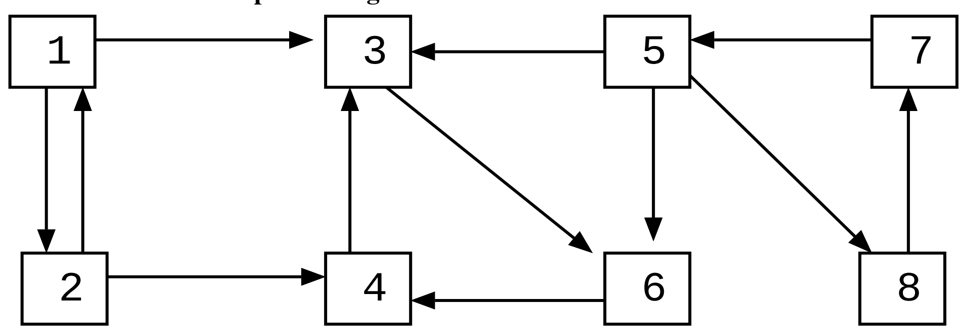
21-54: **Connected Components Eg.**



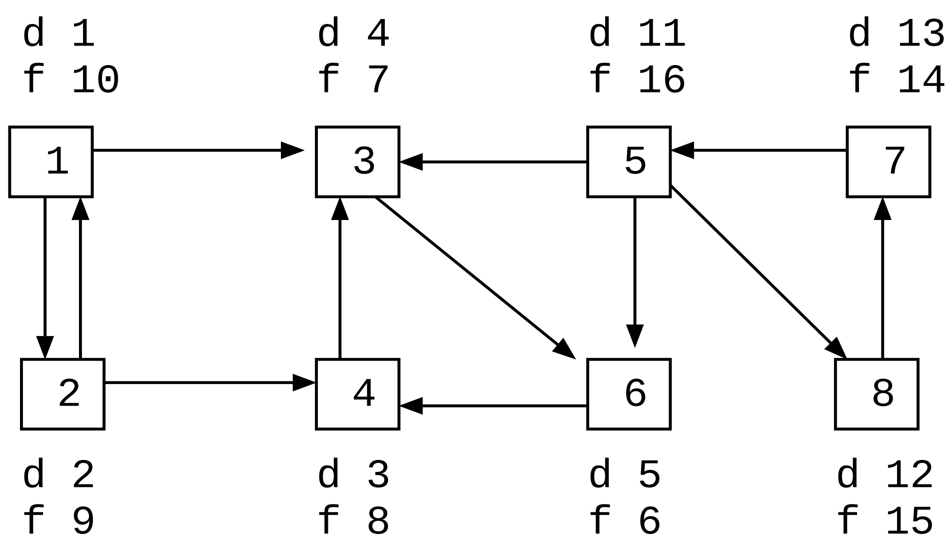
21-55: **Connected Components Eg.**



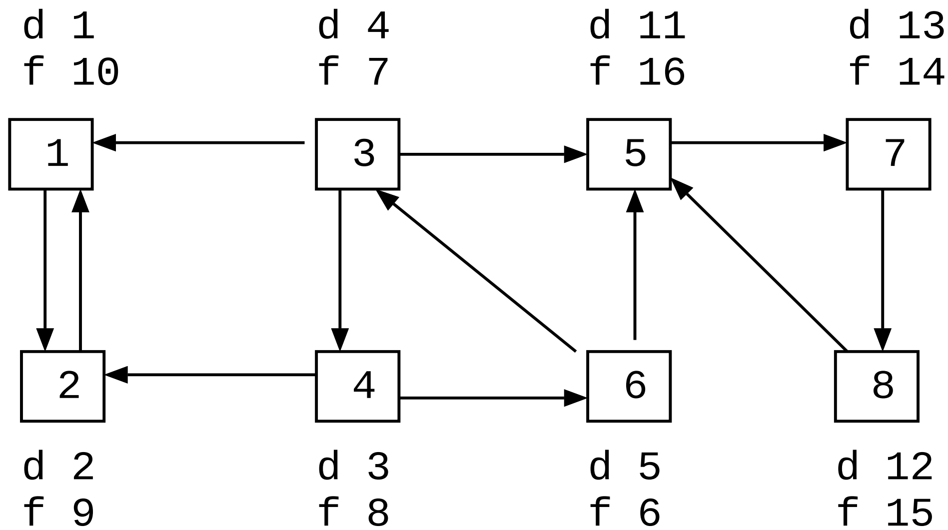
21-56: **Connected Components Eg.**



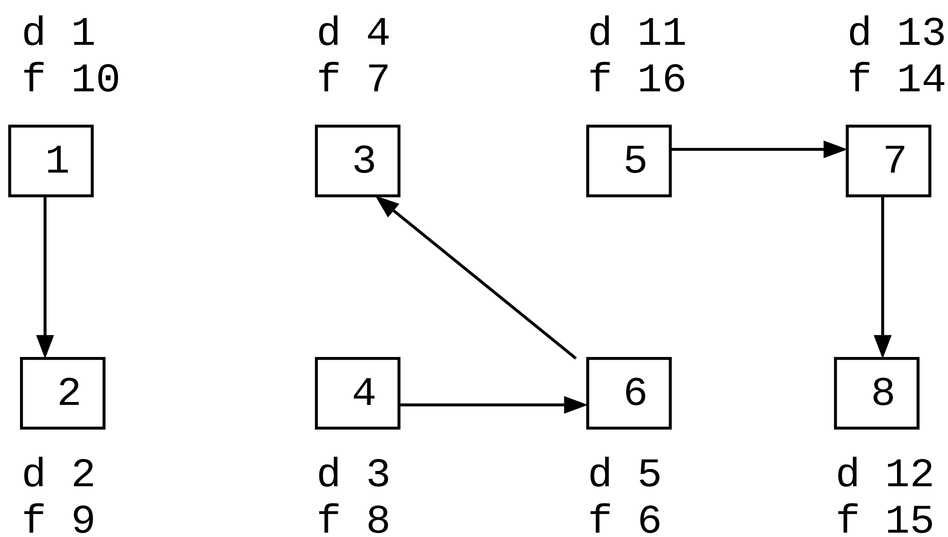
21-57: **Connected Components Eg.**



21-58: **Connected Components Eg.**



21-59: **Connected Components Eg.**



21-60: **Topological Sort**

- How could we use DFS to do a Topological Sort?
  - (Hint – Use discover and/or finish times)

21-61: **Topological Sort**

- How could we use DFS to do a Topological Sort?
  - (Hint – Use discover and/or finish times)
  - (What does it mean if node x finished before node y?)

21-62: **Topological Sort**

- How could we use DFS to do a Topological Sort?

- Do DFS, computing finishing times for each vertex
- As each vertex is finished, add to front of a linked list
- This list is a valid topological sort